25-sept

Assignment

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1) Relation on $\{1,2,3,4,5\}$. If relation is defined as $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (1,5)\ (5,1)\ (3,5)\ (5,3)\ (1,3)\ (3,1)\}$

$\longrightarrow$

$[1] = \{1,5,3\}$ $\quad$ $[3] = \{3,5,1\}$ $\quad$ $[5] = \{5,1,3\}$

$[2] = \{2\}$ $\quad$ $[4] = \{4\}$

$\{\{1,3,5\},\{2\},\{4\}\}$

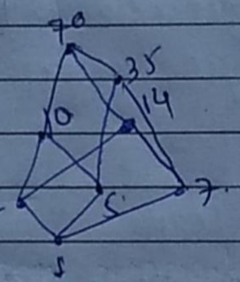
Ans

2) Show that the set of all divisors of 70 for divisibility relation forms a lattice.

$\longrightarrow$ here

$A = \{1,2,5,7,10,14,35,70\}$

Hass diagram representation



If we consider lub @ glb of any two element in hass diagram, we get unique value.
Hence relation forms a lattice.

3) example of relation on set $A = \{1,2,3\}$

$\longrightarrow$ i) $R = \{(1,1)\ (2,2)\ (3,3)\}$

ii) $R = \{(1,2)\ (2,2)\ (3,1)\ (2,1)\}$

iii) ~~Consider relation $R = \{(1,2)\ (2,3)\ (1,3)\ (3,2)\ (1,1)\ (1,2)\}$~~

There is no such relation.

~~such~~ such that R is transitive but

$R \cup R^{-1}$ is not transitive.

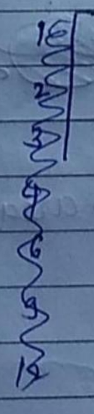
4)

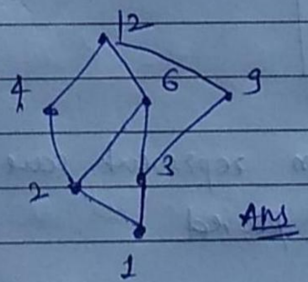
→ here $A = \{1, 2, 3, 4, 6, 9, 12\}$

$R = \{(1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,9)\ (1,12)\ (1,1)$
$(2,4)\ (2,6)\ (2,12)\ (2,2)\ (3,3)\ (3,6)\ (3,9)\ (3,12)$
$(4,4)\ (4,12)\ (6,6)\ (9,9)\ (12,12)\}$

The given relation is
1) reflexive
2) Antisymmetric
3) Transitive
∴ the R is POSET.

Hasse diagram representation ~~of R.~~





Ans

5)

→ given statement $1+2+2^2+\dots+2^n = 2^{n+1}-1$
by induction,
let $P(n): 1+2+2^2+\dots+2^n = 2^{n+1}-1$ is inductive hypo.

base case:
let $n=0$.
$P(0): 1 = 1$ ∴ $P(0)$ is true.

inductive step:

assume that,

P(n) is true. ie. $1+2+2^2+\dots 2^n = 2^{n+1}-1$ —— (1)

consider $P(n+1)$:

$P(n+1): 1+2+2^2+\dots+2^n+2^{n+1} =$

from eqⁿ (1)

$$1+2+2^2+\dots+2^n+2^{n+1} = 2^{n+1}-1+2^{n+1}$$
$$= 2^{n+2}-1$$
$$= 2^{(n+1)+1}-1$$

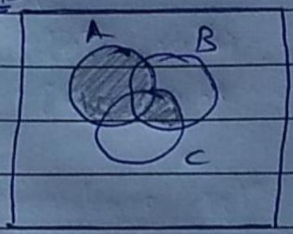
∴ $P(n) \Rightarrow P(n+1)$

hence the predicate is proved by induction

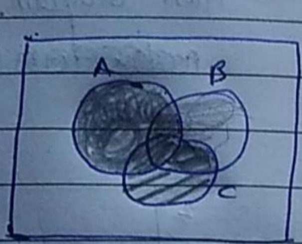
6)

→ 1) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

→ LHS



RHS



both diagram represent same ~~elements~~ area

∴ hence proved.

2) $A\cap(B\oplus C) = (A\cap B)\oplus(A\cap C)$

→ LHS

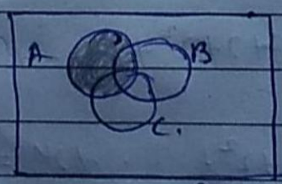


fig1: A. ~~fig~~

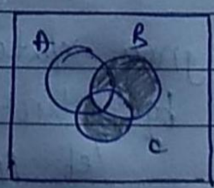


fig2: ~~A⊕B~~ $B\oplus C$

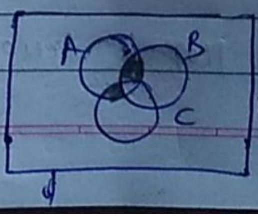


fig3: $A\cap(B\oplus C)$

RHS

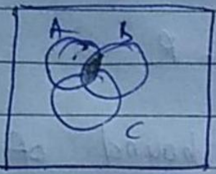


fig. 4. $A \cap B$

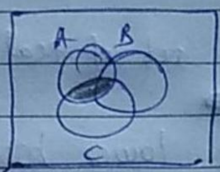


fig 5: $A \cap C$

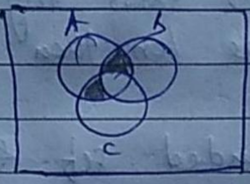


fig 6: $(A \cap B) \oplus (A \cap C)$

from fig 3 & fig:6.

$A \cup (B \oplus C) = (A \cap B) \oplus (A \cap C)$

Hence proved.

7)

→ given conditional statement: "if x is rational, then x is real"

let p = x is rational

q = x is real

~~the given~~ condition the given statement: $p \rightarrow q$

i) converse of given statement is $q \rightarrow p$ i.e. If x is real, then x is rational.

ii) inverse of given statement is $\sim p \rightarrow \sim q$ i.e If x is not real then x is not rational.

iii) contrapositive of given statement is If x is not rational then x is not real.

iv) negation of given statement is $\sim(p \rightarrow q)$ i.e. $\sim(\sim p \vee q)$

by De Morgan's law $\sim(\sim p \vee q) = p \wedge \sim q$.

∴ negation of given statement is

x is rational and x is not real.

8)

→ consider set of real numbers i.e. R.

The lower bound & upper bound of R is undefined which makes R uncountable.

(OR)

we can say that there doesn't exist any number M, m such that all elements of R are less than M & all elements of R are greater than m.

∴ which makes R as unbounded set & hence uncountable.

9)

→ $A = \{a, b, c, d\}$

$\pi = \{\{a, b\}, \{c\}, \{d\}\}$

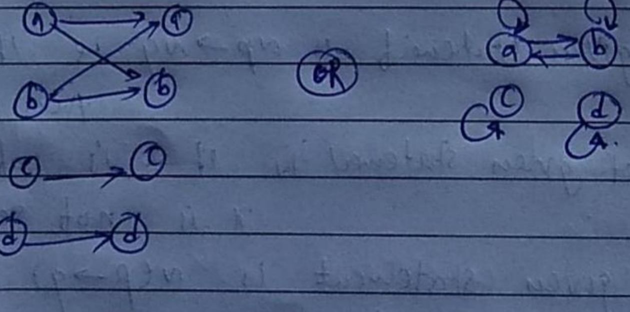
here

$[a] = \{a, b\}$ $\quad$ $[c] = \{c\}$

$[b] = \{a, b\}$ $\quad$ $[d] = \{d\}$

∴ equivalence rel^n is $R = \{\{a,b\}, \{b,a\}, \{a,a\}, \{b,b\}, \{c,c\}, \{d,d\}\}$

digraph for equivalence rel^n is:



10)

→ here $A = \{1, 2, 3, 4\}$

$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. $M_S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$$R \cup S = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Computing transitive closure by Warshall's Algorithm:

$$W_0 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

1st step: (1,1) (1,1) ⇒ (1,1) ⇒ $\underline{W_1 = W_0}$

(1,2) → (1,2)

(1,4) ⇒ (1,4)

2nd step: (1,2) (2,2) ⇒ (1,2) ⇒ $\underline{W_2 = W_1}$

(2,2) (2,2)

(3,2) (3,2)

(4,2) (4,2)

3rd step: (3,3) (3,2) ⇒ (3,2) ⇒ $W_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

(4,3) (3,3) (4,3)

4th step: (1,1) (4,2) ⇒ (1,2) ⇒ $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

(4,1) (4,3) (1,3)

(4,4) (1,4)

(4,2)

(4,3)

(4,4)

∴ transitive closure of $R \cup S$ is $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

Ans