21/18

23-sept

Practice Assignment

Q. perform ckwise 45° rotation of triangle A(2,3), B(5,5) C(4,3) about point (1,1)

→ 1) translate origin T
2) rotation R
3) Translation back to original point T'

$$TRT^{1} = \begin{bmatrix} 1 & 0 & -tx \\ 0 & 1 & -ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ +\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -tx \\ 0 & 1 & -ty \\ 0 & 0 & 1 \end{bmatrix}$$

here $tx = ty = +1$

$\theta = 45°$

$$TRT^{1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ +\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & \sqrt{2} \\ 1 & 1 & \sqrt{2} \\ 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & \sqrt{2} \\ 1 & 1 & -2+\sqrt{2} \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

$$= \cancel{\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & -\sqrt{2}+1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' & B' & C' \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & -\sqrt{2}+1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 \\ 3 & 5 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/\sqrt{2}+1 & 1 & 1/\sqrt{2}+1 \\ \frac{3}{\sqrt{2}}+1 & 4\sqrt{2}+1 & \frac{5}{\sqrt{2}}+1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore A'\left(-\frac{1}{\sqrt{2}}+1, \frac{3}{\sqrt{2}}+1\right)$ $B'\left(1, 4\sqrt{2}+1\right)$ $C'\left(\frac{1}{\sqrt{2}}+1, \frac{5}{\sqrt{2}}+1\right)$

Ans

Que. Perform a 45° rotation of triangle A(0,0) B(1,1) C(5,2)
i) about the origin } assume rotation clkwise.
ii) About P(-1,-1)

$\longrightarrow$

i) $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

here $\theta = 45°$.

$\therefore R = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sqrt{2} & 0 & 1 \\ 7/\sqrt{2} & -3/\sqrt{2} & 1 \end{bmatrix}$$

ii) About (-1,-1)

The rotation matrix for clkwise rotation about arbitrary point:

$$TRT^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ -x_p\cos\theta - y_p\sin\theta & x_p\sin\theta - y_p\cos\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$$

Put $\theta = -45°$ & $x_p = 1$, $y_p = 1$

$$\therefore = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ -x_p\cos\theta - y_p\sin\theta + x_p & x_p\sin\theta - y_p\cos\theta + y_p & 1 \end{bmatrix}$$

here $x_p = -1$, $y_p = -1$, $\theta = 45°$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \sqrt{2}-1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \sqrt{2}-1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}-1 & -1 & 1 \\ 2\sqrt{2}-1 & -1 & 1 \\ 9/\sqrt{2}-1 & -\frac{3}{\sqrt{2}}-1 & 1 \end{bmatrix}$$

$\therefore$ new points are $A'(\sqrt{2}-1, -1)$
$B'(2\sqrt{2}-1, -1)$
$C'(9/\sqrt{2}-1, -\frac{3}{\sqrt{2}}-1)$

Ans

Que Consider the square $A(1,0)$ $B(0,0)$ $C(0,1)$ & $D(1,1)$. Rotate the square by $45°$ anticlockwise direction followed by reflection about x-axis

$\longrightarrow$ $$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ -\sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Page No.
Date

at $\theta = 45°$

$$T = \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ -\sin 45° & -\cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 0 & -\sqrt{2} & 1 \end{bmatrix}$$

∴ New coordinate of square are,

$A'(1/\sqrt{2}, -1/\sqrt{2})$ $B'(0,0)$ $C(-1/\sqrt{2}, -1/\sqrt{2})$ $D(0, -\sqrt{2})$

Que. Give a 3X3 homogeneous transformation matrix for each of the following transformation sequences.

i) rotate anticlockwise about the origin by 45° & then scale the x-direction by one half as large.

ii) Scale the y direction by twice as tall. shift down by 1 unit & then rotate clkwise by 30°.

→ i) rotation → scaling

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} \cos 45° & \sin 45° & 0 \\ -\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [x \ y \ 1] \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Page No.
Date

$$= [x\ y\ 1]\begin{bmatrix} 3/2\sqrt{2} & 1/\sqrt{2} & 0 \\ -3/2\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

117. scaling → translation → rotation.

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{bmatrix}$$

here $tx = 0$, $ty = -1$, $Sx = 1$, $Sy = 2$, $\theta = 30°$

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1 & \sqrt{3} & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Que Prove that two scaling transformation commute i.e. $S_1S_2 = S_2S_1$

→ we have,

$$S_1S_2 = \begin{bmatrix} Sx_1 & 0 \\ 0 & Sy_1 \end{bmatrix}\begin{bmatrix} Sx_2 & 0 \\ 0 & Sy_2 \end{bmatrix} \quad \& \quad S_2S_1 = \begin{bmatrix} Sx_2 & 0 \\ 0 & Sy_2 \end{bmatrix}\begin{bmatrix} Sx_1 & 0 \\ 0 & Sy_1 \end{bmatrix}$$

$$= \begin{bmatrix} Sx_1Sx_2 & 0 \\ 0 & Sy_1Sy_2 \end{bmatrix} — (1) \qquad S_2S_1 = \begin{bmatrix} Sx_1Sx_2 & 0 \\ 0 & Sy_2Sy_1 \end{bmatrix} — (2)$$

from (1) & (2).

$S_1S_2 = S_2S_1$

Hence proved.

Que Prove that two 2D ~~transformation~~ rotations about the origin commute.
i.e. $R_1R_2 = R_2R_1$ (or) prove that $R(\theta_1).R(\theta_2) = R(\theta_1+\theta_2)$.

→ we have,

$$R_1R_2 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix}\begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \quad \& \quad R_2R_1 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix}\begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$R_1R_2 = \begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2 \\ -\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} \quad — ①$$

$$R_2R_1 = \begin{bmatrix} \cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1 & \cos\theta_2\sin\theta_1 + \sin\theta_2\cos\theta_1 \\ -\sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1 & -\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} \quad — ②$$

from eqn ① & ②

$R_1R_2 = R_2R_1$

Hence proved

Que: Fig ① & ② show basic 2D blocks. Apply translation & scaling ~~translations~~ transformations to get fig ③. Draw diagrams of all intermediate steps.

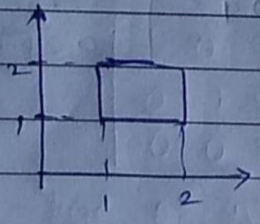


fig 1

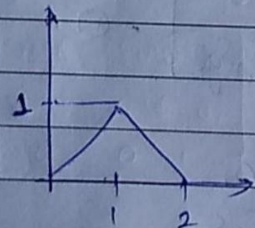
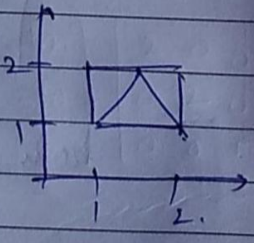


fig 2



1) apply scaling on fig 2 $S_x = 1/2$, $S_y = 1$

2) translate fig 2 $t_x = 1$ $t_y = 1$

∴ overall transformation = S.T

$$= \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ new cordinate $= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 1.5 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

new co-ordinates $(1,1)$ $(3/2, 2)$ $(2,1)$.

Qus Apply the shearing transformation to square with $A(0,0)$ $B(1,0)$ $C(1,1)$ & $D(0,1)$ as given below.

a) Shear parameter value of 0.5 relative to the line $y_{ref} = -1$

b) Shear parameter value of 0.5 relative to the line $x_{ref} = -1$

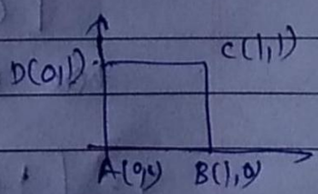
→ shearing relative to other reference line.

xshear with yreference line : $\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ -Sh_x \cdot y_{ref} & 0 & 1 \end{bmatrix}$

yshear with xreference line : $\begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & -Sh_y \cdot x_{ref} & 0 \end{bmatrix}$

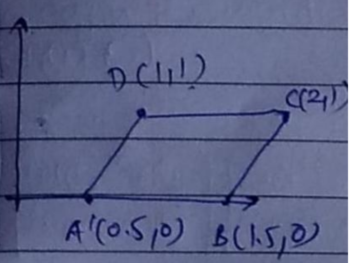
a) here $Sh_x = 3/2$ & $y_{ref} = -1$

$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ -Sh_x y_{ref} & 0 & 1 \end{bmatrix}$



$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 3/2 & 0 & 1 \end{bmatrix}$

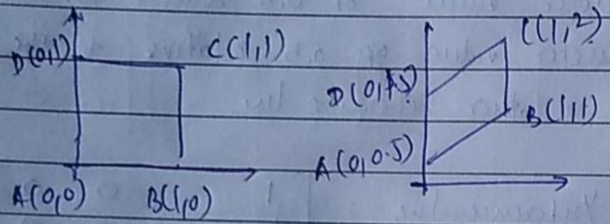
$= \begin{bmatrix} 0.5 & 0 & 1 \\ 1.5 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$



b) here $Sh_y = 0.5$ & $x_{ref} = 1$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & -Sh_y \cdot x_{ref} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 3/2 & 1 \end{bmatrix}$$



Que. Consider a square P(0,0) Q(0,10) R(10,10) S(10,10). Rotate the square about fixed point R(10,10) by an angle 45° (clockwise) followed by scaling by 2 units in both x & y direction

→

$$T = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

here $\theta = -45°$.

$$T = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 5 \\ -1/\sqrt{2} & 1/\sqrt{2} & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Page No.
Date

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -10\sqrt{2}+5 \\ -1/\sqrt{2} & 1/\sqrt{2} & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & \sqrt{2} & -10\sqrt{2}+5 \\ -\sqrt{2} & \sqrt{2} & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[P' \; Q' \; R' \; S'] = \begin{bmatrix} \sqrt{2} & \sqrt{2} & -10\sqrt{2}+5 \\ -\sqrt{2} & \sqrt{2} & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 10 & 10 \\ 0 & 10 & 10 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -10\sqrt{2}+5 & 5 & 10\sqrt{2}+5 & 5 \\ 5 & 10\sqrt{2}+5 & 5 & -10\sqrt{2}+5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ new co-ordinates of square are.

$P'(-10\sqrt{2}+5, 5)$ $\quad$ $R'(10\sqrt{2}+5, 5)$

$Q'(5, 10\sqrt{2}+5)$ $\quad$ $S'(5, -10\sqrt{2}+5)$