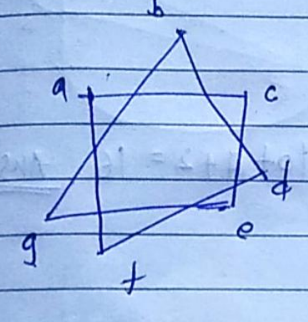
2/1/18

Theory Questions On Unit IV : Graph

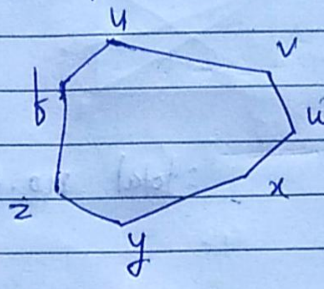
Page No. Date

1)

→ Graph G:



Graph H:



$|V| = 7$

degree of each vertex = 2

$|V| = 7$.

degree of each vertex is 2.

as the no. of vertices & degree of each vertex is also same the given graphs are isomorphic.

mapping of vertices

a → u
c → v
e → w

g → x
b → y
d → z

f → t

2)

→ Given: G is a simple graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 & remaining vertices have degree 3.

find no. of vertices:

Solution: As we know, for any graph (simple):

$$\sum_{i=1}^{n} degree(V_i) = 2|E|$$

here $|E| = 27$ — ①

& $\sum_{i=1}^{n} degree(V_i) = 6 \times 2 + 3 \times 4 + 3.x$, x → remaining vertices (degree 3)

$= 24 + 3x$. — ②

from eqn ① & ②

$24+3x = 2\times27$

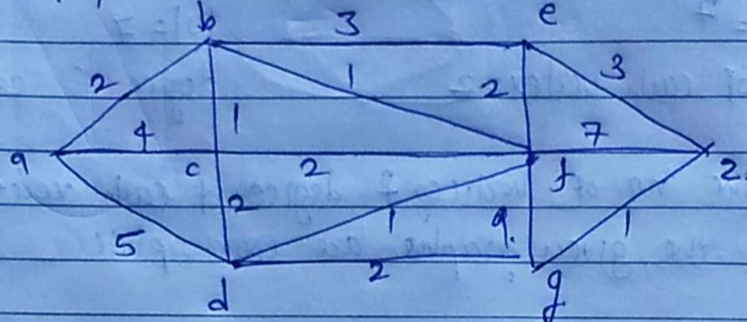
$3x = 54-24$

$x = 10$

∴ total no. of vertices are $10+4+2 = 16$ Ans

3)

→ Given graph:



Solution:

finding shortest path using Dijekstra's Algorithm:

i) $P = \{a\}$

$T = \{b, c, d, e, f, g, z\}$

$l(b) = 2$

$l(c) = 4$

$l(d) = 5$

$l(e) = \infty$

$l(f) = \infty$

$l(g) = \infty$

$l(z) = \infty$

ii) $P = \{a, b\}$

$T = \{c, d, e, f, g, z\}$

$l(c) = \min(4, 2+1) = 3$

$l(d) = \min(5, 2+\infty) = 5$

$l(e) = \min(\infty, 2+3) = 5$

$l(f) = \min(\infty, 2+1) = 3$

$l(g) = \min(\infty, 2+\infty) = \infty$

$l(z) = \min(\infty, 2+\infty) = \infty$

iii) $P = \{a, b, c\}$ $T = \{d, e, f, g, z\}$

$l(d) = \min(5, 3+2) = 5$

$l(e) = \min(5, 3+\infty) = 5$

$l(f) = \min(3, 3+2) = 3$

$l(g) = \min(\infty, 3+\infty) = \infty$

$l(z) = \min(\infty, 3+\infty) = \infty$

iv) $P = \{a, b, c, f\}$ $T = \{d, e, g, z\}$

$l(d) = \min(5, 3+1) = 4$

$l(e) = \min(5, 3+2) = 5$

$l(g) = \min(\infty, 3+4) = 7$

$l(z) = \min(\infty, 3+7) = 10$

v) $P = \{a, b, c, f, d\}$ $T = \{e, g, z\}$

$l(e) = \min(5, 4+\infty) = 5$

$l(g) = \min(7, 4+2) = 6$

$l(z) = \min(10, 4+\infty) = 10$

vi) $P = \{a, b, c, f, d, e\}$ $T = \{g, z\}$

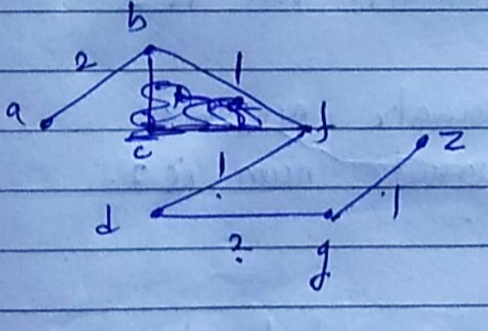
$l(g) = \min(6, 5+\infty) = 6$

$l(z) = \min(10, 5+3) = 8$

vii) $P = \{a, b, c, f, d, e, g\}$ $T = \{z\}$

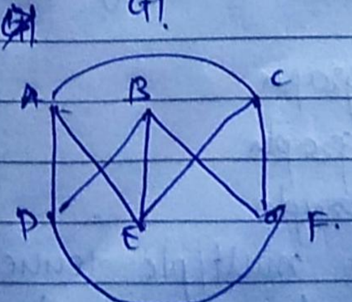
$l(z) = \min(8, 6+1) = 7$

Shortest path length is 7.



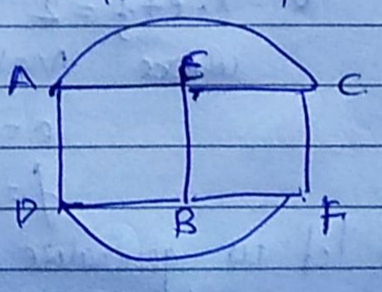
Ans

G!

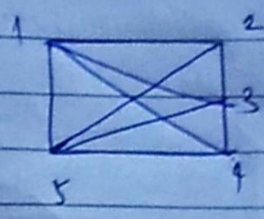


$\Rightarrow$

planar representation

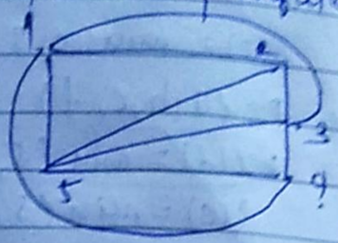


G2.

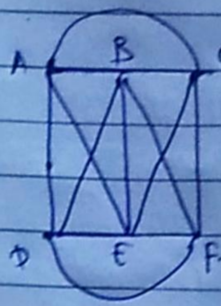


$\Rightarrow$

planar representation.



G3



$\Rightarrow$ the graph is not planar.

5)

$\longrightarrow$ i) chromatic number of complete graph k_n is n.

ii) cycle graph:

If n is even: chromatic num is 2.

if n is odd chromatic nu is 3

iii) wheel graph

If n is even: chromatic nu is 3.

If n is odd: chromatic num is 2.

6)

$\longrightarrow$ According to Euler, for connected planar graph,

$v - e + f = 2$

where v = vertices in graph

e = edges in graph

f = faces in graph

Proof. let us generalize it to allow multiple connected components c. In that case the formula becomes

base case: $v+f = e+c+1$. The proof is by induction over e.
If $e=0$, we have $v=c$, $f=1$ & theorem is true.
inductive step: Suppose we remove an edge.

1) either the number of faces reduced by 1 (or)

2) number of component increased by 1.

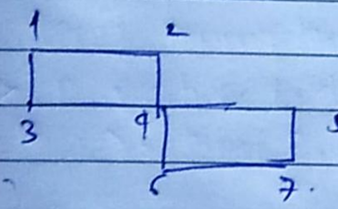
In each case, the formula is true for new graph then it is true for old also.

: Hence proved by induction

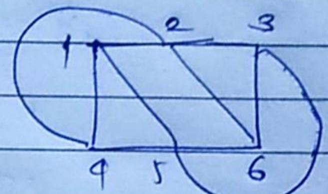
7) Determine whether the following graphs are having Hamiltonian circuit / Hamiltonian path. Justify

→

G1



G2



~~because every vertex is of even degree.~~

~~H.P:~~

H.P: 3 1 2 4 6 7 5

There is no hamiltonian circuit present in the graph.

H.P: 2 6 3 5 1 4

H.C. 2 6 3 5 1 4 2