Assignment - 1

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Problem

1. The co-ordinates of points A,B are ($\mathbf{r}_1$,θ_1) , ($\mathbf{r}_2$,θ_2) referred to O as pole. The internal bisector of angle AOB meets the line AB in D. Find the co-ordinates of D.

Solution

Let the co-ordinates of point D be $\begin{pmatrix} r \\ \theta \end{pmatrix}$.

Now , $\theta = \dfrac{\theta_1 + \theta_2}{2}$

Since ,

$$Area(\triangle AOD) + Area(\triangle DOB) = Area(\triangle AOB) \quad (1)$$

$\therefore$

$$\frac{1}{2}\|(r_1)\times(r\times sin(\frac{\theta}{2}))\|+\frac{1}{2}\|(r_2)\times(r\times sin(\frac{\theta}{2}))\| = \frac{1}{2}\|(r_1)\times(r_2\times sin(\frac{\theta}{2}))\| \quad (2)$$

Now , R.H.S = $\dfrac{1}{2}\|(r_1)\times(r_2\times sin(\theta))\|$

$$= \frac{1}{2}\|(r_1)\times(r_2\times 2\times sin(\frac{\theta}{2})\times cos(\frac{\theta}{2})\| \quad (3)$$

On Comparing (2) and (3),

$$\frac{1}{2}\ \|r\times(r_1+r_2)\| = \|r_1\times r_2\times \cos(\frac{\theta}{2})\| \quad (4)$$

$$\therefore r = |(\frac{2\times r_1\times r_2}{r_1+r_2}\times cos(\frac{\theta}{2}))| \quad (5)$$

Now , D has co-ordinates as $\begin{pmatrix} r \\ \theta \end{pmatrix}$

$\therefore$ *Co-ordinates of D are* $\begin{pmatrix} \dfrac{2r_1\times r_2}{r_1+r_2}\times cos(\dfrac{\theta}{2}) \\ \dfrac{\theta_1+\theta_2}{2} \end{pmatrix}$