Assignment - 1

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SM21MTECH14003

Problem

1. The co-ordinates of points A,B are ($\mathbf{r}_1$,θ_1) , ($\mathbf{r}_2$,θ_2) referred to O as pole. The internal bisector of angle AOB meets the line AB in D. Find the co-ordinates of D.

Solution

Let the co-ordinates of point D be (r,θ).

Now , $\theta = \dfrac{\theta_1 + \theta_2}{2}$

Since , Area of $\triangle$ AOD + Area of $\triangle$ DOB = Area of $\triangle$ AOB

$$\therefore\ \frac{1}{2}r_1 r sin(\frac{\theta}{2}) + \frac{1}{2}r_2 r sin(\frac{\theta}{2}) = \frac{1}{2}r_1 r_2 sin(\theta) \qquad .....(i)$$

Now , R.H.S = $\dfrac{1}{2}r_1 r_2 sin(\theta)$

$$= \frac{1}{2}r_1 r_2 2 sin(\frac{\theta}{2})cos(\frac{\theta}{2}) \qquad .....(ii)$$

On Comparing (i) and (ii)

$$\frac{1}{2}\ r\ (r_1 + r_2) = r_1 r_2 \cos(\frac{\theta}{2})$$

$$\therefore r = \frac{2r_1 r_2}{r_1 + r_2} cos(\frac{\theta}{2})$$

Now , D has co-ordinates as (r,θ)

$\therefore$ *Co-ordinates of D are (* $\dfrac{2r_1 r_2}{r_1 + r_2} cos(\dfrac{\theta}{2}), \dfrac{\theta_1 + \theta_2}{2}$)