Assignment - 1

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PROBLEM

1. The co-ordinates of points A,B are (r_1,θ_1) , (r_2,θ_2) referred to O as pole. The internal bisector of angle AOB meets the line AB in D. Find the co-ordinates of D.

SOLUTION

Let the co-ordinates of point D be $\begin{pmatrix} r \\ \theta \end{pmatrix}$.

Now , $\theta = \dfrac{\theta_1 + \theta_2}{2}$

$\therefore$ Converting into rectangular coordinates,

$$\mathbf{A} = \begin{pmatrix} r_1 sin(\theta_1) \\ r_1 cos(\theta_1) \end{pmatrix}, \mathbf{B} = \begin{pmatrix} r_2 sin(\theta_2) \\ r_2 cos(\theta_2) \end{pmatrix}, \qquad (1)$$

$$\mathbf{O} = \begin{pmatrix} 0 sin(0) \\ 0 cos(0) \end{pmatrix}, \mathbf{D} = \begin{pmatrix} r sin(\theta) \\ r cos(\theta) \end{pmatrix} \qquad (2)$$

For two vectors,

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \qquad (3)$$

$$\|\mathbf{a} \times \mathbf{b}\| = |(a_1 b_2 - a_2 b_1)| \qquad (4)$$

$$\|A - 0\| = \begin{pmatrix} r_1 sin(\theta_1) \\ r_1 cos(\theta_1) \end{pmatrix},$$

$$\|A - D\| = \begin{pmatrix} r_1 sin(\theta_1) - r sin(\theta) \\ r_1 cos(\theta_1) - r cos(\theta) \end{pmatrix},$$

$$\|B - D\| = \begin{pmatrix} r_2 sin(\theta_2) - r sin(\theta) \\ r_2 sin(\theta_2) - r cos(\theta) \end{pmatrix},$$

$$\|B - O\| = \begin{pmatrix} r_2 sin(\theta_2) \\ r_2 cos(\theta_2) \end{pmatrix},$$

$$\|A - B\| = \begin{pmatrix} r_1 sin(\theta_1) - r_2 sin(\theta_2) \\ r_1 cos(\theta_1) - r_2 cos(\theta_2) \end{pmatrix}$$

Since ,

Area $(\triangle AOD) + Area(\triangle DOB) = Area(\triangle AOB)$

$$\frac{1}{2}\|(\mathbf{A} - \mathbf{O}) \times (\mathbf{A} - \mathbf{D})\| + \frac{1}{2}\|(\mathbf{B} - \mathbf{D}) \times (\mathbf{B} - \mathbf{O})\| =$$

$$\frac{1}{2}\|(\mathbf{A} - \mathbf{O}) \times (\mathbf{A} - \mathbf{B})\| \qquad (5)$$

$\therefore$ Substituting A-0 , A-D , B-D , B-O , A-B and simplifying further we get,

$$\frac{1}{2}r_1 r sin(\frac{\theta}{2}) + \frac{1}{2}r_2 r sin(\frac{\theta}{2})$$

$$= \frac{1}{2}r_1 r_2 sin(\frac{\theta}{2}) \qquad (6)$$

Now , $R.H.S = \dfrac{1}{2}r_1(r_2 sin(\theta))$

$$= \frac{1}{2}2r_1 r_2 sin(\frac{\theta}{2})cos(\frac{\theta}{2}) \qquad (7)$$

On Comparing (6) and (7),

$$\therefore \ \frac{1}{2}r(r_1 + r_2) = r_1 r_2 \cos(\frac{\theta}{2}) \qquad (8)$$

$\therefore$

$$r = \frac{2 r_1 r_2}{r_1 + r_2} cos(\frac{\theta}{2}) \qquad (9)$$

Now , D has co-ordinates as $\begin{pmatrix} r \\ \theta \end{pmatrix}$

$$\therefore \text{ Co-ordinates of D are } \begin{pmatrix} \dfrac{2 r_1 r_2}{r_1 + r_2} cos(\dfrac{\theta}{2}) \\[3mm] \dfrac{\theta_1 + \theta_2}{2} \end{pmatrix}$$

To find coordinates of point 'D' which intersects line joining 'A' and 'B' and angle bisector of angle AOD
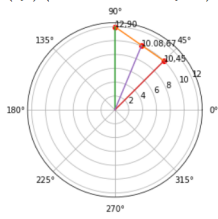A(r1,θ1)=(12,90)
O(r',θ')=(0,0)
B(r2,θ2)=(10,45)
D(r,θ)=(10.078685809214038 ,67.5)

Fig. 1. Triangles AOB,AOD,DOB