Assignment - 1

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Problem

1. The co-ordinates of points A,B are ($\mathbf{r}_1$,θ_1) , ($\mathbf{r}_2$,θ_2) referred to O as pole. The internal bisector of angle AOB meets the line AB in D. Find the co-ordinates of D.

Solution

Let the co-ordinates of point D be $\begin{pmatrix} r \\ \theta \end{pmatrix}$.

Now , $\theta = \dfrac{\theta_1 + \theta_2}{2}$

$\therefore$ Converting into rectangular coordinates,

$$\mathbf{A} = \begin{pmatrix} r_1 sin(\theta_1) \\ r_1 cos(\theta_1) \end{pmatrix}, \mathbf{B} = \begin{pmatrix} r_2 sin(\theta_2) \\ r_2 cos(\theta_2) \end{pmatrix}, \tag{1}$$

$$\mathbf{O} = \begin{pmatrix} 0 sin(0) \\ 0 cos(0) \end{pmatrix}, \mathbf{D} = \begin{pmatrix} r sin(\theta) \\ r cos(\theta) \end{pmatrix} \tag{2}$$

For two vectors,

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{3}$$

$$\|\mathbf{a} \times \mathbf{b}\| = |(a_1 b_2 - a_2 b_1)| \tag{4}$$

A-0=$\begin{pmatrix} r_1 sin(\theta_1) \\ r_1 cos(\theta_1) \end{pmatrix}$, A-D=$\begin{pmatrix} r_1 sin(\theta_1) - r sin(\theta) \\ r_1 cos(\theta_1) - r cos(\theta) \end{pmatrix}$
B-D=$\begin{pmatrix} r_2 sin(\theta_2) - r sin(\theta) \\ r_2 sin(\theta_2) - r cos(\theta) \end{pmatrix}$, B-O=$\begin{pmatrix} r_2 sin(\theta_2) \\ r_2 cos(\theta_2) \end{pmatrix}$
A-B=$\begin{pmatrix} r_1 sin(\theta_1) - r_2 sin(\theta_2) \\ r_1 cos(\theta_1) - r_2 cos(\theta_2) \end{pmatrix}$

Since ,
Area $(\triangle AOD) + Area(\triangle DOB) = Area(\triangle AOB)$

$$\frac{1}{2}\|(\mathbf{A} - \mathbf{O}) \times (\mathbf{A} - \mathbf{D})\| + \frac{1}{2}\|(\mathbf{B} - \mathbf{D}) \times (\mathbf{B} - \mathbf{O})\| = \frac{1}{2}\|(\mathbf{A} - \mathbf{O}) \times (\mathbf{A} - \mathbf{B})\| \tag{5}$$

$\therefore$ Substituting A-0 , A-D , B-D , B-O , A-B and simplifying further we get,

$$\frac{1}{2}\|(r_1) \times (r \times sin(\frac{\theta}{2}))\| + \frac{1}{2}\|(r_2) \times (r \times sin(\frac{\theta}{2}))\| = \frac{1}{2}\|(r_1) \times (r_2 \times sin(\frac{\theta}{2}))\| \tag{6}$$

Now , R.H.S = $\frac{1}{2}\|(r_1) \times (r_2 \times sin(\theta))\|$

$$= \frac{1}{2}\|(r_1) \times (r_2 \times 2 \times sin(\frac{\theta}{2}) \times cos(\frac{\theta}{2})\| \tag{7}$$

On Comparing (6) and (7),

$$\therefore \ \frac{1}{2} \times \|r \times (r_1 + r_2)\| = \|r_1 \times r_2 \times \cos(\frac{\theta}{2})\| \tag{8}$$

$\therefore$

$$r = |(\frac{2 \times r_1 \times r_2}{r_1 + r_2} \times cos(\frac{\theta}{2}))| \tag{9}$$

Now , D has co-ordinates as $\begin{pmatrix} r \\ \theta \end{pmatrix}$

$\therefore$ *Co-ordinates of D are* $\begin{pmatrix} \frac{2r_1 \times r_2}{r_1 + r_2} \times cos(\frac{\theta}{2}) \\ \frac{\theta_1 + \theta_2}{2} \end{pmatrix}$

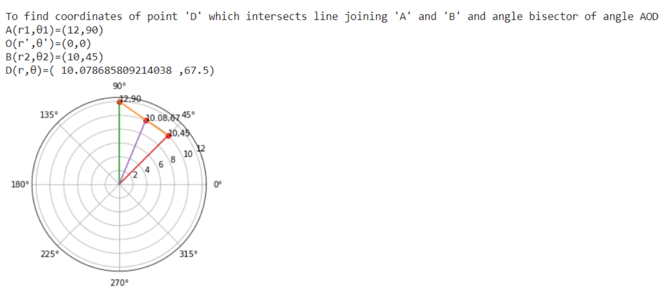

Fig. 1. Triangles AOB,AOD,DOB