Assignment - 1

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Problem

1. The co-ordinates of points A,B are (r_1,θ_1) , (r_2,θ_2) referred to O as pole. The internal bisector of angle AOB meets the line AB in D. Find the co-ordinates of D.

Solution

Let the co-ordinates of point D be $\begin{pmatrix} r \\ \theta \end{pmatrix}$.

Now , $\theta = \dfrac{\theta_1 + \theta_2}{2}$

$$Since, Area(\triangle AOD) + Area(\triangle DOB) = Area(\triangle AOB) \tag{1}$$

$\therefore$

$$\frac{1}{2}\|(r_1) \times (r \times sin(\frac{\theta}{2}))\| + \frac{1}{2}\|(r_2) \times (r \times sin(\frac{\theta}{2}))\| = \frac{1}{2}\|(r_1) \times (r_2 \times sin(\frac{\theta}{2}))\| \tag{2}$$

$$Now\ , R.H.S = \frac{1}{2}\|(r_1) \times (r_2 \times sin(\theta))\|$$

$$= \frac{1}{2}\|(r_1) \times (r_2 \times 2 \times sin(\frac{\theta}{2}) \times cos(\frac{\theta}{2})\| \tag{3}$$

On Comparing (2) and (3),

$$\therefore\ \frac{1}{2} \times \|r \times (r_1 + r_2)\| = \|r_1 \times r_2 \times \cos(\frac{\theta}{2})\| \tag{4}$$

$\therefore$

$$r = |(\frac{2 \times r_1 \times r_2}{r_1 + r_2} \times cos(\frac{\theta}{2}))| \tag{5}$$

Now , D has co-ordinates as $\begin{pmatrix} r \\ \theta \end{pmatrix}$

$$\therefore \text{ Co-ordinates of D are } \begin{pmatrix} \dfrac{2r_1 \times r_2}{r_1 + r_2} \times cos(\dfrac{\theta}{2}) \\ \dfrac{\theta_1 + \theta_2}{2} \end{pmatrix}$$

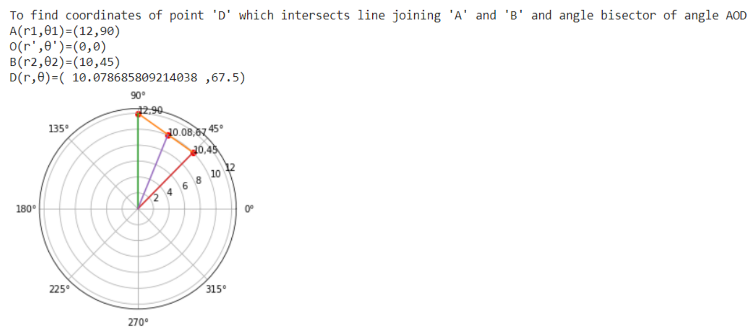

Fig. 1. Triangles AOD,DOB,AOB