Assignment 4

(5) We want to maximize $f^t C f$

Constraint to: i) $f^t f = 1$

ii) $e^t f = 0$ where e is eigenvector with largest eigenvalue.

Using a Lagrange multiplier.

$$\tilde{J}(f) = f^t C f - \mu_1 (f^t f - 1) - \mu_2 (e^t f)$$

$$\frac{\partial}{\partial f} \tilde{J}(f) = 0$$

$$\Rightarrow 2Cf - 2\mu_1 f - \mu_2 e = 0$$

$$\Rightarrow Cf = \mu_1 f + \frac{\mu_2}{2} e \quad \text{——— (i)}$$

$$\Rightarrow e^t C f = \mu_1 \cancel{(e^t f)} + \frac{\mu_2}{2}(e^t e) \quad \text{——— (ii)}$$

As $Ce = \lambda_1 e$ and C is symmetric,

$$e^t C^t = \lambda_1 e^t = e^t C$$

$\therefore$ (ii) $\Rightarrow$

$$e^t C f = \lambda_1 e^t f = \frac{\mu_2}{2} \Rightarrow \boxed{\mu_2 = 0}$$

$\therefore Cf = \mu_1 f$ $\rightarrow$ μ_1 is an eigenvalue ~~corresponding~~ of C, f is corresponding eigenvector.

$f^t C f = \mu_1 f^t f = \mu_1$. $\therefore$ We want to maximise μ_1

$\therefore \mu_1$ is the highest possible eigenvalue s.t. $fe^t = 0$.

~~As~~ As $\mu_1 \neq$ largest eigenvalue (else $fe^t \neq 0$),

($\neq 0$ as rank$(C) \geq 2$)

μ_2 = second largest eigenvalue.

$\Rightarrow f$ is corresponding eigenvector. (As C is real symmetric, $f^t e = 0$)