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Assignment 5

Q1.

We can see that PCA based techniques perform (much!) better than Bilateral Filtering. As compared to the Bilateral Filter, the PCA based techniques are more global and they use image restoration (using that the noise added is a gaussian with known sigma, zero mean). The Bilateral Filter, on the other hand, is a technique for image enhancement only.

Further, constructing the eigenspace using KNN ensures that the mean alpha (squared) is closer to the true value as we use similar patches only. This leads to better restoration.

Q5 Continued...

Note that `Hx(u,v)` will be small when `u` is small. On the other hand, `Hy(u,v)` will be small when `v` is small. Hence, while recomputing `F(u,v)`, we use `Gx/Hx` when u is sufficiently large but `v` is small (and vice versa). However, we are still unable to extract `F(u,v)` when both `(u,v)` are small, as both estimates might blow up.

3. • We observe that there are ringing artifacts in image after applying ideal low pass filter (& not in the case of gaussian low pass filter). This is due to the fact gaussian filter only diminises while ideal low pass filter wipes out the high frequency components required to express the image (edges).

• As D_0 increases ringing artifacts reduce.

• $\sigma \downarrow$ $D \downarrow$ $\Rightarrow$ more blur.

4. Given, $g_1 = f_1 + f_2 * h_2$ $\qquad g_2 = h_1 * f_1 + f_2$

where h_i are blur kernels. (known)

Applying Fourier Transform we get,

$$G_1 = F_1 + H_2 \cdot F_2$$
$$G_2 = H_1 \cdot F_1 + F_2$$

due to linearity of FT & $\because \mathcal{F}(h * f) = \mathcal{F}(h) \cdot \mathcal{F}(f)$ where $\mathcal{F}(h_i) = H_i$.

$$\therefore \hat{F}_1 = \frac{G_2 - \frac{G_1}{H_2}}{H_1 - \frac{1}{H_2}} = \left(\frac{G_2 H_2 - G_1}{H_1 H_2 - 1}\right) \quad \& \quad \hat{F}_2 = \frac{G_1 H_1 - G_2}{H_2 H_1 - 1}$$

$$\therefore f_1 = \mathcal{F}^{-1}\left(\frac{H_2 G_2 - G_1}{H_1 H_2 - 1}\right) \quad \& \quad f_2 = \mathcal{F}^{-1}\left(\frac{H_1 G_1 - G_2}{H_2 H_1 - 1}\right)$$

As h_i are blur kernels, H_i are low pass filters.

$\therefore$ for small frequencies $H_i(u,v) \approx 1$ hence the denominator blows up & we won't be able to extract low frequency components in $\hat{F}_1, \hat{F}_2$ (even the noise (relative) becomes large), ~~[illegible]~~ which are ~~[illegible]~~ having large contributn in natural images.
(large magnitude)

5. Given, $g = h * f$ $\quad \therefore G = H \cdot F$ $\quad$ where $G = \mathcal{F}(g)$ $F = \mathcal{F}(f)$ $H = \mathcal{F}(h)$

$\therefore \hat{F} = \left(\frac{G}{H}\right)$ since h is for gradient operatn

$\therefore H$ is high pass filter $\therefore H(u) \approx 0$. for small u.

$\therefore$ we won't be able to extract low frequency components in image. (even the noise will blow up if present ~~[illegible]~~)

If we have, $g_x = h_x * f$

$g_y = h_y * f$

$\therefore G_x = H_x F$ & $G_y = H_y F$ $\qquad \hat{f} = \mathcal{F}^{-1}(\hat{F})$ where $\hat{F} = \frac{G_x}{H_x}$ or $\frac{G_y}{H_y}$

here we face the same problem, since H_x, H_y are high pass & will be small for low frequencies. $(\longrightarrow 0)$