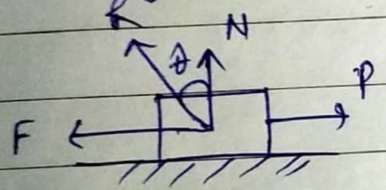
FRICTION

The force which opposes the movement or the tendency of movement is called Frictional force.

→ Limiting friction
→ Static friction
→ Dynamic friction

Sliding   Rolling



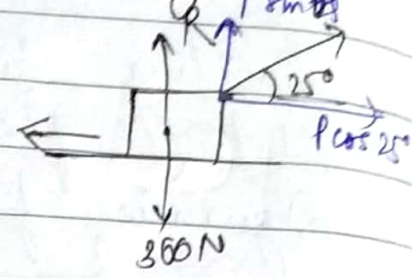
θ = Angle of friction
= It is the angle which resultant makes with normal.

$\tan\theta = \frac{F}{N} = \mu$ (Coeff. of friction).

Angle of Repose :-

It is the angle with an inclined plane makes with the horizontal so that a body placed over it just begins to slide of its own accord.

Q1.) A body of weight 300 N is lying on a rough horizontal ~~surface~~ plane having coeff. of friction 0.3. find the magnitude of the force which can move the body, while acting at an angle of 25 with the horizontal.



Solⁿ Resolving the forces along the horizontal forces —

$F = P\cos 25'$

Resolving the force in vertical direction —

$R + P\sin 25^\circ = 300$

$\mu = \frac{F}{R} = 0.3$

$\Rightarrow 0.3 = \frac{P\cos 25^\circ}{300 - P\sin 25^\circ}$

$\Rightarrow 90 - 0.3 P\sin 25^\circ = P\cos 25^\circ$

$\Rightarrow 90 = P(\cos 25^\circ + 0.3\sin 25^\circ)$

$P = \cancel{\frac{90}{0.779}}\ \frac{90}{1.03}$

$P = 87.1\ N$

Q2.) A body resting on a rough horizontal plane required ~~to~~ a pull of 180N inclined at 30° to the plane just to move it. It was found that a push of 220N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coeff. of the friction.

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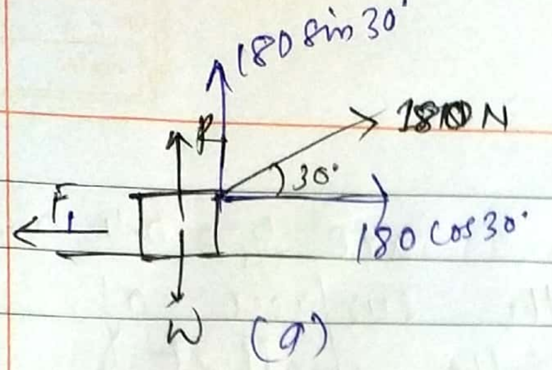

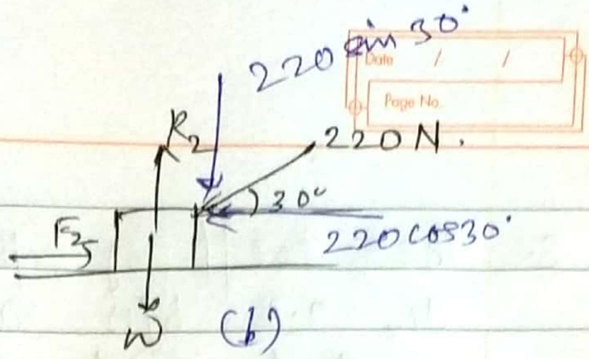

A² (a) Resolving the forces in the horizontal direction –

$F_1 = 180 \cos 30°$

$F_1 = 155.88$ N

Resolving the forces in the vertical direction –

$R_1 + 180 \sin 30° = W$

$R_1 = W - 90$

$\mu = \frac{F_1}{R_1} = \frac{155.88}{W-90}$ — (1)

(b) Resolving forces in horizontal direction –

$F_2 = 220 \cos 30°$

$F_2 = 190.52$ N

In vertical direction,

$W + 220 \sin 30° = R_2$

$W + 110 = R_2$

$\mu = \frac{F_2}{R_2} = \frac{190.52}{W+110}$ — (2)

$\Rightarrow 155.88(W + 110) = 190.52(W - 90)$

$\Rightarrow 155.88W + 17146.8 = 190.52W - 17146.8$

$\Rightarrow 34.64W = 34293.6$

$\Rightarrow W = 990$ N (Ans.)

Put W in eq (1),

$\mu = 0.1732$

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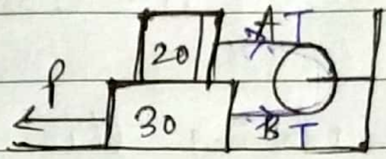
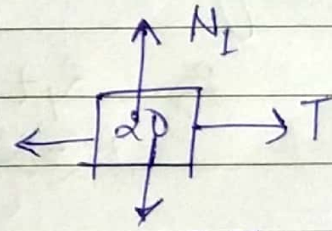
3. The coeff. of friction are $\mu_s = 0.4$ and $\mu_k = 0.3$ b/w all the surfaces of contact. Determine the smallest force P required to start the 30 kg block moving if the cable AB (a) is attached as shown (b) is removed.

Sol: Resolving the forces along the horizontal direction,



$F_1 = T$

$F_1 = \mu_s \times N_1$ —— (1)



Resolving the forces along the vertical direction,

$w_1 = 20 \times 9.81 = 196.2$ N

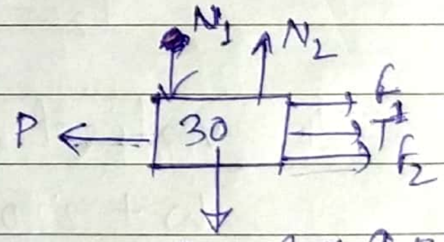
$N_1 = 20 \times 9.81 = 196.2$ N

$F_1 = 0.4 \times 196.2$

$F_1 = 78.48$ N

Block 2,

Resolving the forces along horizontal,



$w_2 = 30 \times 9.81 = 294.3$

$P = F_1 + T + F_2$

$P = 78.48 + 78.48 + 0.4(490.5)$

$P = 156.96 + 196.2$

$P = 353.2$ N

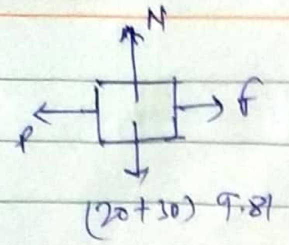
Resolving the vertical forces,

$N_1 + w_2 = N_2$

$N_2 = 490.5$ N

-5

(b)



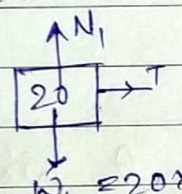
$P = F = \mu_s N$

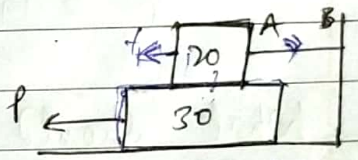
$P = 0.4 \times 50 \times 9.81$

$P = 196.2\,N$ (Ans.)

4. The coefficients of friction are $\mu_s = 0.4$ and $\mu_k = 0.3$ between all the surfaces of contact. Determine the smallest force P required to start the 30 kg block moving if the cable AB (a) is attached (b) is removed.

Soln -





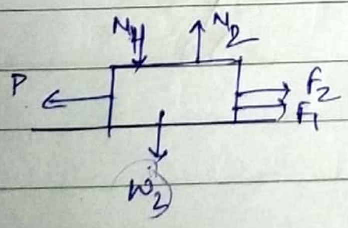
(a) Resolving the forces along horizontal direction,

$F_1 = T$

$= \mu_s N_1$

$= 0.4 \times 196.2$

$F_1 = 78.48\ N$



Resolving the forces in vertical direction,

$N_2 = W_2 + N_1$

$+196.2 + 30\times 9.81 =$

$N_2 = N_1 + W_2$

$= 196.2 + 294.3$

$N_2 = 490.5\ N.$

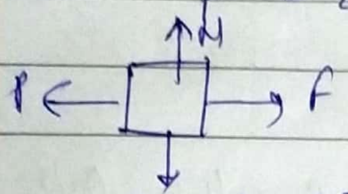
Resolving forces in ~~vertical~~ horizontal direction,

$P = F_1 + F_2$

$= 78.48 + 0.4 \times 490.5$

$P = 274.68\ N$ Ans.

(b)



$W = (20+30) \times 9.81$

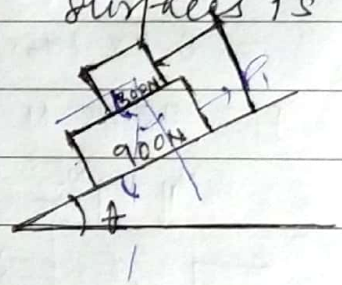
$N_1 = W = 50 \times 9.81 = 490.5$

$P = \mu_s \times N$

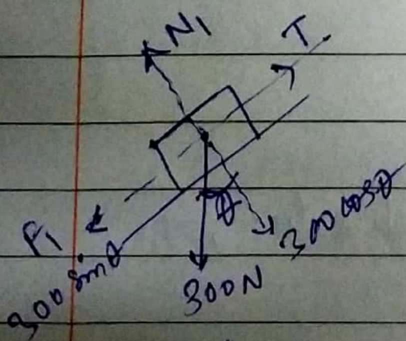
$= 0.4 \times 490.5$

$P = 196.2\ N$ (Ans.)

5. What should be the value of θ which will make the motion of 900 N block down the block to impend? The coefficient of friction force for all contact surfaces is $1/3$.



Soln Free body diagram for 300N Block,



Resolving the forces along the inclined,

$T = F_1 + 300 \sin\theta$

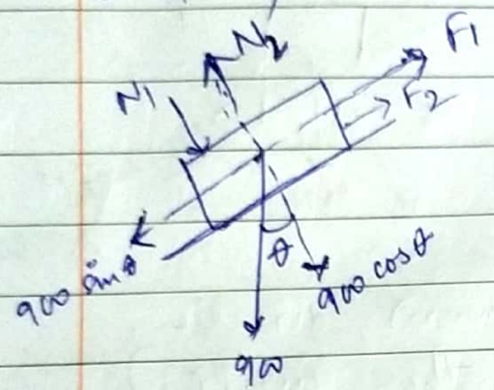
$N_1 = 300 \cos\theta$

$F_1 = \mu_s N_1$

$= \cancel{\frac{1}{3}} \times \cancel{300}^{100} \cos\theta = 100 \cos\theta$

$T = 100 \cos\theta + 300 \sin\theta.$

Block 2,



$N_2 = N_1 + 900 \cos\theta$

$= 300 \cos\theta + 900 \cos\theta$

$= 1200 \cos\theta$

Resolving forces along inclined,

$\cancel{F_1 = \mu_s N_1}$

$= \cancel{1/3}$

$\cancel{F_1 + F_2 = 900 \sin\theta}$

$\cancel{F_2 = 900 \sin\theta - \frac{1}{3} \times 1200^{400} \cos\theta}$

$\cancel{F_2 = 900 \sin\theta - 400 \cos\theta}$

$F_1 + F_2 = 900 \sin\theta$

$900 \sin\theta = 100 \cos\theta + \cancel{\frac{1}{3}} \times \cancel{1200}^{400} \cos\theta$

$900 \sin\theta = 100 \cos\theta + 400 \cos\theta.$

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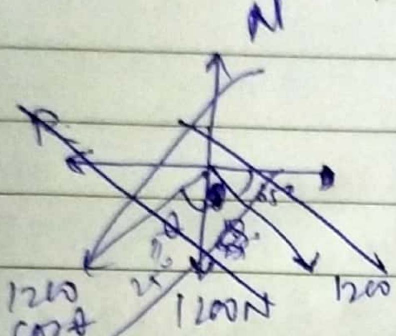
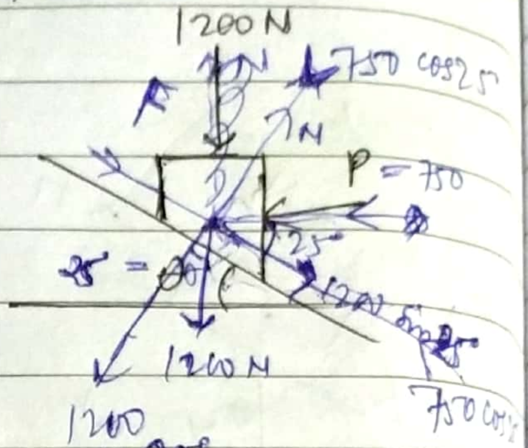
6) Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25$ and $P = 750$ N.
($\mu_s = 0.35$, $\mu_k = 0.25$)

Sol:





Frictional force is taken in downward direction,
Resolving the forces along the inclined,

$$F + 1200 \sin 25^\circ = 750 \cos 25^\circ$$

$$F = 679.73 - 507.14$$

$$F = 172.588 \text{N}$$

Resolving the forces normal to the inclined,

$$N = 1200 \cos 25^\circ + 750 \cos 25^\circ$$

$$N = 1404.53 \text{ N}$$

$$F_{max} = \mu_s N$$

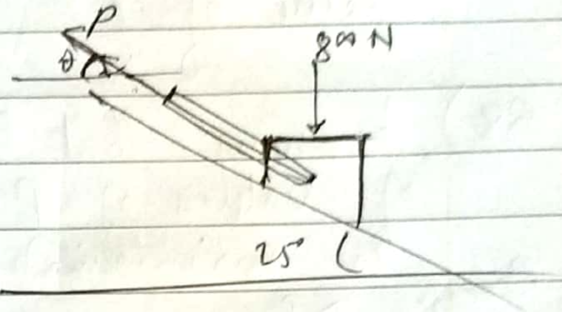
$$= 0.35 \times 1404.53$$

$$F_{max} = 491.53 \text{ N}$$

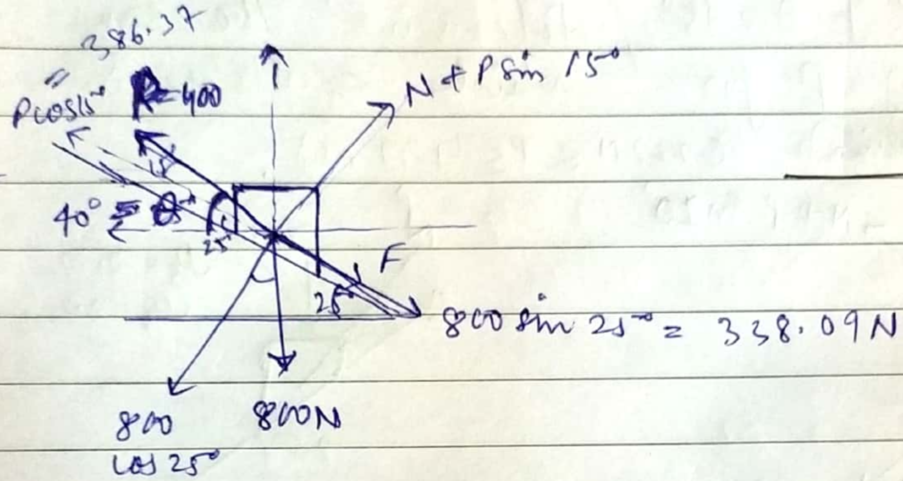
Hence, block is in equilibrium.

Q7) Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 40°$ & $P = 400$ N.

$\mu_s = 0.20$, $\mu_k = 0.15$



Soln



Resolving the forces along the inclined,

$F + 800 \sin 25° = 400 \cos 15°$

$F = 386.37 - 338.09$

$F = 48.28$ N (↘)

Resolving the forces in downward direction,

$N + P \sin 15° = 800 \cos 25°$

$N = 621.57$ N

$F_{max} = \mu_s \cdot N$

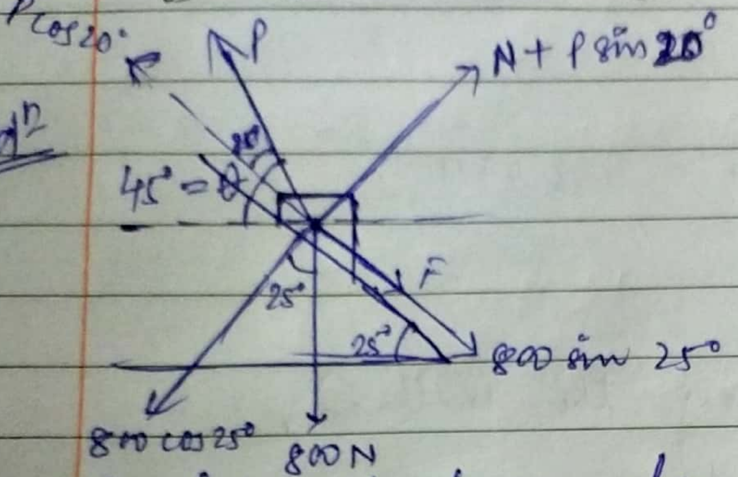
$= 0.20 \times 621.57$

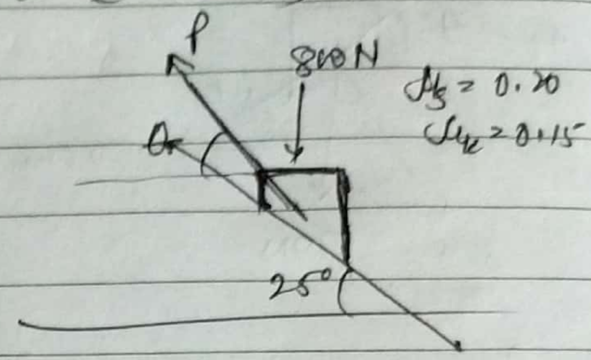
$F_{max} = 124.30$ N

Block is in equilibrium.

Q8.) Knowing that $\theta = 45°$, determine the range of values of P for which equilibrium is maintained ($222N \le P \le 479\,N$).

Soln





Resolving the forces along the inclined.

$F + 800 \sin 25° = P\cos 20°$

Resolving forces along normal to plane,

$N + P\sin 20° = 800\cos 25°$

$\because$ Body is eq^m,

$F < F_{max}$,

$\because \mu_s = 0.2 = F/N$

$$0.2 = \frac{P\cos 20° - 800\sin 25°}{800\cos 25° - P\sin 20°}$$

$\Rightarrow 145.009 - 0.2 \times P\sin 20° = P\cos 20° - 800\sin 25$

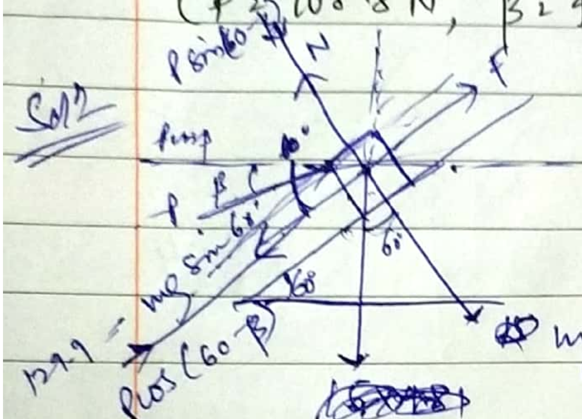
$\Rightarrow P(\cos 20° + 0.2\sin 20) = 483.103$

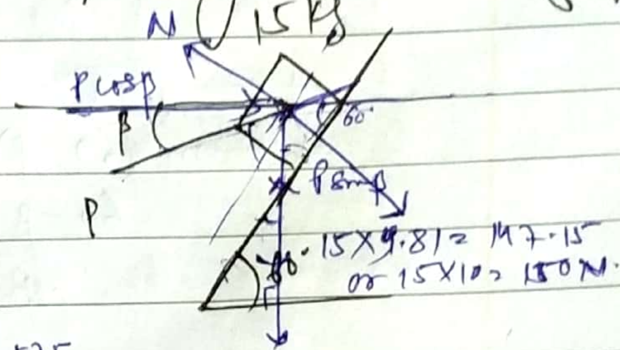
$$P = \frac{483.103}{1.024}$$

$= 471.78$

Q9.) Knowing that the coefficient of friction b/w the 15-kg block and the incline is $\mu_s = 0.25$, determine (a) the smallest value of P required to maintain the block in equilibrium (b) the corresponding value of β.
($P = 108.8$ N, $\beta = 46°$).

Sol2





$mg \cos 60° = 73.575$

$mg = 147.15$ N

Resolving the forces along the inclined,

$$F + P\cos(60-\beta) = 147.15 \sin 60°$$
$$F = 127.43 - P\cos(60-\beta)$$

Resolving the forces in a direction normal to the inclined,

$$P\sin(60-\beta) + mg\cos 60° = N$$
$$N = 73.575 + P\sin(60-\beta)$$

$$\because \mu = \frac{F}{N} = 0.25$$

$$\Rightarrow (0.25)(73.575 + P\sin(60-\beta)) = 127.43 - P\cos(60-\beta)$$

$$\Rightarrow 0.25\, P\sin(60-\beta) + P\cos(60-\beta) = 109.04$$

$$P = \frac{109.04}{0.25\sin(60-\beta) + \cos(60-\beta)}$$

For P to be smallest, the denominator should be maximum.

$$\Rightarrow \frac{d(0.25\sin(60-\beta) + \cos(60-\beta))}{d\beta} = 0$$

$$\Rightarrow \cancel{\cos(60-\beta) - \sin(60-\beta) = 0}$$

$$\Rightarrow -0.25\cos(60-\beta) + \sin(60-\beta) = 0.$$

$$\Rightarrow \tan(60-\beta) = 0.25$$

$$60-\beta = 14.04$$

$$\therefore \boxed{\beta = 45.96^\circ}$$

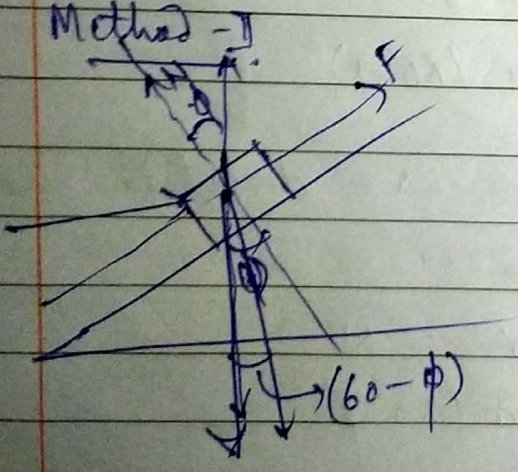
0.25
0.97

1.21

$$P = \frac{109.04}{0.25\sin(14.04) + \cos(14.04)}$$

$$P = 105.86\ N.$$

Method - II



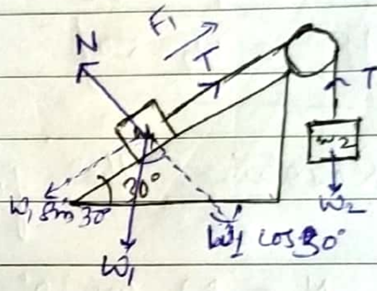
For P to be smallest, it has to be $\perp^r$ to the resultant of normal reaction & the resultant is making an angle of $(60-\phi) = 46^\circ$.

$$\phi = \tan(0.25)$$

$$\phi = 14.03^\circ$$

Q 10: A block of weight $W_1 = 100N$ rests on an inclined plane and another weight W_2 is attached to the first weight through a string as shown in fig. below. If coeff. of friction b/w the block and plane is 0.3, determine the max^m & min^m values of W_2 so that eq^n can exist. $(24 \le W \le 76)$.

Sol^n: ~~(a)~~ Case(a) If body is moving downward.



Resolving the force for weight W_1 along the plane,

$\Rightarrow W_1 \sin 30° - T = F_1$

$\Rightarrow F_1 = 50 - T$

$\Rightarrow F_1 = 50 - W_2$

Resolving the forces normal to inclined,

$N_1 = W_1 \cos 30°$

$= 100 \cos 30°$

$N_1 = 86.60$

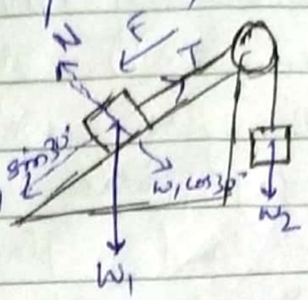
$\mu = \frac{F_1}{N_1} = 0.3$

$\Rightarrow 0.3 = \frac{50 - W_2}{86.60}$

$\Rightarrow W_2 = 50 - 25.98$

$\therefore W_2 = 24.02 N$ (Ans.)

(b) When the body is moving upwards,



Resolving the forces along the plane,

$F = T - W_1 \sin 30^\circ$

$F = T - 50$

$\boxed{F = W_2 - 50}$

Resolving the forces along the normal to plane.

$N = W_1 \cos 30^\circ$

$= 100 \cos 30^\circ$

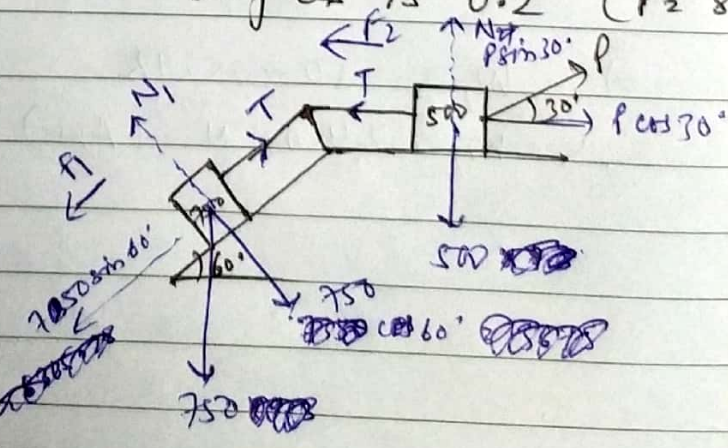
$\mu = 0.3 = \frac{F}{N}$

$0.3 = \frac{W_2 - 50}{86.60}$

$W_2 = 75.98\text{ N}$

Q 11.) What is the value of P in the system shown in figure to cause the motion to impend? Assume the pulley is smooth and coeff. of friction b/w the other contact surfaces is 0.2 ($P = 853.5N$),

Soln



for block 1,

Resolving the forces along the inclined,

$F_1 = T - 750 \sin 60°$

$F_1 = T - 649.51 N$

Resolving the forces along the direction normal to inclined,

$N_1 = 750 \cos 60°$

$N_1 = 375 \ N$

$\mu_s = \frac{F_1}{N_1}$

$0.2 = \frac{T - 649.5}{375}$

$\Rightarrow 75 = T - 649.5$

$\Rightarrow T = 724.52 N$

for block 2,

$T + F_2 = P \cos 30°$

$F_2 = P \cos 30° - T$

$N_2 = 500 - P \sin 30°$

0.966

$\mu = \frac{F_2}{N_2} = \frac{P \cos 30° - T}{500 - P \sin 30°}$

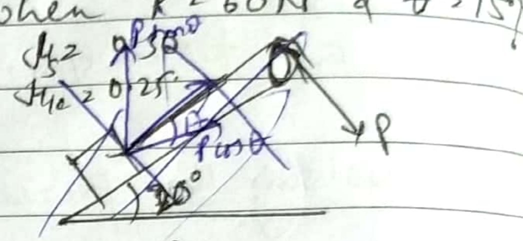
$\Rightarrow 0.2 = \frac{P \cos 30° - 724.52}{500 - P \sin 30°}$

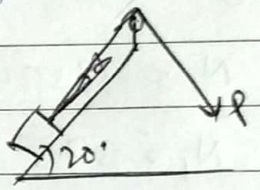
$\Rightarrow 100 - 0.2 P \sin 30° = P \cos 30 - 724.52$

$\Rightarrow 824.52 = P(\cos 30° + 0.2 \sin 30°)$

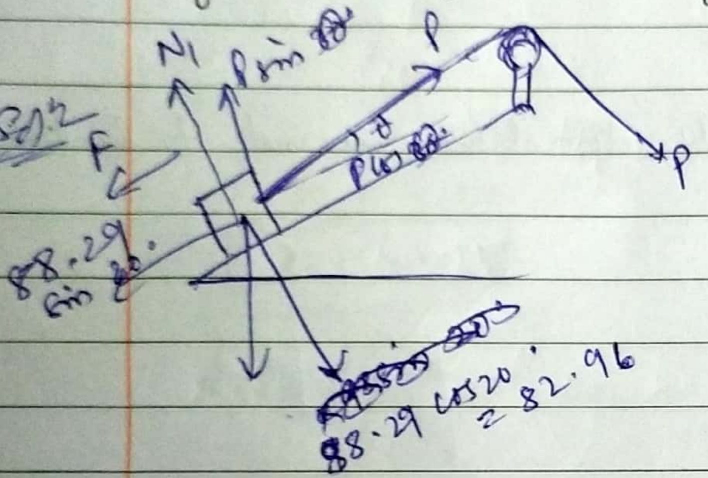
$P = 853.52 \ N$

Q12) Determine whether the 9 kg block shown is in eq^{bm}, find the magnitude and direction of the friction force when $P = 60N$ & $\theta = 15°$)





Sol^n



$N_1 = 82.96 - P\sin\theta$

$N_1 = 82.96 - 15.52 = 67.44$ N

$F_{max} = \mu_s N_1$

$= 0.3(67.44)$

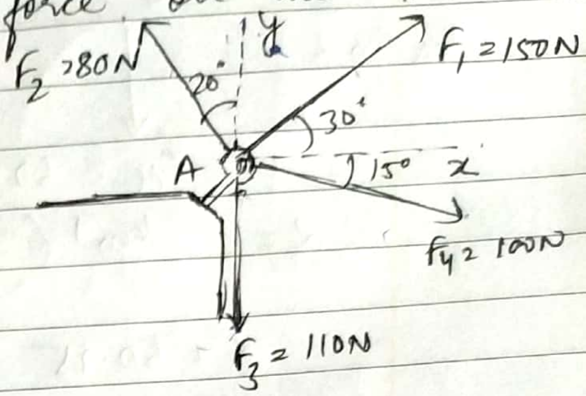
$F_{max} = 20.232$ N

$\therefore F > F_{max}$.

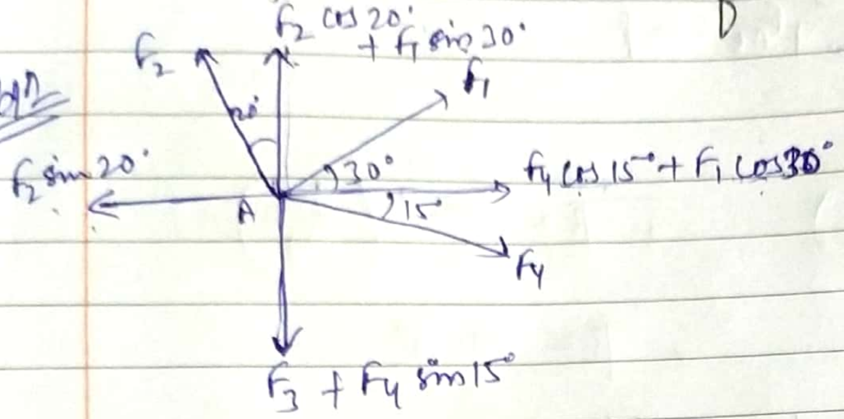
$F = \mu_k N$

$F = 16.86$ N

Q) Four forces act on bolt A as shown. Determine the ~~force~~ resultant of the force on the Bolt.



Soln



Resolving along horizontal,

1613.62

199.62

39856.95

~~$F_2 \sin 20° = F_4 \cos 15° +$~~

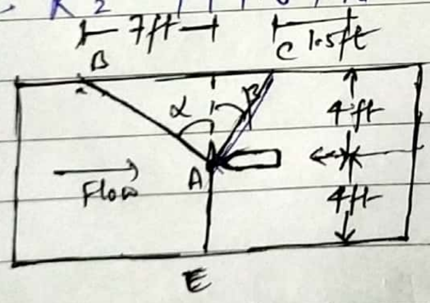
$F_x = \cancel{40.17\,N}$ 14.29 N

$F_y = 199.13\,N$

$\rightarrow R = \sqrt{204.2 + 39652.75}$

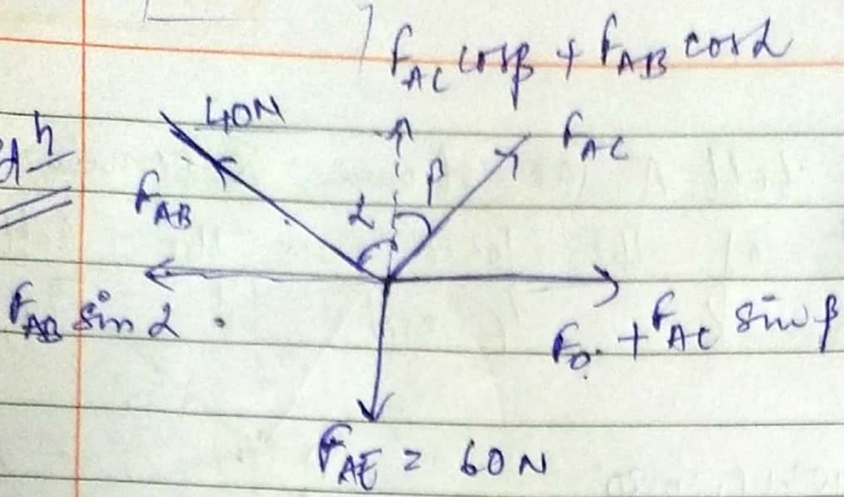
$\therefore R = 199.64\,N$

Q)



It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable AB and 60 lb in cable AE. Determine the drag force exerted on the hull and the tension in cable AC.

Sol^n



$\alpha = \tan^{-1}(7/4)$, $\beta = \tan^{-1}\left(\frac{1.5}{4}\right)$

$\alpha = 60.25$, $\beta = 20.55$

Resolving along horizontal,

$F_{AB}\sin\alpha = F_D + F_{AC}\sin\beta$

Resolving along vertical,

$F_{AC}\cos\beta + F_{AB}\cos\alpha = 60\ lb$

$F_{AC} = \frac{60 - 19.84}{\cos\beta}$

$= \frac{40.15}{\cos\beta}$

$F_{AC} = 42.87\ lb$

$\Rightarrow F_D = F_{AB}\sin\alpha - F_{AC}\sin\beta$

$\therefore F_D = 19.6\ lb.$ (Ans)

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