Working Notes

Your Name

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Aim

State the current question in one sentence.

Lemma 1 (Toy case)**.** *If X is affine and $\mathcal{F}$ is coherent, then …*

Proof. Sketch the toy model / counterexample here. $\square$

Next steps

- Reduce to local statement on $\operatorname{Spec} A$.
- Examine obstruction in Ext^1.

1 Base change sanity check

We record a minimal toy model and the exact point where the argument needs a cohomological vanishing / exactness input.

Lemma 2. *Let $f : X \to Y$ be* (insert hypotheses) *and $\mathcal{F}$ a coherent sheaf* (plus conditions)*. Then formation of $Rf_*\mathcal{F}$ commutes with base change along $g : Y' \to Y$ at the point $y \in Y$ provided* (insert criterion)*.*

Proof. Reduce to an affine chart $Y = \operatorname{Spec} A$, $X = \operatorname{Spec} B$, and phrase the statement as exactness of derived tensor with $A \to A/I$ at y. Identify the obstruction in an Ext^1 group; show it vanishes under your hypotheses. Keep the toy counterexample where it fails. $\square$

Next step. Check the Tor–amplitude hypothesis needed for the fiberwise criterion.

References. Hartshorne III.9, Stacks Project Tag 0A1X; FOAG §.

2 From Čech to derived: a micro-log

Track the exact assumptions under which the Čech complex computes sheaf cohomology and how this feeds a spectral sequence.

Lemma 3. *If $\mathcal{U} = \{U_i\}$ is an acyclic cover for $\mathcal{F}$ (i.e., $H^q(U_{i_0 \cdots i_p}, \mathcal{F}) = 0$ for $q > 0$), then the Čech complex $C^\bullet(\mathcal{U}, \mathcal{F})$ computes $R\Gamma(X, \mathcal{F})$.*

Proof. Sketch the comparison morphism $C^\bullet(\mathcal{U}, \mathcal{F}) \to I^\bullet$ into an injective resolution and note it is a quasi-isomorphism under acyclicity. $\square$

Spectral sequence. State the $E_1^{p,q} = \prod H^q(U_{i_0 \cdots i_p}, \mathcal{F}) \Rightarrow H^{p+q}(X, \mathcal{F})$ and the precise hypotheses.

Next step. Record the smallest condition you actually need in the target application.

References. Godement, Stacks Project Tags 01DW, 01DS; FOAG §.