Take home exam (MM1 queue): We now describe an MM1 queue. Any queueing system is described by arrivals and departures.
The arrivals in an MM1 queue are according to a Poisson process. Here the inter-arrival times are represented by $\{A_1, A_2, \cdots, A_n \cdots, \}$, which are exponentially distributed with parameter λ (mean interarrival time $= 1/\lambda$). Here A_1 is instance at which the first arrival occurs and A_n is the time duration between the $(n-1)$-th and n-th arrival.
Each arrival brings with it a job requirement, and demands server time. These demand times (job processing times) are exponentially distributed with parameter μ, and are independent of all other events.
The server is working conserving, i.e., it keeps working without rest as long as there are customers in the system. If the queue gets empty, it rests, but starts immediately once a new customer arrives.
PASTA (Poisson Arrivals see See Time Averages): This property states the following: "the average number of customers as seen by Poisson arrivals equals the time average of the number of customers (which in turn equals the stationary expected number of customers)". One can estimate the stationary expected number of customers using this property: the sample average of the number of customers present in the system (just before the arrival) across the Poisson arrivals equals the stationary expected number of customers in the system. Basically one needs to consider large number of (Poisson) arrivals (more than 20000/30000) and find the sample average of the number of customers in the system just before the arrivals.
Assume $\lambda = 0.1$ and $\mu = 0.2$.

1. Use PASTA property to estimate the stationary expected number of customers in an MM1 queue. Provide the estimates when you start initially with 1 customer, i.e., when $X_0 = 1$.

2. Provide another estimate when you start with 10 customers, i.e., with $X_0 = 10$. Compare these two estimates.

3. MM1 system is well analysed in the literature. Many standard text books provide the expressions for stationary expected number of customers. Find the formula for the expected number in system and compare it with the estimates that you obtained.

4. Once the system gets empty, a new busy period starts with one arrival. And the busy period is said to end when the system gets empty again. Any busy period is followed by an idle period, which then is followed by another new busy period. And this goes on. Write a program to estimate the expected busy period under stationarity. You can find this by taking sample average of many busy periods. Compare this also with the formula provided by standard text books.