Instructions:

We will practice modeling more optimization problems with integer variables in this lab. In this lab we will only see problems with binary variables. Binary variables are the variables that are allowed to take only two values: 0 or 1. They can be used to model a large number of problems which involve choice between two or more options.

There are 3 exercise problems in this lab. Solve all the problems. Clarifications will be provided for all questions by the instructor/TAs.

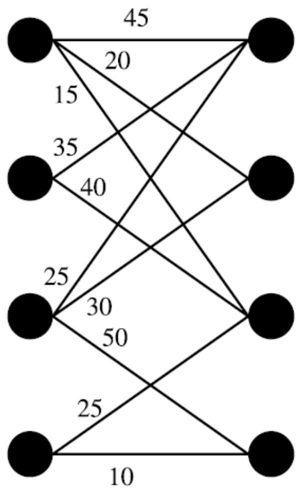
Exercise 1: Assigning Locations
Sind constructions has to build $n \in \mathbb{N}$ different types of factories, one at each of the n locations. The cost of constructing (setup cost) the i^{th} facility at the j^{th} location provided in the Table 1 below. Sind Constructions wants to minimize the sum of the costs of assigning all the facilities to the locations.

1. **[R]** Write a mathematical model of the above assignment problem. Define all the variables and constraints clearly.
2. Write an AMPL model for this problem for a general n. You can assume that the cost matrix is given as data file.
3. Use the table below to make a data file for your model.
4. **[R]** Solve the problem and report which facility must be opened at each location.
5. **[R]** Now change the integer variables in your model to continuous variables, and re-solve the problem. Report the solution (only the non-zero values of the solution).
6. **[R]** Are the optimal costs for both problems same? Are the values of the variables still integer? If yes, explain why.
7. Will the solution to the continuous problem become fractional (non-integer) if the costs are changed to non-integer values? Try changing the costs to my different values and test whether the solution to the LP becomes fractional for any of them.
8. **[R]** Now suppose that, due to some reason, faciliy 1 can not be assigned to location 3, facility 12 can not be assigned to location 12 and facility 7 can not be assigned to location 7. What changes in `.mod` or in `.dat` file will you make? Solve the integer problem and report the solution.

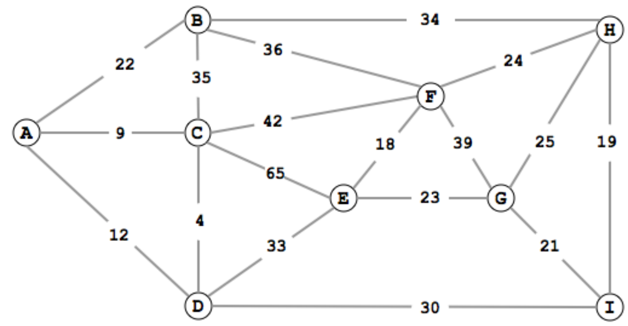
Factory	Location											
	1	2	3	4	5	6	7	8	9	10	11	12
1	19	12	18	19	22	21	17	20	16	15	21	24
2	22	22	19	21	22	24	18	17	21	19	22	23
3	18	23	20	20	21	22	19	18	20	23	19	19
4	18	21	20	18	17	19	24	16	18	16	20	24
5	23	17	16	19	24	21	23	21	20	21	22	21
6	23	20	17	16	20	23	22	25	24	19	17	20
7	22	18	17	15	22	24	23	20	22	19	23	20
8	24	22	21	23	18	17	16	19	24	21	20	23
9	21	20	17	18	16	24	19	17	18	20	21	23
10	19	22	21	24	20	23	19	18	23	24	25	20
11	20	24	22	20	23	19	18	16	22	24	21	24
12	22	23	24	20	21	20	20	19	17	19	20	22

Table 1: Set up cost of factories at different locations

Exercise 2: Another assignment problem



(a) Graph 1



(b) Graph 2

Let $G = (V, E)$ be an undirected graph, where V and E are, respectively, the set of vertices and edges. Let $w_{ij} \geq 0$ be the weight of the edge $(i, j) \in E$. A "*cut*" in a weighted undirected graph is defined as a partition of the vertices into two sets S and $V \setminus S$ (say), and the weight W_S of the cut $(S, V \setminus S)$ is the summation of the weights of the edges that have one end point in S and the other in $V \setminus S$, i.e.,

$$W_S = \sum_{(i,j) \in E : i \in S, j \in V \setminus S} w_{ij}$$

The max-cut problem is defined as the problem to find a cut in the graph G with maximum weight, i.e.,

$$\max_{S \subset V} W_S$$

1. **[R]** Consider the first graph given below. Use your intuition to find the max-cut for Graph-1. Explain why your intuition is correct.

2. **[R]** Write an integer programming formulation to find the max cut of a general graph. Define all the variables and constraints clearly. Your model should have linear constraints and objective function only. You may use binary variables if necessary. (Hint: if you have a nonlinear constraint of the form $y = x_1 x_2$, where x_1, x_2 are binary, then y is also binary, also $x_1 = 0$ should force y to be zero, i.e. $y \leq x_1$. Simiarly we can add more linear constraints to force y to be zero when $x_2 = 0$ and $y = 1$ when $x_1 = x_2 = 1$).

3. Write an AMPL model and data file for Graph-2, and solve the integer problem using CPLEX or Gurobi. You may test your formulation using Graph-1.

4. **[R]** Write down the solution and the value of max-cut for Graph-2. Your solution must only say which vertices of the graph are in each partition.

Exercise 3: More on Assigning Locations
Let us now consider a different variation of the Problem 1. Once the 12 factories are set up and start operating, they will be transporting goods amongst themselves. We would like to assign locations to these factories so that the total cost of transportation between them is minimized. The quantities (in Tonnes) that must be transported every week from factory-i to factory-j, $i, j = 1, \ldots, 12$ are given in the Table 2 below. The cost of transporting goods depends on the location of the source and destination factory. Table 3 below gives the cost of transporting one tonne of goods from location i to location j, $i, j = 1, \ldots, 12$. We can ignore the setup costs of Exercise 1.

Factory (Source)	Factory (Destination)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0	21	21	0	23	20	18	19	0	20	24	25
2	20	18	21	20	19	0	22	20	18	19	0	23
3	21	23	18	19	0	17	16	24	16	18	24	0
4	12	0	18	0	21	22	20	17	15	16	24	21
5	17	18	19	0	16	21	21	23	21	0	21	21
6	22	23	20	24	21	19	20	0	19	17	22	20
7	20	22	0	17	23	0	21	19	24	25	20	17
8	0	0	23	17	24	22	23	0	19	22	20	23
9	22	22	21	21	24	24	18	18	19	22	22	23
10	21	0	22	0	23	16	17	24	19	23	20	23
11	0	22	24	23	20	18	16	19	24	22	22	24
12	17	24	20	16	18	19	0	18	17	21	20	0

Table 2: Quantities that must be transported from a factory to another factory every week

Location (Source)	Location (Destination)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0	11	13	9	19	20	27	13	19	11	19	10
2	11	0	8	9	11	19	10	15	12	20	24	11
3	13	8	0	18	19	20	27	27	19	24	12	17
4	9	9	18	0	19	20	10	20	21	20	20	10
5	19	11	19	19	0	21	17	26	20	14	24	17
6	20	19	20	20	21	0	28	14	22	17	22	20
7	27	10	27	10	17	28	0	12	18	26	10	22
8	13	15	27	20	26	14	12	0	27	10	19	17
9	19	12	19	21	20	22	18	27	0	20	22	16
10	11	20	24	20	14	17	26	10	20	0	25	21
11	19	24	12	20	24	22	10	19	22	25	0	21
12	10	11	17	10	17	20	22	17	16	21	21	0

Table 3: Unit costs of transportation from a location to another location

1. **[R]** Write an integer linear program to model this problem.
2. **[R]** If there are n factories and n locations, how many variables and constraints are there in your model?
3. **[R]** Solve the problem for the given data. Report the location of each factory in the solution and also the running time of your model.