

In this lab, we will use Scilab to perform some more simulations.

**Exercise 1: A virus**

Bindu is a student in a class of  $n$  students. Each student in the class has stored in his/her computer the email addresses of all other students (but not his/her own). Assume that the students have not stored any other email-addresses of people outside of the class. Bindu's computer is infected with a virus. The virus selects exactly one address randomly from the addresses stored on the computer and spreads to the computer of that student. Then it tries to spread in the same fashion from the newly infected computer. If at any stage, the next computer that the virus selects is already infected, the virus stops spreading any further. We are interested in finding how many students are expected to have their computers infected (including Bindu) after the virus has stopped spreading. You may have to use `grand(1,1,'uin',1,n)` to generate a uniform random integer between 1 and  $n$ .

- (a) [R] Explain clearly how you can simulate this situation. Your answer should mention what type of loops and/or arrays will be used and why your logic is correct.
- (b) Implement a function `k = simvirus(n)` which takes  $n$  (the number of students) as an input and returns the number of computers infected in a single simulated run.
- (c) [R] Run your simulation keeping  $n = 20$  for a sufficiently large number of times and report the mean and the variance (and the number of runs used).
- (d) Plot a histogram of the frequency of different values of  $k$ .
- (e) [R] Comment on the histogram and why you think the distribution shown makes sense.

**Exercise 2: Exponential Random variables**

1. Write a program that considers  $(T, \lambda, n)$  as input. First generate a random variable  $Z_1$  as below:

$$Z_1 = \inf\{i : \sum_{s=1}^i X_s \geq T\},$$

where  $\{X_i\}$  are exponential random variables with parameter  $\lambda$ .

2. **[R]** Write an algorithm that can generate the above. Include this algorithm in a sheet of paper.
3. Now generate  $Z_2$  again using the same procedure as above, and using a fresh set of exponential random variables. Repeat this for  $n$  times and generate the random sample  $\{Z_k\} := \{Z_1, Z_2, \dots, Z_n\}$ .
  - (a) **[R]** Plot the histogram of random sample  $\{Z_k\}$ .
  - (b) **[R]** Find the mean of random sample  $\{Z_k\}$ .
  - (c) **[R]** Comment on the histogram and guess the distribution.