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MSC PHD (OR)

Q1)

Exercise 1: LP Reformulation. [4 Marks] Consider the following optimisation problem:

$$\begin{aligned}
\max\ & 0.043x_1 + 0.027x_2 + 0.025x_3 + 0.022x_4 + 0.045x_5 - 0.0275y \\
\text{s.t.}\ & x_1 + x_2 + x_3 + x_4 + x_5 - y \leq 100 \\
& x_2 + x_3 + x_4 \geq 40 \\
& \frac{2x_1 + 2x_2 + x_3 + x_4 + 5x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 1.4 \\
& \frac{9x_1 + 15x_2 + 4x_3 + 3x_4 + 2x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 5 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0 \\
& y \in [0, 10]
\end{aligned}$$

<u>Solution:</u>

<u>Solution to part 1 of the above question:</u>

As $x_i \geq 0$ for all i , we can multiply the denominator of third and forth constraint to R.H.S. and solve the following lpp, so after simplifying 3rd and 4th constraints, we get the lpp as follows:

maximize 0.043*x1 + 0.027*x2 + 0.025*x3 + 0.022*x4 + 0.045*x5-0.0275*y;

subject to constraints:

$x1 + x2 + x3 + x4 + x5 - y \le 100;$

$x2 + x3 + x4 \ge 40;$

$0.6*x1 + 0.6*x2 - 0.4*x3 - 0.4*x4 + 3.6*x5 \le 0;$

$4*x1 + 10*x2 - x3 - 2*x4 - 3*x5 \le 0;$

Where $x1 \ge 0, x2 \ge 0, x3 \ge 0, x4 \ge 0, x5 \ge 0, 0 \le y \le 10;$

Solution to part 3 of the above question:

We have solved the following lpp through ampl:

Console

AMPL

```
ampl: reset;
ampl: include ex1.run;
Gurobi 8.0.0: optimal solution; objective 3.007
5 simplex iterations
x1 = 24
x2 = 0
x3 = 81
x4 = 0
x5 = 5
y = 10
cost = 3.007
con1.lb = -Infinity
con1.ub = 100
con1.body = 100
con2.lb = 40
con2.ub = Infinity
con2.body = 81
con3.lb = -Infinity
con3.ub = 0
con3.body = 0
con4.lb = -Infinity
con4.ub = 0
con4.body = 0

ampl:
```

<u>Solution to part 3 of the above question:</u>

1) The optimal solution came out to be:3.007

2) The values are variables at optimal solution is:

X1=24

X2=0

X3=81

X4=0

X5=5

Y=10

3)we have calculated the upper bound and lower bound of the given constraint 1 through AMPL:

Lower bound= -infinity

Upper bound=100

Body of constraint 1=100

4)we have calculated the upper bound and lower bound of the given constraint 2 through AMPL:

Lower bound= 40

Upper bound= infinity

Body of constraint 2=81

5)we have calculated the upper bound and lower bound of the given constraint 3 through AMPL:

Lower bound= -infinity

Upper bound= 0

Body of constraint 3=0

5)we have calculated the upper bound and lower bound of the given constraint 4 through AMPL:

Lower bound= -infinity

Upper bound= 0

Body of constraint 4=0

Q2)

It is the beginning of monsoon semester at IIT Bombay, and our department needs a system adminstrator to be working every weekday (Mon-Fri) from 8AM to 10PM. There are six candidates available who can do this job, but they are also busy doing other activities during the week. Their availability and wage-rate is listed in the table below

	Wage-rate (per hour)	Availability in hours				
		Mon	Tue	Wed	Thu	Fri
K.C.	150	6	0	6	0	6
D.H.	152	0	6	0	6	0
H.B.	148	4	8	4	0	4
S.C.	146	5	5	5	0	5
K.S.	166	3	0	3	8	0
N.K.	176	0	0	0	6	2

Each candidate has a different qualification and hence they have a different wage-rate. According to the contract, K.C., D.H., H.B. and S.C., must work at least 8 hours every week. K.S. and N.K. must work at least 7 hours every week. There should be exactly one administrator on duty every weekday during the work hours.

<u>Solution:</u>

<u>Solution to part 3 of the above question:</u>

. No of constraints:11

. No variables:18

. The number of variables having non-zero coefficients in each of the constraints are:

1) The number of variables having non-zero coefficients in constraint 1 are:3

2) The number of variables having non-zero coefficients in constraint 2 are:2

3) The number of variables having non-zero coefficients in constraint 3 are:4

4) The number of variables having non-zero coefficients in constraint 4 are:4

5) The number of variables having non-zero coefficients in constraint 5 are:3

6) The number of variables having non-zero coefficients in constraint 6 are:2

7) The number of variables having non-zero coefficients in constraint 7 are:4

8) The number of variables having non-zero coefficients in constraint 8 are:3

9) The number of variables having non-zero coefficients in constraint 9 are:4

10) The number of variables having non-zero coefficients in constraint 10 are:3

11) The number of variables having non-zero coefficients in constraint 11 are:4

Solution to part 1 of the above question:

The lpp of the above problem is :

Minimize

150*x11+150*x13+150*x15+152*x22+152*x24+148*x31+148*x32+148*x33+148*x35+146*x41+146*x42+146*x43+146*x45+166*x51+166*x53+166*x54+176*x64+176*x65;

Subject to constraints:

x11+x13+x15>=8;
x22+x24>=8;
x31+x32+x33+x35>=8;
x41+x42+x43+x45>=8;
x51+x53+x54>=7;
x64+x65>=7;
x11+x31+x41+x51=14;
x22+x32+x42=14;
x13+x33+x43+x53=14;
x24+x54+x64=14;
x15+x35+x45+x65=14;

where

0<=x11<=6;0<=x13<=6;0<=x15<=6;0<=x22<=6;0<=x24<=6;0<=x31<=4;0<=x32<=8;0<=x33<=4;0<=x35<=4;0<=x41<=5;0<=x42<=5;0<=x43<=5;0<=x45<=5;0<=x51<=3;0<=x53<=3;0<=x54<=8;0<=x64<=6;0<=x65<=2;

Solution to part 2 of the above question:

The following problem has been solved by AMPL:

```
Console
AMPL
ampl: reset;
ampl: include ex2.run;
Gurobi 8.0.0: optimal solution; objective 10692
13 simplex iterations
x11 = 4
x13 = 2
x15 = 3
x22 = 2
x24 = 6
x31 = 4
x32 = 7
x33 = 4
x35 = 4
x41 = 5
x42 = 5
x43 = 5
x45 = 5
x51 = 1
x53 = 3
x54 = 3
x64 = 5
x65 = 2

con1.lb = 8
con1.ub = Infinity
con1.body = 9
con2.lb = 8
con2.ub = Infinity
con2.body = 8
con3.lb = 8
con3.ub = Infinity
con3.body = 19
con4.lb = 8
con4.ub = Infinity
con4.body = 20
con5.lb = 7
con5.ub = Infinity
```

```
con5.body = 7
con6.lb = 7
con6.ub = Infinity
con6.body = 7
con7.lb = 14
con7.ub = 14
con7.body = 14
con8.lb = 14
con8.ub = 14
con8.body = 14
con9.lb = 14
con9.ub = 14
con9.body = 14
con10.lb = 14
con10.ub = 14
con10.body = 14
con11.lb = 14
con11.ub = 14
con11.body = 14

ampl:
```

Solution to part 4 of the above question:

1) The optimal solution came out to be:10692

2) The values are variables at optimal solution is:

x11 = 4
x13 = 2

x15 = 3
x22 = 2
x24 = 6
x31 = 4
x32 = 7
x33 = 4
x35 = 4
x41 = 5
x42 = 5
x43 = 5
x45 = 5
x51 = 1
x53 = 3
x54 = 3
x64 = 5
x65 = 2

3)we have calculated the upper bound and lower bound of the given constraints through AMPL:

con1.lb = 8
con1.ub = Infinity
con1.body = 9
con2.lb = 8
con2.ub = Infinity
con2.body = 8
con3.lb = 8
con3.ub = Infinity
con3.body = 19
con4.lb = 8
con4.ub = Infinity

con4.body = 20
con5.lb = 7
con5.ub = Infinity
con5.body = 7
con6.lb = 7
con6.ub = Infinity
con6.body = 7
con7.lb = 14
con7.ub = 14
con7.body = 14
con8.lb = 14
con8.ub = 14
con8.body = 14
con9.lb = 14
con9.ub = 14
con9.body = 14
con10.lb = 14
con10.ub = 14
con10.body = 14
con11.lb = 14
con11.ub = 14
con11.body = 14

Q3)

Exercise 3: Fitting. [7 Marks] (Adapted from Bradley, Hax, and Magnanti, Addison-Wesley, 1977)
The selling prices of a number of warehouses in Powai overlooking the lake are given in the following table, along with the size of the lot and its elevation.

Warehouse i	Selling price P_i	Lot size (sq. ft.) L_i	Elevation (feet) E_i
1	155000	12000	350
2	120000	10000	300
3	100000	9000	100
4	70000	8000	200
5	60000	6000	100
6	100000	9000	200

You have been asked by Sanju Warehousing Company to construct a model to forecast the selling prices of other warehouses in Powai from their lot sizes and elevations. The company feels that a linear model of the form $P = b_0 + b_1 L + b_2 E$ would be reasonably accurate and easy to use. Here b_1 and b_2 would indicate how the price varies with lot size and elevation, respectively, while b_0 would reflect a base price for this section of the city. You would like to select the "best" linear model in some sense. If you knew the three parameters b_0, b_1 and b_2, the six observations in the table would each provide a forecast of the selling price as follows:

$$\hat{P}_i = b_0 + b_1 L_i + b_2 E_i \quad i = 1, 2, \ldots, 6.$$

However, since b_0, b_1 and b_2 cannot, in general, be chosen so that the actual prices P_i are exactly equal to the forecast prices $\hat{P}_i$ for all observations, you would like to minimize the absolute value of the residuals $R_i = P_i - \hat{P}_i$.

1. [R] Formulate a mathematical program to find the "best" values of b_0, b_1 and b_2 by minimizing the linear absolute residual $\sum_{i=1}^{6} \left| P_i - \hat{P}_i \right|$. Write the model in the report.

<u>Solution:</u>

<u>Solution1)</u>

<u>Solution to part 1 of the above question:</u>

minimize z1+z2+z3+z4+z5+z6;

<u>subject to constraints:</u>

z1>=155000-b0-b1*12000-b2*350;
z1>=-155000+b0+b1*12000+b2*350;
z2>=120000-b0-b1*10000-b2*300;

z2>=-120000+b0+b1*10000+b2*300;
z3>=100000-b0-b1*9000-b2*100;
z3>=-100000+b0+b1*9000+b2*100;
z4>=70000-b0-b1*8000-b2*200;
z4>=-70000+b0+b1*8000+b2*200;
z5>=60000-b0-b1*6000-b2*100;
z5>=-60000+b0+b1*6000+b2*100;
z6>=100000-b0-b1*9000-b2*200;
z6>=-100000+b0+b1*9000+b2*200;

where z1>=0;z2>=0;z3>=0;z4>=0;z5>=0;z6>=0;b0>=0;b1&b2 belongs to real;
where zi=$|P_i-\widehat{P\imath}|$, P_i = selling price and $\widehat{P\imath}$ = estimated value of $P_{i.}$

The model of the above problem is thus given as follows:

$\widehat{\boldsymbol{P\imath}} = \mathbf{0 + 8.57143}$***$L_i$ +114.268*E_i**

Solution to part 2 of the above question:

The following problem has been solved by AMPL:

Console

AMPL

```
ampl: reset;
ampl: include ex3.run;
Gurobi 8.0.0: optimal solution; objective 47857.14286
7 simplex iterations
z1 = 12142.9
z2 = 0
z3 = 11428.6
z4 = 21428.6
z5 = 2857.14
z6 = 0
b0 = 0
b1 = 8.57143
b2 = 114.286
residual = 47857.1
con1.lb = 155000
con1.ub = Infinity
con1.body = 155000
con2.lb = -155000
con2.ub = Infinity
con2.body = -130714
con3.lb = 120000
con3.ub = Infinity
con3.body = 120000
con4.lb = -120000
con4.ub = Infinity
con4.body = -120000
con5.lb = 1e+05
con5.ub = Infinity
con5.body = 1e+05
con6.lb = -1e+05
con6.ub = Infinity
con6.body = -77142.9
con7.lb = 70000
con7.ub = Infinity
con7.body = 112857
con8.lb = -70000
con8.ub = Infinity
```

```
con8.body = -70000
con9.lb = 60000
con9.ub = Infinity
con9.body = 65714.3
con10.lb = -60000
con10.ub = Infinity
con10.body = -60000
con11.lb = 1e+05
con11.ub = Infinity
con11.body = 1e+05
con12.lb = -1e+05
con12.ub = Infinity
con12.body = -1e+05
```

Solution to part 3 of the above question:

1) The optimal solution came out to be: 47857.1

2) The values are variables at optimal solution is:

z1 = 12142.9

z2 = 0

z3 = 11428.6

z4 = 21428.6

z5 = 2857.14

z6 = 0

b0 = 0

b1 = 8.57143

b2 = 114.286

3)we have calculated the upper bound and lower bound of the given constraints through AMPL:

```
con1.lb = 155000
con1.ub = Infinity
con1.body = 155000
con2.lb = -155000
con2.ub = Infinity
con2.body = -130714
con3.lb = 120000
con3.ub = Infinity
con3.body = 120000
con4.lb = -120000
con4.ub = Infinity
con4.body = -120000
con5.lb = 1e+05
con5.ub = Infinity
con5.body = 1e+05
con6.lb = -1e+05
con6.ub = Infinity
con6.body = -77142.9
con7.lb = 70000
con7.ub = Infinity
con7.body = 112857
con8.lb = -70000
con8.ub = Infinity
con8.body = -70000
con9.lb = 60000
con9.ub = Infinity
con9.body = 65714.3
```

```
con10.lb = -60000
con10.ub = Infinity
con10.body = -60000
con11.lb = 1e+05
con11.ub = Infinity
con11.body = 1e+05
con12.lb = -1e+05
con12.ub = Infinity
con12.body = -1e+05
```