(MM1 queue): We now describe an MM1 queue. Any queueing system is described by arrivals and departures.

The arrivals in an MM1 queue are according to a Poisson process. Here the inter-arrival times are represented by $\{A_1, A_2, \cdots, A_n \cdots,\}$, which are exponentially distributed with parameter λ (mean interarrival time $= 1/\lambda$). Here A_1 is instance at which the first arrival occurs and A_n is the time duration between the $(n-1)$-th and n-th arrival.

Each arrival brings with it a job requirement, and demands server time. These demand times (job processing times) are exponentially distributed with parameter μ, and are independent of all other events.

The server is working conserving, i.e., it keeps working without rest as long as there are customers in the system. If the queue gets empty, it rests, but starts immediately once a new customer arrives.

PASTA (Poisson Arrivals see See Time Averages): This property states the following: "the average number of customers as seen by Poisson arrivals equals the time average of the number of customers (which in turn equals the stationary expected number of customers)". One can estimate the stationary expected number of customers using this property: the sample average of the number of customers present in the system (just before the arrival) across the Poisson arrivals equals the stationary expected number of customers in the system. Basically one needs to consider large number of (Poisson) arrivals (more than 20000/30000) and find the sample average of the number of customers in the system just before the arrivals.

The above background is same as before. But the questions are different from the take-home part.

1. Use PASTA property to estimate the stationary expected number of customers in an MM1 queue. This time you should use a different algorithm than that using 'JobsDoneInOneArrival'. Now consider arrival as well as departure epochs to construct the algorithm. At every epoch check if the next one is arrival or departure. Construct the evolution process based on arrival/departure (e.g., number increases or decreases based on arrival or departure):

$$X_{n+1} = \begin{cases} X_n - 1 & \text{if departure,} \\ X_n + 1 & \text{if arrival.} \end{cases}$$

   Observe using memoryless property that any residual job is again exponentially distributed with the same parameter μ. Similarly residual inter-arrival time is again exponentially distributed with parameter λ. But remember one needs to consider sample average of number of customers only at arrival epochs owing to PASTA. Also remember the empty system always progresses only with an arrival. Estimate the stationary expected number of customers for the case with $\lambda = 0.1$ and $\mu = 2$

2. **State Dependent Service Rate queue:** Consider that the service rates change with the number in the system. When the number is strictly greater than L (another parameter to the algorithm), the speed doubles while it is served normally in the other cases. Hence the service time, the time taken to complete a job, is exponentially distributed with parameter 2μ when number is strictly greater than L, and with parameter μ otherwise. Remember job sizes are always exponentially distributed with parameter μ, but the time taken to complete such job sizes can become half depending upon the number in the system. Also remember that the residual job sizes are again exponentially distributed with parameter μ, however the time taken to complete the residual job depends upon the number in the system. Now modify the algorithm of step 1 to compute again the stationary expected number of customers for two cases a) $\lambda = 0.1$ and $\mu = 2$ and $L = 1$; and b) $\lambda = 0.1$ and $\mu = 2$ and $L = 20$.

3. Comment on the results obtained