Instructions: In this lab, we will use Scilab to simulate randomness.

You are required to work on your own and submit a report on all exercises other than Exercise 1 by Saturday. Exercise 1 needs to be submitted by 3:30pm (Today) in the lab itself. Late submission of this exercise is allowed with a penalty, you need to submit before Saturday. You need to discuss with your TAs in the class room itself, about any clarifications required on all the exercises. If there is a need assume appropriately and clearly mention your assumptions. Upload all your Scilab files on Moodle. We would reduce the marks by 0.9^d, if there is a delay of d days in submission.

Exercise 1 Distribution of given Random Sample

(a) **[R]** Suppose we want to check whether a given set of numbers (input1.sce) are uniformly randomly distributed. We call such a set of numbers as random sample. Let us consider the input file 'input1.sce' in Moodle. By running this file (`exec('input1.sce', -1)`), you will get an array S. Plot the histogram of its distribution and comment whether it looks randomly uniformly distributed or not.

(b) **[R]** Write another Scilab function `testrand2(k)` that takes an integer k as input and plots histograms of distribution of elements of S when only every k-th element is selected. Does the distribution look uniform when $k = 2$ and $k = 3$?

(c) **[R]** Write your conclusions about this experiment. Are the above two tests sufficient for checking randomness?

Exercise 2: 2D-randomness

Suppose we want to generate a set of uniformly distributed points in a two-dimensional plane, say in the rectangle $(0, 1) \times (0, 1)$.

1. [**R**] Write a Scilab function `twodrand(n)` which takes as input a positive integer `n`. It should create n pairs of random numbers (x, y) so that both x and y are uniformly randomly distributed and are independent of each other. It should then generate a scatterplot of values seen in this set of numbers. In particular, make the plot for $n = 100$, $n = 1000$, and $n = 1000000$. Upload the plot for $n = 1000$ on Moodle. Remember to add proper labels, titles and legends etc. Is the shape of plot as you would expect from a uniform distribution? Explain why or why not.

2. [**R**] Consider the input file 'input2.sce' in Moodle. It loads a matrix S of size 100000x2. Suppose each row of the matrix represents a point (x, y). Check, using some simple logic, whether these points are uniformly randomly distributed or not. Explain the conclusions you make.

3. [**R**] Do the same exercise for the third input file 'input3.sce'.

Exercise 3: Exponential Random variables
Generate a random sample of size n, of the following type of random variables

$$Y = \{Y_k\} \text{ with } Y_k = X_{k,1} + X_{k,2} \text{ for any } k \leq n,$$

where $\{X_{k,1}\}$ and $\{X_{k,2}\}$ are both exponential random variables (independent of others) with parameter λ_1 and λ_2. Plot histograms of distribution of Y.

1. [**R**] Write a program that takes $(n, \lambda_1, \lambda_2)$ as inputs and generates such a random sample and subsequent histogram. Upload the plots for two cases: a) when $\lambda_1 = \lambda_2 = 1$ and b) when $\lambda_1 = 1$, $\lambda_2 = 2$.
2. [**R**] Comment on the plots obtained.

Exercise 4: Exponential Random variables continued .. (For Next week)

1. Write a program that considers (T, λ, n) as input. First generate a random variable Z_1 as below:
$$Z_1 = \inf\{i : \sum_{s=1}^{i} X_i \geq T\},$$
where $\{X_i\}$ are exponential random variables with parameter λ.
2. [**R**] Write an algorithm that can generate the above. Include this algorithm in a sheet of paper.
3. Now generate Z_2 again using the same procedure as above, and using a fresh set of exponential random variables. Repeat this for n times and generate the random sample $\{Z_k\}$.
   (a) [**R**] Plot the histogram of random sample $\{Z_k\}$.
   (b) [**R**] Find the mean of random sample $\{Z_k\}$.