Instructions: We will practice more modeling in this lab.

Answer all the questions in this assignment. Write your report clearly and concisely. Only the questions marked **[R]** need to be answered in a lab-report. Do not include AMPL commands in your report. Upload your report in pdf format, and all your AMPL code/model files as a single zip file (with name lab4_rollno.zip) to Moodle after answering all such questions. Show all your work to TA/instructor before leaving.

There are only 3 exercises. Try to solve all problems on your own. If you have difficulties, ask the instructor or TAs. Please note that clarifications would be provided for all questions.

We first discuss an option in AMPL that is quite useful in modeling. Suppose we have a set in AMPL defined as

```
set S := 'apple' 'boy' 'dog' 'eel' 'fan';
```

Suppose we also have variables associated with this set

```
var x{S};
```

You may recall that if we want to sum over all x in S, then we can do:

```
sum {s in S}x[s];
```

If we want to sum over all x except `eel`, we can do:

```
sum {s in S: s!='eel'}x[s];
```

Similarly, conditions could be added in defining variables and constraints. You might need this syntax in one of the questions of this lab.

Exercise 1: (Adapted from Bradley, Hax, and Magnanti, Addison-Wesley, 1977) [6 Marks]
An imaginary economy has six distinct geographic regions; each has its own specific economic functions, as follows:

Region	Function
A	Food producing
B	Manufacturing machinery
C	Manufacturing machinery and consumer durables
D	Administrative
E	Food producing
F	Manufacturing consumer durables and nondurables

The regions also have the following annual requirements (all quantities measured in tons):

Region	Food	Machinery	Consumer durables	Consumer nondurables
A	5	30	20	10
B	15	100	40	30
C	20	80	50	40
D	30	10	70	60
E	10	60	30	20
F	25	60	60	50

Using the national railroad, transportation costs are ₹40/ton per 100 KM for all hauls over 100 KM. Within 100 KM, all goods are carried by truck at a cost of ₹60/ton per 100 KM. The distances (in KM) between regions are as follows:

	A	B	C	D	E	F
A	—					
B	500	—				
C	200	400	—			
D	75	500	150	—		
E	600	125	350	550	—	
F	300	200	100	400	300	—

Assume producing regions can meet all requirements, but that, due to government regulation, food production in sector A is restricted to half that of sector E (or lesser).

1. [**R**] Formulate a linear program that will meet the requirements and minimize the total transportation costs in the economy. Use simple variables like x and parameters like c in your model.

2. Write the problem in AMPL using a model and a data file. Remember to use human-readable names for variables and parameters.

3. [**R**] Solve the problem and report the solution and the value.

4. [**R**] If the demand for food alone is doubled, how does the optimal cost change? Report the changes in optimal cost when the demand of each item doubles (all other demands are still the original values). Which is the most demand-sensitive item?

Exercise 2: Planning. (Adapted from Bradley, Hax, and Magnanti, Addison-Wesley, 1977) [6 Marks]
From past data, the production manager of a factory knows that, by varying his production rate, he incurs additional costs. He estimates that his cost per unit increases by ₹28 when production is increased from one month to the next. Similarly, reducing production increases costs by ₹12 per unit. A smooth production rate is obviously desirable. Sales forecasts for the next twelve months are (in thousands):

July	4	October	12	January	20	April	6
August	6	November	16	February	12	May	4
September	8	December	20	March	8	June	4

June's production schedule already has been set at 4000 units, and the July 1 inventory level is projected to be 2000 units. Storage is available for only 10,000 units at any one time.

1. [**R**] Ignoring inventory costs, formulate a production schedule for the coming year that will minimize the cost of changing production rates while meeting all sales. Clearly mention the variables and parameters of your model.
2. Write the problem using AMPL model and data files.
3. [**R**] Solve the problem and report the solution and its objective value.

Exercise 3: Multiple Objectives. [8 Marks]
Maya heads the tooling division of Azad Iron Works Incorporated. Her division has to supply tools: Hammers, Spanners and Cutters to the other divisions of the company. The requirements is 100 units of each tool. There are three machines available, and each of the three can be used to manufacture any of the tools. Maya would like to minimise the cost as well as time used in quality-inspection. The cost of production and the quality-inspection time depend on the machine used. The table below provides the data.

	Hammer	Spanner	Cutter
Per unit cost of manufacturing on machine 1	30	25	42
Per unit cost of manufacturing on machine 2	35	22	40
Per unit cost of manufacturing on machine 3	28	27	42
Per unit time of inspection on machine 1	32	32	27
Per unit time of inspection on machine 2	30	30	26
Per unit time of inspection on machine 3	34	32	29

The total amount of raw-material available for production is 900 units. The total capacity of each machine is also limited to 5000 units. In addition, the division must control the total pollution level to 3000 units. The material consumed, the machine capacity consumed and the quantity of pollutants emitted depend upon the product being made and the machine on which it is made. The following tables provide the data.

	Hammer	Spanner	Cutter
Material consumed per unit on machine 1	3	2	2
Material consumed per unit on machine 2	4	1.5	2.5
Material consumed per unit on machine 3	2	3	3
Machine 1 Capacity consumed per unit	48	45	53
Machine 2 Capacity consumed per unit	45	55	50
Machine 3 Capacity consumed per unit	48	50	48
Pollution per unit on machine 1	7	12	10
Pollution per unit on machine 2	9	9	9
Pollution per unit on machine 3	13	12	8

1. **[R]** Write an LP model to minimize the total cost of manufacturing the tools.
2. Solve the LP model to minimize the total cost of manufacturing the tools after writing the model and data files.
3. **[R]** Report the optimal solution value.
4. Solve the LP model to minimize the total time of inspection of the tools after writing the model and data files.
5. **[R]** Report the optimal solution value. Are the two optimal solutions for the two problems different?
6. **[R]** Now suppose we want combinations of these two objective functions (mentioned in 1 and 4). In particular, we will give a weight θ to the first objective and $(1 - \theta)$ to the second objective and sum the two weighted objectives. Then we solve an LP with the new objective function. We can try this for different values of θ. Make a plot (use any relevant software of your choice) with two axes in the following manner. On one axis is the value of the first objective. On the other axis is the value of the second objective. Pick a value of θ and find the optimal solution of the combined objective. Mark the point on the graph at the co-ordinates given by values of the two objective functions at this optimal solution. Now change θ, solve the new LP and plot another point. Try at least 20 different values of θ in the range $[0, 1]$.

Comment on the shape of the plot. Is it piecewise linear? Clearly state all the θ values you have tried. Include all the relevant plots in the report with clear description of axes and with markers for the θ values tried.