Exercise 1: BusyPeriod in a Queueing system . Let $X_0 = 1$ represent the initial number of jobs in a queue and let $\{X_n, J_{n+1}\}$, where represent the remaining jobs and completed jobs sequence as in Exercise 2 of the previous lab session. That is, J_{n+1} equals the number of jobs completed as returned by the program 'JobsDoneInOneArrival (λ, μ, X_n)' when the number of jobs remaining before this equal X_n and $X_{n+1} = (X_n - J_{n+1}) + 1$. Set $\lambda = 0.1$ and $\mu = 0.15$.
Now lets introduce the notion of time to this system. Start with system time, $Time = 0$. Let A_1 represent the time after which first customer arrives in 'JobsDoneInOneArrival (λ, μ, X_0)'. Update system time (immediately after this arrival) to $Time = Time + A_1$. Modify 'JobsDoneInOneArrival' program to also return A (along with J), these (inter) arrival times. In a similar fashion, let A_n represent the time duration (to be more precise inter arrival time) after the $(n-1)$-th arrival, at which n-th customer arrives, i.e., the second result returned by 'JobsDoneInOneArrival (λ, μ, X_n)'. Update the system $Time = Time + A_n$, at each such epoch. Note that the variable $Time$ represents the arrival instance of the n-th arrival, once it is updated after 'JobsDoneInOneArrival (λ, μ, X_n)' is executed.
We say that a busy period ends before arrival epoch n if $J_{n+1} = X_n$ (and if $X_{n+1} = 1$), i.e., if the waiting jobs are completed before the next arrival and if n is the first such epoch. Let τ represent the arrival instance $Time$ of that arrival, at which a busy period ends. In other words,

$$\tau = \inf \left\{ n : \sum_{i=1}^{n} A_i \geq \sum_{i=0}^{n} \sum_{j=1}^{J_i} B_j^i \right\} \text{ (note } J_i \leq X_{i-1}),$$

where B_j^i is the job size of the j-th job among the ones provided service in between $(i-1)$-th and i-th arrival. This τ represents the cycle during which one busy period and idle period elapses. Idle period is the time period, during which system remains idle, i.e., without service.

1. Repeat the above procedure 10000 times (independently) to compute 10000 independent realizations of the 'busy+idle' cycles, $\{\tau_1, \tau_2, \cdots, \tau_k, \cdots, \tau_{10000}\}$.
2. Find the sample mean of the above sequence to estimate the mean value of one cycle (when started with one customer).
3. Do you know how to compute the mean busy period using these estimates ?