Instructions:
In this session, we will learn some more mathematical modeling. We will consider some scenarios where linear programming may be applied to solve a practical problem. Use these exercises to become more comfortable with writing optimisation models and solving them using AMPL.

Answer all the questions in this assignment. Only the questions marked **[R]** need to be answered in a lab-report. Do not include AMPL commands in your report. Upload your report in pdf format, and all your AMPL code/model files as a single zip file (with name lab1_rollno.zip) to Moodle after answering all such questions. Show all your work to TA/instructor before leaving.

Try to solve all problems on your own. If you have difficulties, ask the instructor or TAs.

Exercise 1: LP Reformulation. [4 Marks] Consider the following optimisation problem:

$$\begin{aligned}
\max\ 0.043x_1 + 0.027x_2 + 0.025x_3 + 0.022x_4 + 0.045x_5 - 0.0275y& \\
\text{s.t. } x_1 + x_2 + x_3 + x_4 + x_5 - y \le 100& \\
x_2 + x_3 + x_4 \ge 40& \\
\frac{2x_1 + 2x_2 + x_3 + x_4 + 5x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \le 1.4& \\
\frac{9x_1 + 15x_2 + 4x_3 + 3x_4 + 2x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \le 5& \\
x_1, x_2, x_3, x_4, x_5 \ge 0& \\
y \in [0, 10]&
\end{aligned}$$

1. **[R]** Rewrite this optimisation problem as a linear program, and include it in your report.
2. Create a file 'ex1.mod', containing the linear program in AMPL format.
3. Solve this model using Gurobi solver.
4. **[R]** Report the optimal solution value, the values of variables at the optimal solution and the activities of all constraints of the LP model.

Exercise 2: [6 Marks]
It is the beginning of monsoon semester at IIT Bombay, and our department needs a system adminstrator to be working every weekday (Mon-Fri) from 8AM to 10PM. There are six candidates available who can do this job, but they are also busy doing other activities during the week. Their availability and wage-rate is listed in the table below

	Wage-rate	Availability in hours				
	(per hour)	Mon	Tue	Wed	Thu	Fri
K.C.	150	6	0	6	0	6
D.H.	152	0	6	0	6	0
H.B.	148	4	8	4	0	4
S.C.	146	5	5	5	0	5
K.S.	166	3	0	3	8	0
N.K.	176	0	0	0	6	2

Each candidate has a different qualification and hence they have a different wage-rate. According to the contract, K.C., D.H., H.B. and S.C., must work at least 8 hours every week. K.S. and N.K. must work at least 7 hours every week. There should be exactly one administrator on duty every weekday during the work hours.

1. You are required to find number of hours each candidate must be alloted each day so that the cost of running the facility is minimized. Write a linear optimization model to solve this problem.
2. Solve this problem using Gurobi.
3. [**R**] Report the number of variables, constraints and nonzeros in the constraints of your model.
4. [**R**] Report the solution and the total cost.

Exercise 3: Fitting. [7 Marks] (Adapted from Bradley, Hax, and Magnanti, Addison-Wesley, 1977)
The selling prices of a number of warehouses in Powai overlooking the lake are given in the following table, along with the size of the lot and its elevation.

Warehouse i	Selling price P_i	Lot size (sq. ft.) L_i	Elevation (feet) E_i
1	155000	12000	350
2	120000	10000	300
3	100000	9000	100
4	70000	8000	200
5	60000	6000	100
6	100000	9000	200

You have been asked by Sanju Warehousing Company to construct a model to forecast the selling prices of other warehouses in Powai from their lot sizes and elevations. The company feels that a linear model of the form $P = b_0 + b_1 L + b_2 E$ would be reasonably accurate and easy to use. Here b_1 and b_2 would indicate how the price varies with lot size and elevation, respectively, while b_0 would reflect a base price for this section of the city. You would like to select the "best" linear model in some sense. If you knew the three parameters b_0, b_1 and b_2, the six observations in the table would each provide a forecast of the selling price as follows:

$$\hat{P}_i = b_0 + b_1 L_i + b_2 E_i \quad i = 1, 2, \ldots, 6.$$

However, since b_0, b_1 and b_2 cannot, in general, be chosen so that the actual prices P_i are exactly equal to the forecast prices $\hat{P}_i$ for all observations, you would like to minimize the absolute value of the residuals $R_i = P_i - \hat{P}_i$.

1. **[R]** Formulate a mathematical program to find the "best" values of b_0, b_1 and b_2 by minimizing the linear absolute residual $\sum_{i=1}^{6} \left| P_i - \hat{P}_i \right|$. Write the model in the report.
2. Model the problem in AMPL and solve using Gurobi solver.
3. **[R]** Report the solution.