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Exercise 1):
Q1)(R)

In the given question:

. Food is being produced by two regions- **A and E**. So every region will have two choices for **food**.

.Machines are manufactured at two regions-**B and C**. So every region will have two choices for **machinery**.

.Consumer durables are manufactured at two places **C and F**. So every region will have two choices for **Consumer durables**.

.Consumer non durables are manufactured at **F** only. So every region will have only one choice for **consumer non durables**.

Let us define three sets that will help us formulating the lpp:

1) REGION = {A,B,C,D,E,F}
2) FUNCTION= {fooda,foode,machinery,machinery,c.d_c,c.d_f,c.nd_f}
3) FUNCTION2={food,machinery,c.d,c.nd}

The table of variables:

*	Fooda	Foode	Machineryb	machineryc	c.d_c	c.d_f	c.nd_f
A	$x_{A,Fooda}$	$x_{A,Foode}$	$x_{A,Machineryb}$	$x_{A,Machineryc}$	$x_{A,c.d\_c}$	$x_{A,c.d\_f}$	$x_{A,c.nd\_f}$
B	$x_{B,Fooda}$	$x_{B,Foode}$	$x_{B,Machineryb}$	$x_{B,Machineryc}$	$x_{B,c.d\_c}$	$x_{B,c.d\_f}$	$x_{B,c.nd\_f}$
C	$x_{C,Fooda}$	$x_{C,Foode}$	$x_{C,Machineryb}$	$x_{C,Machineryc}$	$x_{C,c.d\_c}$	$x_{C,c.d\_f}$	$x_{C,c.nd\_f}$
D	$x_{D,Fooda}$	$x_{D,Foode}$	$x_{D,Machineryb}$	$x_{D,Machineryc}$	$x_{D,c.d\_c}$	$x_{D,c.d\_f}$	$x_{D,c.nd\_f}$
E	$x_{A,Fooda}$	$x_{A,Foode}$	$x_{E,Machineryb}$	$x_{E,Machineryc}$	$x_{E,c.d\_c}$	$x_{E,c.d\_f}$	$x_{E,c.nd\_f}$
F	$X_{A,Fooda}$	$X_{A,Foode}$	$x_{F,Machineryb}$	$x_{F,Machineryc}$	$x_{F,c.d\_c}$	$x_{F,c.d\_f}$	$x_{F,c.nd\_f}$

Where,

. $\underline{x_{i,Fooda}}$ is the amount(in tons) of food region 'i' has got from A for all i € {A,B,C,D,E}

• $\underline{x_{i,Foode}}$ is the amount(in tons) of food region 'i' has got from E for all i € {A,B,C,D,E},

. $\underline{x_{i,Machineryb}}$ is the amount(in tons) of Machinery region 'i' has got from B for all i € {A,B,C,D,E}

. $\underline{x_{i,Machineryc}}$ is the amount(in tons) of Machinery region 'i' has got from C for all i € {A,B,C,D,E}

.$x_{i,c.d\_c}$ is the amount(in tons) of Consumer durables region ‘i’ has got from C for all i € {A,B,C,D,E}

.$x_{i,c.d\_f}$ is the amount(in tons) of Consumer durables region ‘i’ has got from F for all i € {A,B,C,D,E}

.$x_{i,c.nd\_f}$ is the amount(in tons) of Consumer non durables region ‘i’ has got from F for all i € {A,B,C,D,E}

<u>Given below is the matrix that represent the distance taken by REGIONS to obtain the resources from OTHER REGIONS:</u>

DISTANCEM = $[d_{ij}]_{REGION*FUNCTION}$

Where d_{ij} = distance travelled when when region i is taking

*	Fooda	Foode	Machineryb	machineryc	c.d_c	c.d_f	c.nd_f
A	0	600	500	200	200	300	300
B	500	125	0	400	400	200	200
C	200	350	400	0	0	100	100
D	75	550	500	150	150	400	400
E	600	0	120	350	350	300	300
F	300	300	200	100	100	0	0

something from other region , where i € REGION and j € FUNCTION

Eg , $d_{E,machineryb}$ is the distance(km) when E is taking machiner from B.

SIMILARLY , we can define another matrix that will tell us the <u>cost per KM per TON</u> when region i is taking something from different regions.

COSTM = $[c_{ij}]_{REGION*FUNCTION}$

Where c_{ij} = cost/ton/km when when region i is taking

*	Fooda	Foode	Machineryb	machineryc	c.d_c	c.d_f	c.nd_f
A	0	0.4	0.4	0.4	0.4	0.4	0.4
B	0.4	0.4	0	0.4	0.4	0.4	0.4
C	0.4	0.4	0.4	0	0	0.6	0.6
D	0.6	0.4	0.4	0.4	0.4	0.4	0.4
E	0.4	0	0.4	0.4	0.4	0.4	0.4
F	0.4	0.4	0.4	0.6	0.6	0	0

something from other region , where i € REGION and j € FUNCTION

Eg , $c_{A,c.d\_c}$ is the cost/ton/km when A is taking costumer durables from C.

Now, with the help of these matrices , we can define our objective function , as my multiplying each term c_{ij} of matrix COSTM to the each term d_{ij} of matrix DISTANCEM, we will get the cost/ton for each movement between the regions.

We are defining one more matrix AVAILM that will tell us requirement of each location for each resource:

AVAILM=$[p_{ij}]_{REGION*FUNCTION2}$

*	food	machinery	c.d	c.nd
A	5	30	20	10
B	15	100	40	30
C	20	80	50	40
D	30	10	70	60
E	10	60	30	20
F	25	60	60	50

where p_{ij} is telling the requirement of every REGION for every RESOURCE.

NOW, THE OBJECTIVE FUNCTION IS:

MINIMIZE: $\sum_{(i \text{ in REGION}, j \text{ in FUNCTION})} d_{ij}*c_{ij}*x_{ij}$

Where, d_{ij}€ DISTANCEM, c_{ij}€ COSTM , and x_{ij} are the decision variables as defined above, where i € REGION, j€ FUNCTION.

Subject to constraints:

a) (we are defining six constrains on food to fulfill the requirement of every region of food using the matrix and notations defined above)
   $x_{i,fooda}+ x_{i,foode=}p_{i,food}$ for all i€ REGION, where p€AVAILM

b) (Now we are defining six constrains on machinery to fulfil the requirement of every region of machinery using the matrix and notations defined above):
$x_{i,machineryb}+x_{i,machineryc}= p_{i,machinery}$ for all i€ REGION, where p€AVAILM

c) (Now we are defining six constrains on consumer durables to fulfill the requirement of every region of consumer durables using the matrix and notations defined above):
$x_{i,c.d\_c}+x_{i,c.d\_f}= p_{i,c.d}$ for all i€ REGION, where p€AVAILM

d) (Now we are defining six constrains on consumer non durables to fulfill the requirement of every region of consumer non durables using the matrix and notations defined above):
$x_{i,c.nd\_f}= p_{i,c.nd}$ for all i€ REGION, where p€AVAILM

e) (As thefood production in sector A is restricted to half that of sector E):
$x_{A,fooda}+ x_{B,fooda}+ x_{C,fooda}+ x_{D,fooda}+ x_{E,fooda}+ x_{F,fooda}\leq 1/2(x_{A,foode}+ x_{B,foode}+ x_{C,foode}+ x_{D,foode}+ x_{E,foode}+ x_{F,foode})$

f) (As the variables are in tons so they cannot be negative):
$x_{ij}\geq 0$ for all i€REGION , J€FUNCTION.

Q3)(R):

The question described above is solved with the help of AMPL and the files are attached in the zip:

```
AMPL
ampl: include ex1.run;
Gurobi 8.0.0: optimal solution; objective 48100
1 simplex iterations
amount [*,*] (tr)
:             a     b     c     d     e     f     :=
c.d_c        20     0    50    70     0     0
c.d_f         0    40     0     0    30    60
c.nd_f       10    30    40    60    20    50
fooda         5     0     0    30     0     0
foode         0    15    20     0    10    25
machineryb    0   100     0     0    60     0
machineryc   30     0    80    10     0    60
;

# $3 = con1.body
# $6 = con2.body
# $7 = con3.lb
# $8 = con3.ub
# $9 = con3.body
# $10 = con4.lb
# $11 = con4.ub
# $12 = con4.body
: con1.lb con1.ub   $3 con2.lb con2.ub   $6   $7   $8   $9  $10  $11  $12
 :=
a     5       5      5     30      30     30   20   20   20   10   10   10
b    15      15     15    100     100    100   40   40   40   30   30   30
c    20      20     20     80      80     80   50   50   50   40   40   40
d    30      30     30     10      10     10   70   70   70   60   60   60
e    10      10     10     60      60     60   30   30   30   20   20   20
f    25      25     25     60      60     60   60   60   60   50   50   50
;

con5.lb = -Infinity
con5.ub = 0
con5.body = 0
```

1) The optimal solution came out to be:48100

2) The values are variables at optimal solution is:

x_{ij} is is the amount in tons and i€REGION and j€FUNCTION

*	Fooda	Foode	Machineryb	machineryc	c.d_c	c.d_f	c.nd_f
A	5	0	0	30	20	2	10
B	0	15	100	0	0	40	30
C	0	20	0	80	50	0	40
D	30	0	0	10	70	0	60
E	0	30	20	0	10	60	0
F	0	60	50	0	25	0	60

3) The Solution is satisfying all the constraints as we can see this the picture attached.

Q3)(R):

If the demand for food alone is doubled, then the only change in the above model will be the change in the requirement matrix in the column of food.

So the new AVAILM is : AVAILM=$[p_{ij}]_{REGION*FUNCTION2}$

*	food	machinery	c.d	c.nd
A	10	30	20	10
B	30	100	40	30
C	40	80	50	40
D	60	10	70	60
E	20	60	30	20
F	50	60	60	50

```
ampl: include ex1b.run;
Gurobi 8.0.0: optimal solution; objective 56000
1 simplex iterations
amount [*,*] (tr)
:             a     b     c     d     e     f      :=
c.d_c        20     0    50    70     0     0
c.d_f         0    40     0     0    30    60
c.nd_f       10    30    40    60    20    50
fooda        10     0     0    60     0     0
foode         0    30    40     0    20    50
machineryb    0   100     0     0    60     0
machineryc   30     0    80    10     0    60
;

ampl: |
```

<u>So , we have solved the above question with changes in AMPL and the cost came out to be : 56000</u>

Now, to find the most sensitive item , we have checked the optimal cost by changing the demand every item (multiplying it by 2 as we have done just now) and they came out to be:

DEMAND DOUBLED FOR:	**OPTIMAL COST:**
FOOD	56000
MACHINERY	57700
CONSUMER DURABLES	60700
CONSUMER NON DURABLES	66100

As we can see from the above table, non-consumer durables is the most sensitive item

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Exercise 2):
Q1)(R)

Defining the decision variables:

Y_i =No of units produced during month i, i=0-12(Here month 0 is considered to be the month June),i.e:

Month 0	June
Month 1	July
Month 2	August
Month 3	September
Month 4	October
Month 5	November
Month 6	December
Month 7	January
Month 8	February
Month 9	March

Month 10	April
Month 11	May
Month 12	june

$Invent_i$ = amount of inventory at the end of month i, i=0-12

Inc_i= amount of increase in total production during month i as compared to i-1.

Dec_i= amount of decrease in total production during month i as compared to i-1.

Defining the parameters:

$SALES_i$ = The sales forcast for month i (in units), i = 0-12

COSTINC = 28 is the cost of increasing production per unit.

COSTDEC = 12 is the cost of decreasing production per unit.

MAXIMUM_INVENTORY = 10000 is the maximum inventory.

Now we define the objective function with the help of the parameters and variables defined above:

Minimize Cost: $(COSTINC*Inc_1)+ (COSTINC*Inc_2)+ (COSTINC*Inc_3)+ (COSTINC*Inc_4)+ (COSTINC*Inc_5)+ (COSTINC*Inc_6)+ (COSTINC*Inc_7)+ (COSTINC*Inc_8)+ (COSTINC*Inc_9)+ (COSTINC*Inc_{10})+ (COSTINC*Inc_{11})+ (COSTINC*Inc_{12})+(COSTDEC*Dec_1) +(COSTDEC*Dec_2) +(COSTDEC*Dec_3) +(COSTDEC*Dec_4) +(COSTDEC*Dec_5) +(COSTDEC*Dec_6) +(COSTDEC*Dec_7) +(COSTDEC*Dec_8)$

$+(COSTDEC*Dec_9) +(COSTDEC*Dec_{10})+$
$+(COSTDEC*Dec_{11}) +(COSTDEC*Dec_{12})$

SUBJECT to the CONSTRAINTS:

1) (As the demand must be met each month)
   $Invent_i+y_{i-1}-Invent_{i-1}=SALES_i$ for all $1 \le i \le 12$.
2) (As at least one of increase or decrease must be zero)
   $y_i-y_{i-1}=Inc_i-Dec_i$ for all $1 \le i \le 12$.
3) (constraint for setting the initial inventory of Month 0 is taken to be zero):
   $Invent_0=0$
4) (Setting the production rate for the preceeding June):

$Y_0=4000$

5) Constraint for setting the maximum inventory:
   $Invent_i \le MAXIMUM\_INVENTORY$

Q3)(R):

The question described above is solved with the help of AMPL and the files are attached in the zip:

```
ampl: include ex2.run;
Gurobi 8.0.0: optimal solution; objective 405360
14 simplex iterations
1 branch-and-cut nodes
cost = 405360

y [*] :=
 0   4000
 1   8222
 2   8222
 3   8223
 4  15333
 5  15333
 6  15334
 7  15333
 8  12000
 9   8000
10   8000
11   8000
12   8000
;

invent [*] :=
 0      0
 1   4222
 2   6444
 3   6667
 4  10000
 5   9333
 6   4667
 7      0
 8      0
 9      0
10   2000
11   6000
12  10000
;
```

```
inc [*] :=
 0     0
 1  4222
 2     0
 3     1
 4  7110
 5     0
 6     1
 7     0
 8     0
 9     0
10     0
11     0
12     0
;

dec [*] :=
 0     0
 1     0
 2     0
 3     0
 4     0
 5     0
 6     0
 7     1
 8  3333
 9  4000
10     0
11     0
12     0
;

# $4 = con2.lb
# $5 = con2.ub
# $6 = con2.body
# $7 = con3.lb
```

```
# $7 = con3.lb
# $8 = con3.ub
# $9 = con3.body
# $12 = con4.body
:  con1.lb con1.ub con1.body  $4  $5  $6  $7  $8  $9 con4.lb con4.ub   $12
 :=
0      .      .       .      .   .   .   0   0   0   4000    4000    4000
1     4000   4000    4000    0   0   0   .   .   .    .       .       .
2     6000   6000    6000    0   0   0   .   .   .    .       .       .
3     8000   8000    8000    0   0   0   .   .   .    .       .       .
4    12000  12000   12000    0   0   0   .   .   .    .       .       .
5    16000  16000   16000    0   0   0   .   .   .    .       .       .
6    20000  20000   20000    0   0   0   .   .   .    .       .       .
7    20000  20000   20000    0   0   0   .   .   .    .       .       .
8    12000  12000   12000    0   0   0   .   .   .    .       .       .
9     8000   8000    8000    0   0   0   .   .   .    .       .       .
10    6000   6000    6000    0   0   0   .   .   .    .       .       .
11    4000   4000    4000    0   0   0   .   .   .    .       .       .
12    4000   4000    4000    0   0   0   .   .   .    .       .       .
;
:     con5.lb  con5.ub con5.body    :=
0    -Infinity   10000       0
1    -Infinity   10000    4222
2    -Infinity   10000    6444
3    -Infinity   10000    6667
4    -Infinity   10000   10000
5    -Infinity   10000    9333
6    -Infinity   10000    4667
7    -Infinity   10000       0
8    -Infinity   10000       0
9    -Infinity   10000       0
10   -Infinity   10000    2000
11   -Infinity   10000    6000
12   -Infinity   10000   10000
;
```

1) The optimal solution came out to be:405360

2) The values are variables at optimal solution is:

i	y_i
0	4000
1	8222
2	8222
3	8223
4	15333
5	15333
6	15334
7	15333
8	12000

9	8000
10	8000
11	8000
12	8000

i	$Invent_i$
0	0
1	4222
2	6444
3	6667
4	10000
5	9333
6	4667
7	0
8	0
9	0
10	2000
11	6000
12	10000

i	Inc_i
0	0
1	4222
2	0
3	1
4	7110
5	0
6	1

7	0
8	0
9	0
10	0
11	0
12	0

i	Dec_i
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	3333
9	4000
10	0
11	0
12	0

3) The Solution is satisfying all the constraints as we can see this the picture attached.

Exercise 2):

Q1)(R):

Defining two sets that will help us in formation of the lpp

Let MACHINE={m1,m2,m3}

Let ITEMS={hammer,spanner,cutter}

Defining the decision variables:-

We are defining a matrix to define the decision variables:-

$X=[x_{ij}]_{MACHINE*ITEMS}$

	hammer	Spanner	cutter
M-1	$X_{M-1,hammer}$	$X_{M-1,spanner}$	$X_{M-1,cutter}$
M-2	$X_{M-2,hammer}$	$X_{M-2,spanner}$	$X_{M-2,cutter}$
M-3	$X_{M-3,hammer}$	$X_{M-3,spanner}$	$X_{M-3,cutter}$

Then x_{ij} represents the amount in units item j made by machine corresponding to i.

Eg $x_{M-2,spanner}$ is the amount(units) of spanner made by the machine M-2

Defining the parameters:

Defining the cost/unit matrix COSTM:

$COSTM=[c_{ij}]_{MACHINE*ITEMS}$

	hammer	Spanner	cutter
M-1	30	25	42
M-2	35	22	40
M-3	28	27	42

Where c_{ij} is the cost/unit machine i is taking while making j

For eg: $c_{M\text{-}1,spanner}$is the cost /unit machine M-1 is taking while making spanner.

Defining a matrix MATM that will tell us the material consumed / unit:

MATM=$[m_{ij}]$ $_{MACHINE*ITEMS}$

	hammer	Spanner	cutter
M-1	3	2	2
M-2	4	1.5	2.5
M-3	2	3	3

Where m_{ij} is the material consumed /unit by machine i while making item j

For eg: $m_{M\text{-}1,spanner}$is the material consumed /unit by machine M-1 while making spanner.

Defining a matrix CAPACITYM that will tell us the capacity consumed /unit:

CAPACITYM=$[a_{ij}]$ $_{MACHINE*ITEMS}$

	hammer	Spanner	cutter
M-1	48	45	53
M-2	45	55	50
M-3	48	50	48

Where a_{ij} is the capacity consumed /unit by machine i on item j.

Now, defining a matrix POLLUTIONM that will tell us the pollution /unit on every machine:

$POLLUTIONM=[p_{ij}]_{MACHINE*ITEMS}$

	hammer	Spanner	cutter
M-1	7	12	10
M-2	9	9	9
M-3	13	12	8

c

Now, we have defined every parameter , Defining the **Objective Function**:

MINIMIZE **COST:** $\sum_{(i \text{ in } MACHINE, J \text{ in } ITEMS)} x_{ij} * c_{ij}$

Where, x_{ij} is the decision variable and c_{ij} € COSTM

Subject to constraints:

1)(As the total amount of raw-material available for production is 900 units):

$\sum_{(i \text{ in } MACHINE, J \text{ in } ITEMS)} x_{ij} * m_{ij} \leq 900$

Where, m_{ij} € MATHM

2) (As the total capacity of each machine is also limited to 5000 units):

$\sum_{(J \text{ in } ITEMS)} x_{ij} * a_{ij} \leq 5000$ for all i€ MACHINE

Where, a_{ij} € CAPACITYM

3) (As the division must control the total pollution level to 3000 units):

$\sum_{(i \text{ in } MACHINE, J \text{ in } ITEMS)} x_{ij} * p_{ij} \leq 3000$

Where p_{ij}€POLLUTIONM

4) (As the requirements is 100 units of each tool):

$\sum_{(i \text{ in MACHINE})} x_{ij}=100$ for all j€ ITEMS

5) (Non negativity constraint):

$x_{ij} \geq 0$ for all i€MACHINE,j€ITEMS

Q3)(R):

The question described above is solved with the help of AMPL and the files are attached in the zip:

```
Gurobi 8.0.0: optimal solution; objective 9312.935001
5 simplex iterations
cost = 9312.94

x :=
m1 cutter    36.0224
m1 hammer    33.1284
m1 spanner   33.3477
m2 cutter    26.6825
m2 hammer     0
m2 spanner   66.6523
m3 cutter    37.2951
m3 hammer    66.8716
m3 spanner    0
;

con1.lb = -Infinity
con1.ub = 900
con1.body = 650.439

:     con2.lb   con2.ub con2.body    :=
m1   -Infinity   5000      5000
m2   -Infinity   5000      5000
m3   -Infinity   5000      5000
;

con3.lb = -Infinity
con3.ub = 3000
con3.body = 3000

:        con4.lb con4.ub con4.body    :=
cutter     100     100       100
hammer     100     100       100
spanner    100     100       100
;
```

1) The optimal solution came out to be:9312.94
2) The values are variables at optimal solution is:
(As the values are in decimals , we can take use the smallest integer functions, as the no of items cannot be in decimals)

X_{ij} is s.t. i€MACHINE, j€ITEMS

X_{ij}	hammer	Spanner	cutter
M-1	33.1284	33.3477	36.0244
M-2	0	66.6523	26.6825
M-3	66.8761	0	37.2951

3) The Solution is satisfying all the constraints as we can see this the picture attached.

Q5)(R):

The question described above is solved with the help of AMPL and the files are attached in the zip:

```
Gurobi 8.0.0: optimal solution; objective 8914.453874
5 simplex iterations
time = 8914.45

x :=
m1 cutter     85.464
m1 hammer      9.80016
m1 spanner     0
m2 cutter      0
m2 hammer     90.1998
m2 spanner    17.1092
m3 cutter     14.536
m3 hammer      0
m3 spanner    82.8908
;

con1.lb = -Infinity
con1.ub = 900
con1.body = 879.072

:     con2.lb   con2.ub con2.body    :=
m1   -Infinity   5000     5000
m2   -Infinity   5000     5000
m3   -Infinity   5000     4842.27
;

con3.lb = -Infinity
con3.ub = 3000
con3.body = 3000

:        con4.lb con4.ub con4.body    :=
cutter     100     100      100
hammer     100     100      100
spanner    100     100      100
;
```

1) The optimal solution came out to be:8914.15

2) The values are variables at optimal solution is:

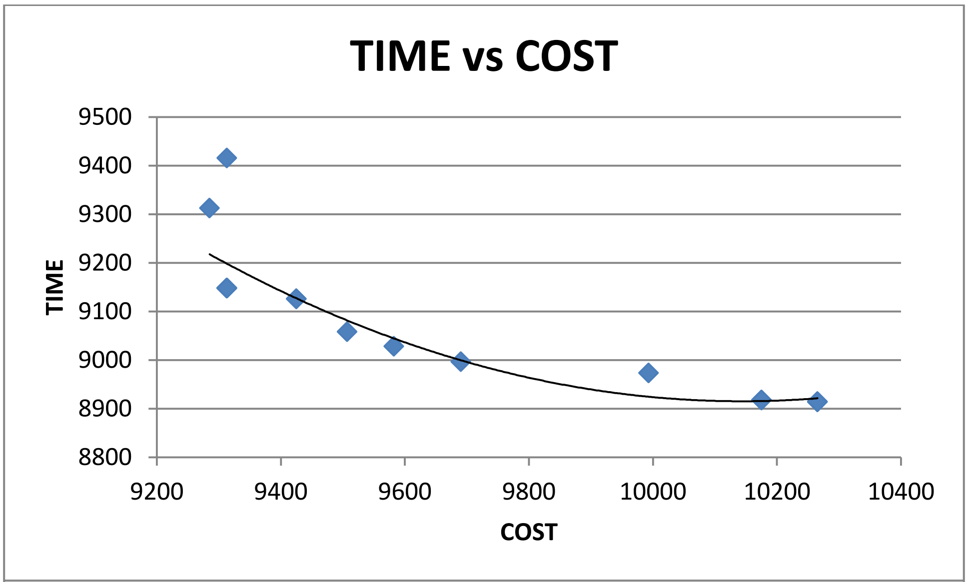
(As the values are in decimals , we can take use the smallest integer functions, as the no of items cannot be in decimals)

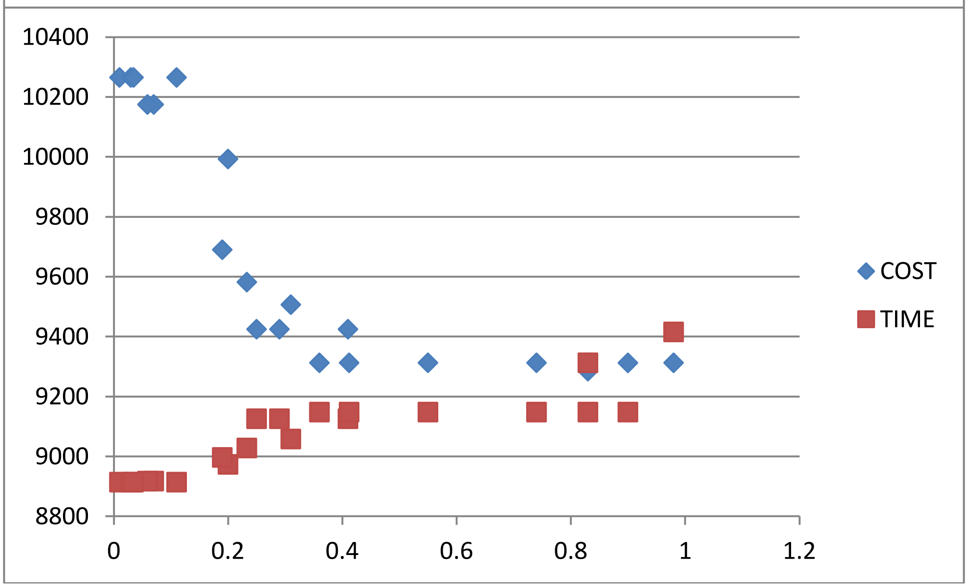
X_{ij} is s.t. i€MACHINE, j€ITEMS

X_{ij}	hammer	Spanner	cutter
M-1	9.80016	0	85.464
M-2	90.1998	17.1092	0
M-3	0	82.8908	14.536

3) The Solution is satisfying all the constraints as we can see this the picture attached(ques3b.dat,ques3b.mod,ques3b.run).

Q6)(R):


TIME vs COST
TIME
COST
9500
9400
9300
9200
9100
9000
8900
8800
9200
9400
9600
9800
10000
10200
10400


10400
10200
10000
9800
9600
9400
9200
9000
8800
0
0.2
0.4
0.6
0.8
1
1.2
COST
TIME

THETA	COST	TIME
0.11	10265.5	8914.45
0.2	9993.14	8973.68
0.25	9425	9126.22
0.31	9506.95	9058.52
0.36	9312.94	9148.35
0.07	10175.3	8917.98
0.9	9312.64	9148.35
0.83	9284.95	9312.94
0.19	9690.39	8996.91
0.55	9312.94	9148.35
0.98	9312.94	9415.97
0.29	9425	9126.22
0.41	9425	9126.22
0.01	10265.5	8914.45
0.74	9312.94	9148.35
0.059	10175.3	8917.98
0.03	10265.5	8914.45
0.83	9312.94	9148.35
0.233	9582.33	9028.51
0.412	9312.94	9148.35
0.0347	10265.5	8914.45