4)

This problem is the variation of Travelling Salesman Problem with minimum cost. We will go from Lander to 3 rocks and then Back.

Let,

Lander → L
Rock-1 → R1
Rock-2 → R2
Rock-3 → R3

The state-space of the problem will be represented by LR1R2 indicating traversal from L → R1 → R2, the entire tree will consist of such states.

Initial state → L

Goal state → L → All 3 rocks → L
(visited once in any order)

Path-cost function is the cost of edges along the path from the start state till current position.

Heuristic function → We will be using Minimum Spanning Tree from current vertex to start vertex.

MST cost will give us the lower bound on cost of path till we have reached

$$TSP_{cost} \geq MST_{cost}$$ (for any number of vertices covered)

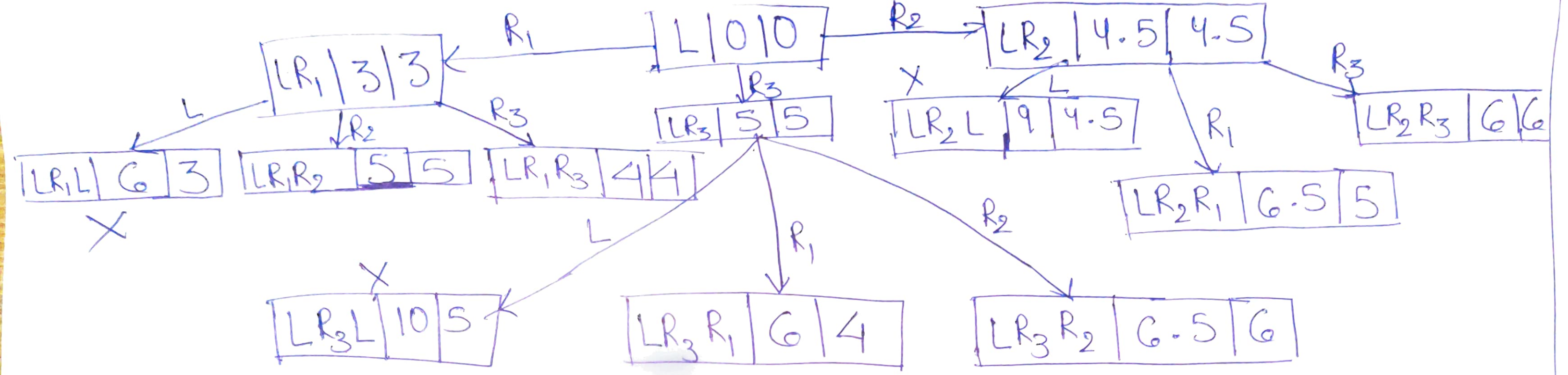
In my solution for the problem, we will be representing the states in the following notation.

State Path (Start → Current)	Path - Cost	MST-Cost

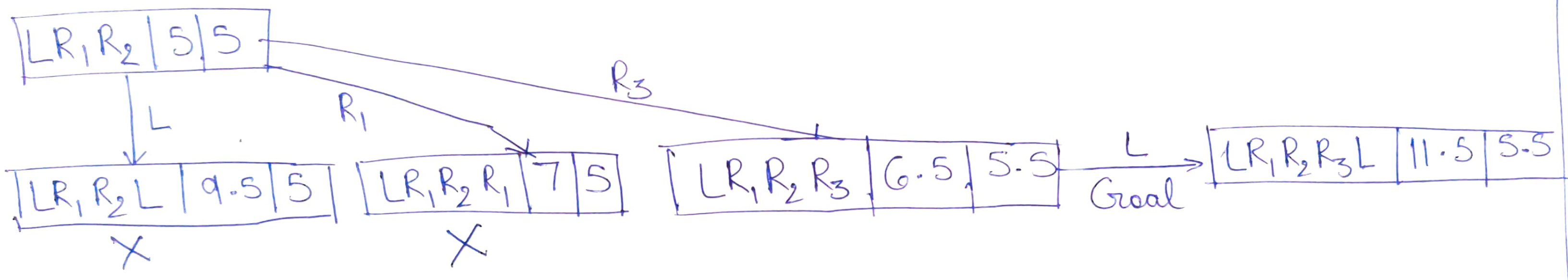
Will represent the tree level by level, such that I do not repeat / reduntant states.

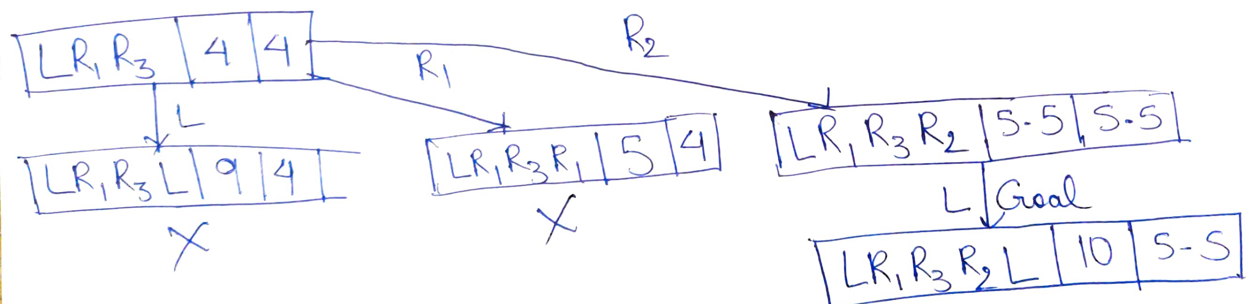
A cross (X) indicates repeated state from where we will not traverse anywhere.

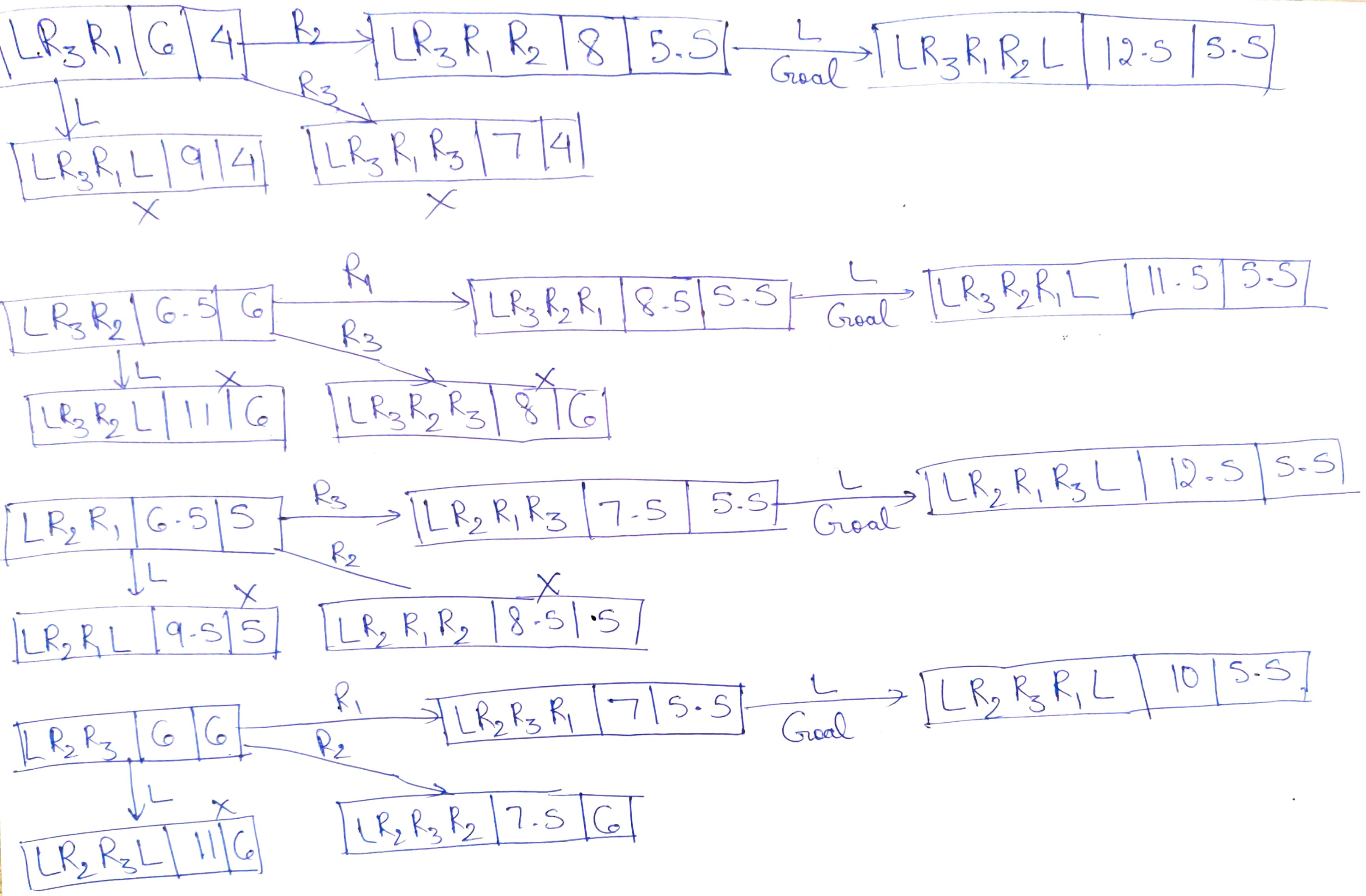
The costs will be represented in terms of number of hours taken



We have 6 states :






LR3R1 | 6 | 4
R2
LR3R1R2 | 8 | 5.5
L
Goal
LR3R1R2L | 12.5 | 5.5
L
LR3R1L | 9 | 4
R3
LR3R1R3 | 7 | 4
LR3R2 | 6.5 | 6
R1
LR3R2R1 | 8.5 | 5.5
L
Goal
LR3R2R1L | 11.5 | 5.5
L
LR3R2L | 11 | 6
R3
LR3R2R3 | 8 | 6
LR2R1 | 6.5 | 5
R3
LR2R1R3 | 7.5 | 5.5
L
Goal
LR2R1R3L | 12.5 | 5.5
L
LR2R1L | 9.5 | 5
R2
LR2R1R2 | 8.5 | .5
LR2R3 | 6 | 6
R1
LR2R3R1 | 7 | 5.5
L
Goal
LR2R3R1L | 10 | 5.5
L
LR2R3L | 11 | 6
R2
LR2R3R2 | 7.5 | 6

As we can see the search tree, the algorithm terminates or does not explore any state the moment it is visited twice. We have reduced to 6 valid and meaningful states which can be enumerated to 6 useful states. These are valid at Level 3 (Level 2 depth) and Level 4 (Level 3 depth).

At level 3 depth, we have traversed all paths between the 3 rocks, so we are left with a final goal state of coming back to the Lander.

In such a case, where the goal is only a step away, it is pointless of enumerating unoptimal states.

We also make another assumption of not backtracking to the lander during the exchange of rocks. It will not validate to an optimal solution.

Once, the goal state has been reached, with 6 possibilities in this case, we need to take the minimal solution (cost must be minimised)

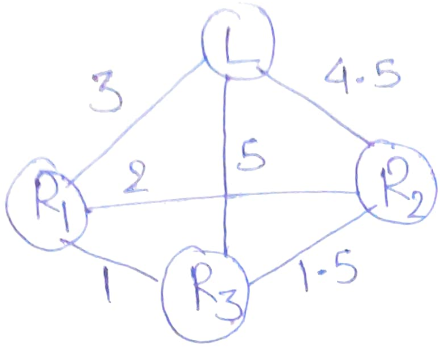
We have 2 solutions with minimal cost of 10

1) $L \rightarrow R_1 \rightarrow R_3 \rightarrow R_2 \rightarrow L$ $(3+1+1.5+4.5=10)$

2) $L \rightarrow R_2 \rightarrow R_3 \rightarrow R_1 \rightarrow L$ $(4.5+1.5+1+3=10)$

There is another aspect that can be looked into solving this problem which is the Triangular Inequality

The graph is (Undirected)



In this graph, consider all possible triangles, (with 3 vertices taken together). In all such triangles, the inequality (Sum of any 2 sides is greater than the 3rd side) is obeyed.

The MST lower bound heuristic also works as this inequality holds always. We have obtained $TSP_{cost} \geq MST_{cost}$ as this inequality holds.

If this inequality failed (we obtained a triangle that does not follow this), then we might have to backtrack to obtain the most optimal solution and MST heuristic would have failed.

As MST is considered, it is the same as assuming this statement is correct giving an optimal lower bound.

This heuristic is very important as backtracking will increase the search space exponentially not giving us a simple tree traversal and we can stop at depth 5 for our solution