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| **[IC200]-Introduction to Astronomy and Astrophysics** | **Fall 2020-21** |

# IC200 Project Report

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Collaborators Names:

**Introduction**

The report deals with the dark matter halo of galaxies. Dark matter haloes are cosmological structures that contain dark matter bounded by gravity. This dark matter is enclosed in subhaloes that consists of one or more galaxies.

The report seeks to enhance our learning and increase our understanding of dark matter haloes. The course of IC200 – Introduction to Astronomy and Astrophysics in the $3^{rd}$ Semester of our B.Tech Program has given immense information and cosmological data, based on observations and statistics.

The mass relation provided to us gives us data of one of the most massive dark haloes present in today's observable and studied universe, so much so, that it contains upto $10^{13}$ stars. The properties of these super massive haloes generally differ from those which are more general with 10% of it's size. Due to the variation from the general size of these galaxies, a detailed research is needed for a deeper and clearer understanding of these haloes.

In the Mathematical Modelling section, based on the mass relation given to us, radius is calculated using the density-volume relation. The relation of velocity (km/s) and radius (kpc) as studied in Class Lectures have been used.

Further on, the method of Stellar Parallax calculation is used to calculate the angular size of stars at very large distances from our Sun. Diagrammatic representations in the report describe the same. Next, the $3^{rd}$ question shows the function of density relation in galaxies with dark matter haloes. We use integration with limits to calculate the radius to which the partial mass is enclosed. At $\alpha$ = 3, we even explore the domain where radius reaches theoretically impossible bounds and logical arguments follow.

Further on, we explore the stellar formation rate relation with that of magnitude of UV Band and it's calculation with the given stellar formation rate for our particular galaxy mass. Nobel Laureate Kroupa's 2001 observation has been used to derive the relation.

The next few questions have explored black holes with it's associated mass, radius and associated parameters. Using the relations of spheroidal bulge in the stars $M_{BH}$ has been calculated. Scientific observations and logical conclusion from various online sites on the web have been used for collection and presentation of data.

At the end, we have found 2 galaxies ESO 146-5 and Tadpole Galaxy that have the desired properties of shape, geometry and orientation based on the mass-orientation relations during the calculation of the stellar geometry and gaseous geometry of the dark matter haloes.

**Mathematical Modelling**

**Solution of problem 1.** The solution for Question 1 is as follows:

Given, $M_{sun}$ = 2×$10^{30}$ kg and $\rho_m$ = 3×$10^{-27}$ kg/$m^3$
$M_h$ = $1140^2$×$10^7$×(2×$10^{30}$) kg = 1299600×2×$10^{37}$ kg = 25992×$10^{39}$ kg
$M_h$= 100×$\rho_m$ × $\frac{4}{3}\pi r^3$ as dark matter is a sphere.
=> 25992×$10^{39}$ kg = 100 × 3×$10^{-27}$ kg/$m^3$ × $\frac{4}{3}\pi r^3$ $m^3$ => 25992×$10^{39}$ = 4$\pi r^3$ ×$10^{-25}$ => 6498×$10^{64}$ = $\pi r^3$
=> $r^3$ = 2068.377×$10^{64}$ = 20.68377×$10^{66}$ => r = 2.745×$10^{22}$ m

Converting m to pc, r = 2.745/3.086$\times 10^6$ pc = 8.89$\times 10^5$ pc = 889 kpc.

In the circular path, velocity as a f(r) is that: $v^2$ = (4/3)$\pi$G$\rho r^2$ if r <= $r_0$ .........eq(1) and $v^2$ = $\frac{GM}{r}$ if r $> r_0$
Since r $< r_0$ in range of r $\in$ (1,200) kpc, so we have a linear graph as follows:
Matching the units with the constants of eq(1), G = 6.674$\times 10^{-11}$ SI Units we get $v^2$ = 83.855$\times 10^{-36} \times r^2$
After taking sqrt on both sides, v = 9.157$\times 10^{-18}\times$r
As v and r are in SI Units, converting v to km/s and r to kpc:
v$\times 10^3$ km/s = 9.157$\times 10^{-18}\times$3.086$\times 10^{19}\times$r kpc i.e. v$\times 10^3$ km/s = 28.258$\times 10^1$r kpc i.e. v km/s = 0.2528$\times$r kpc
Plotting the graph:

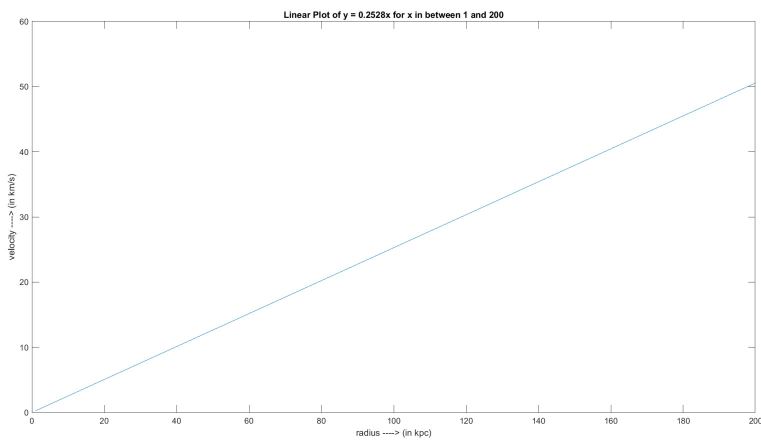


Figure 1.1: Linear Plot of y = (0.2528$\times$x) for x in the range of 1 to 200 kpc

**Solution of problem 2.** The solution for Question 2 is as follows:

Given, distance(d) = 1 Mpc = 1000 kpc and radius of the halo = 889 kpc.
In the method to calculate the stellar parallax: $\tan(\theta/2) = \frac{l}{d}$ i.e $\tan(\theta/2) = \frac{889}{1000}$ = 0.889
=> $\theta$ = 2$\times tan^{-1}$(0.889) = 2 $\times$ 41.63°= 83.26°.
In the night sky, the galaxy will cover an angle of 83.26°$\approx$ 1.4531 radians.
If the distance = 100 Mpc, then with Earth's orbit as baseline, the parallax angle calculated will be:
Distance = 100 Mpc = $10^5$ pc
Using the small-angle formula: D [pc] = 1/[arcsec] we get,
$10^5$ pc = 1/[arcsec] i.e. arcsec = $\theta''$ = $10^{-5}$
The parallax angle will be $\theta''$ = $10^{-5}$ in arcsecond.
In the span of 6 months (half an orbit revolution), we will see that angle moved by the Earth w.r.t Galaxy is the same as angle moved by the Galaxy w.r.t Earth.
The parallax angle uses this approximation to calculate angles with large distances(in Mpc's).

**Solution of problem 3.** The solution for Question 3 is as follows:

Given, $\rho$(r) = $\rho_0 (r/kpc)^{-\alpha}$, where $\rho_0$ = 100 $\times$ $\rho_m$ and $\rho_m$ = 3$\times 10^{-27}$ kg/$m^3$

At $\alpha$ = 1, mass = density $\times$ volume and so m = $\rho$(r) $\times$ $\frac{4}{3}\pi r^3$

Using the differential equation, $\frac{dm}{4\pi r^2 dr}$ = $\rho_0 \times (\frac{r}{3.09\times 10^{19} m})^{-1}$ i.e. dm = 3.09 $\times$ $10^{19}$ $\rho_0 4\pi r\, dr$

Let $r_\alpha$ be the radius that encloses half the mass of the halo ($M_h$/2).
Integrating with the specific limits,
$\int_0^{M_h/2}$ dm = 3.09 $\times$ $10^{19}$ $\rho_0 4\pi \int_0^{r_\alpha} r\, dr$ i.e. $\frac{M_h}{2}$ = 3.09 $\times$ $10^{19}$ $\rho_0 4\pi \times \frac{r_\alpha^2}{2}$ i.e. $r_\alpha^2 = \frac{M_h}{4\pi\rho_0 \times 3.09\times 10^{19}}$

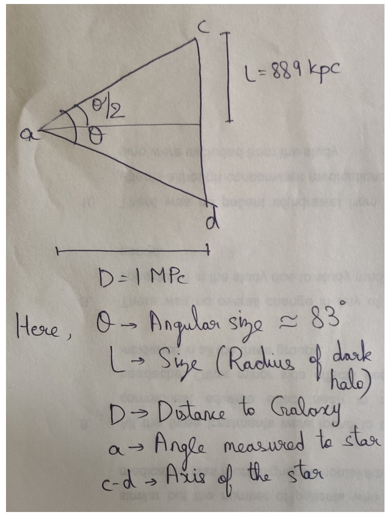


Figure 1.2: Measurement of Angular Size

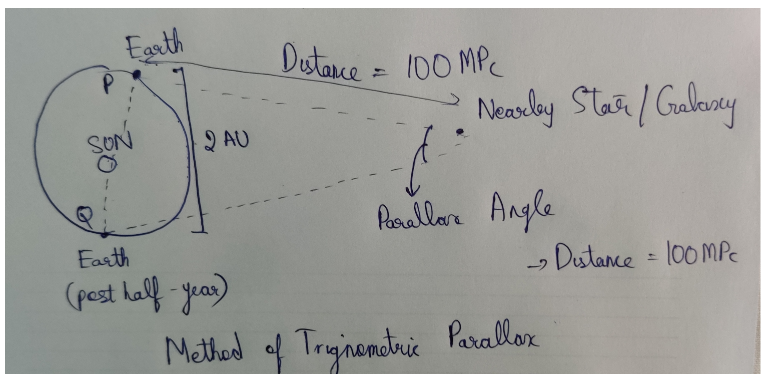


Figure 1.3: Measurement of stellar parallax using Earth's orbit as baseline(PQ)

Substituting the constants,
$r_\alpha^2 = \frac{25992\times10^{39}kg}{4\pi3\times10^{-25}\times3.09\times10^{19}}$ i.e. $r_\alpha$ = 4.723 $\times$ $10^{23}$ m
Convering m to kpc, $r_\alpha$ = 15286.68 kpc

Similarly, At $\alpha$ = 3, with $\rho$(r) = $\rho_0\times(r/kpc)^{-3}$, we get,
dm = $\rho_0 4\pi$ $\frac{dr}{r}$
Let $r_\alpha$ be the radius that encloses half the mass of the halo ($M_h$/2).
Integrating with the specific limits,
$\int_0^{M_h/2}$ dm = $\rho_0 4\pi \int_0^{r_\alpha} \frac{dr}{r}$ i.e. $M_h$ = $8\rho_0\pi$ (ln($r_\alpha$)-ln(0))

As $\ln(0)$ is not defined in the domain of $\log$, so at $\alpha$ = 3, we are not able to calculate $r_\alpha$.

However, the mass distribution of the dark halo is highly non-uniform with almost all the mass concentrated at r = 0 from the above density relation.
So at r $>$ 0, the mass distribution becomes very less compared to 0. So, conclusively for the value of r $>$ 0, $r_\alpha$ will not be possible as the value distribution will be very small to 0 for $\frac{M_h}{2}$.
Thus, for $r_\alpha$ = 0, half the mass is enclosed due to the extreme low distribution for r $>$ 0.
Logically, this can be concluded as the answer for $\alpha$ = 3 as it is impossible to calculate it theoretically.

**Solution of problem 4.** The solution for Question 4 is as follows:

Estimating the relation of $M_h$, $M_g$ and $M_*$

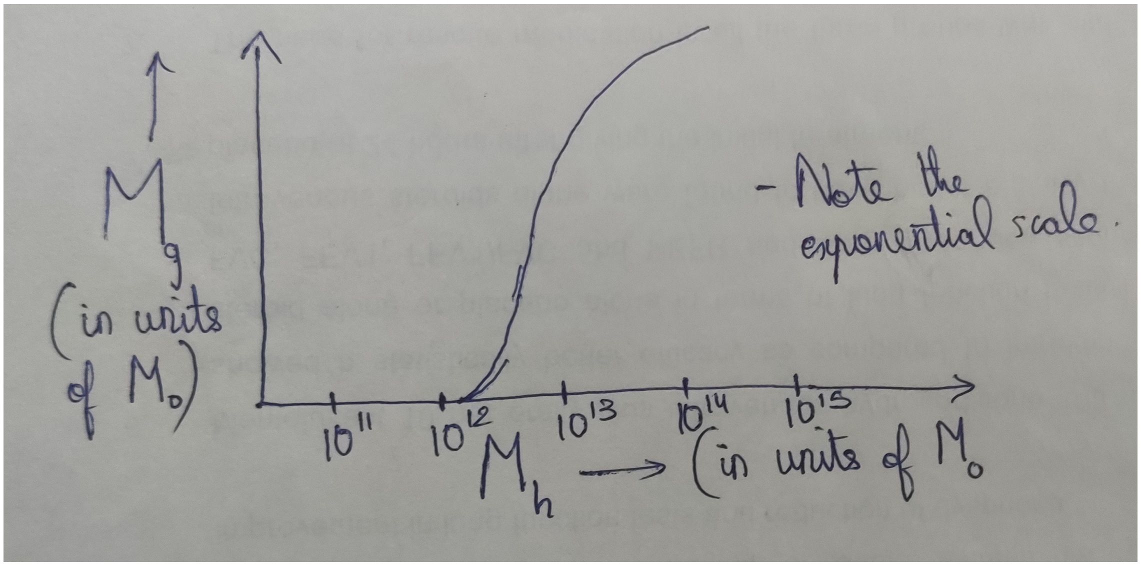


Figure 1.4: Relation of $M_g$ and $M_h$

From our value of $M_h$ at $10^{13}$ $M_0$ and the above graph plot, we obtain the relations that $M_g$ = $10^{11}$ $M_0$ and $M_*$ = $10^{12}$ $M_0$.

The Histogram plots for $M_g/M_h$ and $M_*/M_h$ are as follows:

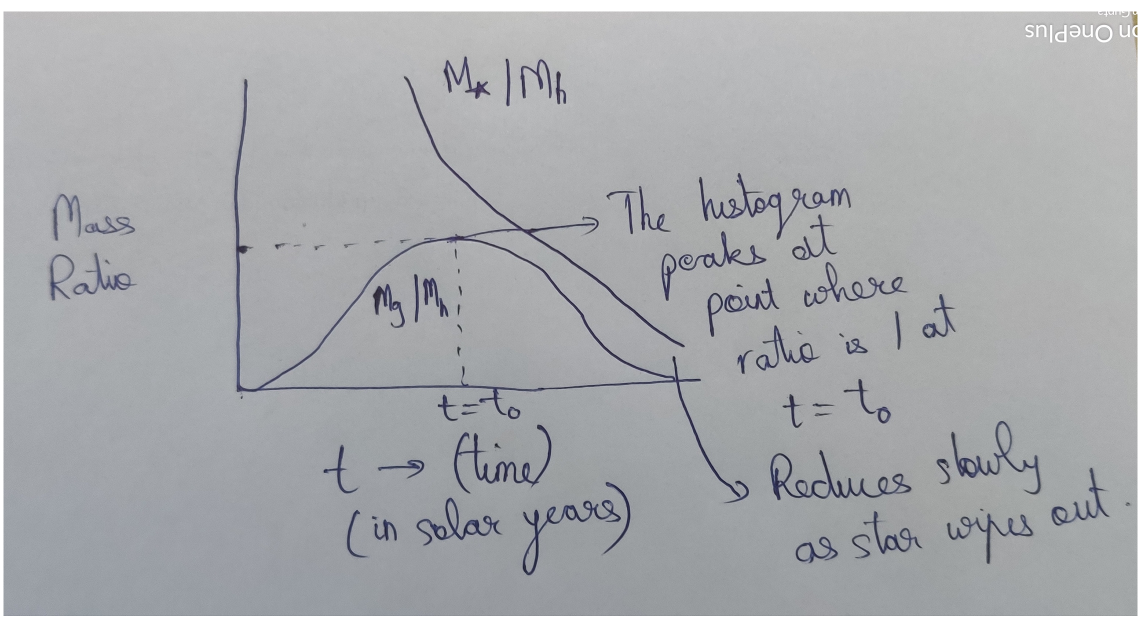


Figure 1.5: Histogram plot of the Mass Ratios with time

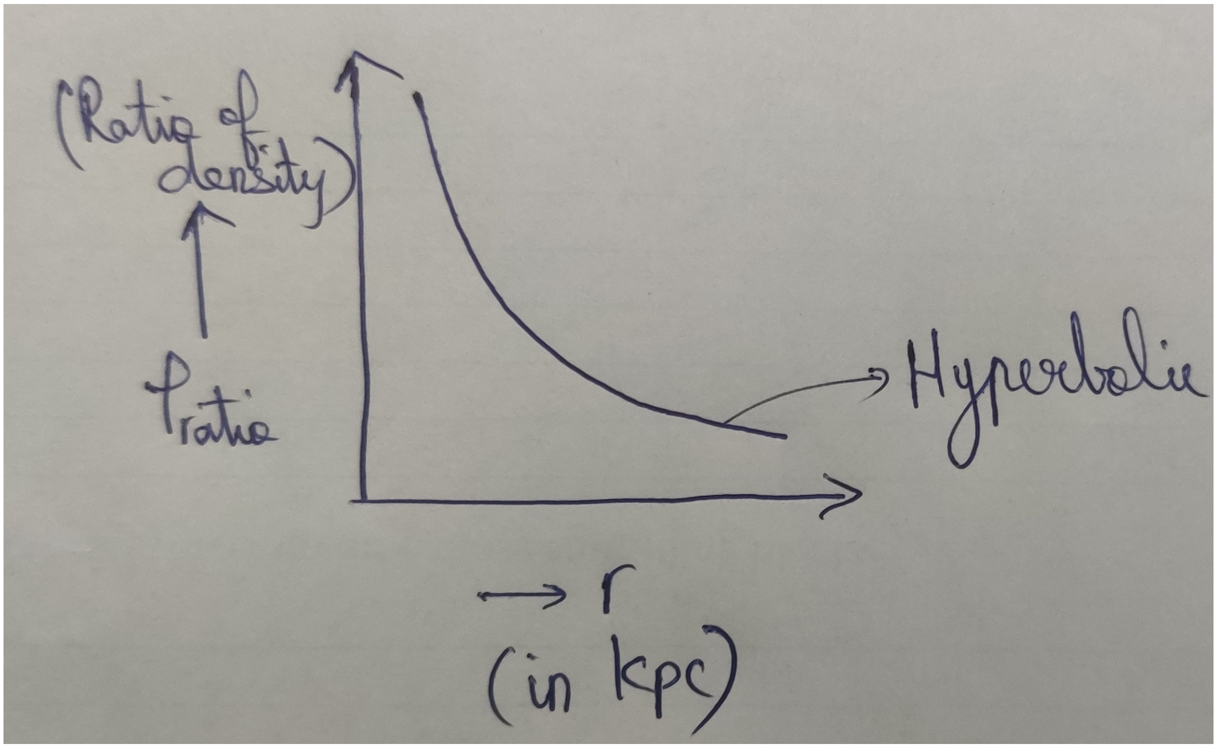


Figure 1.6: Density of air with height of the Earth's Atmosphere

The mathematical relation for the density of air is $\rho_{air} = \frac{PM}{RT} = \frac{\rho_0 M}{RT_0} \times (1 - \frac{2h}{T_0})^{(\frac{M_g}{R_h} - 1)}$ where $\rho_{halo}\ \alpha\ r^{-3}$

At the beginning, the ratio of $M_g/M_h$ is low and with the star formation it increases towards it's current value as the stellar brightness continues to increase. The gaseous mass continues to increase with the decrease in halo mass as the halo mass contributes to the formation of the stellar gas and the ratio moves towards 1.

Once, the ratio becomes 1, the mass of star begins to decrease as the mass of the halo is very low (used up in stellar gas formation) and the star begins to degenerate (degeneration era of the star).

As the star degenerates and moves towards the red giant/supernova stage, all the gases are ejected and burnt out of the system. The ratio begins to decrease (gases burn fast and the halo mass recovers itself as the star along with it's mass collapses).

**Solution of problem 5.** The solution for Question 5 is as follows:

As per scientific observations, 95% of galactic mass is dark matter.
The reason due to which the halo does not collapse to a point even though it has a mass far greater than the baryonic mass is that dark matter interaction proceeds via gravitational forces.

As electromagnetic, strong/weak nuclear forces and any other forces of nature do not interact with the dark matter, the dark matter halo is distributed over finite volume and density.

This distribution obeys the following law:
$\rho$(r) = $\rho_0(1 + (\frac{r}{r_c})^2)^{-1}$ where,
$\rho_0$ is the finite central density of the halo and $r_c$ is the core/virial radius of the dark matter halo.

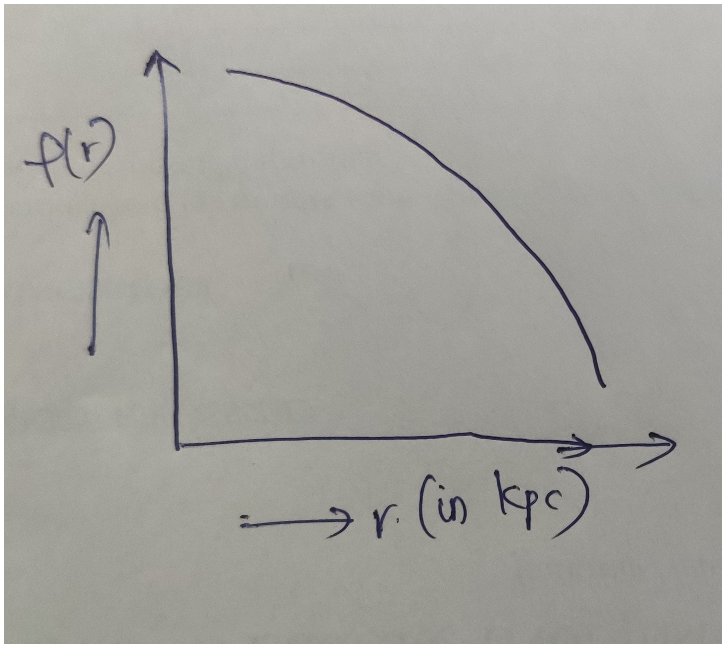


Figure 1.7: Radial density profile of the dark matter halo

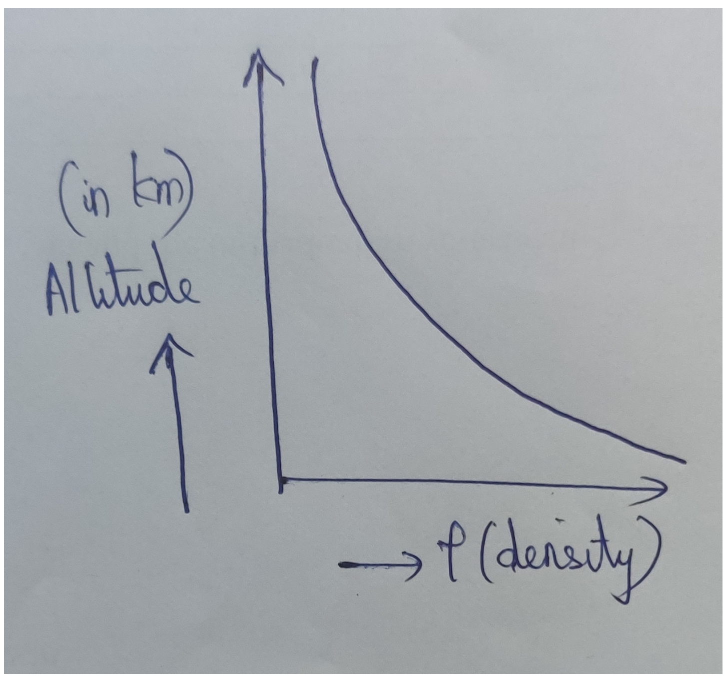


Figure 1.8: Density of air with altitude of the Earth's Atmosphere

Both the above graphs suggest that $\rho$ decreases with the increase in radius/height. However, in air the density decrease is rapid, due to the air friction whereas the halo obeys a smooth decrease due to the surrounding vacuum.

**Solution of problem 6.** The solution for Question 6 is as follows:

With the $M_h$ value of about 13 Trillion $M_0$, the observed galaxies with this mass are very few and all obey an elliptical shape. This category of galaxies are supermassive which is the last scale of galaxy observation based on their shapes.

Gases in these galaxies are mostly present in spherical clumps with masses of the order of $10^{11}$ $M_0$. These clumps collide with the galactic dust and in the process merge with them to form stellar gaseous mass.

The gravitational fields are very high due to the supermassive galactic mass and hence the elliptical shape with eccentricity ($e$) $< 1$ suits the shape. The gases due to their large weights don't tend to move around freely or in rings and stay together as clubbed masses in the shape of clumps (mostly, spherical).

**Solution of problem 7.** The solution for Question 7 is as follows:

The star formation rate ($\psi$) is measured by the flux wavelengths emitted from the stars that track $\psi$ in the recent past. Massive stars are brightest in the UV Band and the data of the UV flux provides most of the information of $\psi$. The UV Band gives direct information of the young bright stars in the galaxy.

Kroupa (2001) had given a relation, of $\psi$ with the luminosity obtained by the UV Band of the star. His observations stated that the wavelengths emitted by stars due to the redshift phenomenon span the spectrum of light, with the maximum in the UV Band. UV flux is marked by the NUV and FUV (Near and Far UV respectively).

Since, the stars are extremely far away with distances $>$ 1 Mpc, it becomes extremely difficulty to calculate luminosity that will give us $\psi$. Hence, absolute magnitude is calculated with the distance at 10 pc. At 10 pc, the NUV only dominates (due to the small distance) and the relation of the absolute magnitude of UV Band is important as it helps in the direct estimation of $\psi$. Once the luminosity(at absolute magnitude) is obtained, it becomes easy to calculate the apparent magnitude.

Using the relation of Initial Mass Function(IMF) of Kroupa(2001), we have
$\psi$ = $10^{-28}L_{v,NUV}$ where $\psi$ is in $M_{sun}(yr^{-1})$ and $L_{v,NUV}$ in ergs $s^{-1}Hz^{-1}$
$\psi$ = $\log_{10}(\frac{M_h}{M_{sun}})$ = $\log_{10}(12996\times10^9)$ = 9 + 4.113 = 13.113.
so, $L_{v,NUV}$ = 13.113$\times10^{35}$ W $Hz^{-1}$ .............W indicates Watt after conversion from ergs to Joules (SI Units)
Given, 10 Mpc = 3.086$\times10^{23}$ m
Apparent Magnitude = L / $4\pi(r^2)$ ...for the flux by the spherical surface area.
=> 13.113$\times10^{35}$ /($4\pi$(9.523$\times10^{46}$)) = 0.3412$\times10^{-11}/\pi$ = 0.1086$\times10^{-11}$ = 1.086$\times10^{-12}$ SI Units.

The Apparent Magnitude (m) of the Galaxy is 1.086$\times10^{-12}$ SI Units

**Solution of problem 8.** The solution for Question 8 is as follows:

The central black hole of the galaxies, is based on $M_h$ and mass of the central spheroidal bulge. Let $M_{bulge}$ be the mass of spheroidal bulge of stars. The formation of bulge and black holes have gone hand in hand since the formation of the Universe.
The mass of black hole is approx proportional to $M_{bulge}$, as $M_{BH} \approx 0.001M_{bulge}$. ......................................eq(1). In

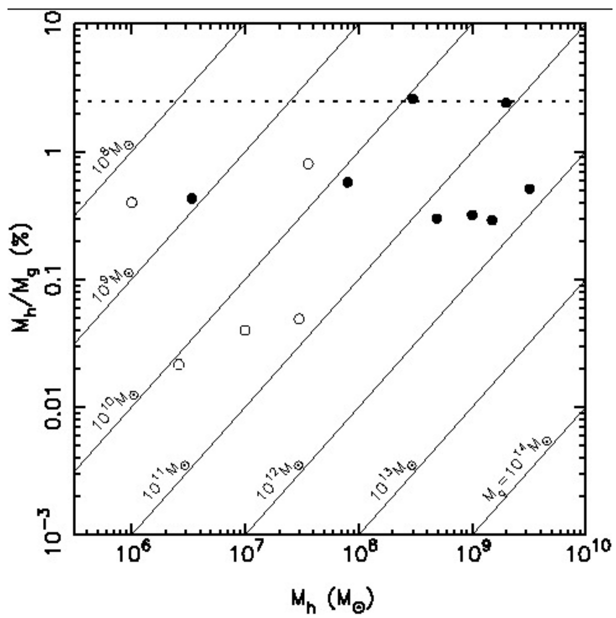


Figure 1.9: Comparison of the ratio of Mass of Black Hole to Spheroidal Bulge

The above figure, shows black hole masses $M_h$ vs. mass ratios $M_h$ / $M_g$ for 13 galaxies; $M_g$ is defined as the total luminous mass in the case of elliptical galaxies and as the bulge mass in the case of disk galaxies. It indicates the relation that percentage of $M_h/M_g$ is about 0.5-3.0% that indicates the relation of eq(1).

Observation:
Dressler & Richstone observed that the ratio of the masses of the dark objects (black holes) in M31 and M32 was closer to the ratio of spheroid luminosities ($\sim$ 15) than it is to the ratio of total luminosities ($\sim$ 70). Since then, the approximate proportionality between black hole mass and bulge mass has held up fairly well.

There is a trend of increasing $M_h/M_g$ with $M_h$ which probably just reflects the shortage of very bright and very faint galaxies in the sample. This trend is reproduced even within the (essentially complete) sample of galaxies from the Local Group (Ratio for the Milky Way is 0.05%). Uncertainties in the determination of $M_h$ are often large, but bulge masses can be very uncertain too.

Nevertheless it is reasonable to expect lower values of $M_h/M_g$ in galaxies with small bulge-to-disk ratios since these galaxies are the least likely to have experienced the strong non-axisymmetric distortions that are believed necessary for driving large amounts of gas into the nucleus and feeding the black hole.

The mean mass of the black holes is of order $10^{-2.5}$ times the mass of their host galaxies and is consistent with the mass in black holes needed to produce the observed energy density in quasar light given reasonable assumptions about the efficiency of quasar energy production.
This logical observation has been used to calculate $M_{BH}$ of our galaxy:
$M_{BH} \approx 10^{-2.5} \times M_h$ i.e. $M_{BH} \approx 25992\times10^{39} \times 10^{-2.5}$ kg
i.e. $M_{BH} \approx (25992/3.1622)\times10^{37} = 8219.594\times10^{37} \approx 8.22\times10^{40}$ kg.

Thus, $M_{BH} = 8.22\times10^{40}$ kg.

**Solution of problem 9.** The solution for Question 9 is as follows:

As the $M_h$ is 13 Trillion Solar masses, the galaxy with closest masses are Tadpole Galaxy which is formed by clumps of dark matter with each being 10 Trillion Solar masses. ESO 146-5 is the next closest with approx 30 Trillion Solaar masses.

ESO 146-5 is one of the most massive interacting elliptical galaxy wheras Tadpole Galaxy is a disrupted and barred spiral galaxy. ESO 146-5 has a very strong gravitational lensing effect with a huge halo of stars with a strong interaction among the nuclei. This has a very unusual shape due to merging of multiple collisions of small galaxies wheras Tadpole has formed after a single merge with a small galaxy. The Tadpole also has a very long trail of stars of about 3 lakh light years due to the weak mutual gravitational attraction. The tidal forces have created the tail of dust and gases.

The difference of shapes is due the distinct form of merging of the stellar halo due to formation of gravitational lensing in both the galaxies. Unusual and indifferent merging of dwarf galaxy clusters have also led to different masses of the galaxies with the ESO 146-5 being an interactive one.

**Conclusion**

Based on the exclusive study of dark matter haloes in the report, we have studied about the orientation of particular haloes with the galaxies enclosed. Relations taught and explained in the course have been particularly useful in major areas of the report.

The report begins at the Mass of Halo ($M_h$) to the mass of stars ($M_*$) and gases ($M_g$) enclosed where we studied the relations of orbital velocity with the radius of the halo. Usage of Calculas was necessary to understand the density distribution of haloes, from theoretically impossible bounds to logical deductions to the properties of the dust particles in the galaxy.

Studying the possible ratio of stellar masses and gas masses with $M_h$ gives a functional density inversely proportional to the square of the core radius. Relation of stellar formation rate and absolute magnitude that was used to measure it's distance is also explored using Kroupa's (2001) observation.

In further depths, exploration of black holes is considered with it's mass ($M_{BH}$) that is dependent on the spheroidal bulge of stars and radius is taken. The study of the 2 cases of the galaxy have also verified the studied observations to a certain extent.

In theoretical calculations with certain observations, there is a deduction of results. However, the cases of the 2 galaxies Tadpole and ESO 146-5 obey few of the observations and disobey the rest. The prior reason for this is that the observations and research still have a long way to go to make more accurate conclusions to deduce the correct observations.

Another reason for this is that everything in nature is not perfect and thus, we derive certain observations with inconclusive results. Overall, the report is a good research work on the dark matter halo fundamentals and explores the unexplored and has helped in deeper and more clearer understanding of the fundamental concepts throughout.