

# C H A P T E R

## 6

### Optimum Design: Numerical Solution Process and Excel Solver

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#### Section 6.5 Excel Solver for Unconstrained Optimization Problems

6.1

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Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.32

The annual operating cost  $U$  for an electrical line system is given by the following expression

$$U = \frac{(21.9E + 07)}{V^2 C} + (3.9E + 06)C + (1.0E + 03)V$$

where  $V$ =line voltage in kilovolts and  $C$ =line conductance in ohms. Find stationary points for the function, and determine  $V$  and  $C$  to minimize the operating cost.

#### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables  $V$  and  $C$  have been renamed  $V\_line$  and  $C\_line$  respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $V\_line=200$  and  $C\_line=0.01$ , a solution of  $V\_line=241.8$  and  $C\_line=0.0310$ , which gives an objective function value of -483,528.61, is obtained.

1

Exercise 6.1

Variables	Value
V_line	200
C_line	0.01

Objective function

$$=(21.9 \times 10^7) / (V\_line \times V\_line \times C\_line) + (3.9 \times 10^6) \times C\_line + (1 \times 10^3) \times V\_line$$

Solver Parameters

Set Target Cell:

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Variable Cells:

Subject to the Constraints:

2

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports

☒ Keep Solver Solution ☐ Restore Original Values

Reports

Answer Sensitivity Limits

3

A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report			
2	Worksheet: [Exercise 6.1.xlsx]Sheet1			
3	Report Created: 6/28/2010 1:31:02 PM			
4				
5				
6	Target Cell (Min)			
7	Cell	Name	Original Value	Final Value
8	\$B\$8	Value	786500	483528.6077
9				
10				
11	Adjustable Cells			
12	Cell	Name	Original Value	Final Value
13	\$B\$4	V_line	200	241.7642712
14	\$B\$5	C_line	0.01	0.030995534
15				

6.2

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.39

$$f(x_1, x_2) = 8x_1^2 + 8x_2^2 - 80\sqrt{x_1^2 + x_2^2 - 20x_2 + 100} - 80\sqrt{x_1^2 + x_2^2 + 20x_2 + 100} - 5x_1 - 5x_2$$

**Solution**

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables  $x_1$  and  $x_2$  have been renamed  $x$  and  $y$  respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x=4$  and  $y=6$ , a solution of  $x=4.15$  and  $y=0.362$ , which gives an objective function value of  $-1616.2$ , is obtained.

1

The Excel worksheet shows the following setup:

Variables	Value
x1 or x	4
x2 or y	6

The Objective function is set to cell \$B\$8 with the formula:  $=8*x*x+8*y*y-80*(x*x+y*y-20*y+100)^(1/2)-80*(x*x+y*y+20*y+100)^(1/2)-5*x-5*y$

The Solver Parameters dialog box is configured as follows:

- Set Target Cell: \$B\$8
- Equal To: Min
- By Changing Variable Cells: \$B\$4:\$B\$5
- Subject to the Constraints: (empty)
- Solve Method: GRG Nonlinear Engine
- Make Variable Non-Negative: unchecked

2

The Solver Results dialog box shows that a solution was found. The "Keep Solver Solution" option is selected. The Reports list includes "Answer", "Sensitivity", and "Limits".

3

	A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report				
2	Worksheet: [Exercise 6.2.xlsx]Sheet1				
3	Report Created: 6/16/2010 2:33:05 PM				
4					
5					
6	Target Cell (Min)				
7		<u>Cell</u>	<u>Name</u>	<u>Original Value</u>	<u>Final Value</u>
8		\$B\$8	Value	-1405.94214	-1616.18353
9					
10					
11	Adjustable Cells				
12		<u>Cell</u>	<u>Name</u>	<u>Original Value</u>	<u>Final Value</u>
13		\$B\$4	x	4	4.144652074
14		\$B\$5	y	6	0.361553621
15					

6.3

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.40

$$f(x_1, x_2) = 9x_1^2 + 9x_2^2 - 100\sqrt{x_1^2 + x_2^2 - 20x_2 + 100} - 64\sqrt{x_1^2 + x_2^2 + 16x_2 + 64} - 5x_1 - 41x_2$$

### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables  $x_1$  and  $x_2$  have been renamed  $x$  and  $y$  respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x=5$  and  $y=2$ , a solution of  $x=3.73$  and  $y=0.341$ , which gives an objective function value of  $-1526.6$ , is obtained.

1

2

3

6.4

Solve the following problem using Excel Solver (choose any reasonable starting point):

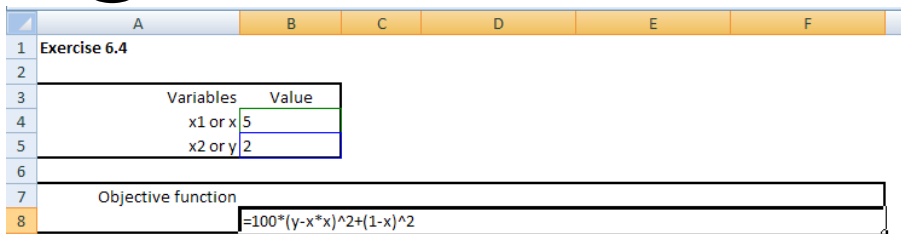
Exercise 4.41

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

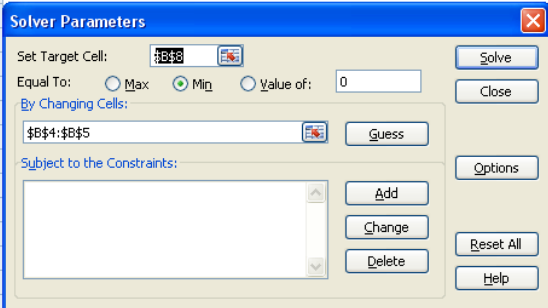
### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables  $x_1$  and  $x_2$  have been renamed  $x$  and  $y$  respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x=5$  and  $y=2$ , a solution of  $x=1.216$  and  $y=1.462$ , which gives an objective function value of 0.0752, is obtained.

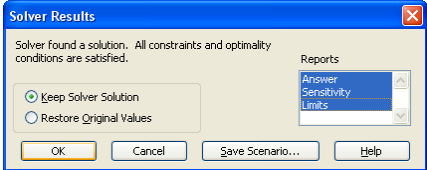
1



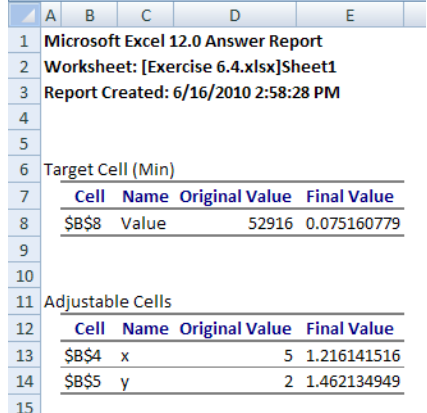
2



3



3



6.5

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.42

$$f(x_1, x_2, x_3, x_4) = (x_1 - 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$

### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables  $x_1$ ,  $x_2$ ,  $x_3$ , and  $x_4$  have been renamed  $x$ ,  $y$ ,  $z$ , and  $v$  respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x=1$ ,  $y=2$ ,  $z=3$ , and  $v=4$ , a solution of  $x=0.246$ ,  $y=0.0257$ ,  $z=0.1808$ , and  $v=0.205$  which gives an objective function value of 0.01578, is obtained.

1

The Excel worksheet shows the following setup:

Variables	Value
x1 or x	1
x2 or y	2
x3 or z	3
x4 or v	4

The Objective function is defined in cell B10 as:  $=(x-10*y)^2+5*(z-v)^2+(y-2*z)^4+10*(x-v)^4$ .

The Solver Parameters dialog box is configured as follows:

- Set Target Cell: \$B\$10
- Equal To: Min
- By Changing Variable Cells: \$B\$4:\$B\$7
- Subject to the Constraints: (empty)
- Options: (checked)

2

The Solver Results dialog box shows the following options:

- Keep Solver Solution (selected)
- Restore Original Values
- Reports: Answer, Sensitivity, Limits

3

A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report			
2	Worksheet: [Exercise 6.5.xlsx]Sheet1			
3	Report Created: 6/16/2010 3:17:13 PM			
4				
5				
6	Target Cell (Min)			
7	Cell	Name	Original Value	Final Value
8	\$B\$10	Value	1432	0.015776556
9				
10				
11	Adjustable Cells			
12	Cell	Name	Original Value	Final Value
13	\$B\$4	x	1	0.245922121
14	\$B\$5	y	2	0.025660894
15	\$B\$6	z	3	0.180763881
16	\$B\$7	v	4	0.204883019
17				

## Section 6.6 Excel Solver for Linear Programming Problems

Solve the following LP problems using Excel Solver:

6.6

Solve the following LP problem using the Excel Solver:

Maximize  $z = x_1 + 2x_2$   
 subject to  $-x_1 + 3x_2 \leq 10$   
 $x_1 + x_2 \leq 6$   
 $x_1 - x_2 \leq 2$   
 $x_1 + 3x_2 \geq 6$   
 $x_1, x_2 \geq 0$

### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x_1=0$  and  $x_2=0$ , a solution of  $x_1=2$  and  $x_2=4$ , which gives an objective function value of 10, is obtained.

1

2

3

3

Cell	Name	Original Value	Final Value
\$E\$12	Objective function: max Sum of LH	0	10

Cell	Name	Original Value	Final Value
\$C\$12	Variable value x1	0	2
\$D\$12	Variable value x2	0	4

6.7

Solve the following LP problem using the Excel Solver:

$$\begin{aligned} \text{Maximize } z &= x_1 + 4x_2 \\ \text{subject to } x_1 + 2x_2 &\leq 5 \\ x_1 + x_2 &= 4 \\ x_1 - x_2 &\geq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x_1=0$  and  $x_2=0$ , a solution of  $x_1=3.67$  and  $x_2=0.667$ , which gives an objective function value of 6.33, is obtained.

1

Exercise 6.7

Problem is to maximize:  $x_1 + 4x_2$   
 subject to  $x_1 + 2x_2 \leq 5$   
 $x_1 + x_2 = 4$   
 $x_1 - x_2 \geq 3$   
 $x_1, x_2 \geq 0$

Problem set up for Solver

Variables	x1	x2	Sum of LH	RHS Limit
Variable value	0	0		
Objective function: max	1	4	$=C12*SC$11+D12*SD$11$	
Constraint 1	1	2	$=C13*SC$11+D13*SD$11$	5
Constraint 2	1	1	$=C14*SC$11+D14*SD$11$	4
Constraint 3	1	-1	$=C15*SC$11+D15*SD$11$	3

Solver Parameters

Set Target Cell:  $SC$12$   
 Equal To: ☒ Max ☐ Min ☐ Value of: 0  
 By Changing Variable Cells:  $SC$11:$D$11$   
 Subject to the Constraints:  
 $SC$11 \geq 0$   
 $SD$11 \geq 0$   
 $SE$13 \leq SF$13$   
 $SE$15 \geq SF$15$

2

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports: Answer, Sensitivity, Limits

☒ Keep Solver Solution  
☐ Restore Original Values

OK Cancel Save Scenario... Help

3

Microsoft Excel 12.0 Answer Report

Worksheet: [Exercise 6.7.xlsx]Sheet1

Report Created: 6/11/2010 10:22:48 AM

Target Cell (Max)			
Cell	Name	Original Value	Final Value
$SE$12$	Objective function: max Sum of LH	0	6.333333333

Adjustable Cells			
Cell	Name	Original Value	Final Value
$SC$11$	Variable value x1	0	3.666666667
$SD$11$	Variable value x2	0	0.666666667



6.8

Solve the following LP problem using the Excel Solver:

Minimize  $f = 5x_1 + 4x_2 - x_3$

subject to  $x_1 + 2x_2 - x_3 \geq 1$

$2x_1 + x_2 + x_3 \geq 4$

$x_1, x_2 \geq 0$ ;  $x_3$  is unrestricted in sign

### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x_1=0$ ,  $x_2=0$ , and  $x_3=0$ , a solution of  $x_1=0$ ,  $x_2=1.67$ , and  $x_3=2.33$ , which gives an objective function value of 4.33, is obtained.

1

Exercise 6.8

Problem is to minimize:  $5x_1 + 4x_2 - x_3$

subject to

$x_1 + 2x_2 - x_3 \geq 1$

$2x_1 + x_2 + x_3 \geq 4$

$x_1, x_2 \geq 0$

$x_3$  is unrestricted

Problem set up for Solver

	$x_1$	$x_2$	$x_3$	Sum of LH	RHS Limit
Variable value	0	0	0		
Objective function: min	5	4	-1	$=C12*SC$11+D12*SD$11+E12*SE$11$	
Constraint 1	1	2	-1	$=C13*SC$11+D13*SD$11+E13*SE$11$	1
Constraint 2	2	1	1	$=C14*SC$11+D14*SD$11+E14*SE$11$	4

Solver Parameters

Set Target Cell:  $\$F\$12$

Equal To: ☐ Max ☒ Min ☐ Value of: 0

By Changing Variable Cells:  $\$C\$11:\$E\$11$

Subject to the Constraints:

$\$C\$11 \geq 0$

$\$D\$11 \geq 0$

$\$F\$13 \geq \$G\$13$

$\$F\$14 \geq \$G\$14$

2

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Keep Solver Solution

Restore Original Values

OK Cancel Save Scenario... Help

Reports

Answer

Sensitivity

Limits

3

A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report			
2	Worksheet: [Exercise 6.8.xlsx]Sheet1			
3	Report Created: 6/11/2010 10:51:29 AM			
4				
5				
6	Target Cell (Min)			
7	Cell	Name	Original Value	Final Value
8	$\$F\$12$	Objective function: min Sum of LH	0	4.333333333
9				
10				
11	Adjustable Cells			
12	Cell	Name	Original Value	Final Value
13	$\$C\$11$	Variable value x1	0	0
14	$\$D\$11$	Variable value x2	0	1.666666667
15	$\$E\$11$	Variable value x3	0	2.333333333
16				

6.9

Solve the following LP problem using the Excel Solver:

$$\text{Maximize } z = 2x_1 + 5x_2 - 4.5x_3 + 1.5x_4$$

$$\text{subject to } 5x_1 + 3x_2 + 1.5x_3 \leq 8$$

$$1.8x_1 - 6x_2 + 4x_3 + x_4 \geq 3$$

$$-3.6x_1 + 8.2x_2 + 7.5x_3 + 5x_4 = 15$$

$$x_i \geq 0; i = 1 \text{ to } 4$$

**Solution**

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x_1=0$ ,  $x_2=0$ ,  $x_3=0$ , and  $x_4=0$ , a solution of  $x_1=1.34$ ,  $x_2=0.441$ ,  $x_3=0$ , and  $x_4=3.24$ , which gives an objective function value of 9.73, is obtained.

1

The Excel worksheet shows the problem setup for Exercise 6.9. The objective function is in cell G12, and the constraints are in cells G13, G14, and G15. The Solver Parameters dialog box is open, showing the target cell G12, the variable cells \$C\$11:\$F\$11, and the constraints.

Variables	x1	x2	x3	x4	Sum of LH	RHS Limit
Variable value	0	0	0	0		
Objective function: max	2	5	-4.5	1.5	=C12*\$C\$11+D12*\$D\$11+E12*\$E\$11+F12*\$F\$11	
Constraint 1	5	3	1.5	0	=C13*\$C\$11+D13*\$D\$11+E13*\$E\$11+F13*\$F\$11	8
Constraint 2	1.8	-6	4	1	=C14*\$C\$11+D14*\$D\$11+E14*\$E\$11+F14*\$F\$11	3
Constraint 3	-3.6	8.2	7.5	5	=C15*\$C\$11+D15*\$D\$11+E15*\$E\$11+F15*\$F\$11	15

2

The Solver Results dialog box shows that the Solver found a solution. The "Keep Solver Solution" option is selected. The "Reports" section shows "Answer, Sensitivity, and Limits" selected.

3

Cell	Name	Original Value	Final Value
\$G\$12	Objective function: max Sum of LH	0	9.732867096

Cell	Name	Original Value	Final Value
\$C\$11	Variable value x1	0	1.335664341
\$D\$11	Variable value x2	0	0.440559431
\$E\$11	Variable value x3	0	0
\$F\$11	Variable value x4	0	3.239160838

6.10

Solve the following LP problem using the Excel Solver:

$$\text{Minimize } f = 8x_1 - 3x_2 + 15x_3$$

$$\text{subject to } 5x_1 - 1.8x_2 - 3.6x_3 \geq 2$$

$$3x_1 + 6x_2 + 8.2x_3 \geq 5$$

$$1.5x_1 - 4x_2 + 7.5x_3 \geq -4.5$$

$$-x_2 + 5x_3 \geq 1.5$$

$$x_1, x_2 \geq 0; x_3 \text{ is unrestricted in sign}$$

### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x_1=0$ ,  $x_2=0$ , and  $x_3=0$ , a solution of  $x_1=0.654$ ,  $x_2=0.0756$ , and  $x_3=0.315$ , which gives an objective function value of 9.73, is obtained.

1

Exercise 6.10

Problem is to minimize:  $8x_1 - 3x_2 + 15x_3$

subject to

$5x_1 - 1.8x_2 - 3.6x_3 \geq 2$

$3x_1 + 6x_2 + 8.2x_3 \geq 5$

$1.5x_1 - 4x_2 + 7.5x_3 \geq -4.5$

$5x_3 - x_2 \geq 1.5$

$x_1, x_2 \geq 0$

$x_3$  is unrestricted

Problem set up for Solver

Variables	$x_1$	$x_2$	$x_3$	Sum of LH	RHS Limit
Variable value	0	0	0		
Objective function: min	8	-3	15	$=C14*SC\$13+D14*SD\$13+E14*SE\$13$	
Constraint 1	5	-1.8	-3.6	$=C15*SC\$13+D15*SD\$13+E15*SE\$13$	2
Constraint 2	3	6	8.2	$=C16*SC\$13+D16*SD\$13+E16*SE\$13$	5
Constraint 3	1.5	-4	7.5	$=C17*SC\$13+D17*SD\$13+E17*SE\$13$	-4.5
Constraint 4	0	-1	5	$=C18*SC\$13+D18*SD\$13+E18*SE\$13$	1.5

Solver Parameters

Set Target Cell:  $\$F\$14$

Equal To: ☐ Max ☒ Min ☐ Value of: 0

By Changing Variable Cells:  $\$C\$13:\$E\$13$

Subject to the Constraints:

$\$C\$13:\$D\$13 \geq 0$

$\$F\$15 \geq \$G\$15$

$\$F\$16 \geq \$G\$16$

$\$F\$17 \geq \$G\$17$

$\$F\$18 \geq \$G\$18$

2

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Keep Solver Solution

Restore Original Values

OK Cancel Save Scenario... Help

3

Microsoft Excel 12.0 Answer Report

Worksheet: [Exercise 6.10.xlsx]Sheet1

Report Created: 6/16/2010 12:17:07 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$\$F\$14$	Objective function: min Sum of LH	0	9.732867134

Adjustable Cells

Cell	Name	Original Value	Final Value
$\$C\$13$	Variable value $x_1$	0	0.654108392
$\$D\$13$	Variable value $x_2$	0	0.075611888
$\$E\$13$	Variable value $x_3$	0	0.315122378

6.11

Solve the following LP problem using the Excel Solver:

$$\begin{aligned} \text{Maximize } z &= 10x_1 + 6x_2 \\ \text{subject to } 2x_1 + 3x_2 &\leq 90 \\ 4x_1 + 2x_2 &\leq 80 \\ x_2 &\geq 15 \\ 5x_1 + x_2 &= 25 \\ x_1, x_2 &\geq 0 \end{aligned}$$

**Solution**

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $x_1=0$ , and  $x_2=0$ , a solution of  $x_1=0$ , and  $x_2=25$ , which gives an objective function value of 150, is obtained.

1

	A	B	C	D	E	F
1	<b>Exercise 6.11</b>					
2						
3	Problem is to maximize:	$10x_1 + 6x_2$				
4	subject to	$2x_1 + 3x_2 \leq 90$				
5		$4x_1 + 2x_2 \leq 80$				
6		$x_2 \geq 15$				
7		$5x_1 + x_2 = 25$				
8		$x_1, x_2 \geq 0$				
9						
10	<b>Problem set up for Solver</b>					
11	Variables	$x_1$	$x_2$	Sum of LH	RHS Limit	
12	Variable value	0	0			
13	Objective function: max	10	6	$=C13*SC12+D13*SD12$		
14	Constraint 1	2	3	$=C14*SC12+D14*SD12$	90	
15	Constraint 2	4	2	$=C15*SC12+D15*SD12$	80	
16	Constraint 3	0	1	$=C16*SC12+D16*SD12$	15	
17	Constraint 4	5	1	$=C17*SC12+D17*SD12$	25	
18						
19	<b>Solver Parameters</b>					
20	Set Target Cell: $SE13$					
21	Equal To: <input checked="" type="radio"/> Max <input type="radio"/> Min <input type="radio"/> Value of: 0					
22	By Changing Cells: $SC12:D12$					
23	Subject to the Constraints:					
24	$SC12:D12 \geq 0$					
25	$SE14 \leq SE15$					
26	$SE15 \leq SE16$					
27	$SE16 \geq SE17$					
28	$SE17 \geq SE18$					
29						
30						
31						

2

**Solver Results**

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports

☒ Keep Solver Solution  
☐ Restore Original Values

OK Cancel Save Scenario... Help

3

	A	B	C	D	E
1	<b>Microsoft Excel 12.0 Answer Report</b>				
2	Worksheet: [Exercise 6.11.xlsx]Sheet1				
3	Report Created: 6/16/2010 12:46:15 PM				
4					
5					
6	Target Cell (Max)				
7	Cell	Name	Original Value	Final Value	
8	$SE13$	Objective function: max Sum of LH	0	150	
9					
10					
11	Adjustable Cells				
12	Cell	Name	Original Value	Final Value	
13	$SC12$	Variable value $x_1$	0	0	
14	$SD12$	Variable value $x_2$	0	25	
15					

## Section 6.7 Excel Solver for Nonlinear Programming

Solve the following problems using Excel Solver:

6.12

Solve the following NLP problem using the Excel Solver:

Exercise 3.35 (Exercise 3.34 using inner and outer diameter as design variables)

Design a hollow torsion rod shown in Fig.E3.34 to satisfy the following requirements (created by J.M. Trummel):

1. The calculated shear stress,  $\tau$ , shall not exceed the allowable shear stress  $\tau_a$  under the normal operation torque  $T_o$  (N·m).
  2. The calculated angle of twist,  $\theta$ , shall not exceed the allowable twist,  $\theta_a$  (radians).
  3. The member shall not buckle under a short duration torque of  $T_{\max}$  (N·m).
- Requirements for the rod and material properties are given in Table E3.34(A) and E3.34(B) (select a material for one rod). Use the following design variables:

$x_1$  = outside diameter of the shaft;  $x_2$  = ratio of inside/outside diameter,  $d_i/d_o$ .

Using graphical optimization, determine the inside and outside diameters for a minimum mass rod to meet the above design requirements. Compare the hollow rod with an equivalent solid rod ( $d_i/d_o = 0$ ). Use consistent set of units (e.g. Newtons and millimeters) and let the minimum and maximum values for design variables be given as

$$0.02 \leq d_o \leq 0.5 \text{ m}, \quad 0.60 \leq \frac{d_i}{d_o} \leq 0.999$$

Useful expressions for the rod are:

Mass of rod:

$$M = \frac{\pi}{4} \rho l (d_o^2 - d_i^2), \text{ kg}$$

Calculated shear stress:

$$\tau = \frac{c}{J} T_o, \text{ Pa}$$

Calculated angle of twist:

$$\theta = \frac{l}{GJ} T_o, \text{ radians}$$

Critical buckling torque:

$$T_{cr} = \frac{\pi d_o^3 E}{12\sqrt{2}(1 - \nu^2)^{0.75}} \left(1 - \frac{d_i}{d_o}\right)^{2.5}, \text{ N.m}$$

### Notation

$M$  = mass of the rod (kg),

$d_o$  = outside diameter of the rod (m),

$d_i$  = inside diameter of the rod (m),

$\rho$  = mass density of material (kg/m<sup>3</sup>),

$l$  = length of the rod (m),

$T_o$  = Normal operation torque (N·m),

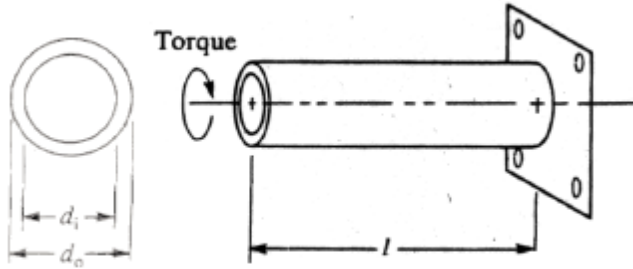
$c$  = Distance from rod axis to extreme fiber (m),

$J$  = Polar moment of inertia (m<sup>4</sup>),

$\theta$  = Angle of twist (radians),

$G$  = Modulus of rigidity (Pa),

$T_{cr}$  = Critical buckling torque ( $\text{N} \cdot \text{m}$ ),  
 $E$  = Modulus of elasticity (Pa), and  
 $\nu$  = Poisson's ratio.



**FIGURE E3-34** Hollow torsion rod.

**TABLE E3-34(A)** Rod Requirements

<i>Torsion rod number</i>	<i>Length, <math>l</math> (m)</i>	<i>Normal torque, <math>T_0</math> (<math>\text{kN} \cdot \text{m}</math>)</i>	<i>Max. torque, <math>T_{max}</math> (<math>\text{kN} \cdot \text{m}</math>)</i>	<i>Allowable twist, <math>\theta_a</math> (degrees)</i>
1	0.50	10.0	20.0	2
2	0.75	15.0	25.0	2
3	1.00	20.0	30.0	2

**TABLE E3-34(B)** Materials and Properties for the Torsion Rod

<i>Material</i>	<i>Density, <math>\rho</math> (<math>\text{kg}/\text{m}^3</math>),</i>	<i>Allowable Shear stress, <math>\tau_a</math> (MPa)</i>	<i>Elastic modulus, <math>E</math> (GPa)</i>	<i>Shear modulus, <math>G</math> (GPa)</i>	<i>Poisson's ratio (<math>\nu</math>)</i>
1. 4140 alloy steel	7850	275	210	80	0.30
2. Aluminum alloy 24 ST4	2750	165	75	28	0.32
3. Magnesium alloy A261	1800	90	45	16	0.35
4. Beryllium	1850	110	300	147	0.02
5. Titanium	4500	165	110	42	0.30

### **Solution**

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $do=400$  and  $di=40$  a solution of  $do=103.0$  and  $di=98.3$  which gives an objective function value of 2.90, is obtained.

	A	B	C	D	E	F
1	Exercise 6.12					
2	1. Design Variables	Lower limit	Symbol	Value	Upper limit	Units
3	Outside diameter of shaft	=0.02*1000	do	102.985279421914	=0.5*1000	mm
4	Inside diameter of shaft	=0.02*1000	di	98.3111555658017	=0.5*1000	mm
5						
6	2. Parameters	Symbol	Value		Units	
7	Length of the rod	L	=0.5*1000		mm	
8	Normal operation torque	To	=10*10^3*10^3		N-mm	
9	Max. torque	Tmax	=2*10^7		N-mm	
10	Allowable twist	θ <sub>a</sub> , theta_a	=2*PI()/180		radians	
11	Mass density of the material	ρ, ro	=7850*10^-9		kg/mm <sup>3</sup>	
12	Allowable shear stress	τ <sub>a</sub> , tau_a	=275		N/mm <sup>2</sup>	
13	Modulus of elasticity	E	=2.1*10^5		N/mm <sup>2</sup>	
14	Modulus of rigidity	G	=8*10^4		N/mm <sup>2</sup>	
15	Poisson's ratio	v	=0.3			
16						
17	3. Analysis Variables	Symbol	Equation		Units	
18	Distance from rod axis to extreme fiber	c_distance	=do/2		m	
19	Polar moment of inertia	J	=PI()/32*(do*do*do*do-di*di*di)		m <sup>4</sup>	
20	Mass of rod	M	=PI()/4*ro*I*(do*do-di*di)		kg	
21	Calculated shear stress	τ, tau	=c_distance/J*To		Pa	
22	Calculated angle of twist:	q, theta	=I/(G*J)*To		radians	
23	Critical buckling torque	Tcr	=(PI()*(do*do*do*E)/(12*SQR(2)*(1-v*v)^0.75))*((1-di/do)^2.5)		N-m	
24						
25	4. Objective Function	Symbol	Equation		Units	
26	Minimize f	f	=(PI()*ro*I/4)*(do*do-di*di)		kg	
27						
28	5. Constraints	Value	</>=		RS	
29	g1 (shear stress constraint)	=(To*c_distance/J)/(tau_a)	<		=tau_a/tau_a	
30	g2 (Allowable twist constraint)	=(To*I)/(G*J)/(theta_a)	<		=theta_a/theta_a	
31	g3 (buckling constraint)	=(Tcr)/(Tmax)	<		=Tmax/Tmax	
32	g6 (design variable constraint)	=(di/do)/0.6	<		=0.6/0.6	
33	g7 (design variable constraint)	=(di/do)/0.999	<		=0.999/0.999	

Solver Parameters

Set Target Cell:

\$D\$26

Solve

Equal To:

☐ Max
☒ Min
☐ Value of:

0

Close

By Changing Variable Cells:

\$D\$3:\$D\$4

Guess

Subject to the Constraints:

\$B\$29:\$B\$33 <= \$D\$29:\$D\$33

\$B\$3:\$B\$4 <= \$D\$3:\$D\$4

\$D\$3:\$D\$4 <= \$E\$3:\$E\$4

Add

Change

Delete

Options

Reset All

Help

6	Target Cell (Min)				
7	Cell	Name	Original Value	Final Value	
8	\$C\$26	f	488.2977461	2.9	
9					
10					
11	Adjustable Cells				
12	Cell	Name	Original Value	Final Value	
13	\$D\$3	do	400	102	
14	\$D\$4	di	40	98	

6.13

Solve the following NLP problem using the Excel Solver:

**Exercise 3.50**

A minimum mass structure (area of member 1 is the same as member 3) three-bar truss is to be designed to support a load  $P$  as shown in Fig. 2.9. The following notation may be used:  $P_u = P \cos \theta$ ,  $P_v = P \sin \theta$ ,  $A_1$  = cross-sectional area of members 1 and 3,  $A_2$  = cross-sectional area of member 2.

The members must not fail under the stress, and deflection at node 4 must not exceed 2cm in either direction. Use Newtons and millimeters as units. The data is given as  $P = 50$  kN;  $\theta = 30^\circ$ ; mass density,  $\rho = 7850$  kg/m<sup>3</sup>; modulus of elasticity,  $E = 210$  GPa; allowable stress,  $\sigma_a = 150$  MPa. The design variables must also satisfy the constraints  $50 \leq A_i \leq 5000$  mm<sup>2</sup>.

**Solution**

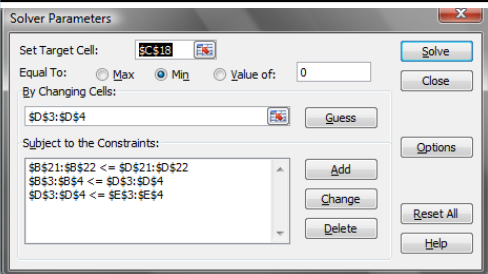
- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $A_1=250$  and  $A_2=100$  a solution of  $A_1=294$  and  $A_2=65.8$  which gives an objective function value of 7.04, is obtained.



Continued.

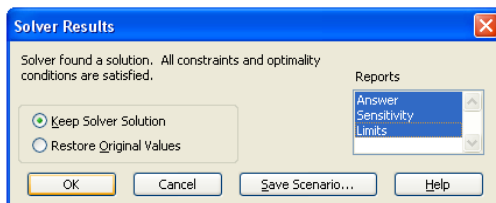
1

A	B	C	D	E	F
Exercise 6.13					
1. Design Variables	Lower limit	Symbol	Value	Upper limit	Units
Cross-sectional area of members 1 and 3	50	A_1	293.631953891414	5000	mm <sup>2</sup>
Cross-sectional area of member 2	50	A_2	65.7466067603963	5000	mm <sup>2</sup>
2. Parameters	Symbol	Value	Units		
Point load	p	=50*10 <sup>3</sup>	N		
Loading angle	θ, theta	=30*PI()/180	radians		
Member length	l	=1*10 <sup>3</sup>	mm		
Mass density	ρ, ro	=7.85*10 <sup>-6</sup>	kg/mm <sup>3</sup>		
Allowable stress	σ <sub>y</sub> , sigma_a	=150	N/mm <sup>2</sup>		
Modulus of elasticity	E	=2.1*10 <sup>5</sup>	N/mm <sup>2</sup>		
3. Analysis variables	Symbol	Equation	Units		
N/A	N/A	N/A	N/A		
4. Objective function	Symbol	Equation	Units		
Minimize f	f	=ro*l*(2*SQRT(2)*A_1+A_2)	kg		
5. Constraints	Value	<=> RS			
g1 (stress constraint 1)	=(((P*COS(theta)/A_1)+(P*SIN(theta)/(A_1+A_2*SQRT(2))))/SQRT(2))/sigma_a	<	=sigma_a/sigma_a		
g2 (stress constraint 2)	=((SQRT(2)*P*SIN(theta))/(A_1+A_2*SQRT(2)))/sigma_a	<	=sigma_a/sigma_a		



3

2



A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report			
2	Worksheet: [Exercise 6.13.xlsx]Sheet1			
3	Report Created: 7/1/2010 9:00:01 AM			
4				
5				
6	Target Cell (Min)			
7	Cell	Name	Original Value	Final Value
8	\$C\$18	f Equation	6.34	7.04
9				
10				
11	Adjustable Cells			
12	Cell	Name	Original Value	Final Value
13	\$D\$3	A_1	250	293.6319539
14	\$D\$4	A_2	100	65.74660676
15				

6.14

Solve the following NLP problem using the Excel Solver:

Exercise 3.51

**Design of a water tower support column.** As a member of the ABC consulting Engineers you have been asked to design a cantilever cylindrical support column of minimum mass for a new water tank. The tank itself has already been designed in the tear-drop shape shown in Fig. E3.51. The height of the base of the tank ( $H$ ), the diameter of the tank ( $D$ ), and wind pressure on the tank ( $w$ ) are given as  $H = 30$  m,  $D = 10$  m, and  $w = 700$  N/m<sup>2</sup>. Formulate the design optimization problem and solve it graphically. (created by G.Baenziger).

In addition to designing for combined axial and bending stresses and buckling, several limitations have been placed on the design. The support column must have an inside diameter of at least 0.70 m ( $d_i$ ) to allow for piping and ladder access to the interior of the tank. To prevent local buckling of the column walls the diameter/thickness ratio ( $d_o/t$ ) shall not be greater than 92. The large mass of water and steel makes deflections critical as they add to the bending moment. The deflection effects as well as an assumed construction eccentricity ( $e$ ) of 10 cm must be accounted for in the design process. Deflection at C.G. of the tank should not be greater than  $\Delta$ .

Limits on the inner radius and wall thickness are  $0.35 \leq R \leq 2.0$  m and  $1.0 \leq t \leq 20$  cm.

*Pertinent constraints and formulas*

Height of water tank,	$h = 10$ m
Allowable deflection,	$\Delta = 20$ cm
Unit weight of water,	$\gamma_w = 10$ kN/m <sup>3</sup>
Unit weight of steel,	$\gamma_s = 80$ kN/m <sup>3</sup>
Modulus of elasticity,	$E = 210$ GPa
Moment of inertia of the column,	$I = \frac{\pi}{64} [d_o^4 - (d_o - 2t)^4]$
Cross-sectional area of column material,	$A = \pi t (d_o - t)$
Allowable bending stress,	$\sigma_b = 165$ MPa
Allowable axial stress,	$\sigma_a = \frac{12\pi^2 E}{92(H/r)^2}$ (calculated using the critical
Radius of gyration,	buckling load with factor of safety of $\frac{23}{12}$
Average thickness of tank wall,	$r = \sqrt{\frac{I}{A}}$
Volume of tank,	$t_t = 1.5$ cm
Surface area of tank,	$V = 1.2\pi D^2 h$
Projected area of tank, for wind loading,	$A_s = 1.25\pi D^2$
Load on the column due to weight of water and steel tank,	$A_p = \frac{2Dh}{3}$
	$P = V\gamma_w + A_s t_t \gamma_s$

Lateral load at the tank C.G due to wind pressure,  
Deflection at C.G. of tank,

$$W = wA_p$$

$$\delta = \delta_1 + \delta_2, \text{ where}$$

$$\delta_1 = \frac{WH^2}{12EI} (4H + 3h)$$

$$\delta_2 = \frac{H}{2EI} (0.5Wh + Pe)(H + h)$$

Moment at base,  
Bending stress,

$$M = W(H + 0.5h) + (\delta + e)P$$

$$f_b = \frac{M}{2I} d_o$$

Axial stress,

$$f_a = (P/A) = \frac{V\gamma_w + A_s t_t \gamma_s}{\pi t (d_o - t)}$$

Combined stress constraint,

$$\frac{f_a}{\sigma_a} + \frac{f_b}{\sigma_b} \leq 1$$

Gravitational acceleration,

$$g = 9.81 \text{ m/s}^2$$

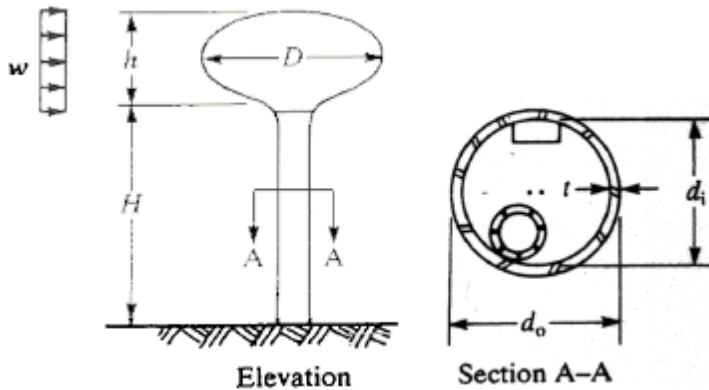


FIGURE E3.51 Water Tower support column.

### Solution

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $t=10$  and  $R_-=100$  a solution of  $t=2.84$  and  $R_-=129.0$  which gives an objective function value of 562, is obtained.

6.14

Continued.

	A	B	C	D	E	F
1	Exercise 6.14					
2	1. Design Variables		Lower limit	Symbol	Value	Upper limit Units
3	Wall thickness	1	t		2.83544442665551	40 cm
4	Mean radius	=0.35*100+0.5*t	R_		129.012713439187	=2.5*100+0.5*t cm
5						
6	2. Parameters		Symbol	Value	Units	
7	Diameter of the tank	D		=1000	cm	
8	Height of the base of the tank	H		=3000	cm	
9	Wind pressure on the tank	w		0.07	N/cm <sup>2</sup>	
10	Height of water tank	h, h_tank		1000	cm	
11	Assumed construction eccentricity	e		10	cm	
12	Allowable deflection	Δ, delta_l		20	cm	
13	Unit weight of water	γ_w, gamma_w		0.01	N/cm <sup>3</sup>	
14	Unit weight of steel	γ_s, gamma_s		0.08	N/cm <sup>3</sup>	
15	Modulus of elasticity	E		=2.1*10^7	N/cm <sup>2</sup>	
16	Allowable bending stress	σ_b, sigma_b		=1.65*10^4	N/cm <sup>2</sup>	
17	Allowable axial stress	σ_a, sigma_a		=(12*Pi()*(1/2*E_)/(92*(H/r_ gyration)^2)	N/cm <sup>2</sup>	
18	Average thickness of tank wall	t, tt		1.5	cm	
19	Gravitational acceleration	g		981	cm/s <sup>2</sup>	
20						
21	3. Analysis Variables		Symbol	Equation	Units	
22	Outside diameter of support column	do		=(2*R_+t)	cm	
23	Inside diameter of support column	di		=(2*R_-t)	cm	
24	Moment of inertia of the column	I		=Pi()*(do^4-di^4)/64	cm <sup>4</sup>	
25	Cross-sectional area of column material	A		=Pi()*(do^2-di^2)/4	cm <sup>2</sup>	
26	Radius of gyration	r, r_ gyration		=(I/A)^(1/2)	cm	
27	Volume of tank	V		=1.2*Pi()*(D^2*h_tank)	cm <sup>3</sup>	
28	Surface area of tank	As		=1.25*Pi()*(D^2)	cm <sup>2</sup>	
29	Projected area of tank, for wind loading	Ap		=D*h_tank/3	cm <sup>2</sup>	
30	Load on column due to weight of water and steel tank	P		=V*gamma_w+As*tt*gamma_s	N	
31	Lateral load at tank C.G. due to wind pressure	W, W_lat		=w*Ap	N	
32	Moment at base	M		=W_lat*(H+0.5*h_tank)+(delta_u+e)*P	N-cm	
33	Bending stress	fb		=(M)/(2*I)*(do)	N/cm <sup>2</sup>	
34	Axial stress	fa		=P/A	N/cm <sup>2</sup>	
35	Deflection at C.G. of tank	δ, delta_u		=(W_lat*h_tank)/(12*E_)*((4*H+3*h_tank)+((H)/(2*E_))*((0.5*w*H+P*e)*(H+h_tank))	cm	
36						
37	4. Objective Function		Symbol	Equation	Units	
38	Minimize mass of support column	f		=(gamma_s/g)*(2*Pi()*(R_-t)^2)*H	kg	
39						
40	5. Constraints		Value	</>=	RS	
41	g1 (combined stress constraint)	=(fa/sigma_a)+(fb/sigma_b)	<	1		
42	g2 (inner diameter constraint)	=70/di	<	=di/di		
43	g3 (diameter/thickness ratio constraint)	=(do/t)/92	<	=92/92		
44	g4 (deflection constraint)	=delta_u/delta_l	<	=delta_l/delta_l		
45						
46	Solver Parameters					
47	Set Target Cell: \$C\$39 To: Value of: 0 By Changing Variable Cells: \$D\$3:\$D\$4 Subject to the Constraints: \$B\$3:\$B\$4 <= \$D\$3:\$D\$4 \$B\$41:\$B\$44 <= \$D\$41:\$D\$44 \$D\$3:\$D\$4 <= \$E\$3:\$E\$4					
48	Solver Results					
49	Solver found a solution. All constraints and optimality conditions are satisfied.					
50	Keep Solver Solution					
51	Restore Original Values					
52	OK Cancel Save Scenario... Help					

3

1	A	B	C	D	E
2	Microsoft Excel 12.0 Answer Report				
3	Worksheet: [Exercise 6.14.xlsx]Sheet1				
4	Report Created: 7/2/2010 11:31:17 AM				
5					
6	Target Cell (Min)				
7	Cell	Name	Original Value	Final Value	
8	\$C\$39	f Equation	1537.170717	562.3099288	
9					
10					
11	Adjustable Cells				
12	Cell	Name	Original Value	Final Value	
13	\$D\$3	t	10	2.835444427	
14	\$D\$4	R_	100	129.0127134	

6.15

Solve the following NLP problem using the Excel Solver:

Exercise 3.54

**Design of a tripod.** Design a minimum mass tripod of height  $H$  to support a vertical load  $W = 60$  kN. The tripod base is an equilateral triangle with sides  $B = 1200$  mm. The struts have a solid circular cross section of diameter  $D$  (Fig. E3.54).

The axial stress in the struts must not exceed the allowable stress in compression, and axial load in the strut  $P$  must not exceed the critical buckling load  $P_{cr}$  divided by a safety factor  $FS = 2$ . Use consistent units of Newtons and centimeters. The minimum and maximum values for design variables are  $0.5 \leq H \leq 5$  m and  $0.5 \leq D \leq 50$  cm. Material properties and other relationship are given below:

*Material:* aluminum alloy 2014-T6

Allowable compressive stress,

Young's modulus,

Mass density,

Strut length,

Critical buckling load,

Moment of inertia,

Strut load,

$$\sigma_a = 150 \text{ MPa}$$

$$E = 75 \text{ GPa}$$

$$\rho = 2800 \text{ kg/m}^3$$

$$l = (H^2 + \frac{1}{3}B^2)^{0.5}$$

$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$I = \frac{\pi}{64} D^4$$

$$P = \frac{Wl}{3H}$$

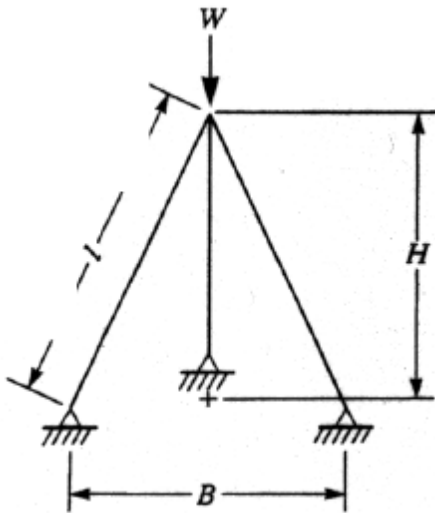


FIGURE E3.54 A tripod.

**Solution**

- (1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.
- (2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.
- (3) The answer report shows that for initial design variable values of  $H=100$  and  $D=20$  a solution of  $H=50$  and  $D=3.42$  which gives an objective function value of 6.61, is obtained.

1

	A	B	C	D	E	F	G	H	I
1	Exercise 6.15								
2	1. Design Variables	Lower limit	Symbol	Value	Upper limit	Units			
3	Tripod height	=0.5*(100)	H	50	=5*(100)	cm			
4	Strut cross-section diameter	0.5	D	3.42322785703146	50	cm			
5									
6	2. Parameters	Symbol	Value	Units					
7	Vertical load	W	=60*1000	N					
8	Tripod sides	B	=1200*(100/1000)	cm					
9	Factor of Safety	FS	2						
10	Allowable compressive stress	$\sigma_a$ , sigma_a	=1.5*10^4	N/cm^2					
11	Young's modulus	E	=7.5*10^6	N/cm^2					
12	Mass density	$\rho$ , ro	=2.8*10^-3	kg/cm^3					
13									
14	3. Analysis variables	Symbol	Equation	Units					
15	Cross-sectional area	A	=PI()*D^2/4	cm^2					
16	Strut length	L	=((H^2+(1/3)*B^2)^(0.5))	cm					
17	Moment of inertia	I	=PI()*D^4/64	cm^4					
18	Critical buckling load	Pcr	=((PI()*2*E*I)/(L^2))	N					
19	Strut load	P	=(W*I)/(3*H)	N					
20									
21	4. Objective function	Symbol	Equation	Units					
22	Minimize mass	f	=3*(ro*A*L)	kg					
23									
24	5. Constraints	Value	</>=		RS				
25	g1 (allowable compressive stress constraint)	=P/A/sigma_a	<		=sigma_a/sigma_a				
26	g2 (buckling load constraint)	=(2*P)/Pcr	<		=Pcr/Pcr				

Solver Parameters

Set Target Cell:

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$25:\$B\$26 <= \$D\$25:\$D\$26

\$B\$3:\$B\$4 <= \$D\$3:\$D\$4

\$D\$3:\$D\$4 <= \$E\$3:\$E\$4

2

3

	A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report				
2	Worksheet: [Exercise 6.15.xlsx]Sheet1				
3	Report Created: 7/2/2010 12:17:47 PM				
4					
5					
6	Target Cell (Min)				
7	Cell	Name	Original Value	Final Value	
8	\$C\$22	f Equation	321.0406429	6.605447007	
9					
10					
11	Adjustable Cells				
12	Cell	Name	Original Value	Final Value	
13	\$D\$3	H	100	50	
14	\$D\$4	D	20	3.423227857	
15					

6.16

Solve the following NLP problem using the Excel Solver:

Solve the spring design problem for the following data: Applied load (P) = 20 lb.

**TABLE 2.2** Information to design a coil spring

<i>Notation</i>	<i>Data</i>
Deflection along spring axis	$\delta$ , in
Mean Coil Diameter	D, in
Wire Diameter	d, in
Number of active coils	N
Gravitational constant	$g = 386 \text{ in/s}^2$
Frequency of surge waves	$\omega$ , Hz
Weight Density of spring material	$\gamma = 0.285 \text{ lb/in}^3$
Shear Modulus	$G = (1.15 \times 10^7) \text{ lb/in}^2$
Mass density of material ( $\rho = \gamma/g$ )	$\rho = (7.38342 \times 10^{-4}) \text{ lb-s}^2/\text{in}^4$
Allowable shear stress	$\tau_a = 80,000 \text{ lb/in}^2$
Number of inactive coils	Q = 2
Applied load	P = 20 lb
Minimum spring deflection	$\Delta = 0.5 \text{ in}$
Lower limit on surge wave frequency	$\omega_o = 100 \text{ Hz}$
Limit on outer diameter of coil	$D_o = 1.5 \text{ in}$

Load deflection equation:  $P = K\delta$

Spring Constant:  $K = \frac{d^4 G}{8D^3 N}$

Shear Stress:  $\tau = \frac{8kPD}{\pi d^3}$

Wahl stress concentration factor:  $k = \frac{(4D - d)}{4(D - d)} + \frac{0.615d}{D}$

Frequency of surge waves:  $\omega = \frac{d}{2\pi ND^2} \sqrt{\frac{G}{2\rho}}$

Design variables for the problem are defined as below:

d = wire diameter, in

D = mean coil diameter, in

N = number of active coils, integer

The problem to minimize the mass of the spring, given as volume\*mass density is:

$$Mass = \left(\frac{\pi d^2}{4}\right)[(N+Q)\pi D]\rho = \frac{\pi d^2 (N+Q)\pi D \rho}{4}$$

The constraints for the spring design problem are formulated as

$$\frac{P}{K} \geq \Delta, \tau \leq \tau_a, \omega \geq \omega_0, D+d \leq D_0, d_{\min} \leq d \leq d_{\max}, D_{\min} \leq D \leq D_{\max}, N_{\min} \leq N \leq N_{\max}$$

### Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below.

The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of  $d=0.2$ ,  $D_{\text{coil}}=1.3$ , and  $N=2$  a solution of  $d=0.0705$ ,  $D_{\text{coil}}=0.444$ , and  $N=10.16$  which gives an objective function value of 0.0268, is obtained.

	A	B	C	D	E	F	G	H
1	Design of Coil Springs							
2	1. Design Variables	Lower limit	Symbol	Value	Upper limit			
3	Wire diameter	=0.05	$d$ , in	0.2	=0.2			
4	Mean coil diameter	=0.25	$D$ , in	1.3	=1.3			
5	Number of active coils	=2	$N$	2	20			
6								
7	2. Parameters	Symbol	Value	Units				
8	Shear modulus	$G$	=11500000	lb/in <sup>2</sup>				
9	Mass density	$\rho$ , ro	=7.38342*10 <sup>-4</sup>	lb-s <sup>2</sup> /in <sup>4</sup>				
10	Allowable shear stress	$\tau_a$ , tau_a	=80000	lb/in <sup>2</sup>				
11	Number of inactive coils	$Q$	=2					
12	Applied load	$P$	=20	lb				
13	Minimum spring deflection	$\Delta$ , Def_min	=0.5	in				
14	Lower limit on surge wave frequency	$\omega_0$ , omega_0	=100	Hz				
15	Limit on outer diameter of the coil	$D_0$	=1.5	in				
16								
17	3. Analysis Variables	Symbol	Equation	Units				
18	Load deflection equation	$\delta$ , Def	=P/K	in				
19	Spring Constant	$K$	=(d <sup>4</sup> *G)/(8*D_coil <sup>3</sup> *N)	lb/in				
20	Shear Stress	$\tau$ , tau	=(8*k_CF*P*D_coil)/(Pi()*d <sup>3</sup> )	lb/in <sup>2</sup>				
21	Wahl stress concentration factor	$k$ , k_CF	=(4*D_coil-d)/(4*(D_coil-d))+(0.615*d)/(D_coil)					
22	Frequency of surge waves	$\omega$ , omega	=(d/(2*Pi()*N*D_coil <sup>2</sup> ))*SQRT(G/(2*ro))	Hz				
23								
24	4. Objective Function	Symbol	Equation	Units				
25	Minimize f	$f$	=(N+2)*D_coil*d <sup>2</sup>					
26	Mass	$M$	=f*ro*Pi()*d <sup>2</sup> /4	lb-s <sup>2</sup> /in				
27								
28	5. Constraints	Value	</>=	RS				
29	Deflection constraint	=1-P/(K*Def_min)	<	0				
30	Shear stress constraint	=tau/tau_a-1	<	0				
31	Frequency constraint	=1-omega/omega_0	<	0				
32	Outer diameter constraint	=(D_coil+d)/(D_0)-1	<	0				
33								
34								

**Solver Parameters**

Set Target Cell:  To: ☐ Max ☒ Min ☐ Value of: 0

By Changing Variable Cells:

Subject to the Constraints:

- Add
- Add
- Add

Change Delete Reset All Help

**Solver Results**

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports: Answer Sensitivity Limits

☒ Keep Solver Solution ☐ Restore Original Values

OK Cancel Save Scenario... Help

Microsoft Excel 12.0 Answer Report

Worksheet: [Exercise 6.16.xlsx]Sheet1

Report Created: 6/17/2010 1:22:09 PM

Target Cell (Min)	Cell	Name	Original Value	Final Value
\$D\$5 f			0.208	0.026777982

Adjustable Cells	Cell	Name	Original Value	Final Value
\$D\$3 d			0.2	0.07047271
\$D\$4 D_coil			1.3	0.443594147
\$D\$5 N			2	10.15486337



6.17

Solve the following NLP problem using the Excel Solver:

Solve the spring design problem for the following data: Number of active coils (N) = 20, limit on outer diameter of the coil ( $D_o$ ) = 1 in, number of inactive coils (Q) = 4.

**TABLE 2.2** Information to design a coil spring

<i>Notation</i>	<i>Data</i>
Deflection along spring axis	$\delta$ , in
Mean Coil Diameter	D, in
Wire Diameter	d, in
Number of active coils	N
Gravitational constant	$g = 386 \text{ in/s}^2$
Frequency of surge waves	$\omega$ , Hz
Weight Density of spring material	$\gamma = 0.285 \text{ lb/in}^3$
Shear Modulus	$G = (1.15 \times 10^7) \text{ lb/in}^2$
Mass density of material ( $\rho = \gamma/g$ )	$\rho = (7.38342 \times 10^{-4}) \text{ lb-s}^2/\text{in}^4$
Allowable shear stress	$\tau_a = 80,000 \text{ lb/in}^2$
Number of inactive coils	Q = 4
Applied load	P = 20 lb
Minimum spring deflection	$\Delta = 0.5 \text{ in}$
Lower limit on surge wave frequency	$\omega_o = 100 \text{ Hz}$
Limit on outer diameter of coil	$D_o = 1 \text{ in}$

Load deflection equation:  $P = K\delta$

Spring Constant:  $K = \frac{d^4 G}{8D^3 N}$

Shear Stress:  $\tau = \frac{8kPD}{\pi d^3}$

Wahl stress concentration factor:  $k = \frac{(4D - d)}{4(D - d)} + \frac{0.615d}{D}$

Frequency of surge waves:  $\omega = \frac{d}{2\pi ND^2} \sqrt{\frac{G}{2\rho}}$

Design variables for the problem are defined as below:

d = wire diameter, in

D = mean coil diameter, in

N = number of active coils, integer

The problem to minimize the mass of the spring, given as volume\*mass density is:

$$Mass = \left(\frac{\pi d^2}{4}\right)[(N+Q)\pi D]\rho = \frac{\pi d^2 (N+Q)\pi D \rho}{4}$$

The constraints for the spring design problem are formulated as

$$\frac{P}{K} \geq \Delta, \tau \leq \tau_a, \omega \geq \omega_0, D+d \leq D_0, d_{\min} \leq d \leq d_{\max}, D_{\min} \leq D \leq D_{\max}, N_{\min} \leq N \leq N_{\max}$$

### Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below.

The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of d=0.2 and D\_coil=1.3 a solution of d=0.05 and D\_coil=0.282 which gives an objective function value of 0.0155, is obtained.

	A	B	C	D	E	F	G	H
1	<b>Design of Coil Springs</b>							
2	<b>1. Design Variables</b>		<b>Lower limit</b>	<b>Symbol</b>	<b>Value</b>	<b>Upper limit</b>		
3	Wire diameter	=0.05	d, in	0.2	=0.2			
4	Mean coil diameter	=0.25	D, in	1.3	=1.3			
5	Number of active coils	N/A	N	20	N/A			
6								
7	<b>2. Parameters</b>		<b>Symbol</b>	<b>Value</b>	<b>Units</b>			
8	Shear modulus	G	=11500000		lb/in <sup>2</sup>			
9	Mass density	ρ, ro	=7.38342*10 <sup>-4</sup>		lb-s <sup>2</sup> /in <sup>4</sup>			
10	Allowable shear stress	τ <sub>a</sub> , tau_a	=80000		lb/in <sup>2</sup>			
11	Number of inactive coils	Q	=4					
12	Applied load	P	=10		lb			
13	Minimum spring deflection	Δ, Def_min	=0.5		in			
14	Lower limit on surge wave frequency	ω <sub>0</sub> , omega_0	=100		Hz			
15	Limit on outer diameter of the coil	D <sub>0</sub>	=1		in			
16								
17	<b>3. Analysis Variables</b>		<b>Symbol</b>	<b>Equation</b>	<b>Units</b>			
18	Load deflection equation	δ, Def	=P/K		in			
19	Spring Constant	K	=(d <sup>4</sup> *G)/(8*D_coil <sup>3</sup> *N)		lb/in			
20	Shear Stress	τ, tau	=(8*k_CF*P*D_coil)/(Pi()*d <sup>3</sup> )		lb/in <sup>2</sup>			
21	Wahl stress concentration factor	k, k_CF	=(4*D_coil-d)/(4*(D_coil-d))+0.615*d/(D_coil)					
22	Frequency of surge waves	ω, omega	=((d)/(2*Pi()*N*D_coil <sup>2</sup> ))*SQRT(G/(2*ro))		Hz			
23								
24	<b>4. Objective Function</b>		<b>Symbol</b>	<b>Equation</b>	<b>Units</b>			
25	Minimize f	f	=(N+2)*D_coil*d <sup>2</sup>					
26	Mass	M	=f*ro*Pi()*2/4		lb-s <sup>2</sup> /in			
27								
28	<b>5. Constraints</b>		<b>Value</b>	<b>&lt;/&gt;=</b>	<b>RS</b>			
29	Deflection constraint	=1-P/(K*Def_min)	<	0				
30	Shear stress constraint	=tau/tau_a-1	<	0				
31	Frequency constraint	=1-omega/omega_0	<	0				
32	Outer diameter constraint	=(D_coil+d)/(D_0)-1	<	0				
33								

**Solver Parameters**

Set Target Cell:

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Cells:

Subject to the Constraints:

\$B\$29:\$B\$32 <= \$D\$29:\$D\$32  
\$B\$33:\$B\$34 <= \$D\$33:\$D\$34  
\$D\$33:\$D\$34 <= \$E\$33:\$E\$34

**Solver Results**

Solver found a solution. All constraints and optimality conditions are satisfied.

☒ Keep Solver Solution ☐ Restore Original Values

Reports: ☒ Answer ☐ Sensitivity ☐ Limits

1 Microsoft Excel 12.0 Answer Report

2 Worksheet: [Exercise 6.17.xlsx]Sheet1

3 Report Created: 6/17/2010 1:41:30 PM

4

5

6 Target Cell (Min)

Cell	Name	Original Value	Final Value
\$C\$25	f	1.144	0.015518106

7

8

9

10

11 Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$3	d	0.2	0.05
\$D\$4	D_coil	1.3	0.28214739

12

13

14

15

6.18

Solve the following NLP problem using the Excel Solver:

Solve the spring design problem for the following data: Aluminum coil with shear modulus ( $G$ ) = 4,000,000 psi, mass density ( $\rho$ ) =  $2.58920 \times 10^{-4}$  lb-s<sup>2</sup>/in<sup>4</sup>, and allowable shear stress ( $\tau_a$ ) = 50,000 lb/in<sup>2</sup>.

**TABLE 2.2** Information to design a coil spring

<i>Notation</i>	<i>Data</i>
Deflection along spring axis	$\delta$ , in
Mean Coil Diameter	$D$ , in
Wire Diameter	$d$ , in
Number of active coils	$N$
Gravitational constant	$g = 386$ in/s <sup>2</sup>
Frequency of surge waves	$\omega$ , Hz
Weight Density of spring material	$\gamma = 0.285$ lb/in <sup>3</sup>
Shear Modulus	$G = (4 \times 10^6)$ lb/in <sup>2</sup>
Mass density of material ( $\rho = \gamma/g$ )	$\rho = 2.58920 \times 10^{-4}$ lb-s <sup>2</sup> /in <sup>4</sup>
Allowable shear stress	$\tau_a = 50,000$ lb/in <sup>2</sup>
Number of inactive coils	$Q = 2$
Applied load	$P = 20$ lb
Minimum spring deflection	$\Delta = 0.5$ in
Lower limit on surge wave frequency	$\omega_o = 100$ Hz
Limit on outer diameter of coil	$D_o = 1.5$ in

Load deflection equation:

$$P = K\delta$$

Spring Constant:

$$K = \frac{d^4 G}{8D^3 N}$$

Shear Stress:

$$\tau = \frac{8kPD}{\pi d^3}$$

Wahl stress concentration factor:

$$k = \frac{(4D - d)}{4(D - d)} + \frac{0.615d}{D}$$

Frequency of surge waves:

$$\omega = \frac{d}{2\pi ND^2} \sqrt{\frac{G}{2\rho}}$$

Design variables for the problem are defined as below:

$d$  = wire diameter, in

$D$  = mean coil diameter, in

$N$  = number of active coils, integer

The problem function to minimize the mass of the spring, given as volume\*mass density is:

$$Mass = \left(\frac{\pi d^2}{4}\right)[(N+Q)\pi D]\rho = \frac{\pi d^2 (N+Q)\pi D \rho}{4}$$

The constraints for the spring design problem are formulated as

$$\frac{P}{K} \geq \Delta, \tau \leq \tau_a, \omega \geq \omega_0, D+d \leq D_0, d_{\min} \leq d \leq d_{\max}, D_{\min} \leq D \leq D_{\max}, N_{\min} \leq N \leq N_{\max}$$

### Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below.

The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of  $d=0.2$ ,  $D_{\text{coil}}=1.3$ , and  $N=2$  a solution of  $d=0.0601$ ,  $D_{\text{coil}}=0.334$ , and  $N=8.74$  which gives an objective function value of 0.0130, is obtained.

The screenshot displays an Excel worksheet for the spring design optimization problem. The worksheet is organized into several sections:

- Design of Coil Springs** (Rows 1-6):
  - 1. Design Variables** (Rows 2-5): A table with columns for Lower limit, Symbol, Value, and Upper limit. Variables include Wire diameter ( $d$ ), Mean coil diameter ( $D$ ), and Number of active coils ( $N$ ).
  - 2. Parameters** (Rows 7-15): A table with columns for Symbol, Value, and Units. Parameters include Shear modulus ( $G$ ), Mass density ( $\rho$ ), Allowable shear stress ( $\tau_a$ ), Number of inactive coils ( $Q$ ), Applied load ( $P$ ), Minimum spring deflection ( $\Delta$ ), Lower limit on surge wave frequency ( $\omega_0$ ), and Limit on outer diameter of the coil ( $D_0$ ).
  - 3. Analysis Variables** (Rows 17-22): A table with columns for Symbol, Equation, and Units. Analysis variables include Load deflection equation ( $\delta$ ), Spring Constant ( $K$ ), Shear Stress ( $\tau$ ), Wahl stress concentration factor ( $k$ ), and Frequency of surge waves ( $\omega$ ).
  - 4. Objective Function** (Rows 24-26): A table with columns for Symbol, Equation, and Units. The objective is to Minimize  $f$  (Mass).
  - 5. Constraints** (Rows 28-32): A table with columns for Value, Comparison Operator, and RS. Constraints include Deflection constraint, Shear stress constraint, Frequency constraint, and Outer diameter constraint.
- Solver Parameters** (Row 6): A dialog box showing the Set Target Cell as  $\$D\$5$ , Equal To: Min, and By Changing Variable Cells as  $\$D\$3:\$D\$5$ . Constraints are listed as  $\$B\$29:\$B\$32 \leq \$D\$29:\$D\$32$ ,  $\$B\$3:\$B\$5 \leq \$D\$3:\$D\$5$ , and  $\$D\$3:\$D\$5 \leq \$E\$3:\$E\$5$ .
- Solver Results** (Row 26): A dialog box showing the Solver found a solution. The Reports section is set to "Keep Solver Solution".
- Answer Report** (Row 6): A table showing the Target Cell (Min) and Adjustable Cells. The Target Cell is  $\$C\$25$  with a value of 0.012955433. The Adjustable Cells are  $\$D\$3$  (d),  $\$D\$4$  (D\_coil), and  $\$D\$5$  (N).

## Section 6.8 Optimum Design of Plate Girders Using Excel Solver

Solve the following problems using Excel Solver:

6.19

Solve the following problem using the Excel Solver:

Solve the plate girder design problem for the following data: Span length (L) = 35 ft.

**TABLE E6-19**

Notation	Data
L	span, 25 m
E	modulus of elasticity, 210 GPa
$\sigma_y$	yield stress, 262 MPa
$\sigma_a$	allowable bending stress, $0.55 \sigma_y = 144.1$ MPa
$\tau_a$	allowable shear stress, $0.33 \sigma_y = 86.46$ MPa
$\sigma_t$	allowable fatigue stress 255 MPa
$D_a$	allowable deflection, $L/800$ , m
$P_m$	concentrated load for moment, 104 kN
$P_s$	concentrated load for shear, 155 kN
LLIF	live load impact factor, $1+50/(L+125)$

Cross-sectional area:

$$A = (ht_w + 2bt_f)$$

Moment of inertia:

$$I = \frac{t_w h^3}{12} + \frac{2bt_f^3}{3} + \frac{bt_f h(h + 2t_f)}{2}$$

Uniform load for the girder:

$$w = (19 + 77A)$$

Bending moment:

$$M = \frac{L(2P_m + wL)}{8}$$

Bending Stress:

$$\sigma = \frac{M(0.5h + t_f)}{1000I}$$

Flange buckling stress limit:

$$\sigma_f = 72,845 \left[ \frac{t_w}{b} \right]^2$$

Web crippling stress limit:

$$\sigma_w = 3,648,276 \left[ \frac{t_w}{h} \right]^2$$

Shear force:

$$S = 0.5(P_s + wL)$$

Deflection:

$$D = \frac{L^3(8P_m + 5wL)}{384 \cdot 10^6 (EI)}$$

Average shear stress:

$$\tau = \frac{S}{1000ht_w}$$

Objective function to minimize the material volume of the girder is defined as:

$$Vol = AL = (ht_w + 2bt_f)L$$

Design variables for the plate girder optimization problem are defined as:

h = web height, m

b = flange weight, m

$t_f$  = flange thickness, m

$t_w$  = web thickness, m

The constraints for the spring design problem are formulated as

$$\sigma \leq \sigma_a, \sigma \leq \sigma_f, \sigma \leq \sigma_w, \tau \leq \tau_a, D \leq D_a, \sigma \leq \frac{\sigma_t}{2},$$

$$0.3 \leq h \leq 2.5, 0.3 \leq b \leq 2.5, 0.01 \leq t_f \leq 0.1, 0.01 \leq t_w \leq 0.1$$

### Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below.

The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of  $h=1$ ,  $b=1$ ,  $t_f=0.1$ , and  $t_w=0.1$  a solution of  $h=2.5$ ,  $b=0.3$ ,  $t_f=0.045$ , and  $t_w=0.013$ , which gives an objective function value of 2.067, is obtained.

	A	B	C	D	E	F	G	H	
1	Plate Girder Design								
2	1. Design variable name	Lower limit	Symbol	Value	Upper limit	Units			
3	web height	0.3	h	1	2.5	m			
4	flange width	0.3	b	1	2.5	m			
5	flange thickness	0.01	$t_f$	0.1	0.1	m			
6	web thickness	0.01	$t_w$	0.1	0.1	m			
7									
8	2. Parameter name	Symbol	Value	Units					
9	Span length	L	35	m					
10	Modulus of elasticity	E	210	Gpa					
11	Yield stress	$\sigma_y$	262	Mpa					
12	Allowable fatigue stress	$\sigma_a$	255	MPa					
13	Concentrated load for moment	Pm	104	kN					
14	Concentrated load for shear	Ps	155	kN					
15	Live load impact factor	LLIF	=1+50/(L+125)	none					
16									
17	3. Dependent variable name	Symbol	Equation	Units					
18	Cross sectional area	A	=h*tw+2*b*tf	m^2					
19	Moment of inertia	I	=(1/12)*tw*h^3+(2/3)*b*tf^3+(1/2)*b*tf*h*(h+2*tf)	m^4					
20	Uniform load	w	=19+77*A	kN/m					
21	Bending moment	M	=L*(2*Pm+w*L)/8	kN-m					
22	Bending stress	$\sigma$	=M*(h/2+tf)/(1000*I)	Mpa					
23	Shear force	S	=(Ps+w*L)/2	kN					
24	Deflection	D	=L^3*(8*Pm+5*w*L)/(384*E*(1000000))	m					
25	Average shear stress	$\tau$	=S/(1000*h*tw)	MPa					
26									
27	4. Objective function name	Symbol	Equation	Units					
28	Volume of material	Vol	=A*L	m^3					
29									
30	5. Constraints	Value/Eq.	</>=	Value/Eq.	Name				
31	Bending stress	= $\sigma$	<	=0.55* $\sigma_y$	Allowable bending stress				
32	Bending stress	= $\sigma$	<	=72845*(tf/b)^2	Flange buckling limit				
33	Bending stress	= $\sigma$	<	=3648276*(tw/h)^2	Web crippling limit				
34	Shear stress	= $\tau$	<	=0.33* $\sigma_y$	Allowable shear stress				
35	Deflection	=D	<	=L/800	Allowable deflection				
36	Bending stress	= $\sigma$	<	= $\sigma_a/2$	Allowable fatigue stress				
37									

**Solver Parameters**

Set Target Cell:

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Variable Cells:

Subject to the Constraints:

**Solver Results**

Solver found a solution. All constraints and optimality conditions are satisfied.

☒ Keep Solver Solution ☐ Restore Original Values

Reports: ☒ Answer ☐ Sensitivity ☐ Limits

6 Target Cell (Min)

Cell	Name	Original Value	Final Value
\$C\$28	Vol	10.5	2.066791187

11 Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$3	h	1	2.5
\$D\$4	b	1	0.3
\$D\$5	$t_f$	0.1	0.044670412
\$D\$6	$t_w$	0.1	0.012899572

6.20

Solve the following problem using the Excel Solver:

Solve the plate girder design problem for the following data: A36 steel with modulus of elasticity (E) = 200 GPa, yield stress ( $\sigma_y$ ) = 250 MPa, allowable fatigue stress ( $\sigma_t$ ) = 243 MPa.

**TABLE E6-20**

<i>Notation</i>	<i>Data</i>
L	span, 25 m
E	modulus of elasticity, 200 GPa
$\sigma_y$	yield stress, 250 MPa
$\sigma_a$	allowable bending stress, $0.55 \sigma_y = 144.1$ MPa
$\tau_a$	allowable shear stress, $0.33 \sigma_y = 86.46$ MPa
$\sigma_t$	allowable fatigue stress 243 MPa
$D_a$	allowable deflection, $L/800$ , m
$P_m$	concentrated load for moment, 104 kN
$P_s$	concentrated load for shear, 155 kN
LLIF	live load impact factor, $1+50/(L+125)$

Cross-sectional area:

$$A = (ht_w + 2bt_f)$$

Moment of inertia:

$$I = \frac{t_w h^3}{12} + \frac{2bt_f^3}{3} + \frac{bt_f h(h + 2t_f)}{2}$$

Uniform load for the girder:

$$w = (19 + 77A)$$

Bending moment:

$$M = \frac{L(2P_m + wL)}{8}$$

Bending Stress:

$$\sigma = \frac{M(0.5h + t_f)}{1000I}$$

Flange buckling stress limit:

$$\sigma_f = 72,845 \left[ \frac{t_w}{b} \right]^2$$

Web crippling stress limit:

$$\sigma_w = 3,648,276 \left[ \frac{t_w}{h} \right]^2$$

Shear force:

$$S = 0.5(P_s + wL)$$

Deflection:

$$D = \frac{L^3(8P_m + 5wL)}{384 \cdot 10^6 (EI)}$$

Average shear stress:

$$\tau = \frac{S}{1000ht_w}$$

Objective function to minimize the material volume of the girder is defined as:

$$Vol = AL = (ht_w + 2bt_f)L$$

Design variables for the plate girder optimization problem are defined as:

h = web height, m

b = flange weight, m

$t_f$  = flange thickness, m

$t_w$  = web thickness, m

The constraints for the spring design problem are formulated as

$$\sigma \leq \sigma_a, \sigma \leq \sigma_f, \sigma \leq \sigma_w, \tau \leq \tau_a, D \leq D_a, \sigma \leq \frac{\sigma_t}{2},$$

$$0.3 \leq h \leq 2.5, 0.3 \leq b \leq 2.5, 0.01 \leq t_f \leq 0.1, 0.01 \leq t_w \leq 0.1$$

### Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below.

The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of  $h=1$ ,  $b=1$ ,  $t_f=0.1$ , and  $t_w=0.1$  a solution of  $h=2.11$ ,  $b=0.403$ ,  $t_f=0.0156$ , and  $t_w=0.0115$ , which gives an objective function value of 0.921, is obtained.

	A	B	C	D	E	F	G	H
1	Plate Girder Design							
2	1. Design variable name	Lower limit	Symbol	Value	Upper limit	Units		
3	web height	0.3	h	1	2.5	m		
4	flange width	0.3	b	1	2.5	m		
5	flange thickness	0.01	tf	0.1	0.1	m		
6	web thickness	0.01	tw	0.1	0.1	m		
7								
8	2. Parameter name	Symbol	Value	Units				
9	Span length	L	25	m				
10	Modulus of elasticity	E	200	Gpa				
11	Yield stress	sigma_y	250	Mpa				
12	Allowable fatigue stress	sigma_t	243	MPa				
13	Concentrated load for moment	Pm	104	kN				
14	Concentrated load for shear	Ps	155	kN				
15	Live load impact factor	LLIF	=1+50/(L+125)	none				
16								
17	3. Dependent variable name	Symbol	Equation	Units				
18	Cross sectional area	A	=h*tw+2*b*tf	m^2				
19	Moment of inertia	I	=(1/12)*tw*h^3+(2/3)*b*tf^3+(1/2)*b*tf*h*(h+2*tf)	m^4				
20	Uniform load	w	=19+77*A	kN/m				
21	Bending moment	M	=L*(2*Pm+w*L)/8	kN-m				
22	Bending stress	sigma	=M*(h/2+tf)/(1000*I)	Mpa				
23	Shear force	S	=(Ps+w*L)/2	kN				
24	Deflection	D	=L^3*(8*Pm+5*w*L)/(384*E*I*1000000)	m				
25	Average shear stress	tau	=S/(1000*h*tw)	MPa				
26								
27	4. Objective function name	Symbol	Equation	Units				
28	Volume of material	Vol	=A*L	m^3				
29								
30	5. Constraints	Value/Eq.	</>=	Value/Eq.	Name			
31	Bending stress	=sigma	<	=0.55*sigma_y	Allowable bending stress			
32	Bending stress	=sigma	<	=72845*(tf/b)^2	Flange buckling limit			
33	Bending stress	=sigma	<	=3648276*(tw/h)^2	Web crippling limit			
34	Shear stress	=tau	<	=0.33*sigma_y	Allowable shear stress			
35	Deflection	=D	<	=L/800	Allowable deflection			
36	Bending stress	=sigma	<	=sigma_t/2	Allowable fatigue stress			

**Solver Parameters**

Set Target Cell:

Equal To: ☐ Max ☒ Min ☐ Value of: 0

By Changing Variable Cells:

Subject to the Constraints:

**Solver Results**

Solver Found a solution. All constraints and optimality conditions are satisfied.

Reports: ☒ Answer ☐ Sensitivity ☐ Limits

☒ Keep Solver Solution ☐ Restore Original Values

5	Target Cell (Min)		
6	Cell	Name	Original Value
7	\$C\$28	Vol	7.5
8			0.921359028
9			
10			
11	Adjustable Cells		
12	Cell	Name	Original Value
13	\$D\$3	h	1
14	\$D\$4	b	1
15	\$D\$5	tf	0.1
16	\$D\$6	tw	0.1
17			0.011525119



6.21

Solve the following problem using the Excel Solver:

Solve the plate girder design problem for the following data: Web height (h) = 1.5 m, flange thickness (tf) = 0.015 m.

**TABLE E6-21**

<i>Notation</i>	<i>Data</i>
L	span, 25 m
E	modulus of elasticity, 200 GPa
$\sigma_y$	yield stress, 250 MPa
$\sigma_a$	allowable bending stress, $0.55 \sigma_y = 144.1$ MPa
$\tau_a$	allowable shear stress, $0.33 \sigma_y = 86.46$ MPa
$\sigma_t$	allowable fatigue stress 243 MPa
$D_a$	allowable deflection, $L/800$ , m
$P_m$	concentrated load for moment, 104 kN
$P_s$	concentrated load for shear, 155 kN
LLIF	live load impact factor, $1+50/(L+125)$

$$\begin{aligned}
 \text{Cross-sectional area:} \quad & A = (ht_w + 2bt_f) \\
 \text{Moment of inertia:} \quad & I = \frac{t_w h^3}{12} + \frac{2bt_f^3}{3} + \frac{bt_f h(h + 2t_f)}{2} \\
 \text{Uniform load for the girder:} \quad & w = (19 + 77A) \\
 \text{Bending moment:} \quad & M = \frac{L(2P_m + wL)}{8} \\
 \text{Bending Stress:} \quad & \sigma = \frac{M(0.5h + t_f)}{1000I} \\
 \text{Flange buckling stress limit:} \quad & \sigma_f = 72,845 \left[ \frac{t_w}{b} \right]^2 \\
 \text{Web crippling stress limit:} \quad & \sigma_w = 3,648,276 \left[ \frac{t_w}{h} \right]^2 \\
 \text{Shear force:} \quad & S = 0.5(P_s + wL) \\
 \text{Deflection:} \quad & D = \frac{L^3(8P_m + 5wL)}{384 \cdot 10^6 (EI)} \\
 \text{Average shear stress:} \quad & \tau = \frac{S}{1000ht_w}
 \end{aligned}$$

Objective function to minimize the material volume of the girder is defined as:

$$Vol = AL = (ht_w + 2bt_f)L$$

Design variables for the plate girder optimization problem are defined as:

h = web height, m

b = flange width, m

tf = flange thickness, m

$t_w$  = web thickness, m

The constraints for the spring design problem are formulated as

$$\sigma \leq \sigma_a, \sigma \leq \sigma_f, \sigma \leq \sigma_w, \tau \leq \tau_a, D \leq D_a, \sigma \leq \frac{\sigma_t}{2},$$

$$0.3 \leq h \leq 2.5, 0.3 \leq b \leq 2.5, 0.01 \leq t_f \leq 0.1, 0.01 \leq t_w \leq 0.1$$

### Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below.

The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of  $b=1$  and  $t_w=0.1$  a solution of  $b=0.4503$  and  $t_w=0.0675$ , which gives an objective function value of 2.87, is obtained.

	A	B	C	D	E	F	G	H
1	Plate Girder Design							
2	1. Design variable name	Lower limit	Symbol	Value	Upper limit	Units		
3	web height	N/A	h	1.5	N/A	m		
4	flange thickness	N/A	tf	0.015	N/A	m		
5	flange width	0.3	b	1	2.5	m		
6	web thickness	0.01	tw	0.1	0.1	m		
7								
8	2. Parameter name	Symbol	Value	Units				
9	Span length	L	25	m				
10	Modulus of elasticity	E	210	Gpa				
11	Yield stress	sigma_y	262	Mpa				
12	Allowable fatigue stress	sigma_t	255	MPa				
13	Concentrated load for moment	Pm	104	kN				
14	Concentrated load for shear	Ps	155	kN				
15	Live load impact factor	LLIF	=1+50/(L+125)	none				
16								
17	3. Dependent variable name	Symbol	Equation	Units				
18	Cross sectional area	A	=h*tw+2*b*tf	m^2				
19	Moment of inertia	I	=(1/12)*tw*h^3+(2/3)*b*tf^3+(1/2)*b*tf*h*(h+2*tf)	m^4				
20	Uniform load	w	=19+77*A	kN/m				
21	Bending moment	M	=L*(2*Pm+w*L)/8	kN-m				
22	Bending stress	sigma	=M*(h/2+tf)/(1000*I)	Mpa				
23	Shear force	S	=(Ps+w*L)/2	kN				
24	Deflection	D	=L^3*(8*Pm+5*w*L)/(384*E*I*1000000)	m				
25	Average shear stress	tau	=S/(1000*h*tw)	MPa				
26								
27	4. Objective function name	Symbol	Equation	Units				
28	Volume of material	Vol	=A*L	m^3				
29								
30	5. Constraints	Value/Eq.	<=>=	Value/Eq.	Name			
31	Bending stress	=sigma	<	=0.55*sigma_y	Allowable bending stress			
32	Bending stress	=sigma	<	=72845*(tf/b)^2	Flange buckling limit			
33	Bending stress	=sigma	<	=3648276*(tw/h)^2	Web crippling limit			
34	Shear stress	=tau	<	=0.33*sigma_y	Allowable shear stress			
35	Deflection	=D	<	=L/800	Allowable deflection			
36	Bending stress	=sigma	<	=sigma_t/2	Allowable fatigue stress			
37								

**Solver Parameters**

Set Target Cell:  To: ☐ Max ☒ Min ☐ Value of: 0

By Changing Variable Cells:

Subject to the Constraints:

**Solver Results**

Solver found a solution. All constraints and optimality conditions are satisfied.

☒ Keep Solver Solution ☐ Restore Original Values

Reports: ☒ Answer ☐ Sensitivity ☐ Limits

6 Target Cell (Min)

Cell	Name	Original Value	Final Value
\$C\$28	Vol	4.5	2.868430337

11 Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$5	b	1	0.450296731
\$D\$6	tw	0.1	0.067485541