C H A P T E R

13

More on Numerical Methods for Constrained Optimum Design

Section 13.3 Approximate Step Size Determination

13.1 ———————————————————————————————————————

Refer to Exercise 12.3 for detailed formulation.

Iteration 1:

1. Initial design is given as $(b^{(0)}, d^{(0)}) = (250, 300)$; set $R_o = 1$, $\gamma = 0.5$, ε_1, $\varepsilon_2 = 0.001$
2. Compute cost and constraint functions:

$f_o = 75000$, $g_1 = 0.0667$, $g_2 = 0.5$, $g_3 = -0.4$, $g_4 = -24$, $g_5 = -0.75$, $g_6 = -29$, $g_7 = -0.7$;

$\nabla f = (300, 250)$

$\nabla g_1 = (-0.004267, -0.007111)$

$\nabla g_2 = (-0.006, -0.005)$;

$\nabla g_3 = (-0.0024, 0.002)$

$\nabla g_4 = (-0.1, 0)$

$\nabla g_5 = (0.001, 0)$

$\nabla g_6 = (0, -0.1)$;

$\nabla g_7 = (0, 0.001)$

$V_o = \max\{0, 0.06667, 0.5, -0.4, -24, -0.75, -29, -0.7\} = 0.5$

3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (49.18033, 40.9836)$ with Lagrange multipliers as $\mathbf{u} = (0, 58196.7, 0, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 58196.7$; $R = \max(R_o, r_o) = 58196.7$
6. Step size at the 4th trial ($\alpha_o = 0.125$) satisfies the descent condition. Design is updated as $b^{(1)} = 256.14754$, $d^{(1)} = 305.12295$
7. $R_1 = 58196.7$, k = 1, go to Step 2.

13.2

Refer to Exercise 12.4 for detailed formulation.

Iteration 1:

1. Initial design is given as $(R^{(0)}, t^{(0)}) = (12, 4)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions:
   $f_o = 1183.752$, $g_1 = -0.99337$, $g_2 = -89.01498$, $g_3 = -1.4$, $g_4 = -0.88$, $g_5 = -7$, $g_6 = -0.2$;
   $\nabla f = (98.646, 295.938)$
   $\nabla g_1 = (-0.0005526, -0.001658)$
   $\nabla g_2 = (-22.50374, -22.50374)$
   $\nabla g_3 = (-0.2, 0)$
   $\nabla g_4 = (0.01, 0)$
   $\nabla g_5 = (0, -2)$
   $\nabla g_6 = (0, 0.2)$;
   $V_o = \max\{0, -0.99337, -89.01498, -1.4, -0.88, -7, -0.2\} = 0$
3. QP subproblem defined using the data given in Step 2 gives the search direction as
   $\mathbf{d}^{(0)} = (-0.45556, -3.5)$ with Lagrange multipliers as $\mathbf{u} = (0, 4.4, 0, 0, 97.1, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 101.5$; $R = \max(R_o, r_o) = 101.5$
6. Step size at the lst trial ($\alpha_o = 1$) satisfies the descent condition. Design is updated as
   $R^{(1)} = 11.54444$, $t^{(1)} = 0.5$
7. $R_1 = 101.5$, k = 1, go to Step 2.

13.3

Refer to Exercise 12.5 for detailed formulation.

Iteration 1:

1. Initial design is given as $(A_1^{(0)}, A_2^{(0)}) = (150, 150)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions:
   $f_o = 13500$, $g_1 = -0.16667$, $g_2 = -0.33333$, $g_3 = -150$, $g_4 = -150$
   $\nabla f = (50, 40)$
   $\nabla g_1 = (-0.005555, 0)$
   $\nabla g_2 = (0, -0.004444)$
   $\nabla g_3 = (-1, 0)$;
   $\nabla g_4 = (0, -1)$
   $V_o = \max\{0, -0.16667, -0.33333, -150, -150\} = 0$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (-30.00306, -40)$ with Lagrange multipliers as $\mathbf{u} = (3599.8, 0, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 3599.8$; $R = \max(R_o, r_o) = 3599.8$
6. Step size at the lst trial ($\alpha_o = 1$) satisfies the descent condition. Design is updated as
   $A_1^{(1)} = 119.99694$, $A_2^{(1)} = 110$
7. $R_1 = 3599.8$, k = 1, go to Step 2.

13.4

Refer to Exercise 12.6 for detailed formulation.

Iteration 1:

1. Initial design is given as $(h^{(0)}, A^{(0)}) = (12, 4000)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions:
   $f_o = 11.2$, $g_1 = 0.3143$, $g_2 = -0.25714$, $g_3 = -2.42857$, $g_4 = -0.42857$, $g_5 = -4000$
   $\nabla f = (0.6, 0.001)$
   $\nabla g_1 = (-0.057143, -1.7143\times10^{-4})$;
   $\nabla g_2 = (0.02857, 1.857\times10^{-4})$
   $\nabla g_3 = (-0.285714, 0)$
   $\nabla g_4 = (0.047619, 0)$
   $\nabla g_5 = (0, -1)$;
   $V_o = \max\{0, 0.3143, -0.25714, -2.42857, -0.42857, -4000\} = 0.3143$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (5.49918, 0.0173)$ with Lagrange multipliers as $\mathbf{u} = (106.7, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 106.7$; $R = \max(R_o, r_o) = 106.7$
6. Step size at the lst trial ($\alpha_o = 1$) satisfies the descent condition. Design is updated as
   $h^{(1)} = 17.49918$, $A^{(1)} = 4000.0173$
7. $R_1 = 106.7$, k = 1, go to Step 2.

13.5

Refer to Exercise 12.7 for detailed formulation.

Iteration 1:

1. Initial design is given as $(R^{(0)}, H^{(0)}) = (6, 15)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = -1696.46$, $g_1 = -0.3717$, $g_2 = -0.2$, $g_3 = -0.7$, $g_4 = -15$, $g_5 = -0.25$,
   $\nabla f = (-565.487, -113.097)$; $\nabla g_1 = (0.10472, 0.04189)$; $\nabla g_2 = (-0.2, 0)$;
   $\nabla g_3 = (0.05, 0)$; $\nabla g_4 = (0, -1)$; $\nabla g_5 = (0, 0.05)$;
   $V_o = \max\{0, -0.3717, -0.2, -0.7, -15, -0.25\} = 0$
2. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (9.54975, -15)$ with Lagrange multipliers as $\mathbf{u} = (5308.8, 0, 0, 94.3, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 5403.1$; $R = \max(R_o, r_o) = 5403.1$
6. Step size at the 2nd trial ($\alpha_o = 0.5$) satisfies the descent condition. Design is updated as $R^{(1)} = 10.774875$, $H^{(1)} = 7.5$
7. $R_1 = 5403.1$, k = 1, go to Step 2.

13.6

Refer to Exercise 12.8 for detailed formulation.

Iteration 1:

1. Initial design is given as $(N^{(0)}, R^{(0)}) = (100, 2)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = -1256.64$, $g_1 = -0.3717$, $g_2 = -3$, $g_3 = -100$;
   $\nabla f = (-12.5664, -628.32)$; $\nabla g_1 = (6.2832\times10^{-3}, 0.62832)$; $\nabla g_2 = (0, -2)$; $\nabla g_3 = (-1, 0)$;
   $V_o = \max\{0, -0.3717, -3, -100\} = 0$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (6, 0.53158)$ with Lagrange multipliers as $\mathbf{u} = (999.2, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 999.2$; $R = \max(R_o, r_o) = 999.2$
6. Step size at the lst trial ($\alpha_o = 1$) satisfies the descent condition. Design is updated as $N^{(1)} = 106$, $R^{(1)} = 2.53158$
7. $R_1 = 999.2$, k = 1, go to Step 2.

13.7

Refer to Exercise 12.9 for detailed formulation.

Iteration 1:

1. Initial design is given as $(W^{(0)}, D^{(0)}) = (100, 100)$; set $R_o = 1$, $\gamma = 0.5$, ε_1, $\varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = 30000$, $g_1 = 0$, $g_2 = -0.5$, $g_3 = 0$, $g_4 = -0.5$, $g_5 = -0.5$, $g_6 = -100$, $g_7 = -100$,
   $\nabla f = (200, 100)$; $\nabla g_1 = (0.01, 0)$; $\nabla g_2 = (0, 0.005)$; $\nabla g_3 = (-0.01, -0.01)$;
   $\nabla g_4 = (-0.005, 0.005)$; $\nabla g_5 = (0.005, -0.005)$; $\nabla g_6 = (-1, 0)$; $\nabla g_7 = (0, -1)$;
   $V_o = \max\{0, 0, -0.5, 0, -0.5, -0.5, -100, -100\} = 0$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (-50, 50)$ with Lagrange multipliers as $\mathbf{u} = (0, 0, 15000, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 15000$; $R = \max(R_o, r_o) = 15000$
6. Step size at the 2nd trial ($\alpha_o = 0.5$) satisfies the descent condition. Design is updated as $W^{(1)} = 75$, $D^{(1)} = 125$
7. $R_1 = 15000$, k = 1, go to Step 2.

13.8

Refer to Exercise 12.10 for detailed formulation.

Iteration 1:

1. Initial design is given as $(r^{(0)}, h^{(0)}) = (6, 16)$; set $R_o = 1$, $\gamma = 0.5$, ε_1, $\varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = 716.283$, $h_1 = 2.01593$, $g_1 = -0.33333$, $g_2 = -0.11111$, $g_3 = -0.2$, $g_4 = -16$, $g_5 = -6$;
   $\nabla f = (138.2301, 37.699)$; $\nabla h_1 = (1.0053, 0.1885)$; $\nabla g_1 = (0.22222, -0.083333)$;
   $\nabla g_2 = (-0.14815, 0.05555)$; $\nabla g_3 = (0, 0.05)$; $\nabla g_4 = (0, -1)$; $\nabla g_5 = (-1, 0)$;
   $V_o = \max\{0, 2.01593, -0.33333, -0.11111, -0.2, -16, -6\} = 2.01593$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (-0.86614, -6.30966)$ with Lagrange multipliers as $\mathbf{u} = (-148.4, 53.4, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 201.8$; $R = \max(R_o, r_o) = 201.8$
6. Step size at the lst trial ($\alpha_o = 1$) satisfies the descent condition. Design is updated as $r^{(1)} = 5.13386$, $h^{(1)} = 9.69034$
7. $R_1 = 201.8$, k = 1, go to Step 2.

13.9

Refer to Exercise 12.11 for detailed formulation.

Iteration 1:

1. Initial design is given as $(b^{(0)}, h^{(0)}) = (5, 10)$; set $R_o = 1$, $\gamma = 0.5$, ε_1, $\varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = 1.06667$, $g_1 = -0.5$, $g_2 = -0.44444$, $g_3 = -5$, $g_4 = -10$, $\nabla f = (-0.17067, -0.021333)$; $\nabla g_1 = (0.1, 0)$; $\nabla g_2 = (0, 0.055555)$; $\nabla g_3 = (-1, 0)$; $\nabla g_4 = (0, -1)$; $V_o = \max\{0, -0.5, -0.44444, -5, -10\} = 0$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (0.17067, 0.02133)$ with Lagrange multipliers as $\mathbf{u} = (0, 0, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 0$; $R = \max(R_o, r_o) = 1$
6. Step size at the lst trial ($\alpha_o = 1$) satisfies the descent condition. Design is updated as $b^{(1)} = 5.17067$, $h^{(1)} = 10.02133$
7. $R_1 = 1$, k = 1, go to Step 2.

13.10

Refer to Exercise 12.12 for detailed formulation.

Iteration 1:

1. Initial design is given as $(b^{(0)}, d^{(0)}, h^{(0)}) = (5, 5, 5)$; set $R_o = 1$, $\gamma = 0.5$, ε_1, $\varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = 18753.306$, $g_1 = 0.16667$, $g_2 = -b+2.5 = -2.5$, $g_3 = -d + 2.5 = -2.5$, $g_4 = -h + 2.5 = -2.5$, (Note: The constraints on design bounds are different from those for Exercise 12.12.)
   $\nabla f = (2367.2867, 2269.8784, 2864.1573)$; $\nabla g_1 = (-0.16667, -0.16667, -0.16667)$; $\nabla g_2 = (-1, 0, 0)$; $\nabla g_3 = (0, -1, 0)$; $\nabla g_4 = (0, 0, -1)$;
   $V_o = \max\{0, 0.16667, -2.5, -2.5, -2.5\} = 0.16667$
2. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (-2.5, 6, -2.5)$ with Lagrange multipliers as $\mathbf{u} = (13655.24, 88.91, 0, 585.78)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 14329.93$; $R = \max(R_o, r_o) = 14329.93$
6. Step size at the 3rd trial ($\alpha_o = 0.25$) satisfies the descent condition. Design is updated as $b^{(1)} = 4.375$, $d^{(1)} = 6.5$, $h^{(1)} = 4.375$
7. $R_1 = 14329.93$, k = 1, go to Step 2.

13.11

Refer to Exercise 12.13 for detailed formulation.

Iteration 1:

1. Initial design is given as $(D^{(0)}, H^{(0)}) = (4, 8)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = 50265.482$, $h_1 = -0.32979$, $g_1 = 0$, $g_2 = -4$, $g_3 = -8$, $\nabla f = (15079.645, 5026.548)$; $\nabla h_1 = (0.3351, 0.08378)$; $\nabla g_1 = (0.05, 0.1)$; $\nabla g_2 = (-1, 0)$; $\nabla g_3 = (0, -1)$; $V_o = \max\{0, 0.32979, 0, -4, -8\} = 0.32979$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (2.98427, -8)$ with Lagrange multipliers as $\mathbf{u} = (-45009.3, 0, 0, 1247.7)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 46257$; $R = \max(R_o, r_o) = 46257$
6. Step size at the 2nd trial ($\alpha_o = 0.5$) satisfies the descent condition. Design is updated as $D^{(1)} = 5.492135$, $H^{(1)} = 4$
7. $R_1 = 46257$, k = 1, go to Step 2.

13.12

Refer to Exercise 12.14 for detailed formulation.

Iteration 1:

1. Initial design is given as $(w^{(0)}, d^{(0)}, h^{(0)}) = (10, 10, 4)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = 2800$, $g_1 = 0.33333$, $g_2 = -10$, $g_3 = -10$, $g_4 = -4$; $\nabla f = (80, 120, 200)$; $\nabla g_1 = (-0.06667, -0.06667, -0.16667)$; $\nabla g_2 = (-1, 0, 0)$; $\nabla g_3 = (0, -1, 0)$; $\nabla g_4 = (0, 0, -1)$; $V_o = \max\{0, 0.33333, -10, -10, -4\} = 0.33333$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (2.0711, -10, 5.17159)$ with Lagrange multipliers as $\mathbf{u} = (1231, 0, 27.9, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 1258.9$; $R = \max(R_o, r_o) = 1258.9$
6. Step size at the 3rd trial ($\alpha_o = 0.25$) satisfies the descent condition. Design is updated as $w^{(1)} = 10.517775$, $d^{(1)} = 7.5$, $h^{(1)} = 5.2928975$
7. $R_1 = 1258.9$, k = 1, go to Step 2.

13.13

Refer to Exercise 12.15 for detailed formulation.

Iteration 1:

1. Initial design is given as $(P_1^{(0)}, P_2^{(0)}) = (2, 1)$; set $R_o = 1$, $\gamma = 0.5$, $\varepsilon_1, \varepsilon_2 = 0.001$
2. Compute cost and constraint functions: $f_o = 5.6$, $g_1 = 0.95$, $g_2 = -2$, $g_3 = -1$; $\nabla f = (3, 2.6)$; $\nabla g_1 = (-0.016667, -0.016667)$; $\nabla g_2 = (-1, 0)$; $\nabla g_3 = (0, -1)$; $V_o = \max\{0, 0.95, -2, -1\} = 0.95$
2. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(0)} = (28.29943, 28.69943)$ with Lagrange multipliers as $\mathbf{u} = (1877.9, 0, 0)$
4. $\|\mathbf{d}^{(0)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_o = 1877.9$; $R = \max(R_o, r_o) = 1877.9$
6. Step size at the 3rd trial ($\alpha_o = 0.25$) satisfies the descent condition. Design is updated as $P_1^{(1)} = 9.0748575$, $P_2^{(1)} = 8.1748575$
7. $R_1 = 1877.9$, k = 1, go to Step 2.

Section 13.4 Constrained Quasi-Newton Methods

13.14

Refer to Exercise 12.3 for detailed formulation.

Iteration 1: Refer to Exercise 13.1.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.1 as:
   $f_o = 78156.49342$; $g_1 = 0.00640156$, $g_2 = 0.4394197$, $g_3 = -0.4044$, $g_4 = -24.61475$, $g_5 = -0.74385$, $g_6 = -29.5123$, $g_7 = -0.69488$: $\nabla f = (305.12295, 256.14754)$;
   $\nabla g_1 = (-0.003929, -0.0065967)$; $\nabla g_2 = (-0.0056195, -0.0047175)$; $\nabla g_3 = (-0.0023252, 0.001952)$; $\nabla g_4 = (-0.1, 0)$; $\nabla g_5 = (0.001, 0)$; $\nabla g_6 = (0, -0.1)$; $\nabla g_7 = (0, 0.001)$;
   $V_1 = \max\{0; 0.00640156, 0.4394197, -0.4044, -24.61475, -0.74385, -29.5123, -0.69488\} = 0.4394197$
   $\mathbf{S}^{(0)} = (6.1475413, 5.12295)$; $\mathbf{Z}^{(0)} = (6.1475413, 5.12295)$; $\mathbf{y}^{(0)} = (27.266794, 22.588108)$;
   $\xi_1 = 283.34149$, $\xi_2 = 64.036881$, $\theta = 1$, $\mathbf{w}^{(0)} = (27.266794, 22.588108)$; $\xi_3 = 283.34149$;
   $$\mathbf{D}^{(0)} = \begin{bmatrix} 2.623965 & 2.173721 \\ 2.173721 & 1.800734 \end{bmatrix};\ \mathbf{E}^{(0)} = \begin{bmatrix} 0.590164 & 0.491803 \\ 0.491803 & 0.409836 \end{bmatrix};\ \mathbf{H}^{(1)} = \begin{bmatrix} 3.033801 & 1.681918 \\ 1.681918 & 2.390898 \end{bmatrix}$$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (44.896387, 39.66602)$ with Lagrange multipliers as $\mathbf{u} = (0, 90407.47, 0, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 90407.47$; $R = \max(R_1, r_1) = 90407.47$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as
   $b^{(2)} = 301.043927$, $d^{(2)} = 344.78897$
7. $R_2 = 90407.47$, k = 2, go to Step 2.

13.15

Refer to Exercise 12.4 for detailed formulation

Iteration 1: Refer to Exercise 13.2.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.2 as:
   $f_o = 142.3516$, $g_1 = -0.944855$, $g_2 = -9.01843$, $g_3 = -1.30889$, $g_4 = -0.88455$, $g_5 = 0$, $g_6 = -0.9$; $\nabla f = (12.33075, 284.70321)$; $\nabla g_1 = (-0.0047768, -0.11029)$; $\nabla g_2 = (-2.60344, -20.03686)$; $\nabla g_3 = (-0.2, 0)$; $\nabla g_4 = (0.01, 0)$; $\nabla g_5 = (0, -2)$; $\nabla g_6 = (0, 0.2)$
   $V_1 = \max\{0; -0.944855, -9.01843, -1.30889, -0.88455, 0, -0.9\} = 0$
   $\mathbf{S}^{(0)} = (-0.45556, -3.5)$; $\mathbf{Z}^{(0)} = (-0.45556, -3.5)$; $\mathbf{y}^{(0)} = (1.24607, -0.380518)$; $\xi_1 = 0.76415$, $\xi_2 = 12.457535$, $\theta = 0.852279$, $\mathbf{w}^{(0)} = (0.994704, -0.841331)$; $\xi_3 = 2.491511$;
   $\mathbf{D}^{(0)} = \begin{bmatrix} 0.397123 & -0.335891 \\ -0.335891 & 0.2841 \end{bmatrix}$; $\mathbf{E}^{(0)} = \begin{bmatrix} 0.016659 & 0.127992 \\ 0.127992 & 0.983341 \end{bmatrix}$; $\mathbf{H}^{(1)} = \begin{bmatrix} 1.380464 & -0.463883 \\ -0.463883 & 0.300759 \end{bmatrix}$

   QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (-3.464044, 0)$ with the Lagrange multipliers as $\mathbf{u} = (0, 2.9, 0, 0, 114.1, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 117$; $R = \max(R_1, r_1) = 117$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as $R^{(2)} = 8.080396$, $t^{(2)} = 0.5$
7. $R_2 = 117$, k = 2, go to Step 2.

13.16

Refer to Exercise 12.5 for detailed formulation

Iteration 1: Refer to Exercise 13.3.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.3 as:
   $f_o = 10399.847$, $g_1 = 0.041693$, $g_2 = -0.09091$, $g_3 = -119.99694$, $g_4 = -110$; $\tilde{\mathbf{N}}f = (50, 40)$;
   $\nabla g_1 = (-0.008681, 0)$; $\nabla g_2 = (0, -0.00826446)$; $\nabla g_3 = (-1, 0)$; $\nabla g_4 = (0, -1)$;
   $V_1 = \max\{0; 0.041693, -0.09091, -119.99694, -110\} = 0.041693$
   $\mathbf{S}^{(0)} = (-30.00306, -40)$; $\mathbf{Z}^{(0)} = (-30.00306, -40)$; $\mathbf{y}^{(0)} = (-11.252975, 0)$; $\xi_1 = 337.62368$,
   $\xi_2 = 2500.1836$, $\theta = 0.924898$, $\mathbf{w}^{(0)} = (-12.661141, -3.00408)$; $\xi_3 = 500.03617$;
   $\mathbf{D}^{(0)} = \begin{bmatrix} 0.320586 & 0.076065 \\ 0.076065 & 0.018048 \end{bmatrix}$; $\mathbf{E}^{(0)} = \begin{bmatrix} 0.360047 & 0.480014 \\ 0.480014 & 0.639953 \end{bmatrix}$; $\mathbf{H}^{(1)} = \begin{bmatrix} 0.960539 & -0.403949 \\ -0.403949 & 0.378095 \end{bmatrix}$
3. QP subproblem defined using the data given in Step 2 gives the search direction as
   $\mathbf{d}^{(1)} = (4.802787, -11.000114)$ with Lagrange multipliers as $\mathbf{u} = (6802.99, 4101.99, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 10904.98$; $R = \max(R_1, r_1) = 10904.98$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as
   $A_1^{(2)} = 124.799727$, $A_2^{(2)} = 98.999886$
7. $R_2 = 10904.98$, k = 2, go to Step 2.

13.17

Refer to Exercise 12.6 for detailed formulation

Iteration 1: Refer to Exercise 13.4.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 11:4 as:

   $f_o = 14.4995$, $g_1 = 4.253\times10^{-5}$, $g_2 = -0.10002$, $g_3 = -3.99976$, $g_4 = -0.166706$, $g_5 = -4000.0173$; $\nabla f = (0.6, 0.001)$; $\nabla g_1 = (-0.057143, -2.499883\times10^{-4})$; $\nabla g_2 = (0.0285715, 2.24994\times10^{-4})$; $\nabla g_3 = (-0.285714, 0)$; $\nabla g_4 = (0.047619, 0)$; $\nabla g_5 = (0, -1)$; $V_1 = \max\{0; 4.253\times10^{-5}, -0.10002, -3.99976, -0.166706, -4000.0173\} = 4.253\times10^{-5}$

   $\mathbf{S}^{(0)} = (5.49918, 0.0173)$; $\mathbf{Z}^{(0)} = (5.49918, 0.0173)$; $\mathbf{y}^{(0)} = (0, -0.008382)$; $\xi_1 = -1.45\times10^{-4}$, $\xi_2 = 30.2413$, $\theta = 0.799997$, $\mathbf{w}^{(0)} = (1.09985, -0.0032455)$; $\xi_3 = 6.048217$; $\mathbf{D}^{(0)} = \begin{bmatrix} 0.200004 & -0.00059 \\ -0.00059 & 0.0000017 \end{bmatrix}$; $\mathbf{E}^{(0)} = \begin{bmatrix} 0.999989 & 0.003146 \\ 0.003146 & 0.0000099 \end{bmatrix}$; $\mathbf{H}^{(1)} = \begin{bmatrix} 0.200015 & -0.003736 \\ -0.003736 & 0.9999918 \end{bmatrix}$

3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (0.000737, 0.001628)$ with the Lagrange multipliers as $\mathbf{u} = (10.5, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| = 0.00179 > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 10.5$; $R = \max(R_1, r_1) = 106.7$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as $h^{(2)} = 17.499917$, $A^{(2)} = 4000.018928$
7. $R_2 = 106.7$, k = 2, go to Step 2.

13.18

Refer to Exercise 12.7 for detailed formulation

Iteration 1: Refer to Exercise 13.5

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.5 as:
   $f_o = -2735.493$, $g_1 = -0.43583$, $g_2 = -1.154975$, $g_3 = -0.46126$, $g_4 = -7.5$, $g_5 = -0.625$;
   $\nabla f = (-507.754, -364.7324)$; $\nabla g_1 = (0.05236, 0.07522)$; $\nabla g_2 = (-0.2, 0)$; $\nabla g_3 = (0.05, 0)$;
   $\nabla g_4 = (0, -1)$; $\nabla g_5 = (0, 0.05)$;
   $V_1 = \max\{0; -0.43583, -1.154975, -0.46126, -7.5, -0.625\} = 0$; $\mathbf{S}^{(0)} = (4.774875, -7.5)$;
   $\mathbf{Z}^{(0)} = (4.774875, -7.5)$; $\mathbf{y}^{(0)} = (-220.23577, -74.693096)$; $\xi_1 = -491.4$, $\xi_2 = 79.04943$,
   $\theta = 0.110859$, $\mathbf{w}^{(0)} = (-20.16958, -14.948959)$; $\xi_3 = 15.809969$; $\mathbf{D}^{(0)} =$
   $\begin{bmatrix} 25.73136 & 19.071146 \\ 19.071146 & 14.134839 \end{bmatrix}$; $\mathbf{E}^{(0)} = \begin{bmatrix} 0.28842 & -0.45303 \\ -0.45303 & 0.71158 \end{bmatrix}$; $\mathbf{H}^{(1)} = \begin{bmatrix} 26.44294 & 19.524176 \\ 19.524176 & 14.423259 \end{bmatrix}$

3. QP subproblem defined using the data given in Step 2 gives the search direction as
   $\mathbf{d}^{(1)} = (9.2252, -0.627512)$ with the Lagrange multipliers as $\mathbf{u} = (2574.7, 0, 2825.06, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 5399.76$; $R = \max(R_1, r_1) = 5403.1$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as
   $R^{(2)} = 20.000075$, $H^{(2)} = 6.872488$
7. $R_2 = 5403.1$, k = 2, go to Step 2.

13.19

Refer to Exercise 12.8 for detailed formulation

Iteration 1: Refer to Exercise 13.6.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.6 as:
   f_o = -1686.081, g_1 = 0.0671097, g_2 = -4.06316, g_3 = -106; ∇f = (-15.90642, -666.0192);
   ∇g_1 = (0.010067, 0.84304); ∇g_2 = (0,-2); ∇g_3 = (-1, 0)
   V_1 = max {0; 0.0671097, - 4.06316, -106} = 0.0671097
   $\mathbf{S}^{(0)}$ = (6, 0.53158); $\mathbf{Z}^{(0)}$ = (6, 0.53158); $\mathbf{y}^{(0)}$ = (0.440953, 176.84902); ξ_1 = 96.655,
   ξ_2 = 36.2826, θ = 1, $\mathbf{w}^{(0)}$ = (0.440953, 176.84902); ξ_3 = 96.655;

$$\mathbf{D}^{(0)} = \begin{bmatrix} 0.00201 & 0.80681 \\ 0.80681 & 323.57949 \end{bmatrix};\ \mathbf{E}^{(0)} = \begin{bmatrix} 0.99221 & 0.08791 \\ 0.08791 & 0.00779 \end{bmatrix};\ \mathbf{H}^{(1)} = \begin{bmatrix} 0.0098 & 0.7189 \\ 0.7189 & 324.5717 \end{bmatrix}$$

3. QP subproblem defined using the data given in Step 2 gives the search direction as
   $\mathbf{d}^{(1)}$ = (163, − 2.026038) with the Lagrange multipliers as **u** = (1431.05, 0, 0)
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. r_1 = 1431.05; R = max (R_1, r_1) = 1431.05
6. Step size at the 4th trial (t_4 = 0.125) satisfies the descent condition. Design is updated as
   $N^{(2)}$ = 126, $R^{(2)}$ = 2.27832525
7. R_2 = 1431.05, k = 2, go to Step 2.

(Note: γ = 0.05 was used in computing β. Also, N is truncated to an integer value)

13.20

Refer to Exercise 12.9 for detailed formulation

Iteration 1: Refer to Exercise 13.7.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.7 as:
   $f_o = 27500$, $g_1 = -0.25$, $g_2 = -0.375$, $g_3 = 0.0625$, $g_4 = -0.166667$, $g_5 = -0.7$, $g_6 = -75$, $g_7 = -125$; $\nabla f = (200, 100)$; $\nabla g_1 = (0.01, 0)$; $\nabla g_2 = (0, 0.005)$; $\nabla g_3 = (-0.0125, -0.0075)$; $\nabla g_4 = (-0.011111, 0.0066667)$; $\nabla g_5 = (0.004, -0.0024)$; $\nabla g_6 = (-1, 0)$; $\nabla g_7 = (0, -1)$;
   $V_1 = \max\{0; -0.25, -0.375, 0.0625, -0.166667, -0.7, -75, -125\} = 0.0625$
   $\mathbf{S}^{(0)} = (-25, 25)$; $\mathbf{Z}^{(0)} = (-25, 25)$; $\mathbf{y}^{(0)} = (-37.5, 37.5)$; $\xi_1 = 1875$, $\xi_2 = 1250$, $\theta = 1$, $\mathbf{w}^{(0)} = (-37.5, 37.5)$; $\xi_3 = 1875$;
   $\mathbf{D}^{(0)} = \begin{bmatrix} 0.75 & -0.75 \\ -0.75 & 0.75 \end{bmatrix}$; $\mathbf{E}^{(0)} = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$; $\mathbf{H}^{(1)} = \begin{bmatrix} 1.25 & -0.25 \\ -0.25 & 1.25 \end{bmatrix}$
3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (-2.5, 12.45)$ with the Lagrange multipliers as $\mathbf{u} = (0, 0, 15500, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 15500$; $R = \max(R_1, r_1) = 15500$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as $w^{(2)} = 72.5$, $D^{(2)} = 137.45$
7. $R_2 = 15500$, k = 2, go to Step 2.

13.21

Refer to Exercise 12.10 for detailed formulation

Iteration 1: Refer to Exercise 13.8.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.8 as:
   $f_o = 395.383$, $h_1 = 0.33729$, $g_1 = 0.056233$, $g_2 = -0.37082$, $g_3 = -0.51548$, $g_4 = -9.69034$, $g_5 = -5.13386$; $\nabla f = (93.143196, 32.256994)$; $\nabla h_1 = (0.520969, 0.138002)$; $\nabla g_1 = (0.183832, -0.097393)$; $\nabla g_2 = (-0.122555, 0.064928)$; $\nabla g_3 = (0, 0.05)$; $\nabla g_4 = (0, -1)$; $\nabla g_5 = (-1, 0)$;
   $V_1 = \max\{0; 0.33729, 0.056233, -0.37082, -0.51548, -9.69034, -5.13386\} = 0.33729$
   $\mathbf{S}^{(0)} = (-0.86614, -6.30966)$; $\mathbf{Z}^{(0)} = (-0.86614, -6.30966)$; $\mathbf{y}^{(0)} = (24.737897, 0.262293)$;
   $\xi_1 = -23.08146$, $\xi_2 = 40.56201$, $\theta = 0.509865$, $\mathbf{w}^{(0)} = (12.18846, -2.95885)$; $\xi_3 = 8.112425$;

$$\mathbf{D}^{(0)} = \begin{bmatrix} 18.31247 & -4.445505 \\ -4.445505 & 1.07918 \end{bmatrix};$$

$$\mathbf{E}^{(0)} = \begin{bmatrix} 0.0185 & 0.13473 \\ 0.13473 & 0.9815 \end{bmatrix}; \quad \mathbf{H}^{(1)} = \begin{bmatrix} 19.29397 & -4.580235 \\ -4.580235 & 1.09768 \end{bmatrix}$$

3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (-0.533584, -0.429772)$ with the Lagrange multipliers as $\mathbf{u} = (-191.22, 80.51, 0, 0, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 271.73$; $R = \max(R_1, r_1) = 271.73$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as $r^{(1)} = 4.600276$, $h^{(1)} = 9.260568$
7. $R_2 = 271.73$, k = 2, go to Step 2.

13.22

Refer to Exercise 12.11 for detailed formulation

Iteration 1: Refer to Exercise 13.9.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.9 as:
   $f_o = 1.038046$, $g_1 = -0.482933$, $g_2 = -0.4432594$, $g_3 = -5.17067$, $g_4 = -10.02133$;
   $\nabla f = (-0.159586, -0.0212426)$; $\nabla g_1 = (0.1, 0)$; $\nabla g_2 = (0, 0.055555)$; $\nabla g_3 = (-1, 0)$;
   $\nabla g_4 = (0, -1)$; $V_1 = \max\{0; -0.482933, -0.4432594, -5.17067, -10.02133\} = 0$
   $\mathbf{S}^{(0)} = (0.17067, 0.02133)$; $\mathbf{Z}^{(0)} = (0.17067, 0.02133)$; $\mathbf{y}^{(0)} = (0.011084, 0.00009)$; $\theta = 0.85471$,
   $\xi_1 = 1.893626\times10^{-3}$, $\xi_2 = 0.0295832$, $\mathbf{w}^{(0)} = (0.03427, 0.00317596)$; $\xi_3 = 5.9166\times10^{-3}$;
   $\mathbf{D}^{(0)} = \begin{bmatrix} 0.198498 & 0.018395 \\ 0.018395 & 0.001705 \end{bmatrix}$; $\mathbf{E}^{(0)} = \begin{bmatrix} 0.984621 & 0.123056 \\ 0.123056 & 0.015379 \end{bmatrix}$; $\mathbf{H}^{(1)} = \begin{bmatrix} 0.21388 & -0.10466 \\ -0.10466 & 0.98633 \end{bmatrix}$

3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (0.798129, 0.106202)$ with the Lagrange multipliers as $\mathbf{u} = (0, 0, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 0$; $R = \max(R_1, r_1) = 1$
6. Step size at the 1st trial ($t_o = 1$) satisfies the descent condition. Design is updated as
   $b^{(2)} = 5.968799$, $h^{(2)} = 10.127532$
7. $R_2 = 1$, k = 2, go to Step 2.

(Note: $\gamma = 0.1$ was used in computing β instead of 0.5)

13.23

Refer to Exercise 12.12 for detailed formulation

Iteration 1: Refer to Exercise 13.10.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.10 as:

$f_o = 18578.5548$, $g_1 = 0.170573$, $g_2 = -1.875$, $g_3 = -4$, $g_4 = -1.875$;

$\nabla f = (2448.14, 1986.1436, 3094.0718)$; $\nabla g_1 = (-0.1895833, -0.1276041, -0.1895833)$;

$\nabla g_2 = (-1, 0, 0)$; $\nabla g_3 = (0, -1, 0)$; $\nabla g_4 = (0, 0, -1)$;

$V_1 = \max\{0; 0.170573, -1.875, -4, -1.875\} = 0.170573$;

$\mathbf{S}^{(0)} = (-0.625, 1.5, -0.625)$; $\mathbf{Z}^{(0)} = (-0.625, 1.5, -0.625)$;

$\mathbf{y}^{(0)} = (-232.07428, 249.67847, -83.01308)$; $\xi_1 = 571.4473$, $\xi_2 = 3.03125$, $\xi_3 = 571.4473$,

$\theta = 1$, $\mathbf{w}^{(0)} = (-232.07428, 249.67847, -83.01308)$;

$$\mathbf{D}^{(0)} = \begin{bmatrix} 94.24924 & -101.39859 & 33.713 \\ & 109.09027 & -36.27032 \\ \textit{symm.} & & 12.05916 \end{bmatrix}; \quad \mathbf{E}^{(0)} = \begin{bmatrix} 0.12887 & -0.30928 & 0.12887 \\ & 0.74227 & -0.30928 \\ \textit{symm.} & & 0.12887 \end{bmatrix};$$

$$\mathbf{H}^{(1)} = \begin{bmatrix} 95.12037 & -101.08931 & 33.58413 \\ & 109.348 & -35.96104 \\ \textit{symm.} & & 12.93029 \end{bmatrix}$$

3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (2.751459, 0.034568, -1.875)$ with the Lagrange multipliers as $\mathbf{u} = (13943.19, 0, 0, 517.59)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 14460.78$; $R = \max(R_1, r_1) = 14460.78$
6. Step size at the 2nd trial ($t_1 = 0.5$) satisfies the descent condition. Design is updated as $b^{(2)} = 5.7507295$, $d^{(2)} = 6.517284$, $h^{(2)} = 3.4375$
7. $R_2 = 14460.78$, k = 2, go to Step 2.

13.24

Refer to Exercise 12.13 for detailed formulation

Iteration 1: Refer to Exercise 13.11.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.11 as:
   $f_o = 46558.798$, $h_1 = -0.368256$; $g_1 = -0.325393$, $g_2 = -5.492135$, $g_3 = -4$;
   $\nabla f = (11928.169, 6901.6204)$; $\nabla h_1 = (0.230054, 0.157936)$; $\nabla g_1 = (0.05, 0.1)$; $\nabla g_2 = (-1, 0)$;
   $\nabla g_3 = (0, -1)$; $V_1 = \max\{0; 0.368256, -0.325393, -5.492135, -4\} = 0.368256$
   $\mathbf{S}^{(0)} = (1.492135, -4)$; $\mathbf{Z}^{(0)} = (1.492135, -4)$; $\mathbf{y}^{(0)} = (1576.5709, -1462.6373)$; $\xi_1 = 8203.0058$,
   $\xi_2 = 18.2265$, $\theta = 1$, $\mathbf{w}^{(0)} = (1576.5709, -1462.6373)$; $\xi_3 = 8203.0058$;
   $$\mathbf{D}^{(0)} = \begin{bmatrix} 303.00793 & -281.11054 \\ -281.11054 & 260.79561 \end{bmatrix}; \mathbf{E}^{(0)} = \begin{bmatrix} 0.12216 & -0.32746 \\ -0.32746 & 0.87784 \end{bmatrix};$$
   $$\mathbf{H}^{(1)} = \begin{bmatrix} 303.88577 & -280.78308 \\ -280.78308 & 260.91777 \end{bmatrix}$$
3. QP subproblem defined using the data given in Step 2 gives the search direction as
   $\mathbf{d}^{(1)} = (-0.199715, 2.621956)$ with the Lagrange multipliers as $\mathbf{u} = (-48385.51, 0, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 48385.51$; $R = \max(R_1, r_1) = 48385.51$
6. Step size at the 1st trial ($\alpha_1 = 1$) satisfies the descent condition. Design is updated as
   $D^{(2)} = 5.29242$, $H^{(2)} = 6.621956$
7. $R_2 = 48385.51$, k = 2, go to Step 2.

13.25

Refer to Exercise 12.14 for detailed formulation

Iteration 1: Refer to Exercise 13.12.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.12 as:
   $f_o = 2800.0015$, $g_1 = 0.304131$, $g_2 = -10.517775$, $g_3 = -7.5$, $g_4 = -5.2928975$;
   $\nabla f = (80, 120, 200)$; $\nabla g_1 = (-0.0661612, -0.0927825, -0.1314721)$; $\nabla g_2 = (-1, 0, 0)$;
   $\nabla g_3 = (0, -1, 0)$; $\nabla g_4 = (0, 0, -1)$;
   $V_1 = \max\{0; 0.304131, -10.517775, -7.5, -5.2928975\} = 0.304131$
   $\mathbf{S}^{(0)} = (0.517775, -2.5, 1.2928975)$; $\mathbf{Z}^{(0)} = (0.517775, -2.5, 1.2928975)$;
   $\mathbf{y}^{(0)} = (0.62264, -32.14818, 43.32492)$; $\xi_1 = 136.70752$, $\xi_2 = 8.189675$, $\xi_3 = 136.70752$;
   $\theta = 1$, $\mathbf{w}^{(0)} = (0.62264, -32.14818, 43.32492)$;

$$\mathbf{D}^{(0)} = \begin{bmatrix} 0.00284 & -0.14642 & 0.19733 \\ & 7.55998 & -10.1883 \\ \textit{symm.} & & 13.7304 \end{bmatrix}; \quad \mathbf{E}^{(0)} = \begin{bmatrix} 0.03274 & -0.15806 & 0.08174 \\ & 0.76316 & -0.39467 \\ \textit{symm.} & & 0.20411 \end{bmatrix};$$

$$\mathbf{H}^{(1)} = \begin{bmatrix} 0.9701 & 0.01164 & 0.11559 \\ & 7.79682 & -9.79363 \\ & & 14.52629 \end{bmatrix}$$

3. QP subproblem defined using the data given in Step 2 gives the search direction as $\mathbf{d}^{(1)} = (13.132463, -2.219396, -2.729153)$ with the Lagrange multipliers as $\mathbf{u} = (1396.57, 0, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 1396.57$; $R = \max(R_1, r_1) = 1396.57$
3. Step size at the 5th trial ($\alpha_1 = 0.0625$) satisfies the descent condition. Design is updated as
   $w^{(2)} = 11.3385539375$, $d^{(2)} = 7.36128775$, $h^{(2)} = 5.1223254375$
7. $R_2 = 1396.57$, k = 2, go to Step 2.

13.26

Refer to Exercise 12.15 for detailed formulation

Iteration 1: Refer to Exercise 13.13.

Iteration 2:

2. Computed cost and constraint functions and their gradients are given in 13.13 as:
   $f_o = 147.0114$, $g_1 = 0.712505$, $g_2 = -9.0748575$, $g_3 = -8.1748575$;
   $\nabla f = (17.149715, 16.949715)$; $\nabla g_1 = (-0.016667, -0.016667)$; $\nabla g_2 = (-1, 0)$; $\nabla g_3 = (0, -1)$;
   $V_1 = \max\{0; 0.712505, -9.0748575, -8.1748575\} = 0.712505$
   $\mathbf{S}^{(0)} = (7.0748575, 7.1748575)$; $\mathbf{Z}^{(0)} = (7.0748575, 7.1748575)$; $\mathbf{y}^{(0)} = (14.149715, 14.349715)$;
   $\xi_1 = 203.0644$, $\xi_2 = 101.5322$, $\theta = 1$, $\mathbf{w}^{(0)} = (14.149715, 14.349715)$; $\xi_3 = 203.0644$;

$$\mathbf{D}^{(0)} = \begin{bmatrix} 0.98597 & 0.9999 \\ 0.9999 & 1.01403 \end{bmatrix}; \quad \mathbf{E}^{(0)} = \begin{bmatrix} 0.49298 & 0.49995 \\ 0.49995 & 0.50702 \end{bmatrix}; \quad \mathbf{H}^{(1)} = \begin{bmatrix} 1.49299 & 0.49995 \\ 0.49995 & 1.50701 \end{bmatrix}$$

3. QP subproblem defined using the data given in Step 2 gives the search direction as
   $\mathbf{d}^{(1)} = (21.424558, 21.324889)$ with Lagrange multipliers as $\mathbf{u} = (3587.79, 0, 0)$
4. $\|\mathbf{d}^{(1)}\| > \varepsilon_2$; Convergence criteria are not satisfied.
5. $r_1 = 3587.79$; $R = \max(R_1, r_1) = 3587.79$
6. Step size at the 1st trial ($t_0 = 1$) satisfies the descent condition. Design is updated as
   $P_1^{(2)} = 30.4994155$, $P_2^{(2)} = 29.4997465$
7. $R_2 = 3587.79$, k = 2, go to Step 2.

13.27 ———————————————————————————————————————

Optimum solution: $x_1^* \doteq 103.0$ mm, $x_2^* \doteq 0.955$, $f^* \doteq 2.9$ kg; shear stress constraint and buckling constraint are active.

13.28 ———————————————————————————————————————

Optimum solution: $d_o^* \doteq 103.0$ mm, $d_i^* \doteq 98.36$ mm, $f^* \doteq 2.9$ kg; shear stress constraint and buckling constraint are active.

13.29 ———————————————————————————————————————

Optimum solution: $R^* = 50.3$ mm, $t^* \doteq 2.35$ mm, $f^* \doteq 2.9$ kg; shearing stress constraint and buckling constraint are active.

13.30———————————————————————————————————————

Optimum solution: $A_1^* \doteq 300$ mm^2, $A_2^* \doteq 50.0$mm^2, $f^* \doteq 7.0$ kg;

Member 1 stress constraint is active.

13.31———————————————————————————————————————

Optimum solution: $R^* \doteq 130$ cm, $t^* \doteq 2.86$ cm, $f^* \doteq 57000$ kg; combined stress constraint and diameter/thickness ratio constraint are active.

13.32* ———————————————————————————————————————

Optimum solution: $d_o^* \doteq 41.56$ cm, $d_i^* \doteq 40.19$ cm, $f^* = 680.0$ kg; top deflection constraint and diameter/thickness ratio constraint are active.

13.33* ———————————————————————————————————————

Optimum solution: $d_o^* \doteq 1310$ mm, $t^* \doteq 14.2$ mm, $f^* \doteq 92{,}500$ N; maximum deflection constraint and diameter/thickness ratio constraint are active.

13.34* ———————————————————————————————————————

Optimum solution: $H^* = 50.0$ cm, $D^* \doteq 3.42$ cm, $f^* \doteq 6.6$ kg; buckling load constraint and maximum height constraint are active.