CHAPTER
14
Practical Applications of Optimization

Note: In all the numerical results presented with IDESIGN (a program based on the SQP method), very severe convergence criteria are used to obtain a precise solution (maximum constraint violation ≤ 0.0001 and convergence parameter ≤ 0.0001). Relaxed convergence criteria can give near optimum solutions in fewer iterations.

14.1

Using the data given in Exercise 3.34 (l = 500mm), the problem is stated as follows after normalizing the constraints:

Find x_1 = outer diameter of the shaft (mm) and x_2 = ratio of inner to outer diameters to minimize:

mass $f(x_1, x_2) = (3.08269 \times 10^{-3})\, x_1^2 (1 - x_2^2)$, kg;

subject to: $g_1 = 1.852\times10^5 / x_1^3(1 - x_2^4) - 1.0 \le 0$

$g_2 = 1.82378\times10^7 / x_1^4(1 - x_2^4) - 1 \le 0;$

$g_3 = 1 - 2.08623\times10^{-3}\, x_1^3 (1 - x_2)^{2.5} \le 0.$

The problem is solved using the IDESIGN program where explicit design variable bound constraints are automatically imposed as $20 \le x_1 \le 500$, $0.6 \le x_2 \le 0.999$. The optimum solution is obtained in 11 iterations with convergence criteria as 0.0001 using the SQP algorithm: $x_1^* = 102.985$ mm, $x_2^* = 0.954614$, $u_1^* = 1.27568$, $u_2^* = 0$, $u_3^* = 0.65795$, $f^* = 2.900453$ kg (starting point x_1 = 50 mm, x_2 = 0.7).

14.2

Using the data given in Exercise 3.35 (l = 500mm), the problem is stated as follows after normalizing the constraints:

Find x_1 = outside diameter of the shaft (mm), x_2 = inside diameter of the shaft(mm) to minimize the mass $f(x_1, x_2) = (3.08269 \times 10^{-3})(x_1^2 - x_2^2)$, kg

subject to $g_1 = (1.852\times10^5)\, x_1/(x_1^4 - x_2^4) - 1 \le 0$;

$g_2 = 1.82378\times10^7/(x_1^4 - x_2^4) - 1 \le 0$

$g_3 = 1 - (2.08623\times10^{-3})\, x_1^3(1 - x_2/x_1)^{2.5} \le 0$;

$g_4 = 1 - x_1 / 20 \le 0$

$g_5 = x_1/500 - 1 \le 0$

$g_6 = 1 - x_2 / 0.6x_1 \le 0$

$g_7 = x_2 / 0.999x_1 - 1 \le 0$.

Starting from the point x_1 = 50 mm, x_2 = 35 mm, the following optimum solution is obtained in 15 iterations with convergence criteria as 0.0001 using the SQP algorithm: x_1^* = 102.974 mm, x_2^* = 98.2999 mm, $\mathbf{u}^*$ = (1.2757, 0, 0.657953, 0, 0, 0, 0), f^* = 2.90017 kg.

14.3

Using the data given in Exercise 3.36 (l = 500mm), the problem is stated as follows after normalizing the constraints:
Find x_1 = mean radius (mm) and x_2 = wall thickness (mm) to minimize the mass

$f(x_1, x_2) = (2.46615\times10^{-2})\, x_1x_2$; subject to $g_1 = (1.157491\times10^{4})(2x_1 + x_2)/(4x_1^3x_2 + x_1x_2^3) - 1 \le 0$; $g_2 = (1.13986\times10^{6}) / (4x_1^3x_2 + x_1x_2^3) - 1 \le 0$; $g_3 = 1 - (1.668985\times10^{-2})(x_1 + 0.5x_2)^{0.5}\, x_2^{2.5} \le 0$, and explicit design variable bounds as $50 \le x_1 \le 200$ mm, $2 \le t \le 40$ mm.

Starting from the point x_1 = 50 mm, x_2 = 2 mm, the optimum solution is obtained in 4 iterations

with convergence criteria as 0.0001 using the SQP algorithm:
x_1^* = 50.3202 mm, x_2^* = 2.33723 mm, $\mathbf{u}^*$ = (1.27761, 0, 0.657082), f^* = 2.90044 kg.

14.4

Referring to Exercise 3.50: Optimum solution: $A_1^* \doteq 300$ mm^2, $A_2^* \doteq 50.0$mm^2, $f^* \doteq 7.0$ kg; member 1 stress constraint is active.

14.5

Using the data and expressions given in Exercise 3.51, the problem is stated as follows after normalizing the constraints:
Find R = mean radius (cm) and t = wall thickness (cm) to minimize the mass of the water tower support column: $f(R, t) = 153.717\ Rt$

subject to $g_1 = \dfrac{4.04488 \times 10^6}{R^3 t + Rt^3/4} + \dfrac{5257.21(2R+t)}{R^3 t + Rt^3/4} + \dfrac{(1.6492 \times 10^{10})(2R+t)}{(R^3 t + Rt^3/4)^2} - 1 \le 0;$

$g_2 = 1 - (2R - t)/70 \le 0$; $g_3 = 2R/91t - 1 \le 0$; $g_4 = (2.2397\times10^6)/(R^3 t + Rt^3/4) - 1 \le 0$;

$g_5 = (R - 0.5t)/250 - 1 \le 0$; and $1 \le t \le 40$ cm.

Starting from the point R = 40 cm, t = 1 cm, the optimum solution is obtained in 15 iterations with convergence criteria as 0.0001 using the SQP algorithm: R^* = 129.184 cm, t^* = 2.83921 cm, $\mathbf{u}^*$ = (273.658, 0, 262.482, 0, 0), f^* = 56380.61 kg.

14.6

Using the data and expressions given in Exercise 3.52, the problem is stated as follows after normalizing the constraints:
Find d_o = outer diameter (cm) and d_i = inner diameter (cm) to minimize the mass of the flag pole:

$f = 6.12611(d_o^2 - d_i^2)$, kg;

subject to $g_1 = 8642.61 d_o / (d_o^4 - d_i^4) - 1 \le 0$; $g_2 = 8.14874(d_o^2 + d_o d_i + d_i^2)/(d_o^4 - d_i^4) - 1 \le 0$;

$g_3 = (3.7186\times10^5)/(d_o^4 - d_i^4) - 1 \le 0$; $g_4 = (d_o + d_i)/60(d_o - d_i) - 1 \le 0$; $g_5 = (d_o - d_i)/4 - 1 \le 0$;

$g_6 = 1 - (d_o - d_i) \le 0$; $5 \le d_o \le 50$; $4 \le d_i \le 45$, cm.

Starting from the point d_o = 40, d_i = 20, the optimum solution is obtained in 11 iterations with convergence criteria as 0.0001, using the SQP algorithm: d_o^* = 41.5442 cm, d_i^* = 40.1821 cm, $\mathbf{u}^*$ = (0, 0, 340.979, 340.789, 0, 0), f^* = 681.957 kg.

14.7

Using the data and expressions given in Exercise 3.53, the problem is formulated as follows after normalizing the constraints: find d_o = outer diameter (mm) and t = wall thickness (mm) to minimize the weight of the sign support: $f = 5.02655t(d_o - t)$, N

subject to $g_1 = \dfrac{1.687973\times10^9}{t(d_o - t)[d_o^2 + (d_o - 2t)^2]} + \dfrac{(4.0976487\times10^7)d_o}{[d_o^4 - (d_o - 2t)^4]} + \dfrac{(1.435461\times10^{17})d_o}{[d_o^4 - (d_o - 2t)^4]^2} - 1.0 \le 0;$

$g_2 = d_o / 92t - 1 \le 0;$ $g_3 = 2.4662052\times10^{11}/[d_o^4 - (d_o - 2t)^4] - 1 \le 0;$
$250 \le d_o \le 1500;$ $5 \le t \le 100$ mm.

Starting from an infeasible point $d_o = 500$, $t = 20$, the optimum solution is obtained in 15 iterations with convergence criteria as 0.0001 using the SQP algorithm: $d_o^* = 1308.36$ mm, $t^* = 14.2213$ mm, $\mathbf{u}^* = (0, 46752.4, 46255.4)$, $f^* = 92510.7$ N. A feasible starting point of (800, 90) also gave the same solution in 12 iterations.

14.8

Using the data and expressions given in Exercise 3.54, the problem is formulated as follows after normalizing the constraints:
Find H = height of the tripod and D = diameter of the cross-section to minimize the mass of the tripod: $f = (6.59734\times10^{-3})\, D^2(H^2 + 4800)^{1/2}$, kg
subject to $g_1 = 1.69765(H^2 + 4800)^{1/2}/D^2H - 1 \le 0;$ $g_2 = 1 - 90.8387HD^4/(H^2 + 4800)^{3/2} \le 0;$
$50 \le H \le 500$, $0.5 \le D \le 50$ cm.
Starting from a feasible point of $H = 200$ cm, $D = 20$ cm, the optimum solution is obtained in 12 iterations with convergence criteria as 0.0001 using the SQP algorithm in IDESIGN: $H^* = 50$ cm, $D^* = 3.4228$ cm, $u^* = (0, 3.30395)$, minimum height constraint is active with Lagrange multipliers as 2.35294, $f^* = 6.603738$ kg.

14.9

Problem formulation: Minimize $f = bh$;
subject to $g_1 = 1.0 - [3gkEI/(3WEI + kWL^3)]^{1/2}/8.0 \le 0$, where $I = bh^3/12$,
and $0.5 \le b \le 1.0$, $0.2 \le h \le 2.0$.
Solution: used program; IDESIGN (SQP algorithm; 6 iterations), initial design: $b = 0.5$, $h = 0.2$, optimum solution; $b^* = 0.5$ in, $h^* = 0.28107$ in, $f^* = 0.140536$ in^2 , active constraints (Lagrange multiplier); $g_1(0.54523)$, lower limit on $b(0.0936936)$.

14.10

<u>Formulation:</u> Units of N and cm are used

1. Design variables: $x_1 = b$, $x_2 = t_1$, $x_3 = t_2$, $x_4 = h$
2. Cost function: $f = L\,(2x_1x_2 + x_3x_4) = 150(2x_1x_2 + x_3x_4)$
3. Constraints: <axial stress> $g_1 = (Mc/I + P\cos\theta/A)/\sigma_a - 1.0 \le 0$, where $M = PL\sin\theta$, $P = 70000$, $L = 150$, $\theta = 45°$, $\sigma_a = 10000$, $c = x_2 + x_4/2$, $I = [x_1(2x_2 + x_4)^3 - (x_1 - x_3)\,x_4^3]\,/\,12$, $A = 2x_1x_2 + x_3x_4$;
<shear stress> $g_2 = (VQ\,/\,I\,x_3)/\tau_a - 1.0 \le 0$, where $V = P\sin\theta$, $Q = x_1x_2(x_2 + x_4)/2 + x_3x_4^2/8$, $\tau_a =$ 6000; <deflection> $g_3 = [P\sin\theta L^3/(3EI)]/\Delta - 1.0 \le 0$, where $\Delta = 1.5$;
<buckling> $g_4 = 1.0 - \pi^2EI/(4L^2P\cos\theta) \le 0$, $g_5 = 1.0 - \pi^2EI'/(4L^2P\cos\theta) \le 0$,
where $I' = x_1^3\,x_2/6 + x_3^3\,x_4/12$; <design limits> $x_1 \ge 10$, $x_2 \le 1$, $x_3 \le 1.5$, $x_4 \le 15$.

<u>Solution</u>: Program IDESIGN (SQP algorithm) is used.
Initial design; $x_1 = 60$, $x_2 = 0.9$, $x_3 = 0.9$, $x_4 = 14$;
Optimum: $x_1^* = 50.4437$ cm, $x_2^* = 1.0$ cm, $x_3^* = 0.52181$ cm, $x_4^* = 15.0$ cm, $f^* = 16307.2$ cm^3, number of iterations: 7, active constraints (Lagrange multipliers): $g_1(15502.0)$, $g_2(805.224)$, upper limit of $x_2(154.797)$, upper limit of $x_4(14641.4)$.

14.11

Formulation:

1. Design variables: $b_i = A_i$, $b_{i+3} = x_i$, i = 1 to 3.
2. Cost function: f = volume of truss members = $\sum_{i=1}^{3} b_i L_i = \sum_{i=1}^{3} b_i \left[L^2 + b_{i+3}^2\right]^{1/2}$
3. Constraints (18 stress constraints):
   $g_j = \sigma_{1j}/5000 - 1.0 \le 0$, j = 1,2,3; $g_{3+j} = -\sigma_{1j}/5000 - 1.0 \le 0$, j = 1,2,3;
   $g_{6+j} = \sigma_{2j}/20000 - 1.0 \le 0$, j = 1,2,3; $g_{9+j} = -\sigma_{2j}/20000 - 1.0 \le 0$, j = 1,2,3;
   $g_{12+j} = \sigma_{3j}/5000 - 1.0 \le 0$, j = 1,2,3; $g_{15+j} = -\sigma_{3j}/5000 - 1.0 \le 0$, j = 1,2,3;
4. design variable limits (arbitrary) $1.E{-}10 \le b_i \le 20$, in^2; $-10.0 \le b_{i+3} \le 10.0$, in, i = 1,2,3;

Solution: Program used; IDESIGN (SQP algorithm; 20 iterations).

Initial design; $b_1 = b_2 = b_3 = 6.0$, $b_4 = b_6 = 0.5$, $b_5 = 0.0$; Optimum solution: $b_1^* = A_1 = 1.4187$, $b_2^* = A_2 = 2.0458$, $b_3^* = A_3 = 2.9271$ in^2, $b_4^* = x_1 = -4.6716$, $b_5^* = x_2 = 8.9181$, $b_6^* = x_3 = 4.6716$ in, $f^* = 75.3782$ in^3; active constraints (Lagrange multipliers): $g_3(27.411)$, $g_7(4.86191)$, $g_{11}(0.0)$, $g_{13}(20.5489)$, $g_{17}(22.5562)$.

14.12

Formulation: Minimize $f = (x_2 - x_3)^2$;

subject to $h_1 = \phi^2 x_1(x_1 - x_2 + x_3)/x_2 x_3 - 1 = 0$, $h_2 = 1.0 - x_2(1 - x_1 + x_2)/\phi^3 x_1 = 0$, and design bounds $1.0E{-}10 \le x_1, x_2, x_3 \le 1000.0$

Solution for $\phi = \sqrt{2}$; Program used: IDESIGN (SQP algorithm; 8 iterations), $x^{(0)} = (1, 1, 1)$.

Optimum: $x_1^* = 2.4138$, $x_2^* = 3.4138$, $x_3^* = 3.4141$, $f^* = 1.2877\times10^{-7}$; active constraints (Lagrange multipliers): $h_1(-0.007119)$, $h_2(0.003528)$

Solution for $\phi = 2^{1/3}$; Program used: IDESIGN (SQP algorithm; 8 iterations), $x^{(0)} = (1, 1, 1)$.

optimum; $x_1^* = 2.2606$, $x_2^* = 2.8481$, $x_3^* = 2.8472$, $f^* = 8.03\times10^{-7}$. active constraints (Lagrange multipliers); $h_1(-5.47\times10^{-6})$, $h_2(1.6236\times10^{-6})$.

14.13

Formulation: units of N, kg, and cm are used.

1. Design variables: x_1 = outside diameter of the pole at the base, x_2 = outside diameter of the pole at the top, x_3 = thickness

2. Cost function: $f = \rho\int_0^H A(x)\mathrm{d}x = \rho\pi H x_3(x_1/2 + x_2/2 - x_3)$, where $A(x) = \pi x_3[d(x) - x_3]$ = cross-sectional area of the pole at a distance x from the base, and $d(x) = x_1 - (x_1 - x_2)\, x/H$ = outside diameter of the pole at a distance x from the base. Note that $d(0) = x_1$ and $d(H) = x_2$.

3. Constraints: <bending stress> $g_1 = \sigma(x)/\sigma_b - 1.0 \le 0$, where $\sigma(x) = M(x)\, d(x)/2I(x)$, $M(x)$ = bending moment at $x = P(H - x) + w(H - x)^2/2$, $I(x)$ = moment of inertia of the cross-section at $x = \pi[\{d(x)\}^4 - \{d(x) - 2x_3\}^4]/64 = A(x)[\{d(x)\}^2 + \{d(x) - 2x_3\}^2]/16$. Since the constraint g_1 depends on the distance x, we impose it at $x = 0$ and five other points that give local maximum of g_1;
<shearing stress> $g_7 = \tau(x)/\tau_s - 1.0 \le 0$, where $\tau(x) = S(x)[d^2(x) + d(x)\{d(x) - 2x_3\} + d(x) - 2x_3\}^2] / 24I(x)x_3$, $S(x)$ = shear force at $x = P + w(H - x)$. This constraint is also a function of x and it is imposed at $x = 0$ and $x = H$;
<deflection> $g_9 = \delta/10 - 1.0 \le 0$, where δ = deflection at the top = $\int_0^H [M(x)(H - x)/EI(x)]\mathrm{d}x$. δ is evaluated numerically using the Gaussian quadrature for the numerical integration;
<ratio> $g_{10} = (x_1 - x_3)/60x_3 - 1.0 \le 0$,
$g_{11} = (x_2 - x_3)/60x_3 - 1.0 \le 0$;
<design bounds> $5 \le x_1 \le 50$, $5 \le x_2 \le 50$, $0.5 \le x_3 \le 2$.

Solution: Program IDESIGN (SQP algorithm) is used. Initial design: $x_1 = 30.0$, $x_2 = 30.0$, $x_3 = 1.0$; Optimum design: $x_1^* = 48.6727$, $x_2^* = 16.7117$, $x_3^* = 0.797914$ cm, $f^* = 623.611$ kg ; (11 iterations); Active constraints (Lagrange multipliers): $g_9(311.805)$, $g_{10}(311.481)$.

14.14

Formulation: units of N, kg, and mm are used.

1. Definition of variables; x_1 = outer diameter of the pole at the base (x = 0), x_2 = outside diameter of the pole at the top ($x = H$), x_3 = thickness, x = distance from the base ($0 \le x \le H + h$), $A(x)$ = cross-sectional area of the pole = $\pi x_3[x_1 - x(x_1 - x_2)/H - x_3]$, $d(x)$ = outer diameter = $x_1 - x(x_1 - x_2)/H$, $I(x)$ = moment of inertia of the cross-section = $A(x)[2x^2(x_1 - x_2)^2/H^2 + x(x_1 - x_2)(x_3 - x_1)/H + 2x_1^2 - 4x_1x_3 + 4x_3^2]/16$, $\delta(x)$ = horizontal deflection, $M(x)$ = bending moment of the pole = $F(H + h/2 - x) + W[\delta(H) - \delta(x)]$, F = wind force = pbh, W = weight of sign = wbh, $f_a(x)$ = axial stress = $W/A(x)$, $f_b(x)$ = bending stress = $M(x)\,d(x)/2I(x)$, σ_a = allowable axial stress (approximated) = $(12/23)P_{cre}/A_e$, P_{cre} = effective critical buckling load (approximated as linear combination of P_{cr} at $x = 0$ and P_{cr} at $x = H$) = $c_1\,\pi^2 EI(0)/4H^2 + c_2\,\pi^2 EI(H)/4H^2$, (in the current computation, $c_1 = c_2 = 0.5$ are used), A_e = effective cross-sectional area (approximated as linear combination of $A(0)$ and $A(H)$) = $c_3\,A(0) + c_4\,A(H)$, where $c_3 = 1$ and $c_4 = 0$ are assumed.
2. Cost function:

f = weight of the pole = $\gamma \int_0^H A(x)dx = \gamma\pi H x_3(x_1/2 + x_2/2 - x_3) = 5.02655x_3(x_1/2 + x_2/2 - x_3)$;

3. Constraints: $g_1 = f_a(x)/\sigma_a + f_b(x)/\sigma_b - 1.0 \le 0$; Note that g_1 may have 6 maximum points, thus g_1 implies 6 constraints according to x.
4. The maximum points x are obtained numerically. $g_7 = x_1/92x_3 - 1.0 \le 0$;
5. $g_8 = x_2/92x_3 - 1.0 \le 0$;
6. $g_9 = \delta(H + h/2)/\Delta - 1.0 = \delta(H + h/2)/100 - 1.0 \le 0$, where δ

$$\left(H + h/2\right) = \int_0^H \int_0^x \frac{M'(y)}{EI(y)}dy\,dx + \frac{h}{2}\int_0^H \frac{M'(x)}{EI(x)}dx, \text{ with } M'(x) = F(h/2 + H - x).$$

4. Design bounds; $250 \le x_1 \le 1500$; $250 \le x_2 \le 1500$; $5 \le x_3 \le 100$

Solution (tapered): Program used: IDESIGN (SQP) (18 iterations with an initial design $x_1 = x_2 = 500$, $x_3 = 20$); Optimum: $x_1^* = 1419$, $x_2^* = 956.5$, $x_3^* = 15.42$, $f^* = 90894$; Active constraints (Lagrange multipliers): $g_7(45932.1)$, $g_9(45447.0)$.

Solution (constant): Additional equality constraint $h_1 = x_1 - x_2 = 0$ is imposed. Program used: IDESIGN (SQP) (15 iterations with the same initial design as above); Optimum; $x_1^* = x_2^* = 1308.4$, $x_3^* = 14.22$ mm, $f^* = 92510.8$, active constraints (Lagrange multipliers): $h_1(29531)$, $g_7(25847)$, $g_8(21265)$, $g_9(46255)$.

14.15

Formulation: x_1 = outer width of the pole at the base, x_2 = outer width of the pole at the top, x_3 = thickness, $d(x)$ = outer width of the pole at the distance x from the base = $x_1 - x(x_1 - x_2)/H$; Note that $d(0) = x_1$ and $d(H) = x_2$. $A(x)$ = cross-sectional area at $x = 4x_3[x_1 - x(x_1 - x_2)/H - x_3]$; Note that $A(0) = 4x_3(x_1 - x_3)$ and $A(H) = 4x_3(x_2 - x_3)$. $I(x)$ = moment of inertia = $[(d(x))^4 - (d(x) - 2x_3)^4]/12$, $M(x)$ = bending moment = $P(H - x) + w(H - x)^2/2$, $S(x)$ = shearing force = $P + w(H - x)$, δ = deflection at the top = $\int_0^H [M(x)(H - x)/EI(x)]dx$;

cost function: $f = \rho\int_0^H A(x)dx = 4\rho Hx_3(x_1/2 + x_2/2 - x_3)$;

constraints: $g_1 = M(x)\, d(x)/[2I(x)\sigma_b] - 1.0 \le 0$ (6 constraints are imposed referring to Exercise 14.13),

$g_7 = S(x)\, Q(x)/2x_3I(x)\tau_s - 1.0 \le 0$ (2 constraints are imposed at $x = 0$ and $x = H$), where $Q(x)= [d^3(x) - \{d(x) - 2x_3\}^3]/8$,

$g_9 = \delta/10 - 1.0 \le 0$,

$g_{10} = (x_1 - x_3)/60x_3 - 1.0 \le 0$,

$g_{11} = (x_2 - x_3)/60x_3 - 1.0 \le 0$,

$5 \le x_1 \le 50$, $5 \le x_2 \le 50$, $0.5 \le x_3 \le 2$.

Solution: Program IDESIGN (SQP algorithm) is used. Initial design: $x_1 = 30.0$, $x_2 = 30.0$, $x_3 = 1.0$. Optimum design: $x_1^* = 42.6407$ cm, $x_2^* = 14.6403$ cm, $x_3^* = 0.699028$ cm, $f^* = 609.396$ kg. (9 iterations); Active constraints (Lagrange multipliers): $g_9(304.698)$, $g_{10}(304.381)$.

14.16

Formulation: Problem formulation is exactly the same as Exercise 14.14 except the definitions of following variables and expressions. x_1 = outer width of the hollow square at the base; x_2 = outer width of the hollow square at the top; x_3 = thickness; $d(x)$ = outer width of the hollow square at a distance x from the base = $x_1 - x(x_1 - x_2)/H$; $A(x) = d^2(x) - [d(x) - 2x_3]^2$;

$I(x) = [d^4(x) - (d(x) - 2x_3)^4]/12$; f^* = cost function = $4\gamma Hx_3(x_1/2 + x_2/2 - x_3)$;

Solution: IDESIGN is used (SQP); Initial design: x_1 = 500, x_2 = 500, x_3 = 20; Optimum design: x_1^* = 1243.2, x_2^* = 837.97, x_3^* = 13.513 mm, f^* = 88822.2 kg (17 iterations); active constraints (Lagrange multipliers): g_7(44885.1), g_9(44411.1).

14.17

Case 1: u_a = 25; f_1 = 1.07301E–06, f_2 = 1.83359E–02, f_3 = 2.49977E+01, f_4 = 0.10, NIT = 39, NCF = 39, NGE = 104.

Case 2: u_a = 35; f_1 = 6.88503E–07, f_2 = 1.55413E–02, f_3 = 3.78253E+01, f_4 = 0.10, NIT = 43, NCF = 43, NGE = 109.

14.18

Case 1: u_a = 25; f_1 = 2.31697E–06, f_2 = 2.74712E–02, f_3 = 7.54602, f_4 = 0.10, NIT = 11, NCF = 11, NGE = 62.

Case 2: u_a = 35; f_1 = 2.31097E–06, f_2 = 2.72567E–02, f_3 = 7.48359, f_4 = 0.10, NIT = 9, NCF = 9, NGE = 53.

14.19

Case 1: u_a = 25; f_1 = 1.11707E–06, f_2 = 1.52134E–02, f_3 = 19.8150, f_4 = 3.3052E–02, NIT = 18, NCF = 18, NGE = 77, CPU = 220

Case 2: u_a = 35; f_1 = 6.90972E–07, f_2 = 1.36872E–02, f_3 = 31.4790, f_4 = 2.3974E–02, NIT = 20, NCF = 25, NGE = 60.

14.20

Formulation 1: M = 0.05 kg; f_1 = 1.12618E–06, f_2 = 1.79800E–02, f_3 = 33.5871, f_4 = 0.10, NIT = 48, NCF = 48, NGE = 122.

14.21

Formulation 2: M = 0.05; f_1 = 2.34615E–06, f_2 = 2.60131E–02, f_3 = 10.6663, f_4 = 0.10, NIT = 12, NCF = 12, NGE = 66.

14.22

Formulation 3: $M = 0.05$; $f_1 = 1.15097\text{E}{-06}$, $f_2 = 1.56229\text{E}{-02}$, $f_3 = 28.7509$, $f_4 = 3.2547\text{E}{-02}$, NIT = 19, NCF = 19, NGE = 73.

14.23

$f_1 = 8.53536\text{E}{-07}$, $f_2 = 1.68835\text{E}{-02}$, $f_3 = 31.7081$, $f_4 = 0.10$, NIT = 76, NCF = 78, NGE = 182.

14.24

$f_1 = 2.32229\text{E}{-06}$, $f_2 = 2.73706\text{E}{-02}$, $f_3 = 7.48085$, $f_4 = 0.10$, NIT = 16, NCF = 16, NGE = 84.

14.25

$f_1 = 8.65157\text{E}{-07}$, $f_2 = 1.45560\text{E}{-02}$, $f_3 = 25.9761$, $f_4 = 2.9336\text{E}{-02}$, NIT = 17, NCF = 18, NGE = 62.

14.26

$f_1 = 8.27815\text{E}{-07}$, $f_2 = 1.65336\text{E}{-02}$, $f_3 = 28.2732$, $f_4 = 0.10$, NIT = 35, NCF = 35, NGE = 89.

14.27

$f_1 = 2.31300\text{E}{-06}$, $f_2 = 2.72300\text{E}{-02}$, $f_3 = 6.86705$, $f_4 = 0.10$, NIT = 9, NCF = 9, NGE = 49.

14.28

$f_1 = 8.39032\text{E}{-07}$, $f_2 = 1.43298\text{E}{-02}$, $f_3 = 25.5695$, $f_4 = 2.9073\text{E}{-02}$, NIT = 19, NCF = 20, NGE = 62.

14.29

Formulation: Use the artificial variable as a cost function; design variables: k = spring constant, c = damping coefficient, A = artificial variable;

cost function: $f = A$;

constraints: $5\ddot{x} + c\dot{x} + kx = 0$, with $x(0) = 0$ and $\dot{x}(0) = 5$; $|x(t)| \le 0.05$ and $|\ddot{x}(t)| \le A$ for $0 \le t \le 10$; $1000 \le k \le 3000$, $0 \le c \le 300$, $0.01 \le A \le 1000$ (arbitrarily chosen)

Solution: IDESIGN is used. Dynamic constraints are imposed at 100 time grid points. Initial design: $k^{(0)} = 2000$, $c^{(0)} = 200$, $A^{(0)} = 50$. Optimum: $k* = 2084.08$, $c* = 300$ (upper limit), $f^* = A* = 1.64153$ (22 iterations).

Section 14.8 Optimum Design of Tension Members

14.31

Solve the following problem using the Excel Solver:

Solve the W-shape optimization problem of Section 14.8 where a W14 shape is desired.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

$\mathbf{x} = (d, b_f, t_f, t_w)$

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.31

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, $A_e = UA_n$, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, in
P_n	Nominal axial strength, kips
P_a	Required strength, kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

<u>**Solution**</u>

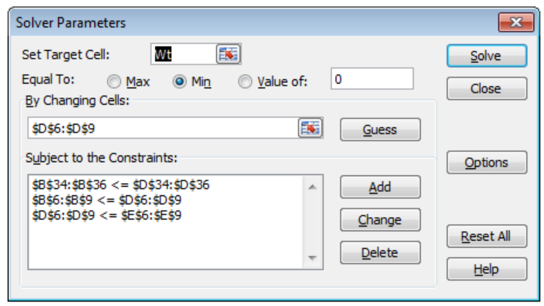
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

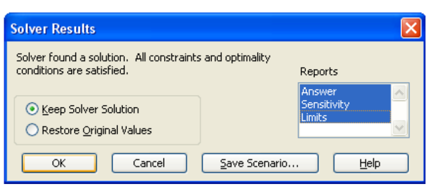
(3) The answer report shows that for initial design variable values of d=15.05, bf=10.5, tf=1.1125, and tw=0.705 a solution of d=13.7, bf=5, tf=0.335, and tw=0.23, which gives an objective function value of 21.6, is obtained. A W14x22 shape is selected which has allowable strengths of 194 kips and 158 kips in the yielding and rupture limit states, respectively.

	A	B	C	D	E	F
1	**Exercise 6.22**					
2						
3	**Shape:**	W14				
4						
5	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
6	Depth	13.7	d	13.7	16.4	in
7	Flange width	5	bf	5	16	in
8	Flange thickness	0.335	tf	0.335	1.89	in
9	Web thickness	0.23	tw	0.23	1.18	in
10						
11	**2. Parameter name**	**Symbol**	**Value**	**Units**		
12	Unsupported length	L	=25*12	in		
13	Yield stress	Fy	=50	ksi		
14	Ultimate stress	Fu	65	ksi		
15	Required strength	Pa	150	kips		
16	Factor of safety for tension (yielding)	FSt	=5/3			
17	Factor of safety for tension (rupture)	FSr	2			
18	Density of steel	gamma	=0.283	lb/in^3		
19						
20	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
21	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
22	Effective net area	Ae	=0.75*Ag	in^2		
23	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
24	Least radius of gyration	ry	=SQRT(Iy/Ag)	in		
25	Nominal axial yield strength	Pny	=Fy*Ag	kips		
26	Allowable axial yield strength	Pay	=Pny/FSt	kips		
27	Nominal axial rupture strength	Pnr	=Fu*Ae	kips		
28	Allowable axial rupture strength	Par	=Pnr/FSr	kips		
29						
30	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
31	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
32						
33	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
34	Yield limit state	=Pa/Pa	<	=(Pny)/(FSt*Pa)		
35	Rupture limit state	=Pa/Pa	<	=(Pnr)/(FSr*Pa)		
36	Slenderness ratio	=(L/ry)/300	<	=300/300		

1



2



3

	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**			
2	**Worksheet: [Exercise 6.22.xlsx]Sheet1**			
3	**Report Created: 7/10/2010 11:39:16 AM**			
4				
5				
6	Target Cell (Min)			
7	**Cell**	**Name**	**Original Value**	**Final Value**
8	$C$31 Wt		110.0444085	21.5540724
9				
10				
11	Adjustable Cells			
12	**Cell**	**Name**	**Original Value**	**Final Value**
13	$D$6 d		15.05	13.7
14	$D$7 bf		10.5	5
15	$D$8 tf		1.1125	0.335
16	$D$9 tw		0.705	0.23

14.32

Solve the following problem using the Excel Solver:

Solve the W-shape optimization problem of Section 14.8 where a W12 shape is desired.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

$\mathbf{x} = (d, b_f, t_f, t_w)$

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.32

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, A_e=UA_n, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, in
P_n	Nominal axial strength, kips
P_a	Required strength, kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6 F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5 F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

Solution

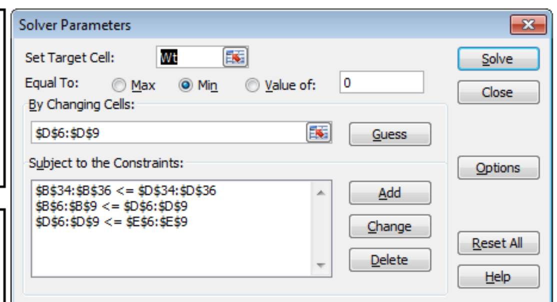
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

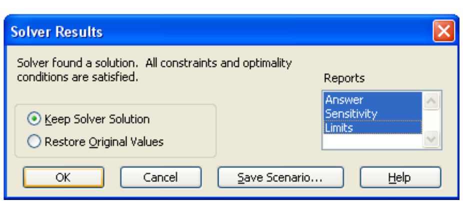
(3) The answer report shows that for initial design variable values of d=13.3, bf=8.385, tf=1.0625, and tw=0.69 a solution of d=11.9, bf=4.85, tf=0.322, and tw=0.269, which gives an objective function value of 20.9, is obtained. A W12x22 shape is selected which has allowable strengths of 194kips and 158kips in the yielding and rupture limit states, respectively.

	A	B	C	D	E	F
1	**Exercise 6.23**					
2						
3	**Shape:**	W12				
4						
5	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
6	Depth	11.9	d	11.9000000158262	14.7	in
7	Flange width	3.97	bf	4.85397031182653	12.8	in
8	Flange thickness	0.225	tf	0.321898946083892	1.9	in
9	Web thickness	0.2	tw	0.269084317785002	1.18	in
10						
11	**2. Parameter name**	**Symbol**	**Value**	**Units**		
12	Unsupported length	L	=25*12	in		
13	Yield stress	Fy	=50	ksi		
14	Ultimate stress	Fu	65	ksi		
15	Required strength	Pa	150	kips		
16	Factor of safety for tension (yielding)	FSt	=5/3			
17	Factor of safety for tension (rupture)	FSr	2			
18	Density of steel	gamma	=0.283	lb/in^3		
19						
20	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
21	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
22	Effective net area	Ae	=0.75*Ag	in^2		
23	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
24	Least radius of gyration	ry	=SQRT(Iy/Ag)	in		
25	Nominal axial yield strength	Pny	=Fy*Ag	kips		
26	Allowable axial yield strength	Pay	=Pny/FSt	kips		
27	Nominal axial rupture strength	Pnr	=Fu*Ae	kips		
28	Allowable axial rupture strength	Par	=Pnr/FSr	kips		
29						
30	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
31	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
32						
33	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
34	Yield limit state	=Pa/Pa	<	=(Pny)/(FSt*Pa)		
35	Rupture limit state	=Pa/Pa	<	=(Pnr)/(FSr*Pa)		
36	Slenderness ratio	=(L/ry)/300	<	=300/300		

1



2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.23.xlsx]Sheet1**				
3	**Report Created: 7/10/2010 11:44:32 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$31	Wt	86.6960595	20.89845193
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$6	d	13.3	11.90000002
14		$D$7	bf	8.385	4.853970312
15		$D$8	tf	1.0625	0.321898946
16		$D$9	tw	0.69	0.269084318

14.33

Solve the following problem using the Excel Solver:

Solve the W-shape optimization problem of Section 14.8 where a W8 shape is desired, the required strength P_a for the member is 200 kips, the length of the member is 13 ft, and the material is A992 Grade 50 steel.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

$\mathbf{x} = (d, b_f, t_f, t_w)$

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.33

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, A_e=UA_n, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, 156 in
P_n	Nominal axial strength, kips
P_a	Required strength, 200 kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

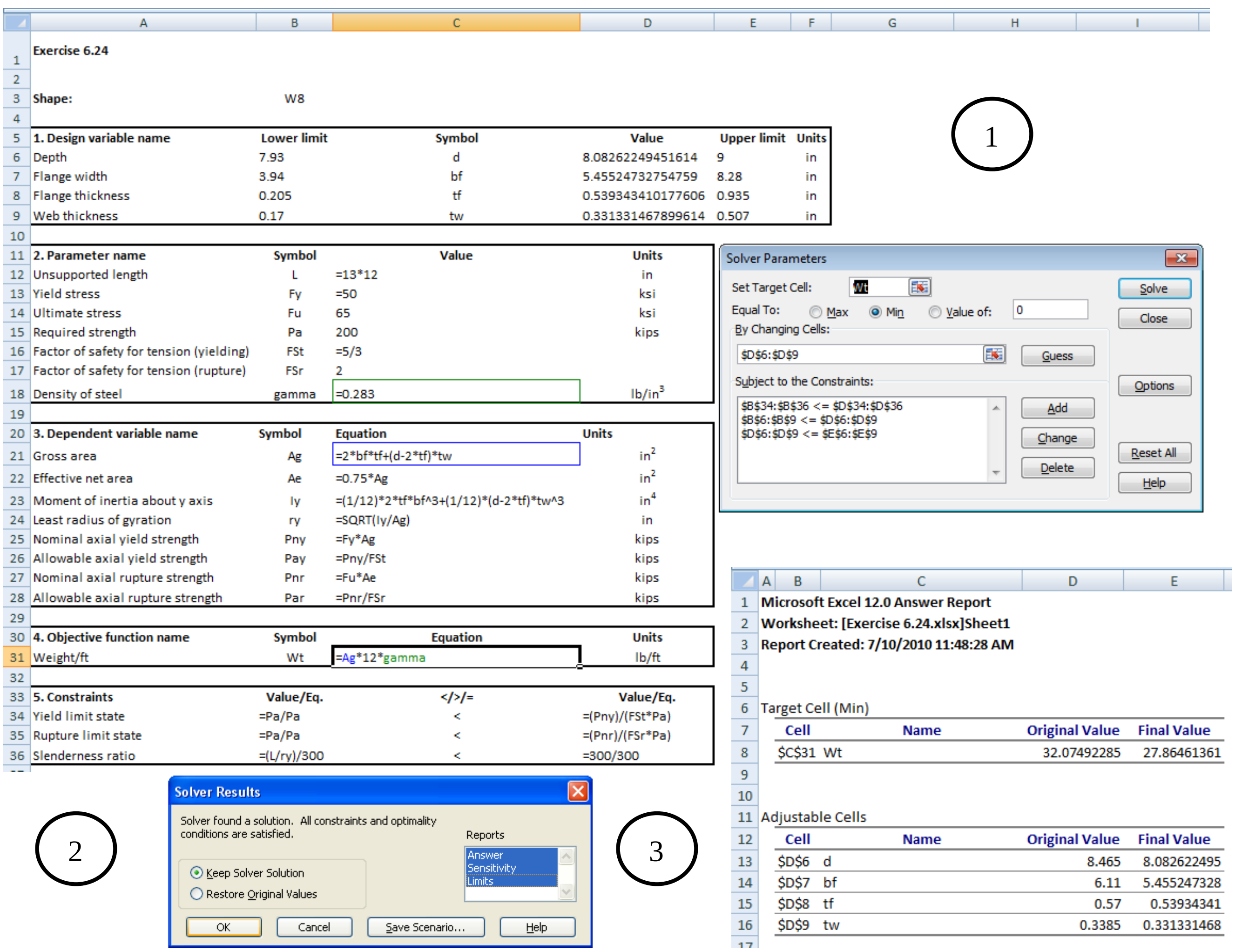
$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6 F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5 F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=8.465, bf=6.11, tf=0.57, and tw=0.3385 a solution of d=8.083, bf=5.46, tf=0.539, and tw=0.331, which gives an objective function value of 27.9, is obtained. A W8x28 shape is selected which has allowable strengths of 247kips and 201kips in the yielding and rupture limit states, respectively.



	A	B	C	D	E	F
1	Exercise 6.24					
2						
3	Shape:	W8				
4						
5	1. Design variable name	Lower limit	Symbol	Value	Upper limit	Units
6	Depth	7.93	d	8.08262249451614	9	in
7	Flange width	3.94	bf	5.45524732754759	8.28	in
8	Flange thickness	0.205	tf	0.539343410177606	0.935	in
9	Web thickness	0.17	tw	0.331331467899614	0.507	in
10						
11	2. Parameter name	Symbol	Value	Units		
12	Unsupported length	L	=13*12	in		
13	Yield stress	Fy	=50	ksi		
14	Ultimate stress	Fu	65	ksi		
15	Required strength	Pa	200	kips		
16	Factor of safety for tension (yielding)	FSt	=5/3			
17	Factor of safety for tension (rupture)	FSr	2			
18	Density of steel	gamma	=0.283	lb/in^3		
19						
20	3. Dependent variable name	Symbol	Equation	Units		
21	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
22	Effective net area	Ae	=0.75*Ag	in^2		
23	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
24	Least radius of gyration	ry	=SQRT(Iy/Ag)	in		
25	Nominal axial yield strength	Pny	=Fy*Ag	kips		
26	Allowable axial yield strength	Pay	=Pny/FSt	kips		
27	Nominal axial rupture strength	Pnr	=Fu*Ae	kips		
28	Allowable axial rupture strength	Par	=Pnr/FSr	kips		
29						
30	4. Objective function name	Symbol	Equation	Units		
31	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
32						
33	5. Constraints	Value/Eq.	</>/=	Value/Eq.		
34	Yield limit state	=Pa/Pa	<	=(Pny)/(FSt*Pa)		
35	Rupture limit state	=Pa/Pa	<	=(Pnr)/(FSr*Pa)		
36	Slenderness ratio	=(L/ry)/300	<	=300/300		

	A	B	C	D	E
1		Microsoft Excel 12.0 Answer Report			
2		Worksheet: [Exercise 6.24.xlsx]Sheet1			
3		Report Created: 7/10/2010 11:48:28 AM			
4					
5					
6		Target Cell (Min)			
7		Cell	Name	Original Value	Final Value
8		$C$31	Wt	32.07492285	27.86461361
9					
10					
11		Adjustable Cells			
12		Cell	Name	Original Value	Final Value
13		$D$6	d	8.465	8.082622495
14		$D$7	bf	6.11	5.455247328
15		$D$8	tf	0.57	0.53934341
16		$D$9	tw	0.3385	0.331331468

14.34

Solve the following problem using the Excel Solver:

Solve the W-shape optimization problem of Section 14.8 where a W10 shape is desired, the required strength P_a for the member is 200 kips, the length of the member is 13 ft, and the material is A992 Grade 50 steel.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

$\mathbf{x} = (d, b_f, t_f, t_w)$

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.38

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, A_e=UA_n, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, 156 in
P_n	Nominal axial strength, kips
P_a	Required strength, 200 kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

Solution

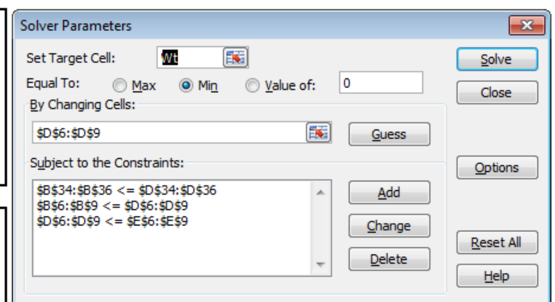
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

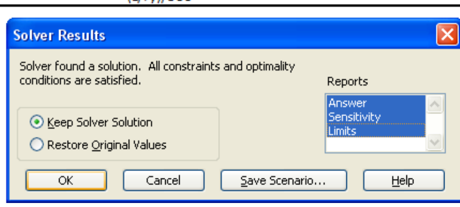
(3) The answer report shows that for initial design variable values of d=10.565, bf=7.18, tf=0.73, and tw=0.4725 a solution of d=9.73, bf=4.59, tf=0.516, and tw=0.398, which gives an objective function value of 27.9, is obtained. A W10x30 shape is selected which has allowable strengths of 265kips and 215kips in the yielding and rupture limit states, respectively.

	A	B	C	D	E	F
1	**Exercise 6.25**					
2						
3	**Shape:**	W10				
4						
5	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
6	Depth	9.73	d	9.73000030684637	11.4	in
7	Flange width	3.96	bf	4.59349665252135	10.4	in
8	Flange thickness	0.21	tf	0.516320360969869	1.25	in
9	Web thickness	0.19	tw	0.398016283929889	0.755	in
10						
11	**2. Parameter name**	**Symbol**	**Value**	**Units**		
12	Unsupported length	L	=13*12	in		
13	Yield stress	Fy	=50	ksi		
14	Ultimate stress	Fu	65	ksi		
15	Required strength	Pa	200	kips		
16	Factor of safety for tension (yielding)	FSt	=5/3			
17	Factor of safety for tension (rupture)	FSr	2			
18	Density of steel	gamma	=0.283	lb/in^3		
19						
20	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
21	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
22	Effective net area	Ae	=0.75*Ag	in^2		
23	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
24	Least radius of gyration	ry	=SQRT(Iy/Ag)	in		
25	Nominal axial yield strength	Pny	=Fy*Ag	kips		
26	Allowable axial yield strength	Pay	=Pny/FSt	kips		
27	Nominal axial rupture strength	Pnr	=Fu*Ae	kips		
28	Allowable axial rupture strength	Par	=Pnr/FSr	kips		
29						
30	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
31	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
32						
33	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
34	Yield limit state	=Pa/Pa	<	=(Pny)/(FSt*Pa)		
35	Rupture limit state	=Pa/Pa	<	=(Pnr)/(FSr*Pa)		
36	Slenderness ratio	=(L/ry)/300	<	=300/300		

1



2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.25.xlsx]Sheet1**				
3	**Report Created: 7/10/2010 11:51:48 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$31	Wt	50.20956285	27.86459581
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$6	d	10.565	9.730000307
14		$D$7	bf	7.18	4.593496653
15		$D$8	tf	0.73	0.516320361
16		$D$9	tw	0.4725	0.398016284

Section 14.9 Optimum Design of Compression Members

14.35

Solve the following problem using the Excel Solver:

Solve the W-shape optimization problem of Section 14.9 where a W14 shape is desired.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

x = (d, b_f, t_f, t_w)

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.35

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, $A_e=UA_n$, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, in
P_n	Nominal axial strength, kips
P_a	Required strength, kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

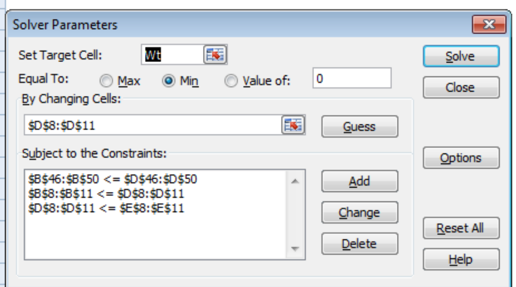
(3) The answer report shows that for initial design variable values of d=15.05, bf=10.5, tf=1.1125, and tw=0.705 a solution of d=16.4, bf=16, tf=1.663, and tw=0.970, which gives an objective function value of 224, is obtained. A W14x233 shape is selected which has an available strength of 1529 kips.

	A	B	C	D	E	F
1	**Exercise 6.26**					
2						
3	**Shape:**	W14		1		
4	**Assume Buckling Type:**	Inelastic	*Correct Assumption*			
5	**Assume Buckling Axis:**	x_axis	*Correct Assumption*			
6						
7	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
8	Depth	13.7	d	16.4	16.4	in
9	Flange width	5	bf	16	16	in
10	Flange thickness	0.335	tf	1.66244562582188	1.89	in
11	Web thickness	0.23	tw	0.969490735792893	1.18	in
12						
13	**2. Parameter name**	**Symbol**	**Value**	**Units**		
14	Yield stress	Fy	=50	ksi		
15	Mudulus of elasticity	E	=29000	ksi		
16	Effective length factor (x) axis	Kx	1			
17	Effective length factor (y) axis	Ky	1			
18	Laterally unsupported length (x) axis	Lx	420	in		
19	Laterally unsupported length (y) axis	Ly	180	in		
20	Required strength	Pa	1500	kips		
21	Factor of safety for compression (yielding)	FSc	=5/3			
22	Density of steel	γ, gamma	0.283	lb/in^3		
23						
24	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
25	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
26	Moment of inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4		
27	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
28	Radius of gyration about (x) axis	rx_	=SQRT(Ix/Ag)	in		
29	Radius of gyration about (y) axis	ry_	=SQRT(Iy/Ag)	in		
30	Limiting value of slenderness ratio	λ_e, lamda_e	=4.71*SQRT(E/Fy)			
31	Slenderness ratio about (x) axis	λ_x, lamda_x	=(Kx*Lx)/(rx_)			
32	Slenderness ratio about (y) axis	λ_y, lamda_y	=(Ky*Ly)/(ry_)			
33	Slenderness ratio	λ, lamda	=MAX(lamda_x,lamda_y)			
34	Euler stress about the (x) axis	Fex	=(PI()^2*E)/(lamda_x^2)	ksi		
35	Euler stress about the (y) axis	Fey	=(PI()^2*E)/(lamda_y^2)	ksi		
36	Euler stress	Fe	=(PI()^2*E)/(lamda^2)	ksi		
37	Critical stress for the member about (x) axis	Fcrx	=(0.658^(Fy/Fex))*Fy	ksi		

6.26

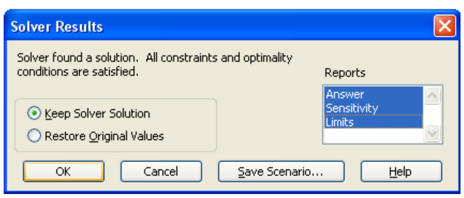
Continued.

	A	B	C	D
38	Critical stress for the member about (y) axis	Fcry	=(0.658^(Fy/Fey))*Fy	ksi
39	Critical stress for member	Fcr	=(0.658^(Fy/Fe))*Fy	ksi
40	Nominal axial strength	Pn	=Fcr*Ag	kips
41				
42	**4. Objective function name**	**Symbol**	**Equation**	**Units**
43	Weight/ft	Wt	=Ag*12*gamma	lb/ft
44				
45	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
46	Slenderness ratio	=lamda_x/lamda_e	<	=1
47	Slenderness ratio	=lamda_x/lamda_e	<	=1
48	Strength	1	<	=0.6*Fcrx*Ag/Pa
49	local buckling	=((d-2*tf)/(tw))/(0.56*SQRT((E)/(Fy)))	<	=1
50	local buckling	=((bf)/(2*tf))/(1.49*SQRT((E)/(Fy)))	<	=1



1

2



3

Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.26.xlsx]Sheet1
Report Created: 7/12/2010 3:46:50 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$43	Wt	110.0444085	223.7096554

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$8	d	15.05	16.4
$D$9	bf	10.5	16
$D$10	tf	1.1125	1.662445626
$D$11	tw	0.705	0.969490736

14.36

Solve the following problem using the Excel Solver:

Solve the W-shape optimization problem of Section 14.9 where a W12 shape is desired and required strength P_a is 1000 kips.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

x = (d, b_f, t_f, t_w)

The optimization function for the mass minimization problem is given as

$$f = 12\gamma A_g \text{, lbs/ft}$$

TABLE E14.36

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, A_e=UA_n, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, in
P_n	Nominal axial strength, kips
P_a	Required strength, 1000 kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=13.3, bf=8.385, tf=1.0625, and tw=0.69 a solution of d=14.7, bf=12.8, tf=1.441, and tw=0.876, which gives an objective function value of 160.4, is obtained. A W12x170 shape is selected which has an available strength of 1012 kips.

1

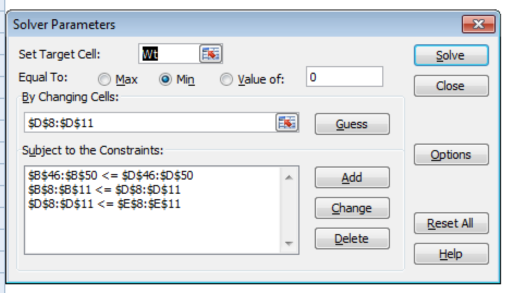
	A	B	C	D	E	F
1	**Exercise 6.27**					
2						
3	**Shape:**	W12				
4	**Assume Buckling Type:**	Inelastic	*Correct Assumption*			
5	**Assume Buckling Axis:**	x_axis	*Correct Assumption*			
6						
7	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
8	Depth	11.9	d	14.7	14.7	in
9	Flange width	3.97	bf	12.8	12.8	in
10	Flange thickness	0.225	tf	1.44049676371642	1.9	in
11	Web thickness	0.2	tw	0.876353492718897	1.18	in
12						
13	**2. Parameter name**	**Symbol**	**Value**	**Units**		
14	Yield stress	Fy	=50	ksi		
15	Mudulus of elasticity	E	=29000	ksi		
16	Effective length factor (x) axis	Kx	1			
17	Effective length factor (y) axis	Ky	1			
18	Laterally unsupported length (x) axis	Lx	420	in		
19	Laterally unsupported length (y) axis	Ly	180	in		
20	Required strength	Pa	1000	kips		
21	Factor of safety for compression (yielding)	FSc	=5/3			
22	Density of steel	γ, gamma	0.283	lb/in^3		
23						
24	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
25	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
26	Moment of inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4		
27	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
28	Radius of gyration about (x) axis	rx_	=SQRT(Ix/Ag)	in		
29	Radius of gyration about (y) axis	ry_	=SQRT(Iy/Ag)	in		
30	Limiting value of slenderness ratio	λ_e, lamda_e	=4.71*SQRT(E/Fy)			
31	Slenderness ratio about (x) axis	λ_x, lamda_x	=(Kx*Lx)/(rx_)			
32	Slenderness ratio about (y) axis	λ_y, lamda_y	=(Ky*Ly)/(ry_)			
33	Slenderness ratio	λ, lamda	=MAX(lamda_x,lamda_y)			
34	Euler stress about the (x) axis	Fex	=(PI()^2*E)/(lamda_x^2)	ksi		
35	Euler stress about the (y) axis	Fey	=(PI()^2*E)/(lamda_y^2)	ksi		

6.27

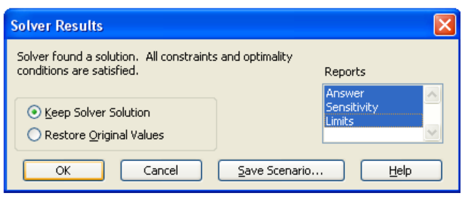
Continued.

	A	B	C	D	E	F
36	Euler stress	Fe	=(PI()^2*E)/(lamda^2)	ksi		
37	Critical stress for the member about (x) axis	Fcrx	=(0.658^(Fy/Fex))*Fy	ksi		
38	Critical stress for the member about (y) axis	Fcry	=(0.658^(Fy/Fey))*Fy	ksi		
39	Critical stress for member	Fcr	=(0.658^(Fy/Fe))*Fy	ksi		
40	Nominal axial strength	Pn	=Fcr*Ag	kips		
41						
42	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
43	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
44						
45	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
46	Slenderness ratio	=lamda_x/lamda_e	<	=1		
47	Slenderness ratio	=lamda_x/lamda_e	<	=1		
48	Strength	1	<	=0.6*Fcrx*Ag/Pa		
49	local buckling	=((d-2*tf)/(tw))/(0.56*SQRT((E)/(Fy)))	<	=1		
50	local buckling	=((bf)/(2*tf))/(1.49*SQRT((E)/(Fy)))	<	=1		

1



2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.27.xlsx]Sheet1**				
3	**Report Created: 7/13/2010 8:55:18 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$43	Wt	86.6960595	160.4078348
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$8	d	13.3	14.7
14		$D$9	bf	8.385	12.8
15		$D$10	tf	1.0625	1.440496764
16		$D$11	tw	0.69	0.876353493
17					

14.37

Solve the following problem using the Excel Solver:

Design a compression member to carry a load of 400 kips. The length of the member is 26 feet, and the material is A572 Grade 50 steel. The member is not braced. Select W18 shape.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

x = (d, b_f, t_f, t_w)

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.37

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, A_e=UA_n, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, 312 in
P_n	Nominal axial strength, kips
P_a	Required strength, 400 kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

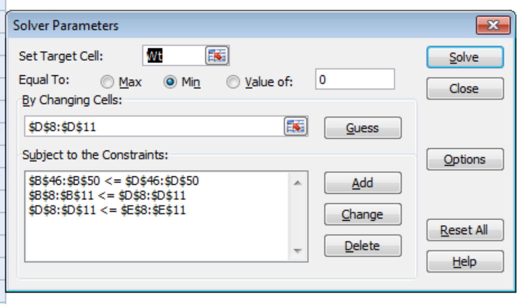
(3) The answer report shows that for initial design variable values of d=19.4, bf=8.85, tf=1.2675, and tw=0.73 a solution of d=17.7, bf=11.7, tf=1.376, and tw=1.108, which gives an objective function value of 165.6, is obtained. A W18x175 shape is selected which has an available strength of 603 kips.

	A	B	C	D	E	F
1	**Exercise 6.28**			1		
2						
3	**Shape:**	W18				
4	**Assume Buckling Type:**	Inelastic	*Correct Assumption*			
5	**Assume Buckling Axis:**	y_axis	*Correct Assumption*			
6						
7	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
8	Depth	17.7	d	17.7	21.1	in
9	Flange width	6	bf	11.7	11.7	in
10	Flange thickness	0.425	tf	1.37553174458809	2.11	in
11	Web thickness	0.3	tw	1.10842969293487	1.16	in
12						
13	**2. Parameter name**	**Symbol**	**Value**	**Units**		
14	Yield stress	Fy	=50	ksi		
15	Mudulus of elasticity	E	=29000	ksi		
16	Effective length factor (x) axis	Kx	1			
17	Effective length factor (y) axis	Ky	1			
18	Laterally unsupported length (x) axis	Lx	=26*12	in		
19	Laterally unsupported length (y) axis	Ly	=26*12	in		
20	Required strength	Pa	400	kips		
21	Factor of safety for compression (yielding)	FSc	=5/3			
22	Density of steel	γ, gamma	0.283	lb/in^3		
23						
24	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
25	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
26	Moment of inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4		
27	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
28	Radius of gyration about (x) axis	rx_	=SQRT(Ix/Ag)	in		
29	Radius of gyration about (y) axis	ry_	=SQRT(Iy/Ag)	in		
30	Limiting value of slenderness ratio	λ_e, lamda_e	=4.71*SQRT(E/Fy)			
31	Slenderness ratio about (x) axis	λ_x, lamda_x	=(Kx*Lx)/(rx_)			
32	Slenderness ratio about (y) axis	λ_y, lamda_y	=(Ky*Ly)/(ry_)			
33	Slenderness ratio	λ, lamda	=MAX(lamda_x,lamda_y)			
34	Euler stress about the (x) axis	Fex	=(PI()^2*E)/(lamda_x^2)	ksi		
35	Euler stress about the (y) axis	Fey	=(PI()^2*E)/(lamda_y^2)	ksi		

6.28

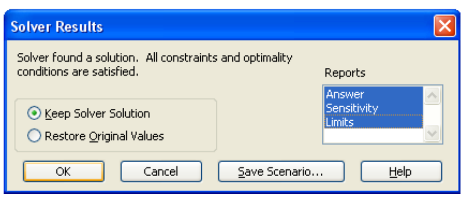
Continued.

	A	B	C	D
36	Euler stress	Fe	=(PI()^2*E)/(lamda^2)	ksi
37	Critical stress for the member about (x) axis	Fcrx	=(0.658^(Fy/Fex))*Fy	ksi
38	Critical stress for the member about (y) axis	Fcry	=(0.658^(Fy/Fey))*Fy	ksi
39	Critical stress for member	Fcr	=(0.658^(Fy/Fe))*Fy	ksi
40	Nominal axial strength	Pn	=Fcr*Ag	kips
41				
42	**4. Objective function name**	**Symbol**	**Equation**	**Units**
43	Weight/ft	Wt	=Ag*12*gamma	lb/ft
44				
45	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
46	Slenderness ratio	=lamda_y/lamda_e	<	=1
47	Slenderness ratio	=lamda_y/lamda_e	<	=1
48	Strength	1	<	=0.6*Fcry*Ag/Pa
49	local buckling	=((d-2*tf)/(tw))/(0.56*SQRT((E)/(Fy)))	<	=1
50	local buckling	=((bf)/(2*tf))/(1.49*SQRT((E)/(Fy)))	<	=1



1

2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.28.xlsx]Sheet1**				
3	**Report Created: 7/13/2010 9:29:06 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$43	Wt	117.9980952	165.5797498
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$8	d	19.4	17.7
14		$D$9	bf	8.85	11.7
15		$D$10	tf	1.2675	1.375531745
16		$D$11	tw	0.73	1.108429693

14.38

Solve the following problem using the Excel Solver:

Design a compression member to carry a load of 200 kips. The length of the member is 13 feet, and the material is A572 Grade 50 steel. The member is not braced. Select W14 shape.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

$\mathbf{x} = (d, b_f, t_f, t_w)$

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.38

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, A_e=UA_n, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, 156 in
P_n	Nominal axial strength, kips
P_a	Required strength, 200 kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

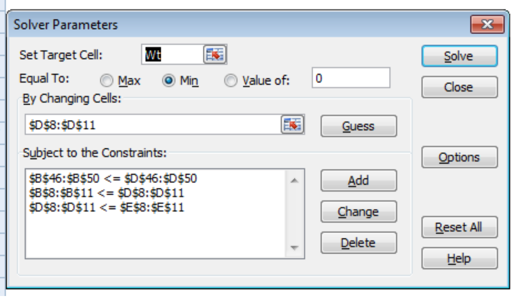
(3) The answer report shows that for initial design variable values of d=15.05, bf=10.5, tf=1.1125, and tw=0.705 a solution of d=13.7, bf=8.44, tf=0.335, and tw=0.966, which gives an objective function value of 62.0, is obtained. A W14x68 shape is selected which has an available strength of 446.3 kips.

	A	B	C	D	E	F
1	**Exercise 6.29**			1		
2						
3	**Shape:**	W14				
4	**Assume Buckling Type:**	Inelastic	*Correct Assumption*			
5	**Assume Buckling Axis:**	y_axis	*Correct Assumption*			
6						
7	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
8	Depth	13.7	d	13.7	16.4	in
9	Flange width	5	bf	8.43614286494759	16	in
10	Flange thickness	0.335	tf	0.335	1.89	in
11	Web thickness	0.23	tw	0.966145180798421	1.18	in
12						
13	**2. Parameter name**	**Symbol**	**Value**	**Units**		
14	Yield stress	Fy	=50	ksi		
15	Mudulus of elasticity	E	=29000	ksi		
16	Effective length factor (x) axis	Kx	1			
17	Effective length factor (y) axis	Ky	1			
18	Laterally unsupported length (x) axis	Lx	=13*12	in		
19	Laterally unsupported length (y) axis	Ly	=13*12	in		
20	Required strength	Pa	200	kips		
21	Factor of safety for compression (yielding)	FSc	=5/3			
22	Density of steel	γ, gamma	0.283	lb/in^3		
23						
24	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
25	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
26	Moment of inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4		
27	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
28	Radius of gyration about (x) axis	rx_	=SQRT(Ix/Ag)	in		
29	Radius of gyration about (y) axis	ry_	=SQRT(Iy/Ag)	in		
30	Limiting value of slenderness ratio	λ_e, lamda_e	=4.71*SQRT(E/Fy)			
31	Slenderness ratio about (x) axis	λ_x, lamda_x	=(Kx*Lx)/(rx_)			
32	Slenderness ratio about (y) axis	λ_y, lamda_y	=(Ky*Ly)/(ry_)			
33	Slenderness ratio	λ, lamda	=MAX(lamda_x,lamda_y)			
34	Euler stress about the (x) axis	Fex	=(PI()^2*E)/(lamda_x^2)	ksi		
35	Euler stress about the (y) axis	Fey	=(PI()^2*E)/(lamda_y^2)	ksi		

6.29

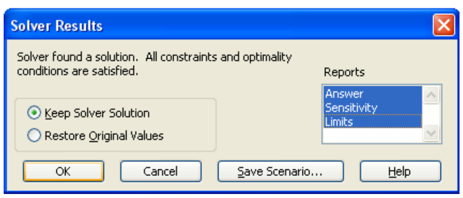
Continued.

	A	B	C	D	E	F
36	Euler stress	Fe	=(PI()^2*E)/(lamda^2)	ksi		
37	Critical stress for the member about (x) axis	Fcrx	=(0.658^(Fy/Fex))*Fy	ksi		
38	Critical stress for the member about (y) axis	Fcry	=(0.658^(Fy/Fey))*Fy	ksi		
39	Critical stress for member	Fcr	=(0.658^(Fy/Fe))*Fy	ksi		
40	Nominal axial strength	Pn	=Fcr*Ag	kips		
41						
42	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
43	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
44						
45	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
46	Slenderness ratio	=lamda_y/lamda_e	<	=1		
47	Slenderness ratio	=lamda_y/lamda_e	<	=1		
48	Strength	1	<	=0.6*Fcry*Ag/Pa		
49	local buckling	=((d-2*tf)/(tw))/(0.56*SQRT((E)/(Fy)))	<	=1		
50	local buckling	=((bf)/(2*tf))/(1.49*SQRT((E)/(Fy)))	<	=1		



1

2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.29.xlsx]Sheet1**				
3	**Report Created: 7/13/2010 9:40:35 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$43	Wt	110.0444085	61.9467329
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$8	d	15.05	13.7
14		$D$9	bf	10.5	8.436142865
15		$D$10	tf	1.1125	0.335
16		$D$11	tw	0.705	0.966145181
17					

14.39

Solve the following problem using the Excel Solver:

Design a compression member to carry a load of 200 kips. The length of the member is 13 feet, and the material is A572 Grade 50 steel. The member is not braced. Select W12 shape.

$$A_g = 2b_f t_f + (d - 2t_f)t_w$$

$$I_y = 2\frac{(t_f b_f^3)}{12} + \frac{(d - 2t_f)t_w^3}{12}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

Design variables for the W-shape optimization problem are defined as a vector

x = (d, b_f, t_f, t_w)

The optimization function for the mass minimization problem is given as

$f = 12\gamma A_g$, lbs/ft

TABLE E14.39

Notation	*Data*
A_g	Gross area of the section, in^2
A_n	Net area (gross area less cross-sectional areas due to bolt holes), in^2
A_e	Effective net area, A_e=UA_n, in^2
b_f	Width of flange, in
d	Depth of section, in
F_y	Specified minimum yield stress, 50 ksi for A992 steel, ksi
F_u	Specified minimum ultimate stress, 65 ksi for A992 steel, ksi
L	Laterally supported length of member, 156 in
P_n	Nominal axial strength, kips
P_a	Required strength, 200 kips
r_y	Least radius of gyration, in
t_f	Thickness of flange, in
t_w	Thickness of web, in
U	Shear lag coefficient: reduction coefficient for net area
x	Distance for plane of shear transfer to centroid of tension member cross section, in
Ω_t	Factor of safety for tension, 1.67 and 2.00, for yielding and rupture, respectively
γ	Density of steel, 0.283 lb/in^3

The constraints for the W-shape design problem are given as

$$P_a \le \frac{P_{ny}}{\Omega_t};\ P_{ny} = F_y A_g \rightarrow P_a \le 0.6F_y A_g\ ,\ \ P_a \le \frac{P_{nr}}{\Omega_t};\ P_{nr} = F_u A_e \rightarrow P_a \le 0.5F_u A_e\ ,\ \ \frac{L}{r_y} \le 300$$

<u>Solution</u>

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

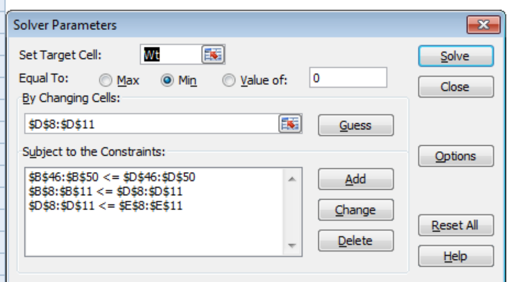
(3) The answer report shows that for initial design variable values of d=13.7, bf=8.44, tf=0.335, and tw=0.966 a solution of d=13.2, bf=8.28, tf=0.333, and tw=0.926, which gives an objective function value of 58.0, is obtained. A W14x61 shape is selected which has an available strength of 398 kips.

	A	B	C	D	E	F
1	**Exercise 6.30**			1		
2						
3	**Shape:**	W12				
4	**Assume Buckling Type:**	Elastic	*Correct Assumption*			
5	**Assume Buckling Axis:**	y_axis	*Correct Assumption*			
6						
7	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
8	Depth	11.9	d	13.1595465967123	14.7	in
9	Flange width	3.97	bf	8.27943085601131	12.8	in
10	Flange thickness	0.225	tf	0.332898667892536	1.9	in
11	Web thickness	0.2	tw	0.926383395130776	1.18	in
12						
13	**2. Parameter name**	**Symbol**	**Value**	**Units**		
14	Yield stress	Fy	=50	ksi		
15	Mudulus of elasticity	E	=29000	ksi		
16	Effective length factor (x) axis	Kx	1			
17	Effective length factor (y) axis	Ky	1			
18	Laterally unsupported length (x) axis	Lx	=13*12	in		
19	Laterally unsupported length (y) axis	Ly	=13*12	in		
20	Required strength	Pa	200	kips		
21	Factor of safety for compression (yielding)	FSc	=5/3			
22	Density of steel	γ, gamma	0.283	lb/in^3		
23						
24	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
25	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
26	Moment of inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4		
27	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
28	Radius of gyration about (x) axis	rx_	=SQRT(Ix/Ag)	in		
29	Radius of gyration about (y) axis	ry_	=SQRT(Iy/Ag)	in		
30	Limiting value of slenderness ratio	λ_e, lamda_e	=4.71*SQRT(E/Fy)			
31	Slenderness ratio about (x) axis	λ_x, lamda_x	=(Kx*Lx)/(rx_)			
32	Slenderness ratio about (y) axis	λ_y, lamda_y	=(Ky*Ly)/(ry_)			
33	Slenderness ratio	λ, lamda	=MAX(lamda_x,lamda_y)			
34	Euler stress about the (x) axis	Fex	=(PI()^2*E)/(lamda_x^2)	ksi		
35	Euler stress about the (y) axis	Fey	=(PI()^2*E)/(lamda_y^2)	ksi		

6.30

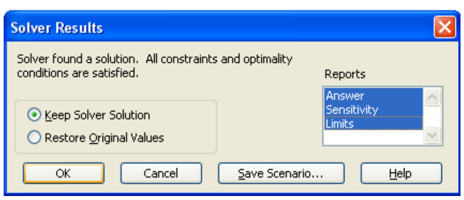
Continued.

	A	B	C	D	E	F
36	Euler stress	Fe	=(PI()^2*E)/(lamda^2)	ksi		
37	Critical stress for the member about (x) axis	Fcrx	=0.877*Fex	ksi		
38	Critical stress for the member about (y) axis	Fcry	=0.877*Fey	ksi		
39	Critical stress for member	Fcr	=0.877*Fe	ksi		
40	Nominal axial strength	Pn	=Fcr*Ag	kips		
41						
42	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
43	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
44						
45	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
46	Slenderness ratio	=lamda_e/lamda_y	<	=1		
47	Slenderness ratio	=lamda_y/200	<	=1		
48	Strength	1	<	=0.6*Fcry*Ag/Pa		
49	local buckling	=((d-2*tf)/(tw))/(0.56*SQRT((E)/(Fy)))	<	=1		
50	local buckling	=((bf)/(2*tf))/(1.49*SQRT((E)/(Fy)))	<	=1		



1

2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.30.xlsx]Sheet1**				
3	**Report Created: 7/13/2010 9:52:30 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$43	Wt	61.9467329	58.02549884
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$8	d	13.7	13.1595466
14		$D$9	bf	8.436142865	8.279430856
15		$D$10	tf	0.335	0.332898668
16		$D$11	tw	0.966145181	0.926383395

Section 14.10 Optimum Design of Members for Flexure

14.40

Solve the following problem using the Excel Solver:

Solve the problem of Example 14.7 for a beam of span 40 ft. Assume compact shape and inelastic LTB.

$$M_a = 337.5 \text{ kip-ft}, \; V_a = 37.5 \text{ kips}$$
$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$
$$L_p < L_b \le L_r$$
$$M_n \le M_p, \;\; M_a \le 0.6\,\mathrm{M}_n$$
$$13.7 \le d \le 16.4, \;\; 5.0 \le b_f \le 16.0, \; 0.335 \le t_f \le 1.89, \;\; 0.23 \le t_w \le 1.18$$

<u>Solution</u>

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=15.05, bf=10.5, tf=1.1125, and tw=0.705 a solution of d=16.4, bf=16, tf=0.944, and tw=0.23, which gives an objective function value of 113.9, is obtained. A W14x145 shape is selected in order to satisfy the moment strength constraint.

	A	B	C	D	E	F
1	**Exercise 6.31**					
2						
3	**Shape:**	W14				
4	**Support Condition:**	Simple				
5	**Assume Shape Type:**	Compact	Correct Assumption			
6	**Assume LTB/FLB Type:**	Inelastic	Correct Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	Yes				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	13.7	d	14.8	16.4	in
12	Flange width	5	bf	15.5	16	in
13	Flange thickness	0.335	tf	1.09	1.89	in
14	Web thickness	0.23	tw	0.68	1.18	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=40*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	=15	kips		
23	Uniformly distributed dead load	w	=2	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)-R_2	kips		
28	Reaction 2	R_2	=(P*(Lb/12)/2+w*(Lb/12)^2/2)*(1/(Lb/12))	kips		
29	Required moment strength	Ma	=ABS((R_1)*((Lb/12)/2)-(w*(Lb/12)^2)/(8))	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2-P*((Lb/12)*(3/4)-(Lb/12)/2))	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			

1

14.40

Continued.

	A	B	C	D
37	Constant C1	C1	=SQRT(E/Fy)	
38	Lambda f	LMf	=0.5*bf/tf	
39	Lambda pf	LMpf	=0.38*_C1	
40	Lambda rf	LMrf	=_C1	
41	Lambda w	LMw	=h/tw	
42	Lambda pw	LMpw	=3.76*_C1	
43	Lambda rw	LMrw	=5.7*_C1	
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3
46	Yield moment	My	=Fy*Sx/12	kip-ft
47	Height of web	h	=d-2*tf	in
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in
50	Distance a between centroids	a	=d-2*ybar	in
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in
55	Limiting length Lp	Lp	=1.76*ry*_C1	in
56	Flange centroid distance	h0	=d-tf	in
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in
60	Constant C2	C2	=J*c_/(Sx*h0)	
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in
62	Constant C3	C3	=(Lb/rts)^2	
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)	
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi
65	Nominal strength (LTB)	Mn_LTB	=Cb*(Mp-(Mp-0.7*My)*_C4)	kip-ft
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)	
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/tw)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))	
68	Nominal strength (FLB)	Mn_FLB	=MIN((Mp-(Mp-0.7*My)*_C5),Mp)	kip-ft
69	Nominal strength	Mn	=MIN(Mp,Mn_LTB)	kip-ft
70	Maximum shear	Va	=R_1	kips
71	Shear area (i.e. web area)	Aw	=d*tw	in^2
72	Critical ratio Cv	Cv	=1	
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips
74	Allowable shear strength	V_allowable	=Vn/FSv	kips
75	Allowable deflection	D_allowable	=Lb/240	in
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(48*E*Ix)	in
77				
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft
80				
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
82	Compact/Noncompact flange	=LMf/LMpf	<	1
83	Compact/Noncompact web	=LMw/LMpw	<	1
84	Compact/Noncompact flange	=LMf/LMpf	<	1
85	Compact/Noncompact web	=LMw/LMpw	<	1
86	Moment strength	1	<	=0.6*Mn/Ma
87	First bound on Lb	1	<	=Lr/Lb
88	Second bound on Lb	=Lp/Lb	<	1
89	Limit on Mn	=Mn/Mp	<	1
90				
91	A W14x145, which has an allowable moment strength of			
92	Note: allowable moment strength must be greater than or equal to			
93				

1

2

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports: Answer, Sensitivity, Limits

Keep Solver Solution

Restore Original Values

OK | Cancel | Save Scenario... | Help

3

Solver Parameters

Set Target Cell: Wt

Equal To: Max / Min (selected) / Value of: 0

By Changing Cells: $D$11:$D$14

Subject to the Constraints:

$B$11:$B$14 <= $D$11:$D$14
$B$82:$B$89 <= $D$82:$D$89
$D$11:$D$14 <= $E$11:$E$14

Solve | Close | Guess | Options | Add | Change | Delete | Reset All | Help

Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.31.xlsx]Sheet1
Report Created: 7/15/2010 2:30:30 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$79	Wt	110.0444085	113.9078671

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$11	d	15.05	16.4
$D$12	bf	10.5	16
$D$13	tf	1.1125	0.943873645
$D$14	tw	0.705	0.23

14.41

Solve the following problem using the Excel Solver:

Solve the problem of Example 14.7 for a beam of span 40 ft. Assume compact shape and elastic LTB.

$$M_a = 337.5 \text{ kip-ft},\ V_a = 37.5 \text{ kips}$$
$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$
$$L_p < L_b \le L_r$$
$$M_n \le M_p,\ \ M_a \le 0.6\,\mathrm{M}_n$$
$$13.7 \le d \le 16.4,\ 5.0 \le b_f \le 16.0,\ 0.335 \le t_f \le 1.89,\ 0.23 \le t_w \le 1.18$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=15.05, bf=10.5, tf=1.1125, and tw=0.705 a solution of d=19.69, bf=11.7, tf=1.254, and tw=0. 3, which gives an objective function value of 117.2, is obtained. A W18x143 shape is selected in order to satisfy the moment strength constraint.

	A	B	C	D	E	F
1	**Exercise 6.32**					
2						
3	**Shape:**	W18				
4	**Support Condition:**	Simple				
5	**Assume Shape Type:**	Compact	Correct Assumption			
6	**Assume LTB/FLB Type:**	Elastic	Correct Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	Yes				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	17.7	d	19.5	21.1	in
12	Flange width	6	bf	11.2	11.7	in
13	Flange thickness	0.425	tf	1.32	2.11	in
14	Web thickness	0.3	tw	0.73	1.16	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=40*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	=15	kips		
23	Uniformly distributed dead load	w	=2	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)-R_2	kips		
28	Reaction 2	R_2	=(P*(Lb/12)/2+w*(Lb/12)^2/2)*(1/(Lb/12))	kips		
29	Required moment strength	Ma	=ABS((R_1)*((Lb/12)/2)-(w*(Lb/12)^2)/(8))	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2-P*((Lb/12)*(3/4)-(Lb/12)/2))	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			

1

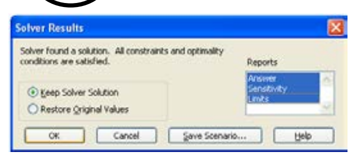
14.41

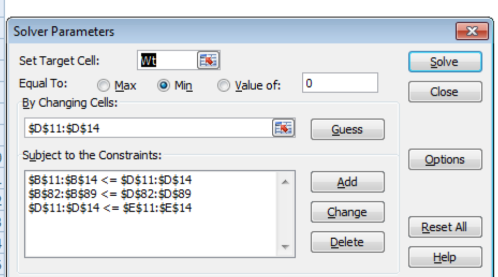
Continued.

	A	B	C	D
37	Constant C1	C1	=SQRT(E/Fy)	
38	Lambda f	LMf	=0.5*bf/tf	
39	Lambda pf	LMpf	=0.38*_C1	
40	Lambda rf	LMrf	=_C1	
41	Lambda w	LMw	=h/tw	
42	Lambda pw	LMpw	=3.76*_C1	
43	Lambda rw	LMrw	=5.7*_C1	
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3
46	Yield moment	My	=Fy*Sx/12	kip-ft
47	Height of web	h	=d-2*tf	in
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in
50	Distance a between centroids	a	=d-2*ybar	in
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in
55	Limiting length Lp	Lp	=1.76*ry*_C1	in
56	Flange centroid distance	h0	=d-tf	in
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in
60	Constant C2	C2	=J*c_/(Sx*h0)	
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in
62	Constant C3	C3	=(Lb/rts)^2	
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)	
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi
65	Nominal strength (LTB)	Mn_LTB	=MIN(Fcr*Sx*(1/12),Mp)	kip-ft
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)	
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw),4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/tw)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))	
68	Nominal strength (FLB)	Mn_FLB	=MIN((0.9*E*kc*Sx*(1/12))/(LMf^2),Mp)	kip-ft
69	Nominal strength	Mn	=MIN(Mp,Mn_LTB)	kip-ft
70	Maximum shear	Va	=R_1	kips
71	Shear area (i.e. web area)	Aw	=d*tw	in^2
72	Critical ratio Cv	Cv	=1	
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips
74	Allowable shear strength	V_allowable	=Vn/FSv	kips
75	Allowable deflection	D_allowable	=Lb/240	in
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(48*E*Ix)	in
77				
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft
80				
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
82	Compact/Noncompact flange	=LMf/LMpf	<	1
83	Compact/Noncompact web	=LMw/LMpw	<	1
84	Compact/Noncompact flange	=LMf/LMpf	<	1
85	Compact/Noncompact web	=LMw/LMpw	<	1
86	Moment strength	=1	<	=0.6*Mn/Ma
87	First bound on Lb	=-Lb/Lr	<	-1
88	Second bound on Lb	=-Lb/Lr	<	-1
89	Limit on Mn	=Mn/Mp	<	1
90				
91	A W18x143, which has an allowable moment strength of			
92	Note: allowable moment strength must be greater than or equal to			
93				

1

2





3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.32.xlsx]Sheet1**				
3	**Report Created: 7/15/2010 2:58:06 PM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$79	Wt	110.0444085	117.1513721
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$11	d	15.05	19.68583536
14		$D$12	bf	10.5	11.7
15		$D$13	tf	1.1125	1.253996485
16		$D$14	tw	0.705	0.3
17					

14.42

Solve the following problem using the Excel Solver:

Solve the problem of Example 14.7 for a beam of span 10 ft. Assume noncompact shape and inelastic LTB.

$$M_a = 337.5 \text{ kip-ft}, \ V_a = 37.5 \text{ kips}$$
$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$
$$L_p < L_b \le L_r$$
$$M_n \le M_p, \ \ M_a \le 0.6\,\mathrm{M}_n$$
$$13.7 \le d \le 16.4, \ 5.0 \le b_f \le 16.0, \ 0.335 \le t_f \le 1.89, \ 0.23 \le t_w \le 1.18$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=19.6, bf=9, tf=1.223, and tw=0.695 a solution of d=21.5, bf=6.13, tf=0.335, and tw=0.23, which gives an objective function value of 30.2, is obtained. A W21x44 shape is selected in order to satisfy the moment strength constraint.

1

	A	B	C	D	E	F
1	**Exercise 6.33**					
2						
3	**Shape:**	W21 or W24				
4	**Support Condition:**	Simple				
5	**Assume Shape Type:**	Noncompact	Incorrect Assumption			
6	**Assume LTB/FLB Type:**	Inelastic	Correct Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	Yes				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	13.7	d	20.7	25.5	in
12	Flange width	5	bf	6.5	13	in
13	Flange thickness	0.335	tf	0.45	2.11	in
14	Web thickness	0.23	tw	0.35	1.16	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=10*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	=15	kips		
23	Uniformly distributed dead load	w	=2	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)-R_2	kips		
28	Reaction 2	R_2	=(P*(Lb/12)/2+w*(Lb/12)^2/2)*(1/(Lb/12))	kips		
29	Required moment strength	Ma	=ABS((R_1)*((Lb/12)/2)-(w*(Lb/12)^2)/(8))	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2-P*((Lb/12)*(3/4)-(Lb/12)/2))	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			

14.42

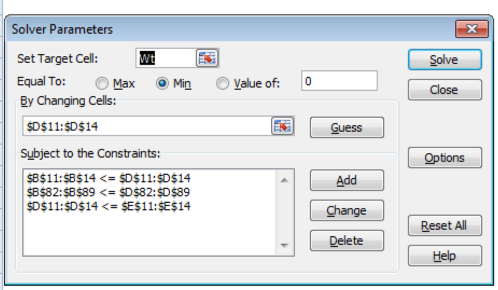
Continued.

	A	B	C	D	E	F
37	Constant C1	C1	=SQRT(E/Fy)			
38	Lambda f	LMf	=0.5*bf/tf			
39	Lambda pf	LMpf	=0.38*_C1			
40	Lambda rf	LMrf	=_C1			
41	Lambda w	LMw	=h/tw			
42	Lambda pw	LMpw	=3.76*_C1			
43	Lambda rw	LMrw	=5.7*_C1			
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4		
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3		
46	Yield moment	My	=Fy*Sx/12	kip-ft		
47	Height of web	h	=d-2*tf	in		
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2		
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in		
50	Distance a between centroids	a	=d-2*ybar	in		
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3		
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft		
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4		
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in		
55	Limiting length Lp	Lp	=1.76*ry*_C1	in		
56	Flange centroid distance	h0	=d-tf	in		
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6		
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4		
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in		
60	Constant C2	C2	=J*c_/(Sx*h0)			
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in		
62	Constant C3	C3	=(Lb/rts)^2			
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)			
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi		
65	Nominal strength (LTB)	Mn_LTB	=Cb*(Mp-(Mp-0.7*My)*_C4)	kip-ft		
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)			
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw),4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/tw)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))			
68	Nominal strength (FLB)	Mn_FLB	=MIN((Mp-(Mp-0.7*My)*_C5),Mp)	kip-ft		
69	Nominal strength	Mn	=MIN(Mn_LTB,Mn_FLB)	kip-ft		
70	Maximum shear	Va	=R_1	kips		
71	Shear area (i.e. web area)	Aw	=d*tw	in^2		
72	Critical ratio Cv	Cv	=1			
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips		
74	Allowable shear strength	V_allowable	=Vn/FSv	kips		
75	Allowable deflection	D_allowable	=Lb/240	in		
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(48*E*Ix)	in		
77						
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft		
80						
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
82	Compact/Noncompact flange	=LMpf/LMf	<	1		
83	Compact/Noncompact web	=LMpw/LMw	<	1		
84	Compact/Noncompact flange	=LMf/LMrf	<	1		
85	Compact/Noncompact web	=LMw/LMrw	<	1		
86	Moment strength	=1	<	=0.6*Mn/Ma		
87	First bound on Lb	1	<	=Lr/Lb		
88	Second bound on Lb	=Lp/Lb	<	1		
89	Limit on Mn	=Mn/Mp	<	1		
90						
91	A W21x44, which has an allowable moment strength of					
92	Note: allowable moment strength must be greater than or equal to					

1

2

3



Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.33.xlsx]Sheet1
Report Created: 7/15/2010 3:25:14 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$79	Wt	115.2185541	30.2189501

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$11	d	19.6	21.49714198
$D$12	bf	9	6.131570961
$D$13	tf	1.2225	0.335
$D$14	tw	0.695	0.23

14.43

Solve the following problem using the Excel Solver:

Solve the problem of Example 14.7 for a beam of span 40 ft. Assume noncompact shape and elastic LTB.

$$M_a = 337.5 \text{ kip-ft}, \; V_a = 37.5 \text{ kips}$$
$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$
$$L_p < L_b \le L_r$$
$$M_n \le M_p, \;\; M_a \le 0.6\,\mathrm{M}_n$$
$$13.7 \le d \le 16.4, \;\; 5.0 \le b_f \le 16.0, \; 0.335 \le t_f \le 1.89, \;\; 0.23 \le t_w \le 1.18$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=24.08, bf=10.97, tf=2.24, and tw=1.275 a solution of d=27.8, bf=17.44, tf=0.566, and tw=0.1944, which gives an objective function value of 84.7, is obtained. A W18x143 shape is selected in order to satisfy the moment strength constraint.

	A	B	C	D	E	F
1	**Exercise 6.34**					
2						
3	**Shape:**	All W				
4	**Support Condition:**	Simple				
5	**Assume Shape Type:**	Noncompact	Incorrect Assumption			
6	**Assume LTB/FLB Type:**	Elastic	Correct Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	Yes				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	4.16	d	19.5	44	in
12	Flange width	3.94	bf	11.2	18	in
13	Flange thickness	0.195	tf	1.32	4.29	in
14	Web thickness	0.17	tw	0.73	2.38	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=40*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	15	kips		
23	Uniformly distributed dead load	w	2	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)-R_2	kips		
28	Reaction 2	R_2	=(P*(Lb/12)/2+w*(Lb/12)^2/2)*(1/(Lb/12))	kips		
29	Required moment strength	Ma	=ABS((R_1)*((Lb/12)/2)-(w*(Lb/12)^2)/(8))	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2-P*((Lb/12)*(3/4)-(Lb/12)/2))	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			

1

14.43

Continued

	A	B	C	D
37	Constant C1	C1	=SQRT(E/Fy)	
38	Lambda f	LMf	=0.5*bf/tf	
39	Lambda pf	LMpf	=0.38*_C1	
40	Lambda rf	LMrf	=_C1	
41	Lambda w	LMw	=h/tw	
42	Lambda pw	LMpw	=3.76*_C1	
43	Lambda rw	LMrw	=5.7*_C1	
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3
46	Yield moment	My	=Fy*Sx/12	kip-ft
47	Height of web	h	=d-2*tf	in
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in
50	Distance a between centroids	a	=d-2*ybar	in
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in
55	Limiting length Lp	Lp	=1.76*ry*_C1	in
56	Flange centroid distance	h0	=d-tf	in
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in
60	Constant C2	C2	=J*c_/(Sx*h0)	
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in
62	Constant C3	C3	=(Lb/rts)^2	
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)	
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi
65	Nominal strength (LTB)	Mn_LTB	=MIN(Fcr*Sx*(1/12),Mp)	kip-ft
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)	
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw),4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/t w)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))	
68	Nominal strength (FLB)	Mn_FLB	=MIN((0.9*E*kc*Sx*(1/12))/(LMf^2),Mp)	kip-ft
69	Nominal strength	Mn	=MIN(Mn_LTB,Mn_FLB)	kip-ft
70	Maximum shear	Va	=R_1	kips
71	Shear area (i.e. web area)	Aw	=d*tw	in^2
72	Critical ratio Cv	Cv	=1	
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips
74	Allowable shear strength	V_allowable	=Vn/FSv	kips
75	Allowable deflection	D_allowable	=Lb/240	in
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(48*E*Ix)	in
77				
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft
80				
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
82	Compact/Noncompact flange	=LMpf/LMf	<	1
83	Compact/Noncompact web	=LMpw/LMw	<	1
84	Compact/Noncompact flange	=LMf/LMrf	<	1
85	Compact/Noncompact web	=LMw/LMrw	<	1
86	Moment strength	=1	<	=0.6*Mn/Ma
87	First bound on Lb	=-Lb/Lr	<	-1
88	Second bound on Lb	=-Lb/Lr	<	-1
89	Limit on Mn	=Mn/Mp	<	1
90				
91	A W18x143, which has an allowable moment strength of			
92	Note: allowable moment strength must be greater than or equal to			
93				

1

2

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports: Answer, Sensitivity, Limits

- Keep Solver Solution
- Restore Original Values

OK | Cancel | Save Scenario... | Help

3

Solver Parameters

Set Target Cell: Wt

Equal To: Max / Min (selected) / Value of: 0

By Changing Cells: $D$11:$D$14

Subject to the Constraints:

$B$11:$B$14 <= $D$11:$D$14
$B$82:$B$89 <= $D$82:$D$89
$D$11:$D$14 <= $E$11:$E$14

Solve | Close | Guess | Options | Add | Change | Delete | Reset All | Help

Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.34.xlsx]Sheet1
Report Created: 7/23/2010 2:54:21 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$79	Wt	251.9291187	84.66975156

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$11	d	24.08	27.82099165
$D$12	bf	10.97	17.43968205
$D$13	tf	2.2425	0.566046271
$D$14	tw	1.275	0.194420389

14.44

Solve the following problem using the Excel Solver:

Design a cantilever beam of span 15 ft subjected to a dead load of 3 kips/ft and a point live load of 20 kips at the end. The material of the beam is A572 Grade 50 steel. Assume compact shape and inelastic LTB.

$$M_a = 337.5 \text{ kip-ft}, \; V_a = 37.5 \text{ kips}$$

$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$

$$L_p < L_b \le L_r$$

$$M_n \le M_p, \;\; M_a \le 0.6\,\mathrm{M}_n$$

$$13.7 \le d \le 16.4, \;\; 5.0 \le b_f \le 16.0, \; 0.335 \le t_f \le 1.89, \;\; 0.23 \le t_w \le 1.18$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=15.05, bf=10.5, tf=1.1125, and tw=0.705 a solution of d=16.4, bf=15.46, tf=1.023, and tw=0.23, which gives an objective function value of 118.6, is obtained. A W14x145 shape is selected in order to satisfy the moment strength constraint.

1

	A	B	C	D	E	F
1	**Example 6.35**					
2						
3	**Shape:**	W14				
4	**Support Condition:**	Cantilever				
5	**Assume Shape Type:**	Compact	Correct Assumption			
6	**Assume LTB/FLB Type:**	Inelastic	Correct Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	No				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	13.7	d	14.8	16.4	in
12	Flange width	5	bf	15.5	16	in
13	Flange thickness	0.335	tf	1.09	1.89	in
14	Web thickness	0.23	tw	0.68	1.18	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=15*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	=20	kips		
23	Uniformly distributed dead load	w	3	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)	kips		
28	Reaction 2	R_2	=P*(Lb/12)+w*(Lb/12)^2/2	ft-kips		
29	Required moment strength	Ma	=ABS(R_2)	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(-R_2+R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(-R_2+R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(-R_2+R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2)	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			

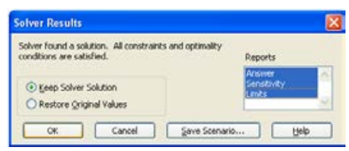
14.44

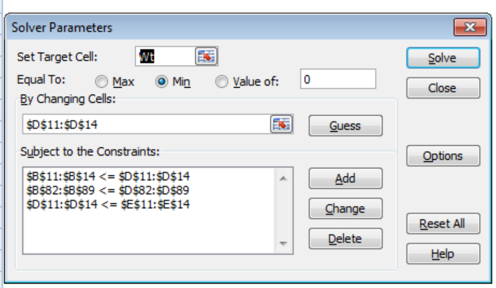
Continued.

1

	A	B	C	D
37	Constant C1	C1	=SQRT(E/Fy)	
38	Lambda f	LMf	=0.5*bf/tf	
39	Lambda pf	LMpf	=0.38*_C1	
40	Lambda rf	LMrf	=_C1	
41	Lambda w	LMw	=h/tw	
42	Lambda pw	LMpw	=3.76*_C1	
43	Lambda rw	LMrw	=5.7*_C1	
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3
46	Yield moment	My	=Fy*Sx/12	kip-ft
47	Height of web	h	=d-2*tf	in
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in
50	Distance a between centroids	a	=d-2*ybar	in
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in
55	Limiting length Lp	Lp	=1.76*ry*_C1	in
56	Flange centroid distance	h0	=d-tf	in
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in
60	Constant C2	C2	=J*c_/(Sx*h0)	
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in
62	Constant C3	C3	=(Lb/rts)^2	
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)	
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi
65	Nominal strength (LTB)	Mn_LTB	=Cb*(Mp-(Mp-0.7*My)*_C4)	kip-ft
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)	
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw),4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/tw)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))	
68	Nominal strength (FLB)	Mn_FLB	=MIN((Mp-(Mp-0.7*My)*_C5),Mp)	kip-ft
69	Nominal strength	Mn	=MIN(Mp,Mn_LTB)	kip-ft
70	Maximum shear	Va	=R_1	kips
71	Shear area (i.e. web area)	Aw	=d*tw	in^2
72	Critical ratio Cv	Cv	=1	
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips
74	Allowable shear strength	V_allowable	=Vn/FSv	kips
75	Allowable deflection	D_allowable	=Lb/240	in
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(3*E*Ix)	in
77				
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft
80				
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
82	Compact/Noncompact flange	=LMf/LMpf	<	1
83	Compact/Noncompact web	=LMw/LMpw	<	1
84	Compact/Noncompact flange	=LMf/LMpf	<	1
85	Compact/Noncompact web	=LMw/LMpw	<	1
86	Moment strength	1	<	=0.6*Mn/Ma
87	First bound on Lb	1	<	=Lr/Lb
88	Second bound on Lb	=Lp/Lb	<	1
89	Limit on Mn	1	<	1
90				
91	A W14x145, which has an allowable moment strength of			
92	Note: allowable moment strength must be greater than or equal to			

2





3

Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.35.xlsx]Sheet1
Report Created: 7/15/2010 3:53:07 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$79	Wt	110.0444085	118.6108205

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$11	d	15.05	16.4
$D$12	bf	10.5	15.45943325
$D$13	tf	1.1125	1.02284258
$D$14	tw	0.705	0.23

14.45

Solve the following problem using the Excel Solver:

Design a cantilever beam of span 15 ft subjected to a dead load of 3 kips/ft and a point live load of 20 kips at the end. The material of the beam is A572 Grade 50 steel. Assume compact shape and elastic LTB.

$$M_a = 337.5 \text{ kip-ft}, \; V_a = 37.5 \text{ kips}$$

$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$

$$L_p < L_b \le L_r$$

$$M_n \le M_p, \;\; M_a \le 0.6\,\mathrm{M}_n$$

$$13.7 \le d \le 16.4, \;\; 5.0 \le b_f \le 16.0, \; 0.335 \le t_f \le 1.89, \;\; 0.23 \le t_w \le 1.18$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of d=19.6, bf=9, tf=1.223, and tw=0.695 a solution of d=25.5, bf=5, tf=1.56, and tw=0.548, which gives an objective function value of 94.5, is obtained. A W21x111 shape is selected in order to satisfy the moment strength constraint.

	A	B	C	D	E	F
1	**Exercise 6.36**					
2						
3	**Shape:**	W21 or W24				
4	**Support Condition:**	Cantilever				
5	**Assume Shape Type:**	Compact	Correct Assumption			
6	**Assume LTB/FLB Type:**	Elastic	Incorrect Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	Yes				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	13.7	d	21.5	25.5	in
12	Flange width	5	bf	12.3	13	in
13	Flange thickness	0.335	tf	0.875	2.11	in
14	Web thickness	0.23	tw	0.55	1.16	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=15*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	20	kips		
23	Uniformly distributed dead load	w	3	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)	kips		
28	Reaction 2	R_2	=P*(Lb/12)+w*(Lb/12)^2/2	ft-kips		
29	Required moment strength	Ma	=ABS(R_2)	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(-R_2+R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(-R_2+R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(-R_2+R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2)	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			

1

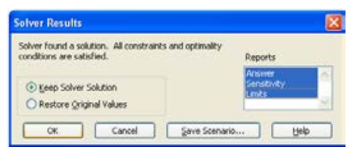
14.45

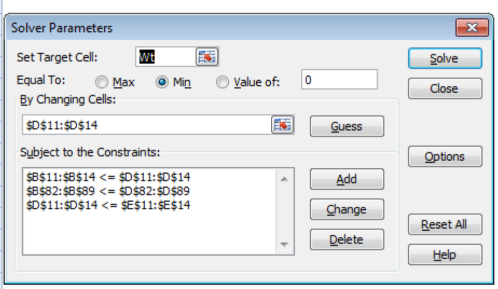
Continued.

	A	B	C	D
37	Constant C1	C1	=SQRT(E/Fy)	
38	Lambda f	LMf	=0.5*bf/tf	
39	Lambda pf	LMpf	=0.38*_C1	
40	Lambda rf	LMrf	=_C1	
41	Lambda w	LMw	=h/tw	
42	Lambda pw	LMpw	=3.76*_C1	
43	Lambda rw	LMrw	=5.7*_C1	
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3
46	Yield moment	My	=Fy*Sx/12	kip-ft
47	Height of web	h	=d-2*tf	in
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in
50	Distance a between centroids	a	=d-2*ybar	in
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in
55	Limiting length Lp	Lp	=1.76*ry*_C1	in
56	Flange centroid distance	h0	=d-tf	in
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in
60	Constant C2	C2	=J*c_/(Sx*h0)	
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in
62	Constant C3	C3	=(Lb/rts)^2	
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)	
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi
65	Nominal strength (LTB)	Mn_LTB	=MIN(Fcr*Sx*(1/12),Mp)	kip-ft
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)	
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw),4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/tw)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))	
68	Nominal strength (FLB)	Mn_FLB	=MIN((0.9*E*kc*Sx*(1/12))/(LMf^2),Mp)	kip-ft
69	Nominal strength	Mn	=MIN(Mp,Mn_LTB)	kip-ft
70	Maximum shear	Va	=R_1	kips
71	Shear area (i.e. web area)	Aw	=d*tw	in^2
72	Critical ratio Cv	Cv	=1	
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips
74	Allowable shear strength	V_allowable	=Vn/FSv	kips
75	Allowable deflection	D_allowable	=Lb/240	in
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(3*E*Ix)	in
77				
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft
80				
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
82	Compact/Noncompact flange	=LMf/LMpf	<	1
83	Compact/Noncompact web	=LMw/LMpw	<	1
84	Compact/Noncompact flange	=LMf/LMpf	<	1
85	Compact/Noncompact web	=LMw/LMpw	<	1
86	Moment strength	=1	<	=0.6*Mn/Ma
87	First bound on Lb	=-Lb/Lr	<	-1
88	Second bound on Lb	=-Lb/Lr	<	-1
89	Limit on Mn	=Mn/Mp	<	1
90				
91	A W21x111, which has an allowable moment strength of			
92	Note: allowable moment strength must be greater than or equal to			

1

2





3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.36.xlsx]Sheet1**				
3	**Report Created: 7/16/2010 7:21:23 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$79	Wt	115.2185541	94.53617787
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$11	d	19.6	25.5
14		$D$12	bf	9	5
15		$D$13	tf	1.2225	1.55596923
16		$D$14	tw	0.695	0.548409065
17					

14.46

Solve the following problem using the Excel Solver:

Design a cantilever beam of span 15ft subjected to a dead load of 3kips/ft and a point live load of 20kips at the end. The material of the beam is A572 Grade 50 steel. Assume noncompact shape and inelastic LTB.

$$M_a = 337.5 \text{ kip-ft}, \ V_a = 37.5 \text{ kips}$$

$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$

$$L_p < L_b \le L_r$$

$$M_n \le M_p, \ \ M_a \le 0.6\,\mathrm{M}_n$$

$$13.7 \le d \le 16.4, \ 5.0 \le b_f \le 16.0, \ 0.335 \le t_f \le 1.89, \ 0.23 \le t_w \le 1.18$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of d=19.6, bf=9, tf=1.223, and tw=0.695 a solution of d=25.5, bf=12.79, tf=0.699, and tw=0.23, which gives an objective function value of 79.5, is obtained. A W21x111 shape is selected in order to satisfy the moment strength constraint.

	A	B	C	D	E	F
1	**Exercise 6.37**					
2						
3	**Shape:**	W21 or W24				
4	**Support Condition:**	Cantilever				
5	**Assume Shape Type:**	Noncompact	Incorrect Assumption			
6	**Assume LTB/FLB Type:**	Inelastic	Correct Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	Yes				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	13.7	d	21.5	25.5	in
12	Flange width	5	bf	12.3	13	in
13	Flange thickness	0.335	tf	0.875	2.11	in
14	Web thickness	0.23	tw	0.55	1.16	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=15*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	20	kips		
23	Uniformly distributed dead load	w	3	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)	kips		
28	Reaction 2	R_2	=P*(Lb/12)+w*(Lb/12)^2/2	ft-kips		
29	Required moment strength	Ma	=ABS(R_2)	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(-R_2+R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(-R_2+R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(-R_2+R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2)	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			

1

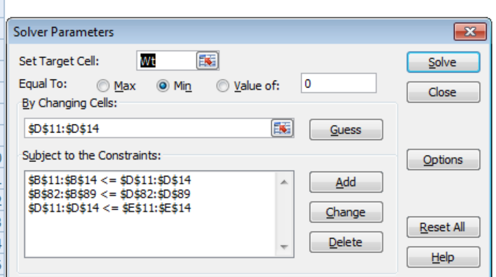
14.46

Continued.

	A	B	C	D
37	Constant C1	C1	=SQRT(E/Fy)	
38	Lambda f	LMf	=0.5*bf/tf	
39	Lambda pf	LMpf	=0.38*_C1	
40	Lambda rf	LMrf	=_C1	
41	Lambda w	LMw	=h/tw	
42	Lambda pw	LMpw	=3.76*_C1	
43	Lambda rw	LMrw	=5.7*_C1	
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3
46	Yield moment	My	=Fy*Sx/12	kip-ft
47	Height of web	h	=d-2*tf	in
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in
50	Distance a between centroids	a	=d-2*ybar	in
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in
55	Limiting length Lp	Lp	=1.76*ry*_C1	in
56	Flange centroid distance	h0	=d-tf	in
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in
60	Constant C2	C2	=J*c_/(Sx*h0)	
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in
62	Constant C3	C3	=(Lb/rts)^2	
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)	
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi
65	Nominal strength (LTB)	Mn_LTB	=Cb*(Mp-(Mp-0.7*My)*_C4)	kip-ft
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)	
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw),4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/tw)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))	
68	Nominal strength (FLB)	Mn_FLB	=MIN((Mp-(Mp-0.7*My)*_C5),Mp)	kip-ft
69	Nominal strength	Mn	=MIN(Mn_LTB,Mn_FLB)	kip-ft
70	Maximum shear	Va	=R_1	kips
71	Shear area (i.e. web area)	Aw	=d*tw	in^2
72	Critical ratio Cv	Cv	=1	
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips
74	Allowable shear strength	V_allowable	=Vn/FSv	kips
75	Allowable deflection	D_allowable	=Lb/240	in
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(3*E*Ix)	in
77				
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft
80				
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
82	Compact/Noncompact flange	=LMpf/LMf	<	1
83	Compact/Noncompact web	=LMpw/LMw	<	1
84	Compact/Noncompact flange	=LMf/LMrf	<	1
85	Compact/Noncompact web	=LMw/LMrw	<	1
86	Moment strength	1	<	=0.6*Mn/Ma
87	First bound on Lb	1	<	=Lr/Lb
88	Second bound on Lb	=Lp/Lb	<	1
89	Limit on Mn	1	<	1
90				
91	A W21x111, which has an allowable moment strength of			
92	Note: allowable moment strength must be greater than or equal to			

1

2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.37.xlsx]Sheet1**				
3	**Report Created: 7/16/2010 7:41:13 AM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		$C$79	Wt	115.2185541	79.51160667
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		$D$11	d	19.6	25.5
14		$D$12	bf	9	12.78814337
15		$D$13	tf	1.2225	0.698682569
16		$D$14	tw	0.695	0.23
17					

14.47

Solve the following problem using the Excel Solver:

Design a cantilever beam of span 15 ft subjected to a dead load of 1 kips/ft and a point live load of 10 kips at the end. The material of the beam is A572 Grade 50 steel. Assume noncompact shape and elastic LTB.

$$M_a = 337.5 \text{ kip-ft},\ V_a = 37.5 \text{ kips}$$

$$\lambda_f \le \lambda_{pf} \text{ and } \lambda_w \le \lambda_{pw}$$

$$L_p < L_b \le L_r$$

$$M_n \le M_p,\ \ M_a \le 0.6\,\mathrm{M}_n$$

$$13.7 \le d \le 16.4,\ 5.0 \le b_f \le 16.0,\ 0.335 \le t_f \le 1.89,\ 0.23 \le t_w \le 1.18$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of d=19.6, bf=9, tf=1.223, and tw=0.695 a solution of d=25.5, bf=7.71, tf=0.360, and tw=0.23, which gives an objective function value of 38.2, is obtained. A W21x57 shape is selected in order to satisfy the moment strength constraint.

	A	B	C	D	E	F
1	**Exercise 6.38**					
2					1	
3	**Shape:**	W21 or W24				
4	**Support Condition:**	Cantilever				
5	**Assume Shape Type:**	Noncompact	Incorrect Assumption			
6	**Assume LTB/FLB Type:**	Elastic	Correct Assumption			
7	**Shear Satisfied?**	Yes				
8	**Deflection Satisfied?**	Yes				
9						
10	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
11	Depth	13.7	d	21.1	25.5	in
12	Flange width	5	bf	6.56	13	in
13	Flange thickness	0.335	tf	0.65	2.11	in
14	Web thickness	0.23	tw	0.405	1.16	in
15						
16	**2. Parameter name**	**Symbol**	**Value**	**Units**		
17	Unbraced length	Lb	=15*12	in		
18	Mudulus of elasticity	E	=29000	ksi		
19	Yield stress	Fy	=50	ksi		
20	Factor of safety for bending	FSb	=5/3			
21	Factor of safety for shear	FSv	=1.5			
22	Concentrated live load	P	10	kips		
23	Uniformly distributed dead load	w	1	kips/ft		
24	Density of steel	gamma	=0.283	lb/in^3		
25						
26	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
27	Reaction 1	R_1	=P+w*(Lb/12)	kips		
28	Reaction 2	R_2	=P*(Lb/12)+w*(Lb/12)^2/2	ft-kips		
29	Required moment strength	Ma	=ABS(R_2)	kip-ft		
30	Maximum moment	Mmax	=Ma	kip-ft		
31	Quarter-point moment	MA_1	=ABS(-R_2+R_1*((Lb/12)/4)-w*((Lb/12)/4)^2/2)	kip-ft		
32	Mid-point moment	MB_1	=ABS(-R_2+R_1*((Lb/12)/2)-w*((Lb/12)/2)^2/2)	kip-ft		
33	Three-quarter point moment	MC_1	=ABS(-R_2+R_1*((Lb/12)*(3/4))-w*((Lb/12)*(3/4))^2/2)	kip-ft		
34	Doubly symm. shape parameter	c_	1			
35	Monosymmetry parameter Rm	Rm_1	1			
36	Beam bending coefficient Cb	Cb	=MIN(((12.5*Mmax)/(2.5*Mmax+3*MA_1+4*MB_1+3*MC_1))*Rm_1,3)			
37	Constant C1	C1	=SQRT(E/Fy)			
38	Lambda f	LMf	=0.5*bf/tf			
39	Lambda pf	LMpf	=0.38*_C1			

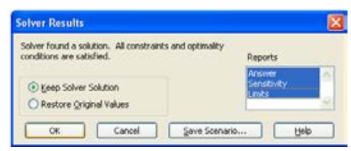
14.47

Continued.

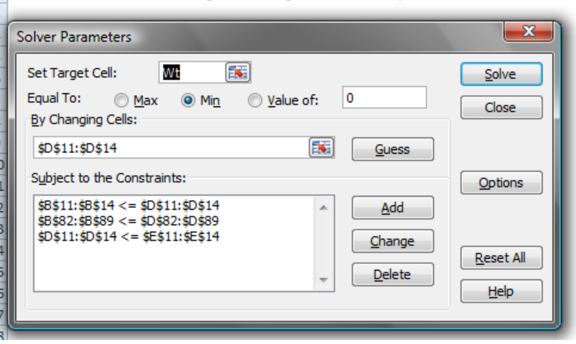
	A	B	C	D
41	Lambda w	LMw	=h/tw	
42	Lambda pw	LMpw	=3.76*_C1	
43	Lambda rw	LMrw	=5.7*_C1	
44	Moment of Inertia about x axis	Ix	=2*(1/12)*bf*tf^3+(1/12)*tw*(d-2*tf)^3+2*bf*tf*(d/2-tf/2)^2	in^4
45	Section modulus	Sx	=(Ix)/(0.5*d)	in^3
46	Yield moment	My	=Fy*Sx/12	kip-ft
47	Height of web	h	=d-2*tf	in
48	Gross area	Ag	=2*bf*tf+(d-2*tf)*tw	in^2
49	Distance ybar	ybar	=(1/Ag)*(bf*tf*tf+h*tw*(0.25*h+tf))	in
50	Distance a between centroids	a	=d-2*ybar	in
51	Plastic section modulus	Zx	=0.5*a*Ag	in^3
52	Plastic moment	Mp	=Fy*Zx/12	kip-ft
53	Moment of inertia about y axis	Iy	=(1/12)*2*tf*bf^3+(1/12)*(d-2*tf)*tw^3	in^4
54	Radius of gyration ry	ry	=SQRT(Iy/Ag)	in
55	Limiting length Lp	Lp	=1.76*ry*_C1	in
56	Flange centroid distance	h0	=d-tf	in
57	Warping constant (doubly symm.)	Cw	=(1/4)*Iy*h0^2	in^6
58	Torsional constant	J	=(2*bf*tf*tf*tf+h0*tw*tw*tw)/3	in^4
59	Radius rts	rts	=SQRT((SQRT(Iy*Cw))/(Sx))	in
60	Constant C2	C2	=J*c_/(Sx*h0)	
61	Limiting length Lr	Lr	=1.95*rts*(E/(0.7*Fy))*SQRT(_C2)*SQRT(1+SQRT(1+6.76*((0.7*Fy/E)*(1/_C2))^2))	in
62	Constant C3	C3	=(Lb/rts)^2	
63	Constant C4	C4	=(Lb-Lp)/(Lr-Lp)	
64	Critical stress Fcr	Fcr	=(Cb*PI()^2*E/_C3)*SQRT(1+0.078*_C2*_C3)	ksi
65	Nominal strength (LTB)	Mn_LTB	=MIN(Fcr*Sx*(1/12),Mp)	kip-ft
66	Constant C5	C5	=(LMf-LMpf)/(LMrf-LMpf)	
67	Constant kc	kc	<=IF(AND(0.35<=4/SQRT(h/tw),4/SQRT(h/tw)<=0.76),4/SQRT(h/tw),IF(4/SQRT(h/tw)<0.35,0.35,IF(4/SQRT(h/tw)>0.76,0.76,"Error")))	
68	Nominal strength (FLB)	Mn_FLB	=MIN((0.9*E*kc*Sx*(1/12))/(LMf^2),Mp)	kip-ft
69	Nominal strength	Mn	=MIN(Mn_LTB,Mn_FLB)	kip-ft
70	Maximum shear	Va	=R_1	kips
71	Shear area (i.e. web area)	Aw	=d*tw	in^2
72	Critical ratio Cv	Cv	=1	
73	Nominal shear strength Vn	Vn	=0.6*Fy*Aw*Cv	kips
74	Allowable shear strength	V_allowable	=Vn/FSv	kips
75	Allowable deflection	D_allowable	=Lb/240	in
76	Deflection due to live loads Δ	Def	=P*(Lb)^3/(3*E*Ix)	in
77				
78	**4. Objective function name**	**Symbol**	**Equation**	**Units**
79	Weight/ft	Wt	=Ag*12*gamma	lb/ft
80				
81	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**
82	Compact/Noncompact flange	=LMpf/LMf	<	1
83	Compact/Noncompact web	=LMpw/LMw	<	1
84	Compact/Noncompact flange	=LMf/LMrf	<	1
85	Compact/Noncompact web	=LMw/LMrw	<	1
86	Moment strength	1	<	=0.6*Mn/Ma
87	First bound on Lb	=-Lb/Lr	<	-1
88	Second bound on Lb	=-Lb/Lr	<	-1
89	Limit on Mn	1	<	1
90				
91	A W21x57, which has an allowable moment strength of			
92	Note: allowable moment strength must be greater than or equal to			

1

2



3



Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.38.xlsx]Sheet1
Report Created: 7/16/2010 7:52:24 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$79	Wt	115.2185541	38.18335232

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$11	d	19.6	25.5
$D$12	bf	9	7.710374352
$D$13	tf	1.2225	0.3595158
$D$14	tw	0.695	0.23

Section 14.11 Optimum Design of Telecommunication Poles

14.48

Solve the following problem using the Excel Solver:

Solve the problem of Example 14.10 for a pole of height 40 m.

Solution

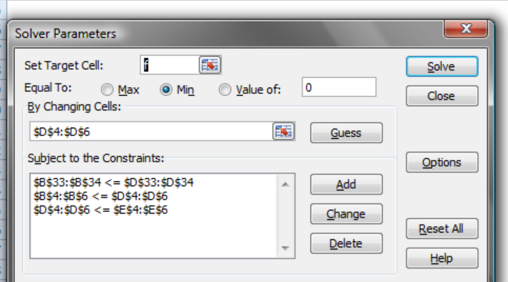
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of dt=0.4, t=0.005, and tau=0.02 a solution of dt=0.3, t=0.0032, and tau=0.01874, which gives an objective function value of 33030.7, is obtained.

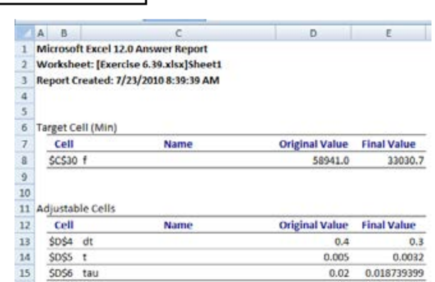
	A	B	C	D	E	F
1	**Exercise 6.39**					
2						
3	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
4	External diameter pole tip	0.3	dt	0.3	1	m
5	Thickness of the pole wall	0.0032	t	0.0032	0.0254	m
6	Taper of the pole, τ	0	tau	0.0187393990165856	0.05	m/m
7						
8	**2. Parameter name**	**Symbol**	**Value**	**Units**		
9	Height of the pole	H	40	m		
10	Element length	L_elem	1	m		
11	Projected area on a vertical plane	At	10	m^2		
12	Drag coefficient area At	Ct	1			
13	Distributed area ladder and cables	Alc	0.3	m^2/m		
14	Drag coefficient ladder and cables	Clc	1			
15	Drag coefficient pole body	Cp	0.75			
16	Self-weight ladder and cables	plc	400	N/m		
17	Specific weight of steel, γ	gamma	78500	N/m^3		
18	Dead load at top	Fv	10400	N		
19	Modulus of elasticity	E	=210*10^9	Pa		
20	Steel allowable stress, σ_a	sigma_a	=150*10^6	Pa		
21	Maximum wind velocity	Vo	40	m/s		
22	Allowable rotation, v'a	vpa	=(30/60)*(PI()/180)	radians		
23						
24	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
25	Wind load at top	Fh	=Ct*At*(463.1*H^0.25)	N		
26	Allowable tip deflection	va	=0.02*H	m		
27	Operational wind velocity	Voper	=0.55*Vo	m/s		
28						
29	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
30	Minimize weight	f	=(PI()*gamma)/(24*tau)*(((dt+2*tau*(H))^3-(dt)^3)-((dt+2*tau*(H)-2*t)^3-(dt-2*t)^3))	N		
31						
32	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
33	Stress constraint	=sigma_0/sigma_a	<	1		
34	Tip deflection constraint	=ABS(v_H)/va	<	1		

1



2

3



Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.39.xlsx]Sheet1
Report Created: 7/23/2010 8:39:39 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$30	f	58941.0	33030.7

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$4	dt	0.4	0.3
$D$5	t	0.005	0.0032
$D$6	tau	0.02	0.018739399

14.48

Continued.

1

	A	B	C	D	E	F	G	H	I
1	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.
2	0	1	2	3	4	5	6	7	8
3	Node	Height above ground, z [m]	Effective wind velocity pressure, q(z) [N/m^2]	External diameter pole section, de(z) [m]	Internal diameter pole section, di(z) [m]	Cross-sectional area at height z, A(z) [m^2]	Moment of inertia at height z, I(z) [m^4]	Section modulus at height z, S(z) [m^3]	Distance from pole tip to given section, y [m]
4		<=IF(A5*L_elem<=H,A5*L_elem,"-")	<=IF(A5*L_elem<=H,463.1*B5^0.25,"-")	<=IF(A5*L_elem<=H,dt+2*tau*(H-B5),"-")	<=IF(A5*L_elem<=H,D5-2*t,"-")	<=IF(A5*L_elem<=H,PI()/4*(D5^2-E5^2),"-")	<=IF(A5*L_elem<=H,PI()/64*(D5^4-E5^4),"-")	<=IF(A5*L_elem<=H,(2*G5)/(D5),"-")	<=IF(A5*L_elem<=H,H-B5,"-")
5	0	0	0.0	1.799151921	1.792751921	0.0181	0.0073	0.0081	40
6	1	1	463.1	1.761673123	1.755273123	0.0177	0.0068	0.0078	39
7	2	2	550.7	1.724194325	1.717794325	0.0173	0.0064	0.0074	38
8	3	3	609.5	1.686715527	1.680315527	0.0169	0.0060	0.0071	37
44	39	39	1157.3	0.337478798	0.331078798	0.0034	0.0000	0.0003	1
45	40	40	1164.6	0.3	0.2936	0.0030	0.0000	0.0002	0
46	41	-	-	-	-	-	-	-	-
56	Value at base	0	0.0	1.799151921	1.792751921	0.0181	0.0073	0.0081	40
57	Value at top	40	1164.6	0.3	0.2936	0.0030	0.0000	0.0002	0

A_0

A_H

1

	J	K	L	M	N	O	P	Q	R
1	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.
2	0	9	10	11	12	13	14	15	16
3	Node	Horizontal distributed load, ph(y) [N/m]	Vertical distributed load, pv(y) [N/m]	Axial load at height z, N(z) [N]	Bending moment at height z, M(z) [N-m]	Axial stress at height z, σ(z) [Pa]	Curvature in the vertical plane, v''(z) [m^{-1}]	Rotation in the vertical plane, v'(z) [radians]	Horizontal displacement height z, v(z) [m]
4		<=IF(A5*L_elem<=H,(Alc*Clc+D5*Cp)*C5,"-")	<=IF(A5*L_elem<=H,plc+F5*gamma,"-")	<=IF(A5*L_elem<=H,Fv+plc*(H-B5)+(PI()*gamma)/(24*tau)*(((dt+2*tau*(H-B5))^3-(dt)^3)-((dt+2*tau*(H-B5)-2*t)^3-(dt-2*t)^3)),"-")	<=IF(A5*L_elem<=H,Alc*Clc*(205.822*H^2.25-370.48*H^1.25*B5+164.658*B5^2.25)+Cp*(dt*(205.822*H^2.25-370.48*H^1.25*B5+164.658*B5^2.25)+tau*(126.66*H^3.25-329.316*H^2.25*B5+329.316*H*B5^2.25-126.66*B5^3.25))+Fh*(H-B5),"-")	<=IF(A5*L_elem<=H,(M5)/(F5)+(N5)/(H5),"-")	<=IF(A5*L_elem<=H,-(N5)/(E*G5),"-")	<=0 <=IF(A6*L_elem<=H,Q5+(P6+P5)/(2)*(B6-B5),"-")	<=0 <=IF(A6*L_elem<=H,R5+(Q6+Q5)/(2)*(B6-B5),"-")
5	0	0.0	1817.3	59430.7	1187168.1	149999975.3	-7.77E-04	0.00E+00	0.00E+00
6	1	750.8	1787.7	57628.1	1137602.1	149904839.4	-7.93E-04	-7.85E-04	-3.92E-04
7	2	877.4	1758.2	55855.2	1088771.3	149763999.3	-8.09E-04	-1.59E-03	-1.58E-03
8	3	953.8	1728.6	54111.8	1040814.1	149590873.7	-8.27E-04	-2.40E-03	-3.57E-03
44	39	640.1	663.8	11049.0	11956.8	46266909.6	-1.21E-03	-5.36E-02	-8.55E-01
45	40	611.4	634.2	10400.0	0.0	3485536.2	5.57E-17	-5.43E-02	-9.09E-01
46	41	-	-	-	-	-	-	-	-
56	Value at base	0.0	1817.3	59430.7	1187168.1	149999975.3	-7.77E-04	0.00E+00	0.00E+00
57	Value at top	40.0	611.4	634.2	10400.0	0.0	3.49E+06	5.57E-17	-5.43E-02

sigma_0

v_H

14.49

Solve the following problem using the Excel Solver:

Solve the problem of Example 14.11 for a pole of height 40 m.

Solution

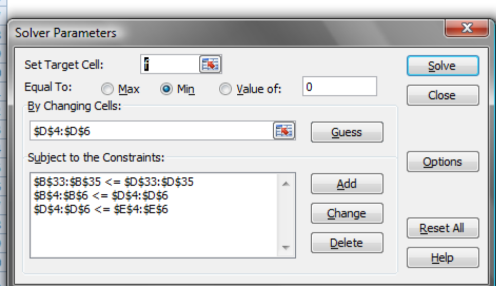
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

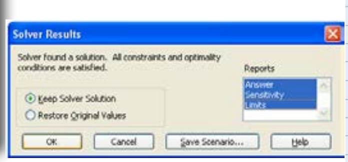
(3) The answer report shows that for initial design variable values of dt=0.4, t=0.005, and tau=0.02 a solution of dt=0.3, t=0.0032, and tau=0.01874, which gives an objective function value of 33030.7, is obtained.

	A	B	C	D	E	F
1	Exercise 6.40					
2						
3	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
4	External diameter pole tip	0.3	dt	0.3	1	m
5	Thickness of the pole wall	0.0032	t	0.0032	0.0254	m
6	Taper of the pole, τ	0	tau	0.0187393990165856	0.05	m/m
7						
8	**2. Parameter name**	**Symbol**	**Value**	**Units**		
9	Height of the pole	H	40	m		
10	Element length	L_elem	1	m		
11	Projected area on a vertical plane	At	10	m^2		
12	Drag coefficient area At	Ct	1			
13	Distributed area ladder and cables	Alc	0.3	m^2/m		
14	Drag coefficient ladder and cables	Clc	1			
15	Drag coefficient pole body	Cp	0.75			
16	Self-weight ladder and cables	plc	400	N/m		
17	Specific weight of steel, γ	gamma	78500	N/m^3		
18	Dead load at top	Fv	10400	N		
19	Modulus of elasticity	E	=210*10^9	Pa		
20	Steel allowable stress, σ_a	sigma_a	=150*10^6	Pa		
21	Maximum wind velocity	Vo	40	m/s		
22	Allowable rotation, v'a	vpa	=(30/60)*(PI()/180)	radians		
23						
24	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
25	Wind load at top	Fh	=Ct*At*(463.1*H^0.25)	N		
26	Allowable tip deflection	va	=0.02*H	m		
27	Operational wind velocity	Voper	=0.55*Vo	m/s		
28						
29	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
30	Minimize weight	f	=(PI()*gamma)/(24*tau)*(((dt+2*tau*(H))^3-(dt)^3)-((dt+2*tau*(H)-2*t)^3-(dt-2*t)^3))	N		
31						
32	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**		
33	Stress constraint	=sigma_0/sigma_a	<	1		
34	Tip deflection constraint	=ABS(v_H)/va	<	1		
35	Tip rotation constraint	=(ABS(vp_H)*0.55^2)/(vpa)	<	1		

1



2



3

Microsoft Excel 12.0 Answer Report
Worksheet: [Exercise 6.40.xlsx]Sheet1
Report Created: 7/23/2010 9:00:15 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$C$30	f	58941.0	33030.7

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$4	dt	0.4	0.3
$D$5	t	0.005	0.0032
$D$6	tau	0.02	0.018739399

14.49

Continued.

1

	A	B	C	D	E	F	G	H	I
1	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.
2	0	1	2	3	4	5	6	7	8
3	Node	Height above ground, z [m]	Effective wind velocity pressure, q(z) [N/m^2]	External diameter pole section, de(z) [m]	Internal diameter pole section, di(z) [m]	Cross-sectional area at height z, A(z) [m^2]	Moment of inertia at height z, I(z) [m^4]	Section modulus at height z, S(z) [m^3]	Distance from pole tip to given section, y [m]
4		<=IF(A5*L_elem<=H,A5*L_elem,"-")	<=IF(A5*L_elem<=H,463.1*B5^0.25,"-")	<=IF(A5*L_elem<=H,dt+2*tau*(H-B5),"-")	<=IF(A5*L_elem<=H,D5-2*t,"-")	<=IF(A5*L_elem<=H,PI()/4*(D5^2-E5^2),"-")	<=IF(A5*L_elem<=H,PI()/64*(D5^4-E5^4),"-")	<=IF(A5*L_elem<=H,(2*G5)/(D5),"-")	<=IF(A5*L_elem<=H,H-B5,"-")
5	0	0	0.0	1.799151921	1.792751921	0.0181	0.0073	0.0081	40
6	1	1	463.1	1.761673123	1.755273123	0.0177	0.0068	0.0078	39
7	2	2	550.7	1.724194325	1.717794325	0.0173	0.0064	0.0074	38
8	3	3	609.5	1.686715527	1.680315527	0.0169	0.0060	0.0071	37
45	40	40	1164.6	0.3	0.2936	0.0030	0.0000	0.0002	0
46	41	-	-	-	-	-	-	-	-
56	Value at base	0	0.0	1.799151921	1.792751921	0.0181	0.0073	0.0081	40
57	Value at top	40	1164.6	0.3	0.2936	0.0030	0.0000	0.0002	0

A_0

A_H

1

	J	K	L	M	N	O	P	Q	R
1	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.
2	0	9	10	11	12	13	14	15	16
3	Node	Horizontal distributed load, ph(y) [N/m]	Vertical distributed load, pv(y) [N/m]	Axial load at height z, N(z) [N]	Bending moment at height z, M(z) [N-m]	Axial stress at height z, σ(z) [Pa]	Curvature in the vertical plane, v''(z) [m^{-1}]	Rotation in the vertical plane, v'(z) [radians]	Horizontal displacement height z, v(z) [m]
4		<=IF(A5*L_elem<=H,(Alc*Clc+D5*Cp)*C5,"-")	<=IF(A5*L_elem<=H,plc+F5*gamma,"-")	<=IF(A5*L_elem<=H,Fv+plc*(H-B5)+(PI()*gamma)/(24*tau)*(((dt+2*tau*(H-B5))^3-(dt)^3)-((dt+2*tau*(H-B5)-2*t)^3-(dt-2*t)^3)),"-")	<=IF(A5*L_elem<=H,Alc*Clc*(205.822*H^2.25-370.48*H^1.25*B5+164.658*B5^2.25)+Cp*(dt*(205.822*H^2.25-370.48*H^1.25*B5+164.658*B5^2.25)+tau*(126.66*H^3.25-329.316*H^2.25*B5+329.316*H*B5^2.25-126.66*B5^3.25))+Fh*(H-B5),"-")	<=IF(A5*L_elem<=H,(M5)/(F5)+(N5)/(H5),"-")	<=IF(A5*L_elem<=H,-(N5)/(E*G5),"-")	<=0 <=IF(A6*L_elem<=H,Q5+(P6+P5)/(2)*(B6-B5),"-")	<=0 <=IF(A6*L_elem<=H,R5+(Q6+Q5)/(2)*(B6-B5),"-")
5	0	0.0	1817.3	59430.7	1187168.1	149999975.3	-7.77E-04	0.00E+00	0.00E+00
6	1	750.8	1787.7	57628.1	1137602.1	149904839.4	-7.93E-04	-7.85E-04	-3.92E-04
7	2	877.4	1758.2	55855.2	1088771.3	149763999.3	-8.09E-04	-1.59E-03	-1.58E-03
8	3	953.8	1728.6	54111.8	1040814.1	149590873.7	-8.27E-04	-2.40E-03	-3.57E-03
45	40	611.4	634.2	10400.0	0.0	3485536.2	5.57E-17	-5.43E-02	-9.09E-01
46	41	-	-	-	-	-	-	-	-
56	Value at base	0.0	1817.3	59430.7	1187168.1	149999975.3	-7.77E-04	0.00E+00	0.00E+00
57	Value at top	40.0	611.4	634.2	10400.0	0.0	3.49E+06	5.57E-17	-5.43E-02

sigma_0

vp_H

v_H

14.50

Solve the following problem using the Excel Solver:

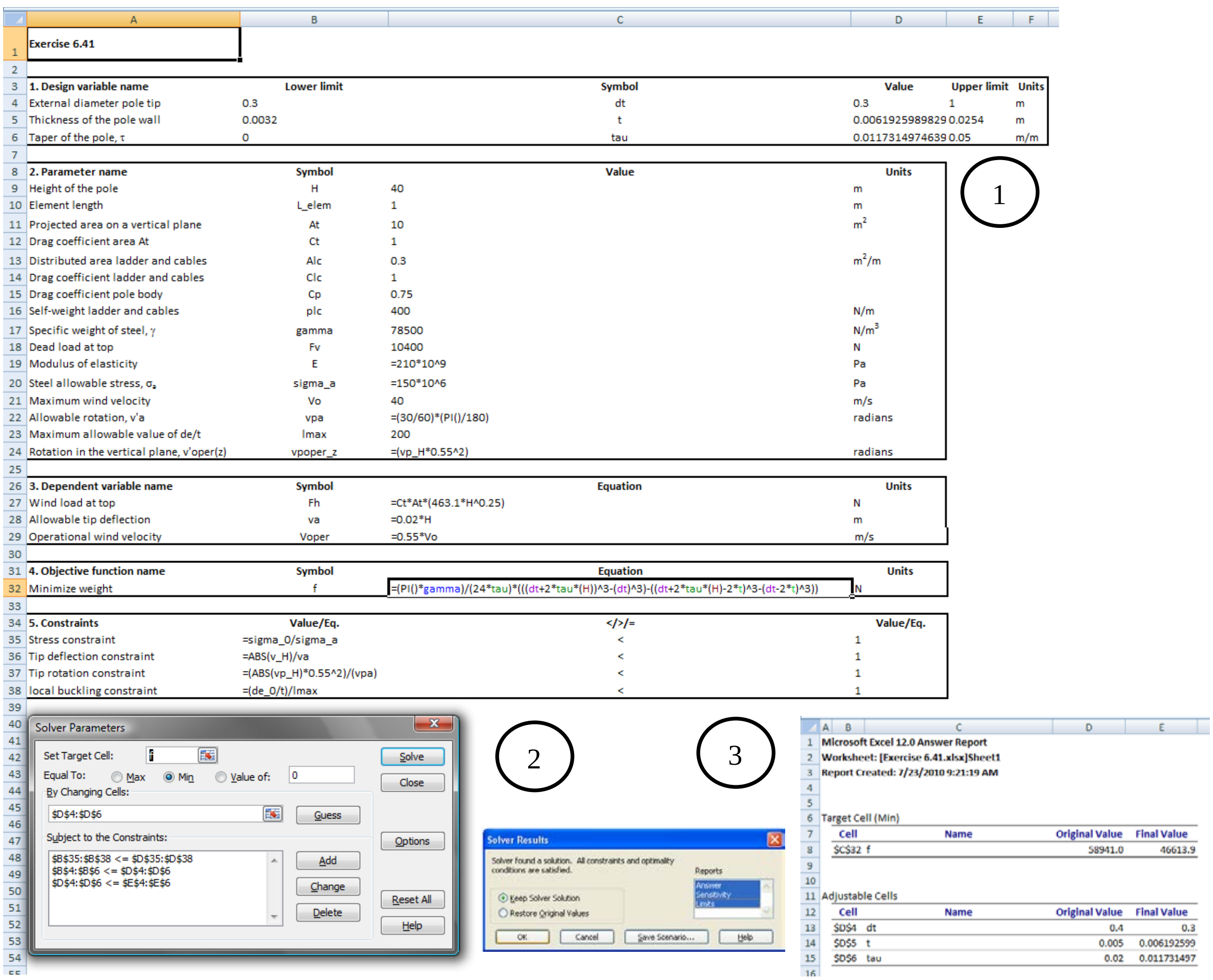
Solve the problem of Example 14.12 for a pole of height 40m.

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of dt=0.4, t=0.005, and tau=0.02 a solution of dt=0.3, t=0.00619, and tau=0.01173, which gives an objective function value of 46614, is obtained.



	A	B	C	D	E	F
1	Exercise 6.41					
2						
3	1. Design variable name	Lower limit	Symbol	Value	Upper limit	Units
4	External diameter pole tip	0.3	dt	0.3	1	m
5	Thickness of the pole wall	0.0032	t	0.0061925989829	0.0254	m
6	Taper of the pole, τ	0	tau	0.0117314974639	0.05	m/m
7						
8	2. Parameter name	Symbol	Value	Units		
9	Height of the pole	H	40	m		
10	Element length	L_elem	1	m		
11	Projected area on a vertical plane	At	10	m^2		
12	Drag coefficient area At	Ct	1			
13	Distributed area ladder and cables	Alc	0.3	m^2/m		
14	Drag coefficient ladder and cables	Clc	1			
15	Drag coefficient pole body	Cp	0.75			
16	Self-weight ladder and cables	plc	400	N/m		
17	Specific weight of steel, γ	gamma	78500	N/m^3		
18	Dead load at top	Fv	10400	N		
19	Modulus of elasticity	E	=210*10^9	Pa		
20	Steel allowable stress, σ_a	sigma_a	=150*10^6	Pa		
21	Maximum wind velocity	Vo	40	m/s		
22	Allowable rotation, v'a	vpa	=(30/60)*(PI()/180)	radians		
23	Maximum allowable value of de/t	lmax	200			
24	Rotation in the vertical plane, v'oper(z)	vpoper_z	=(vp_H*0.55^2)	radians		
25						
26	3. Dependent variable name	Symbol	Equation	Units		
27	Wind load at top	Fh	=Ct*At*(463.1*H^0.25)	N		
28	Allowable tip deflection	va	=0.02*H	m		
29	Operational wind velocity	Voper	=0.55*Vo	m/s		
30						
31	4. Objective function name	Symbol	Equation	Units		
32	Minimize weight	f	=(PI()*gamma)/(24*tau)*(((dt+2*tau*(H))^3-(dt)^3)-((dt+2*tau*(H)-2*t)^3-(dt-2*t)^3))	N		
33						
34	5. Constraints	Value/Eq.	</>/=	Value/Eq.		
35	Stress constraint	=sigma_0/sigma_a	<	1		
36	Tip deflection constraint	=ABS(v_H)/va	<	1		
37	Tip rotation constraint	=(ABS(vp_H)*0.55^2)/(vpa)	<	1		
38	local buckling constraint	=(de_0/t)/lmax	<	1		

	A	B	C	D	E
1	Microsoft Excel 12.0 Answer Report				
2	Worksheet: [Exercise 6.41.xlsx]Sheet1				
3	Report Created: 7/23/2010 9:21:19 AM				
4					
5					
6	Target Cell (Min)				
7		Cell	Name	Original Value	Final Value
8		$C$32	f	58941.0	46613.9
9					
10					
11	Adjustable Cells				
12		Cell	Name	Original Value	Final Value
13		$D$4	dt	0.4	0.3
14		$D$5	t	0.005	0.006192599
15		$D$6	tau	0.02	0.011731497

14.50

Continued.

1

	A	B	C	D	E	F	G	H	I
1	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.
2	0	1	2	3	4	5	6	7	8
3	Node	Height above ground, z [m]	Effective wind velocity pressure, q(z) [N/m^2]	External diameter pole section, de(z) [m]	Internal diameter pole section, di(z) [m]	Cross-sectional area at height z, A(z) [m^2]	Moment of inertia at height z, I(z) [m^4]	Section modulus at height z, S(z) [m^3]	Distance from pole tip to given section, y [m]
4		<=IF(A5*L_elem<=H,A5*L_elem,"-")	<=IF(A5*L_elem<=H,463.1*B5^0.25,"-")	<=IF(A5*L_elem<=H,dt+2*tau*(H-B5),"-")	<=IF(A5*L_elem<=H,D5-2*t,"-")	<=IF(A5*L_elem<=H,PI()/4*(D5^2-E5^2),"-")	<=IF(A5*L_elem<=H,PI()/64*(D5^4-E5^4),"-")	<=IF(A5*L_elem<=H,(2*G5)/(D5),"-")	<=IF(A5*L_elem<=H,H-B5,"-")
5	0	0	0.0	1.238519797	1.226134599	0.0240	0.0046	0.0073	40
6	1	1	463.1	1.215056802	1.202671604	0.0235	0.0043	0.0071	39
7	2	2	550.7	1.191593807	1.179208609	0.0231	0.0041	0.0068	38
8	3	3	609.5	1.168130812	1.155745614	0.0226	0.0038	0.0065	37
45	40	40	1164.6	0.3	0.287614802	0.0057	0.0001	0.0004	0
46	41	-	-	-	-	-	-	-	-
56	Value at base	0	0.0	1.238519797	1.226134599	0.0240	0.0046	0.0073	40
57	Value at top	40	1164.6	0.3	0.287614802	0.0057	0.0001	0.0004	0

de_0

A_0

A_H

1

	J	K	L	M	N	O	P	Q	R
1	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.	Column No.
2	0	9	10	11	12	13	14	15	16
3	Node	Horizontal distributed load, ph(y) [N/m]	Vertical distributed load, pv(y) [N/m]	Axial load at height z, N(z) [N]	Bending moment at height z, M(z) [N-m]	Axial stress at height z, σ(z) [Pa]	Curvature in the vertical plane, v''(z) [m^{-1}]	Rotation in the vertical plane, v'(z) [radians]	Horizontal displacement height z, v(z) [m]
4		<=IF(A5*L_elem<=H,(Alc*Clc+D5*Cp)*C5,"-")	<=IF(A5*L_elem<=H,plc+F5*gamma,"-")	<=IF(A5*L_elem<=H,Fv+plc*(H-B5)+(PI()*gamma)/(24*tau)*(((dt+2*tau*(H-B5))^3-(dt)^3)-((dt+2*tau*(H-B5)-2*t)^3-(dt-2*t)^3)),"-")	<=IF(A5*L_elem<=H,Alc*Clc*(205.822*H^2.25-370.48*H^1.25*B5+164.658*B5^2.25)+Cp*(dt*(205.822*H^2.25-370.48*H^1.25*B5+164.658*B5^2.25)+tau*(126.66*H^3.25-329.316*H^2.25*B5+329.316*H*B5^2.25-126.66*B5^3.25))+Fh*(H-B5),"-")	<=IF(A5*L_elem<=H,(M5)/(F5)+(N5)/(H5),"-")	<=IF(A5*L_elem<=H,-(N5)/(E*G5),"-")	<=0 <=IF(A6*L_elem<=H,Q5+(P6+P5)/(2)*(B6-B5),"-")	<=0 <=IF(A6*L_elem<=H,R5+(Q6+Q5)/(2)*(B6-B5),"-")
5	0	0.0	2282.0	73013.9	1080020.2	149999999.7	-1.13E-03	0.00E+00	0.00E+00
6	1	560.9	2246.2	70749.8	1037350.2	149703284.3	-1.15E-03	-1.14E-03	-5.70E-04
7	2	657.4	2210.3	68521.6	995229.6	149350915.7	-1.17E-03	-2.30E-03	-2.29E-03
8	3	716.8	2174.5	66329.1	953763.6	148952780.4	-1.19E-03	-3.48E-03	-5.18E-03
45	40	611.4	848.7	10400.0	0.0	1819482.1	3.27E-17	-6.16E-02	-1.17E+00
46	41	-	-	-	-	-	-	-	-
56	Value at base	0.0	2282.0	73013.9	1080020.2	149999999.7	-1.13E-03	0.00E+00	0.00E+00
57	Value at top	40.0	611.4	848.7	10400.0	0.0	1.82E+06	3.27E-17	-6.16E-02

sigma_0

vp_H

v_H