C H A P T E R

6

Optimum Design: Numerical Solution Process and Excel Solver

Section 6.5 Excel Solver for Unconstrained Optimization Problems

6.1

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.32

The annual operating cost *U* for an electrical line system is given by the following expression

$$U = \frac{(21.9E+07)}{V^2C} + (3.9E+06)C + (1.0E+03)V$$

where *V*=line voltage in kilovolts and *C*=line conductance in ohms. Find stationary points for the function, and determine *V* and *C* to minimize the operating cost.

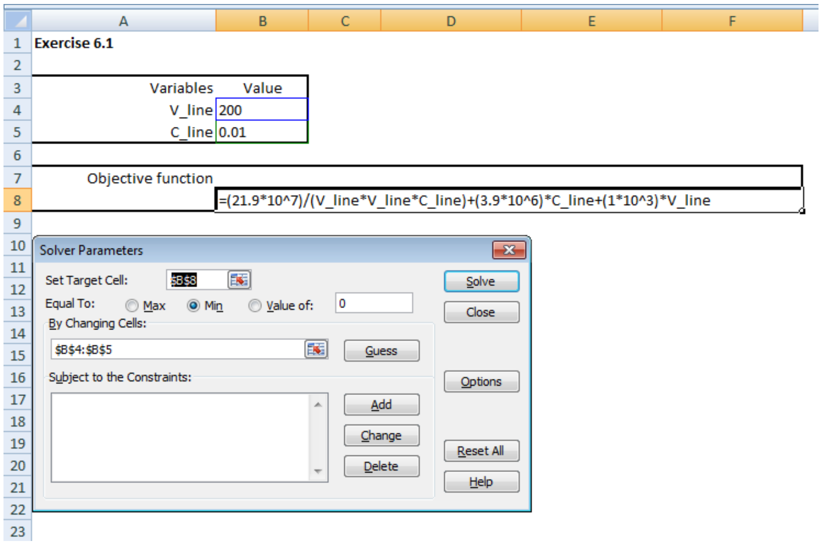
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables V and C have been renamed V_line and C_line respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of V_line=200 and C_line=0.01, a solution of V_line=241.8 and C_line=0.0310, which gives an objective function value of -483,528.61, is obtained.

1



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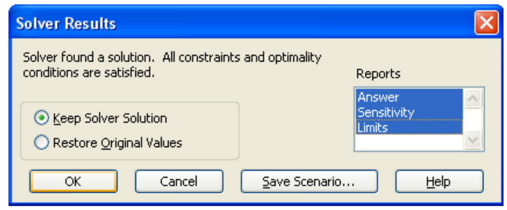

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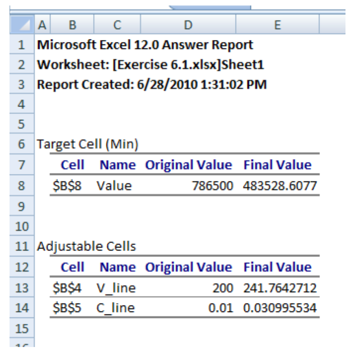
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Adjustable Cells

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B4	V_line	200	241.7642712
B5	C_line	0.01	0.030995534

6.2

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.39

$$f(x_1, x_2) = 8x_1^2 + 8x_2^2 - 80\sqrt{x_1^2 + x_2^2 - 20x_2 + 100} - 80\sqrt{x_1^2 + x_2^2 + 20x_2 + 100} - 5x_1 - 5x_2$$

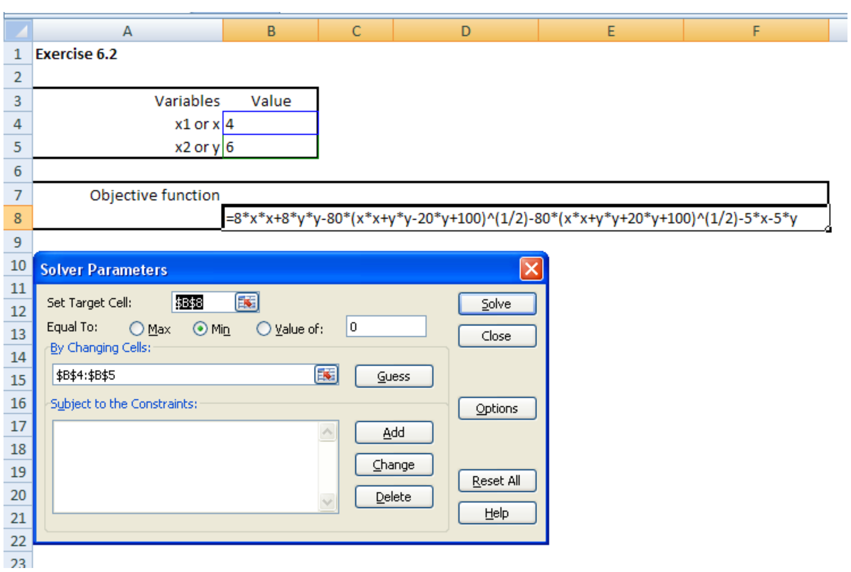
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables x1 and x2 have been renamed x and y respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of x=4 and y=6, a solution of x=4.15 and y=0.362, which gives an objective function value of -1616.2, is obtained.

1



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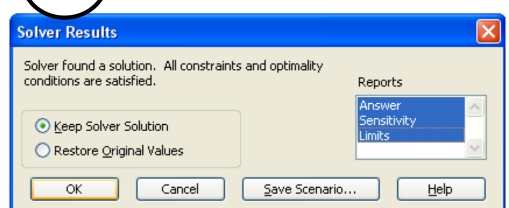

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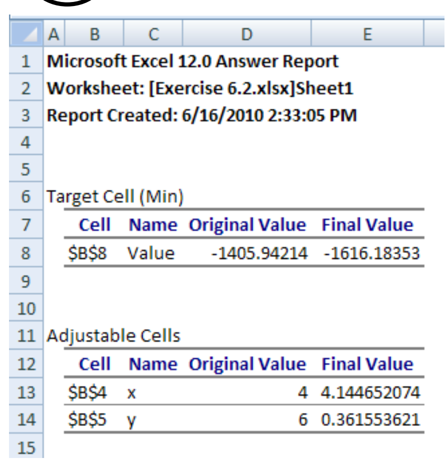

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Adjustable Cells

Cell	Name	Original Value	Final Value
B4	x	4	4.144652074
B5	y	6	0.361553621

6.3

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.40

$$f(x_1, x_2) = 9x_1^2 + 9x_2^2 - 100\sqrt{x_1^2 + x_2^2 - 20x_2 + 100} - 64\sqrt{x_1^2 + x_2^2 + 16x_2 + 64} - 5x_1 - 41x_2$$

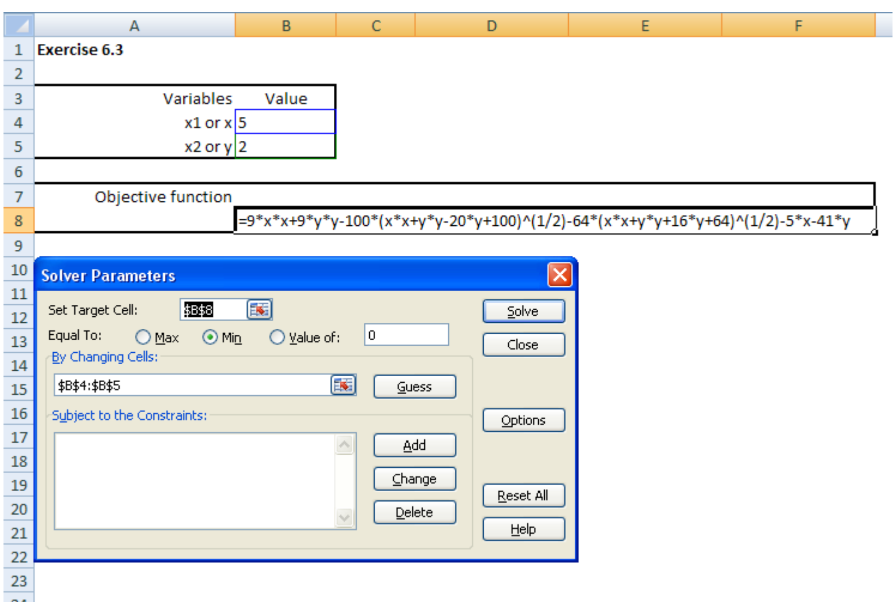
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables x1 and x2 have been renamed x and y respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of x=5 and y=2, a solution of x=3.73 and y=0.341, which gives an objective function value of -1526.6, is obtained.

1



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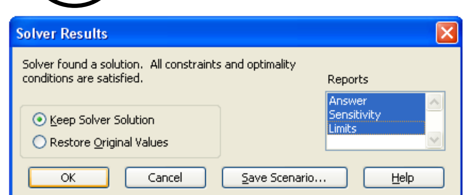

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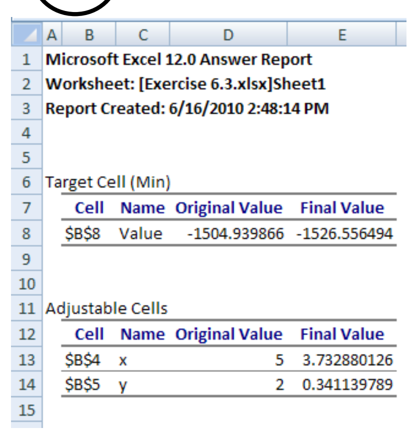


6.4

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.41

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

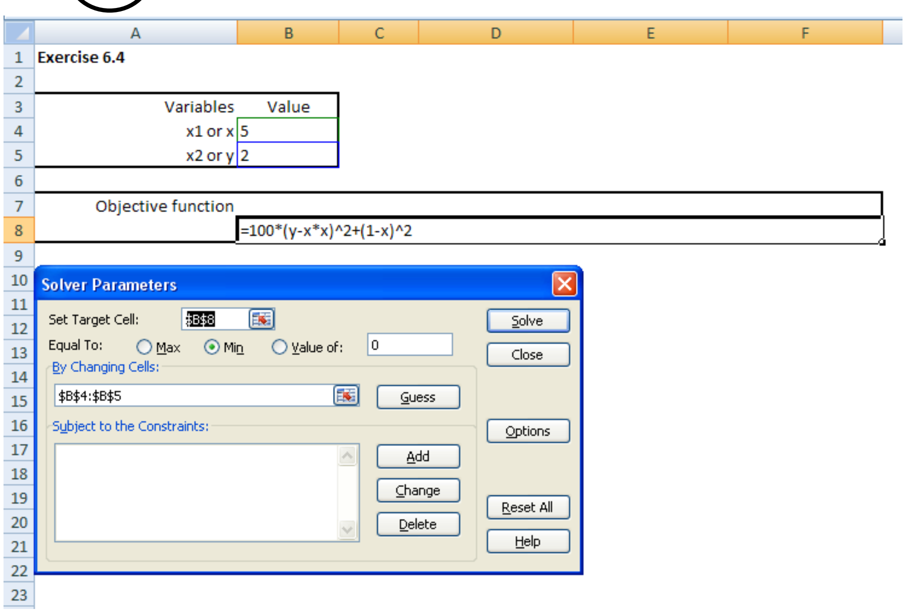
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables x1 and x2 have been renamed x and y respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of x=5 and y=2, a solution of x=1.216 and y=1.462, which gives an objective function value of 0.0752, is obtained.

1



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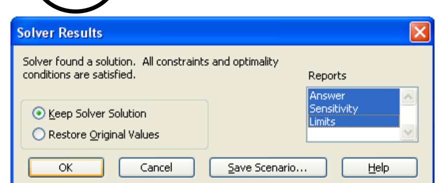

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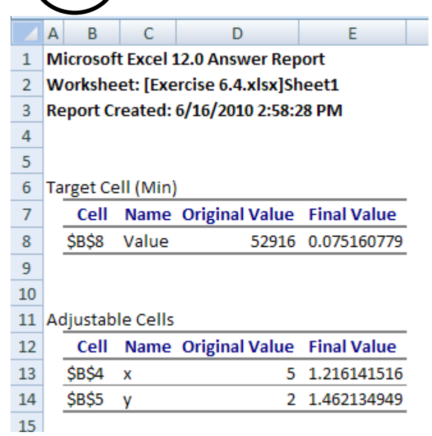


6.5

Solve the following problem using Excel Solver (choose any reasonable starting point):

Exercise 4.42

$$f(x_1, x_2, x_3, x_4) = (x_1 - 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$

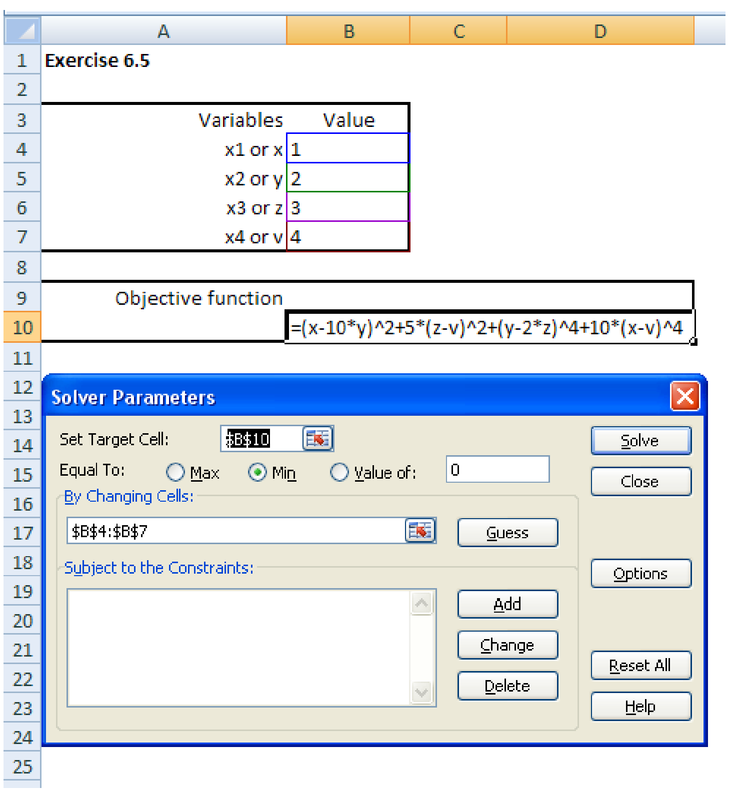
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. Variables x1, x2, x3, and x4 have been renamed x, y, z, and v respectively. The objective function and variables are input into the Solver Parameters dialog box as shown. In the options dialog box, conjugate gradient method and forward finite difference were selected. However, Newton method and/or central difference could have been selected instead. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of x=1, y=2, z=3, and v=4, a solution of x=0.246, y=0.0257, z=0.1808, and v=0.205which gives an objective function value of 0.01578, is obtained.

1



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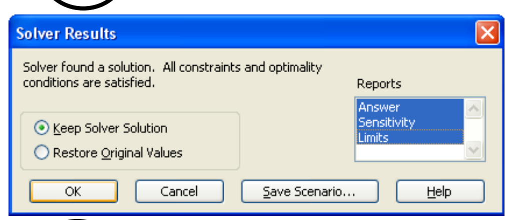

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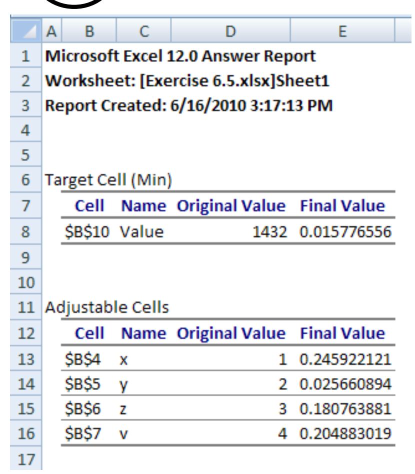


Section 6.6 Excel Solver for Linear Programming Problems

Solve the following LP problems using Excel Solver:

6.6

Solve the following LP problem using the Excel Solver:

Maximize $z = x_1 + 2x_2$

subject to $-x_1 + 3x_2 \le 10$

$x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$

$x_1 + 3x_2 \ge 6$

$x_1, x_2 \ge 0$

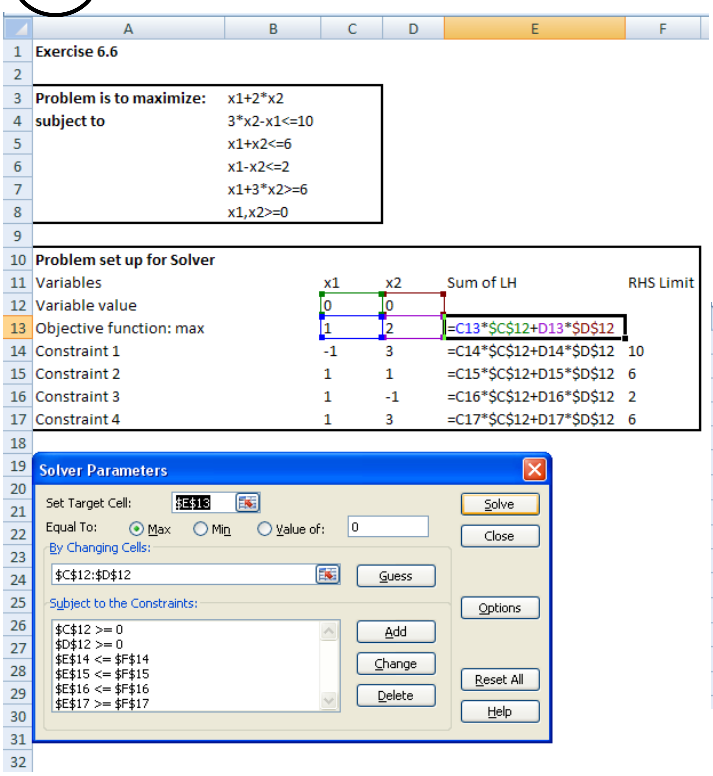
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of x1=0 and x2=0, a solution of x1=2 and x2=4, which gives an objective function value of 10, is obtained.

1



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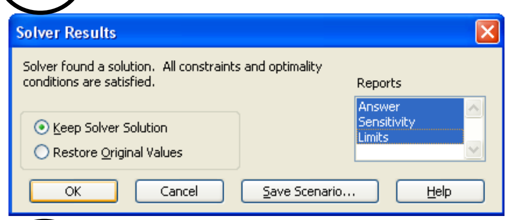

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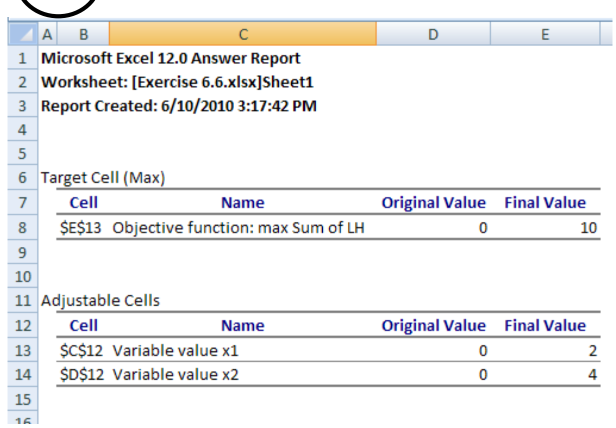


6.7

Solve the following LP problem using the Excel Solver:

Maximize $z = x_1 + 4x_2$

subject to $x_1 + 2x_2 \le 5$

$x_1 + x_2 = 4$

$x_1 - x_2 \ge 3$

$x_1, x_2 \ge 0$

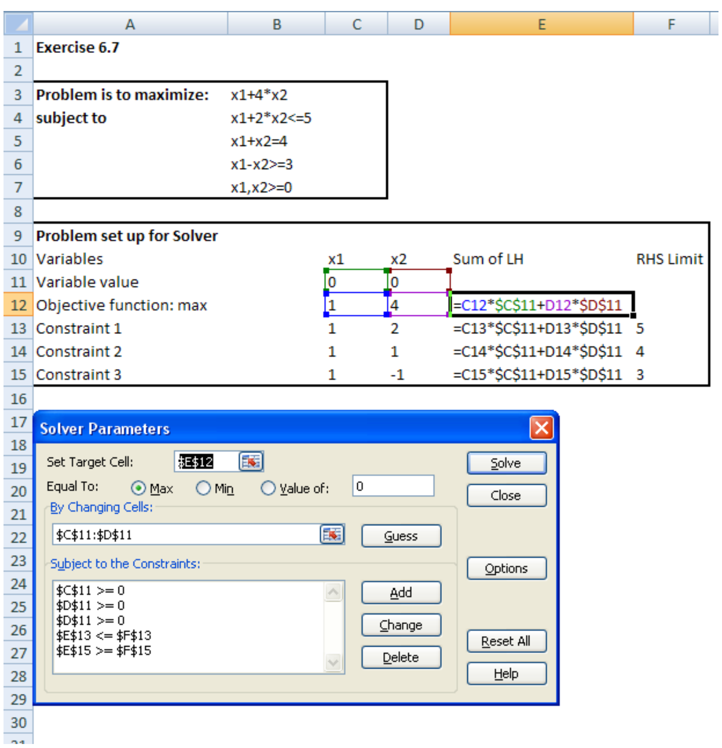
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of x1=0 and x2=0, a solution of x1=3.67 and x2=0.667, which gives an objective function value of 6.33, is obtained.

1



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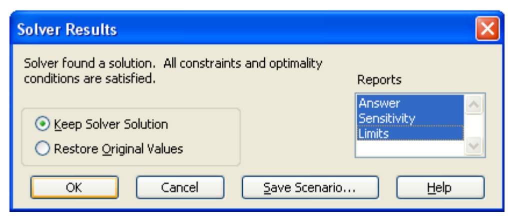

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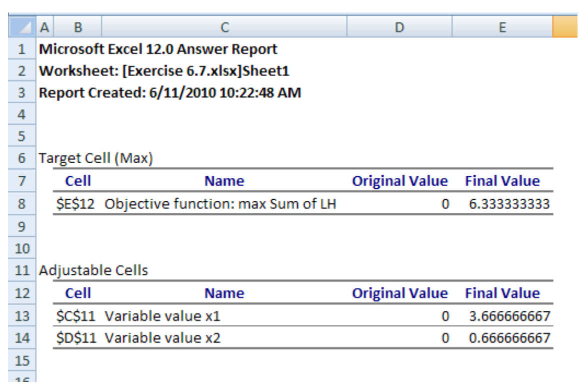


6.8

Solve the following LP problem using the Excel Solver:

Minimize $f = 5x_1 + 4x_2 - x_3$

subject to $x_1 + 2x_2 - x_3 \geq 1$

$2x_1 + x_2 + x_3 \geq 4$

$x_1, x_2 \geq 0$; x_3 is unrestricted in sign

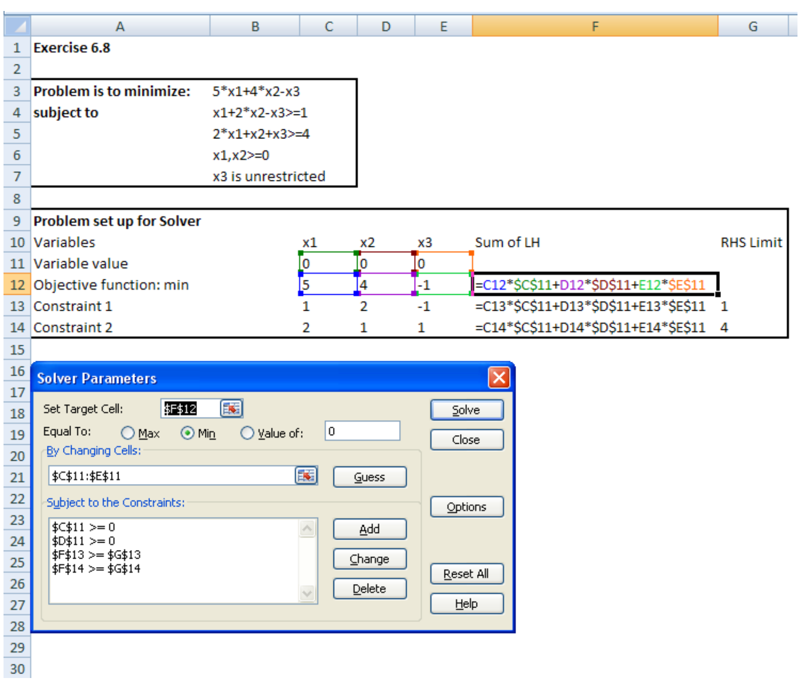
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of x1=0, x2=0, and x3=0, a solution of x1=0, x2=1.67, and x3=2.33, which gives an objective function value of 4.33, is obtained.

1



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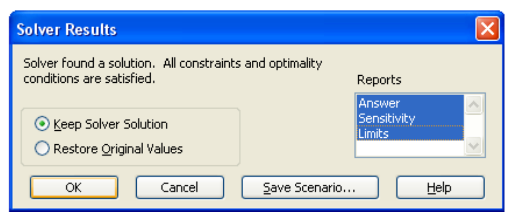

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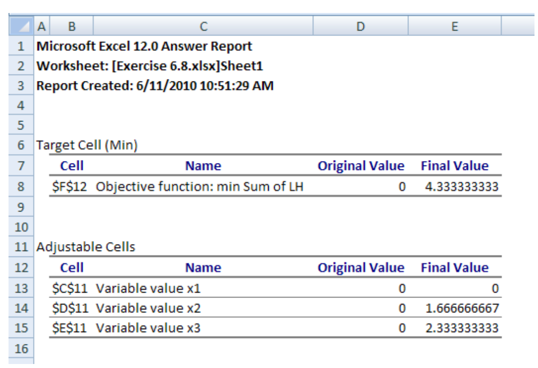


6.9

Solve the following LP problem using the Excel Solver:

Maximize $z = 2x_1 + 5x_2 - 4.5x_3 + 1.5x_4$

subject to $5x_1 + 3x_2 + 1.5x_3 \le 8$

$1.8x_1 - 6x_2 + 4x_3 + x_4 \ge 3$

$-3.6x_1 + 8.2x_2 + 7.5x_3 + 5x_4 = 15$

$x_i \ge 0;\ i = 1 \text{ to } 4$

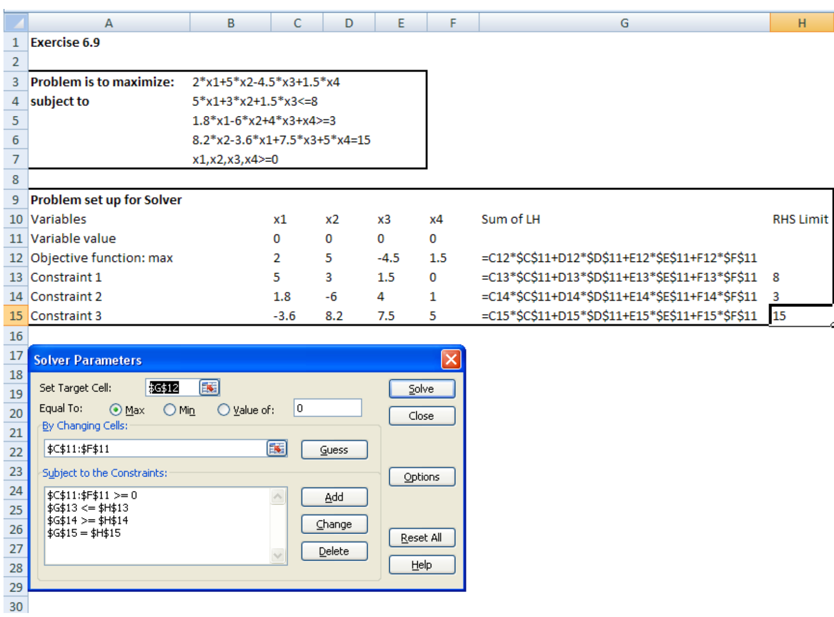
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of x1=0, x2=0, x3=0, and x4=0, a solution of x1=1.34, x2=0.441, x3=0, and x4=3.24, which gives an objective function value of 9.73, is obtained.

1



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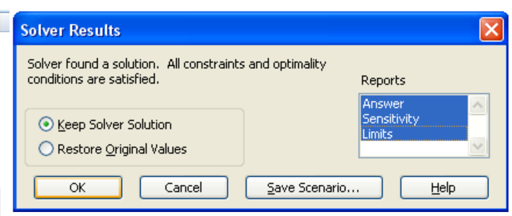

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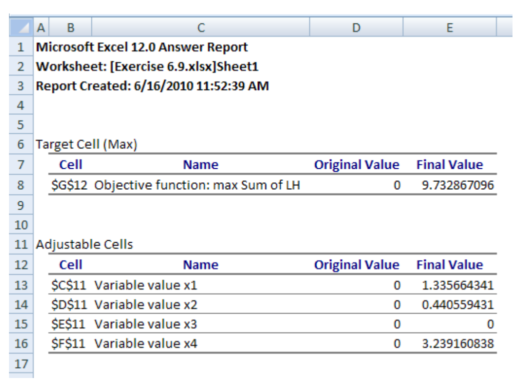


6.10

Solve the following LP problem using the Excel Solver:

Minimize $f = 8x - 3x_2 + 15x_3$

subject to $5x_1 - 1.8x_2 - 3.6x_3 \geq 2$

$3x_1 + 6x_2 + 8.2x_3 \geq 5$

$1.5x_1 - 4x_2 + 7.5x_3 \geq -4.5$

$-x_2 + 5x_3 \geq 1.5$

$x_1, x_2 \geq 0$; x_3 is unrestricted in sign

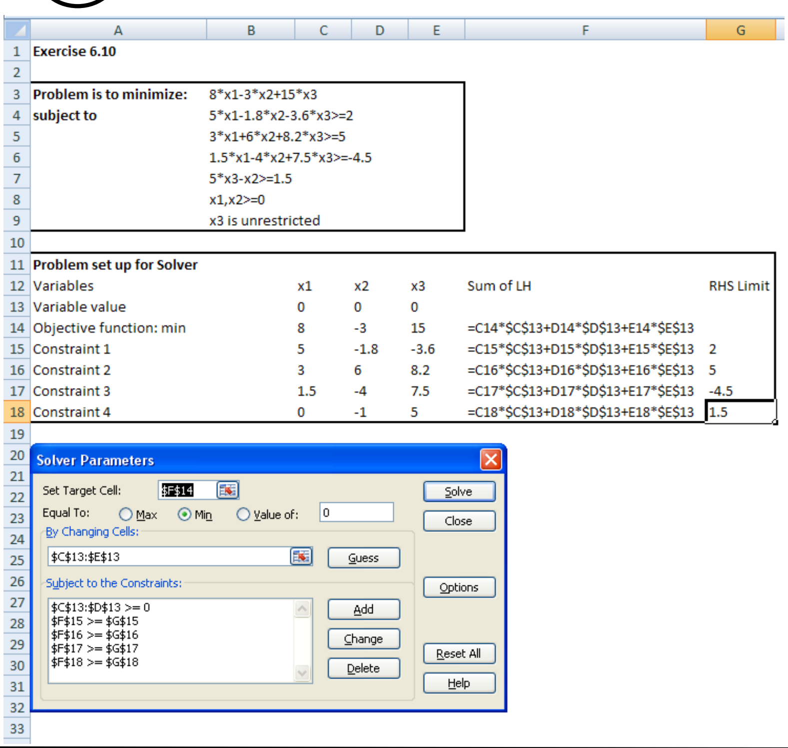
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of x1=0, x2=0, and x3=0, a solution of x1=0.654, x2=0.0756, and x3=0.315, which gives an objective function value of 9.73, is obtained.

1



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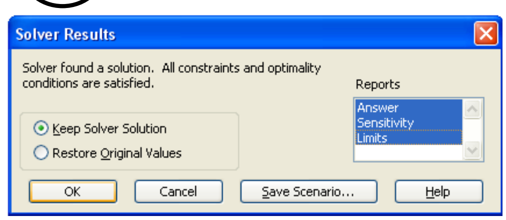

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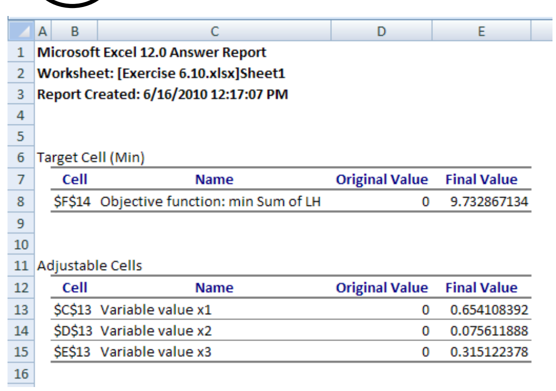


6.11

Solve the following LP problem using the Excel Solver:

Maximize $z = 10x_1 + 6x_2$

subject to $2x_1 + 3x_2 \le 90$

$4x_1 + 2x_2 \le 80$

$x_2 \ge 15$

$5x_1 + x_2 = 25$

$x_1, x_2 \ge 0$

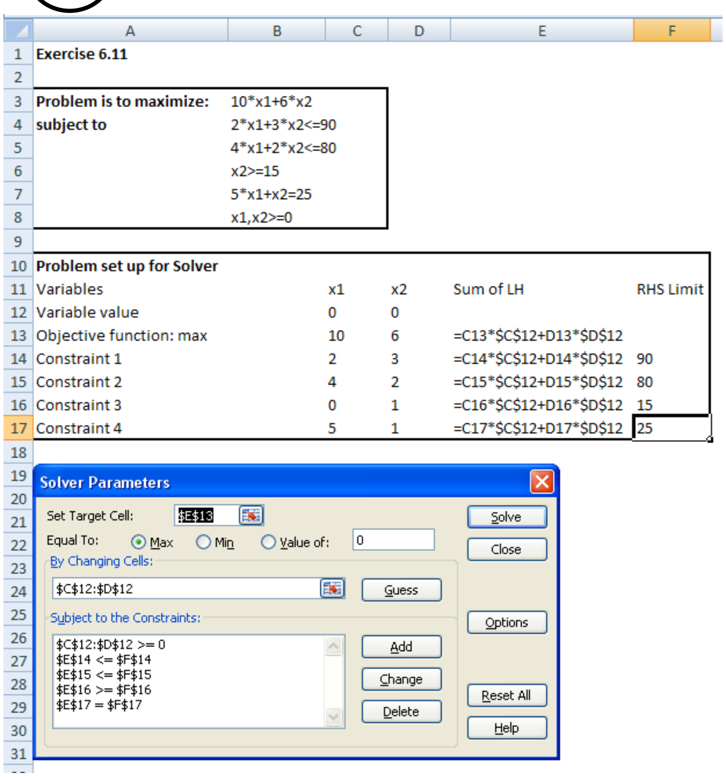
Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

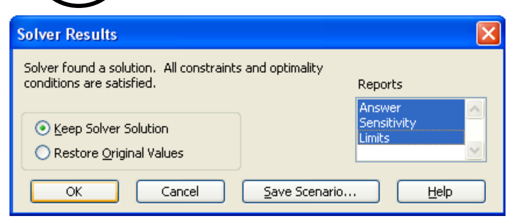
(3) The answer report shows that for initial design variable values of x1=0, and x2=0, a solution of x1=0, and x2=25, which gives an objective function value of 150, is obtained.

1



	A	B	C	D	E	F
1	Exercise 6.11					
2						
3	Problem is to maximize:	10*x1+6*x2				
4	subject to	2*x1+3*x2<=90				
5		4*x1+2*x2<=80				
6		x2>=15				
7		5*x1+x2=25				
8		x1,x2>=0				
9						
10	Problem set up for Solver					
11	Variables		x1	x2	Sum of LH	RHS Limit
12	Variable value		0	0		
13	Objective function: max		10	6	=C13*C12+D13*D12	
14	Constraint 1		2	3	=C14*C12+D14*D12	90
15	Constraint 2		4	2	=C15*C12+D15*D12	80
16	Constraint 3		0	1	=C16*C12+D16*D12	15
17	Constraint 4		5	1	=C17*C12+D17*D12	25

2



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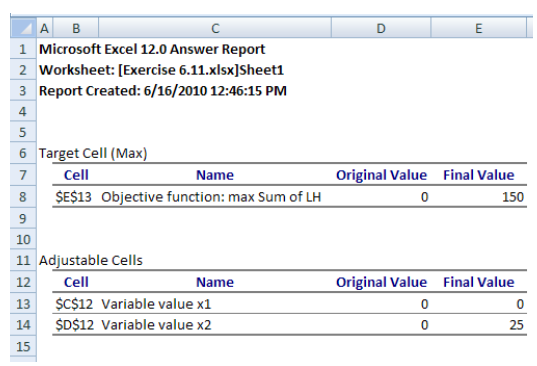

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Report Created: 6/16/2010 12:46:15 PM

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E13	Objective function: max Sum of LH	0	150

Adjustable Cells

Cell	Name	Original Value	Final Value
C12	Variable value x1	0	0
D12	Variable value x2	0	25

Section 6.7 Excel Solver for Nonlinear Programming

Solve the following problems using Excel Solver:

6.12

Solve the following NLP problem using the Excel Solver:

Exercise 3.35 (Exercise 3.34 using inner and outer diameter as design variables)
Design a hollow torsion rod shown in Fig.E3.34 to satisfy the following requirements (created by J.M. Trummel):

1. The calculated shear stress, τ , shall not exceed the allowable shear stress τ_a under the normal operation torque T_o (N·m).
2. The calculated angle of twist, θ, shall not exceed the allowable twist, θ_a (radians).
3. The member shall not buckle under a short duration torque of T_{max} (N·m).

Requirements for the rod and material properties are given in Table E3.34(A) and E3.34(B) (select a material for one rod). Use the following design variables:
x_1 = outside diameter of the shaft; x_2 = ratio of inside/outside diameter, d_i/d_o.

Using graphical optimization, determine the inside and outside diameters for a minimum mass rod to meet the above design requirements. Compare the hollow rod with an equivalent solid rod (d_i/d_o = 0). Use consistent set of units (e.g. Newtons and millimeters) and let the minimum and maximum values for design variables be given as

$$0.02 \leq d_o \leq 0.5 \text{ m}, \quad 0.60 \leq \frac{d_i}{d_o} \leq 0.999$$

Useful expressions for the rod are:

Mass of rod: $$M = \frac{\pi}{4}\rho l(d_o^2 - d_i^2), kg$$

Calculated shear stress: $$\tau = \frac{c}{J}T_o, Pa$$

Calculated angle of twist: $$\theta = \frac{l}{GJ}T_o, radians$$

Critical buckling torque: $$T_{cr} = \frac{\pi d_o^3 E}{12\sqrt{2}(1-\nu^2)^{0.75}}(1 - \frac{d_i}{d_o})^{2.5}, \text{N.m}$$

Notation

M = mass of the rod (kg),
d_o = outside diameter of the rod (m),
d_i = inside diameter of the rod (m),
ρ = mass density of material (kg/m^3),
l = length of the rod (m),
T_0 = Normal operation torque (N · m),
c = Distance from rod axis to extreme fiber (m),
J = Polar moment of inertia (m^4),
θ = Angle of twist (radians),
G = Modulus of rigidity (Pa),

T_{cr} = Critical buckling torque (N · m),
E = Modulus of elasticity (Pa), and
ν = Poisson's ratio.

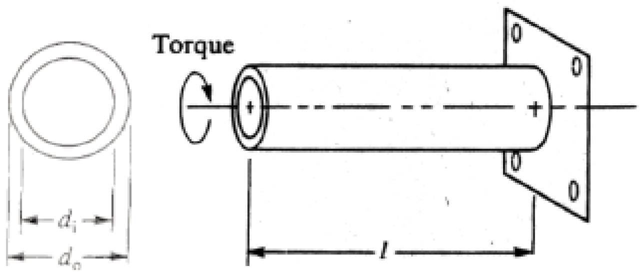


FIGURE E3-34 Hollow torsion rod.

TABLE E3-34(A) Rod Requirements

Torsion rod number	*Length, l* (m)	*Normal torque,* T_0 (kN · m)	*Max. torque,* T_{max} (kN · m)	*Allowable twist,* θ_a *(degrees)*
1	0.50	10.0	20.0	2
2	0.75	15.0	25.0	2
3	1.00	20.0	30.0	2

TABLE E3-34(B) Materials and Properties for the Torsion Rod

Material	*Density,* ρ (kg/m^3),	*Allowable Shear stress,* τ_a (MPa)	*Elastic modulus, E* (GPa)	*Shear modulus, G* (GPa)	*Poisson's ratio* (ν)
1. 4140 alloy steel	7850	275	210	80	0.30
2. Aluminum alloy 24 ST4	2750	165	75	28	0.32
3. Magnesium alloy A261	1800	90	45	16	0.35
4. Berylium	1850	110	300	147	0.02
5. Titanium	4500	165	110	42	0.30

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

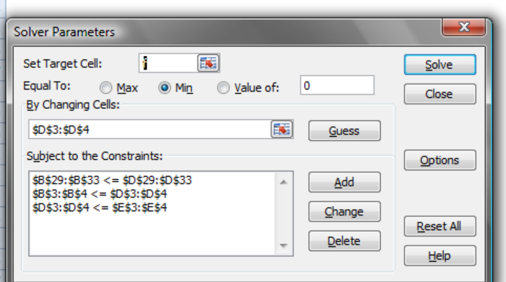
(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of do=400 and di=40 a solution of do=103.0 and di=98.3 which gives an objective function value of 2.90, is obtained.

1

	A	B	C	D	E	F
1	Exercise 6.12					
2	1. Design Variables	Lower limit	Symbol	Value	Upper limit	Units
3	Outside diameter of shaft	=0.02*1000	do	102.985279421914	=0.5*1000	mm
4	Inside diameter of shaft	=0.02*1000	di	98.3111555658017	=0.5*1000	mm
5						
6	2. Parameters	Symbol	Value	Units		
7	Length of the rod	l	=0.5*1000	mm		
8	Normal operation torque	To	=10*10^3*10^3	N-mm		
9	Max. torque	Tmax	=2*10^7	N-mm		
10	Allowable twist	θ_a, theta_a	=2*PI()/180	radians		
11	Mass density of the material	ρ, ro	=7850*10^-9	kg/mm^3		
12	Allowable shear stress	τ_a, tau_a	=275	N/mm^2		
13	Modulus of elasticity	E	=2.1*10^5	N/mm^2		
14	Modulus of rigidity	G	=8*10^4	N/mm^2		
15	Poisson's ratio	v	=0.3			
16						
17	3. Analysis Variables	Symbol	Equation	Units		
18	Distance from rod axis to extreme fiber	c_distance	=do/2	m		
19	Polar moment of inertia	J	=PI()/32*(do*do*do*do-di*di*di*di)	m^4		
20	Mass of rod	M	=PI()/4*ro*l*(do*do-di*di)	kg		
21	Calculated shear stress	τ, tau	=c_distance/J*To	Pa		
22	Calculated angle of twist:	q, theta	=l/(G*J)*To	radians		
23	Critical buckling torque	Tcr	=((PI()*do*do*do*E)/(12*SQRT(2)*(1-v*v)^0.75))*((1-di/do)^2.5)	N-m		
24						
25	4. Objective Function	Symbol	Equation	Units		
26	Minimize f	f	=(PI()*ro*l/4)*(do*do-di*di)	kg		
27						
28	5. Constraints	Value	</>/=	RS		
29	g1 (shear stress constraint)	=(To*c_distance/J)/(tau_a)	<	=tau_a/tau_a		
30	g2 (Allowable twist constraint)	=((To*l)/(G*J))/(theta_a)	<	=theta_a/theta_a		
31	g3 (buckling constraint)	=(-Tcr)/(Tmax)	<	=-Tmax/Tmax		
32	g6 (design variable constraint)	=(-di/do)/0.6	<	=-0.6/0.6		
33	g7 (design variable constraint)	=(di/do)/0.999	<	=0.999/0.999		

3



2

Target Cell (Min)

Cell	Name	Original Value	Final Value
C26	f	488.2977461	2.900453065

Adjustable Cells

Cell	Name	Original Value	Final Value
D3	do	400	102.9852794
D4	di	40	98.31115557

6.13

Solve the following NLP problem using the Excel Solver:

Exercise 3.50

A minimum mass structure (area of member 1 is the same as member 3) three-bar truss is to be designed to support a load P as shown in Fig. 2.9. The following notation may be used: $P_u = P\cos\theta$, $P_v = P\sin\theta$, A_1 = cross-sectional area of members 1 and 3, A_2 = cross-sectional area of member 2.

The members must not fail under the stress, and deflection at node 4 must not exceed 2cm in either direction. Use Newtons and millimeters as units. The data is given as P = 50 kN; θ = 30°; mass density, ρ = 7850 kg/m^3; modulus of elasticity, E = 210 GPa; allowable stress, σ_a = 150 MPa. The design variables must also satisfy the constraints $50 \leq A_i \leq 5000$ mm^2 .

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

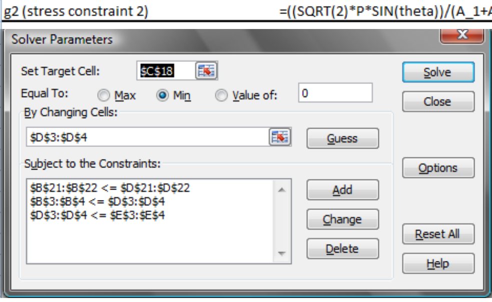
(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of A_1=250 and A_2=100 a solution of A_1=294 and A_2=65.8 which gives an objective function value of 7.04, is obtained.

Continued.

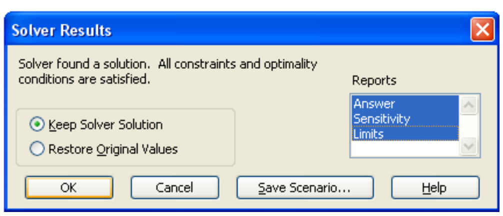
1

	A	B	C	D	E	F
1	**Exercise 6.13**					
2	**1. Design Variables**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
3	Cross-sectional area of members 1 and 3	50	A_1	293.631953891414	5000	mm^2
4	Cross-sectional area of member 2	50	A_2	65.7466067603963	5000	mm^2
5						
6	**2. Parameters**	**Symbol**	**Value**	**Units**		
7	Point load	P	=50*10^3	N		
8	Loading angle	θ, theta	=30*PI()/180	radians		
9	Member length	l	=1*10^3	mm		
10	Mass density	ρ, ro	=7.85*10^-6	kg/mm^3		
11	Allowable stress	σ_a, sigma_a	=150	N/mm^2		
12	Modulus of elasticity	E	=2.1*10^5	N/mm^2		
13						
14	**3. Analysis variables**	**Symbol**	**Equation**	**Units**		
15	N/A	N/A	N/A	N/A		
16						
17	**4. Objective function**	**Symbol**	**Equation**	**Units**		
18	Minimize f	f	=ro*l*(2*SQRT(2)*A_1+A_2)	kg		
19						
20	**5. Constraints**	**Value**	**</>/=**	**RS**		
21	g1 (stress constraint 1)	=(((P*COS(theta)/A_1)+(P*SIN(theta)/(A_1+A_2*SQRT(2))))/SQRT(2))/sigma_a	<	=sigma_a/sigma_a		
22	g2 (stress constraint 2)	=((SQRT(2)*P*SIN(theta))/(A_1+A_2*SQRT(2)))/sigma_a	<	=sigma_a/sigma_a		



3

2



	A	B	C	D	E
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5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		C18	f Equation	6.34	7.04
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		D3	A_1	250	293.6319539
14		D4	A_2	100	65.74660676
15					

6.14

Solve the following NLP problem using the Excel Solver:

Exercise 3.51

Design of a water tower support column. As a member of the ABC consulting Engineers you have been asked to design a cantilever cylindrical support column of minimum mass for a new water tank. The tank itself has already been designed in the tear-drop shape shown in Fig. E3.51. The height of the base of the tank (*H*), the diameter of the tank (*D*), and wind pressure on the tank (*w*) are given as *H* = 30 m, *D* = 10 m, and *w* = 700 N/m^2. Formulate the design optimization problem and solve it graphically. (created by G.Baenziger).

In addition to designing for combined axial and bending stresses and buckling, several limitations have been placed on the design. The support column must have an inside diameter of at least 0.70 m (d_i) to allow for piping and ladder access to the interior of the tank. To prevent local buckling of the column walls the diameter/thickness ratio (d_0/t) shall not be greater than 92. The large mass of water and steel makes deflections critical as they add to the bending moment. The deflection effects as well as an assumed construction eccentricity (*e*) of 10 cm must be accounted for in the design process. Deflection at C.G. of the tank should not be greater than Δ.

Limits on the inner radius and wall thickness are $0.35 \le R \le 2.0$ m and $1.0 \le t \le 20$ cm.

Pertinent constraints and formulas

Height of water tank,	h = 10 m
Allowable deflection,	Δ = 20 cm
Unit weight of water,	γ_w= 10 kN/m^3
Unit weight of steel,	γ_s= 80 kN/m^3
Modulus of elasticity,	E = 210 GPa
Moment of inertia of the column,	$I = \frac{\pi}{64}[d_o^4 - (d_o - 2t)^4]$
Cross-sectional area of column material,	$A = \pi t(d_o - t)$
Allowable bending stress,	σ_b = 165 MPa
Allowable axial stress,	$\sigma_a = \frac{12\pi^2 E}{92(H/r)^2}$ (calculated using the critical buckling load with factor of safety of $\frac{23}{12}$
Radius of gyration,	$r = \sqrt{\frac{I}{A}}$
Average thickness of tank wall,	t_t = 1.5 cm
Volume of tank,	$V = 1.2\pi D^2 h$
Surface area of tank,	$A_s = 1.25\pi D^2$
Projected area of tank, for wind loading,	$A_p = \frac{2Dh}{3}$
Load on the column due to weight of water and steel tank,	$P = V\gamma_w + A_s t_t \gamma_s$

Lateral load at the tank C.G due to wind pressure,	$W = wA_p$
Deflection at C.G. of tank,	$\delta = \delta_1 + \delta_2$,where $\delta_1 = \frac{WH^2}{12EI}(4H + 3h)$ $\delta_2 = \frac{H}{2EI}(0.5Wh + Pe)(H + h)$
Moment at base,	$M = W(H + 0.5h) + (\delta + e)P$
Bending stress,	$f_b = \frac{M}{2I} d_o$
Axial stress,	$f_a = \left({}^{P}\!/_{A}\right) = \frac{V\gamma_w + A_s t_t \gamma_s}{\pi t(d_o - \mathrm{t})}$
Combined stress constraint,	$\frac{f_a}{\sigma_a} + \frac{f_b}{\sigma_b} \leq 1$
Gravitational acceleration,	g = 9.81 m/s^2

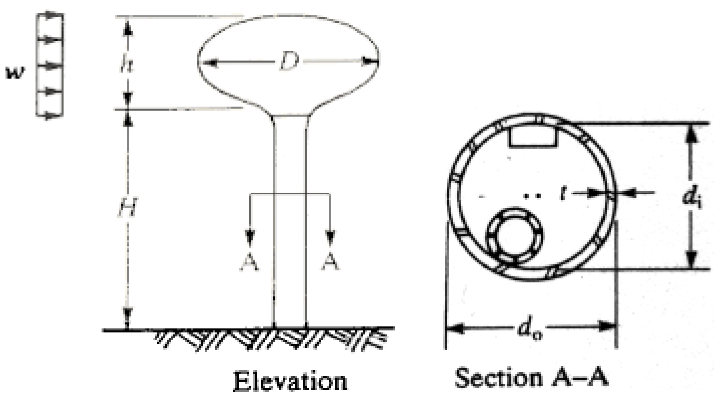


FIGURE E3.51 Water Tower support column.

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

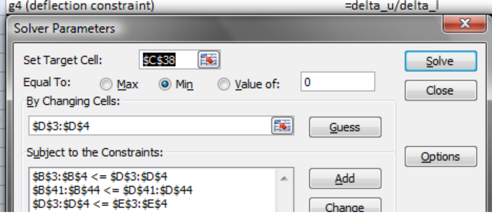
(3) The answer report shows that for initial design variable values of t=10 and R_=100 a solution of t=2.84 and R_=129.0 which gives an objective function value of 562, is obtained.

6.14

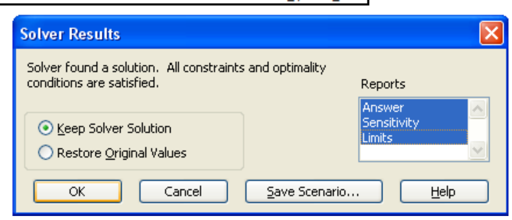
Continued.

	A	B	C	D	E	F
1	**Exercise 6.14**					
2	**1. Design Variables**	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
3	Wall thickness	1	t	2.83544442665551	40	cm
4	Mean radius	=0.35*100+0.5*t	R_	129.012713439187	=2.5*100+0.5*t	cm
5						
6	**2. Parameters**	**Symbol**	**Value**	**Units**		
7	Diameter of the tank	D	=1000	cm		
8	Height of the base of the tank	H	=3000	cm		
9	Wind pressure on the tank	w	0.07	N/cm^2		
10	Height of water tank	h, h_tank	1000	cm		
11	Assumed construction eccentricity	e	10	cm		
12	Allowable deflection	Δ, delta_l	20	cm		
13	Unit weight of water	γ_w, gamma_w	0.01	N/cm^3		
14	Unit weight of steel	γ_s, gamma_s	0.08	N/cm^3		
15	Modulus of elasticity	E	=2.1*10^7	N/cm^2		
16	Allowable bending stress	σ_b, sigma_b	=1.65*10^4	N/cm^2		
17	Allowable axial stress	σ_a, sigma_a	=(12*PI()^2*E_)/(92*(H/r_gyration)^2)	N/cm^2		
18	Average thickness of tank wall	t_t, tt	1.5	cm		
19	Gravitational acceleration	g	981	cm/s^2		
20						
21	**3. Analysis Variables**	**Symbol**	**Equation**	**Units**		
22	Outside diameter of support column	do	=(2*R_+t)	cm		
23	Inside diameter of support column	di	=(2*R_-t)	cm		
24	Moment of inertia of the column	I	=PI()*(do^4-di^4)/64	cm^4		
25	Cross-sectional area of column material	A	=PI()*(do^2-di^2)/4	cm^2		
26	Radius of gyration	r, r_gyration	=(I/A)^(1/2)	cm		
27	Volume of tank	V	=1.2*PI()*D^2*h_tank	cm^3		
28	Surface area of tank	As	=1.25*PI()*1000^2	cm^2		
29	Projected area of tank, for wind loading	Ap	=2*D*h_tank/3	cm^2		
30	Load on column due to weight of water and steel tank	P	=V*gamma_w+As*tt*gamma_s	N		
31	Lateral load at tank C.G. due to wind pressure	W, W_lat	=w*Ap	N		
32	Moment at base	M	=W_lat*(H+0.5*h_tank)+(delta_u+e)*P	N-cm		
33	Bending stress	fb	=(M)/(2*I)*(do)	N/cm^2		
34	Axial stress	fa	=P/A	N/cm^2		
35	Deflection at C.G. of tank	δ, delta_u	=((W_lat*H^2)/(12*E_*I))*(4*H+3*h_tank)+((H)/(2*E_*I))*(0.5*w*H+P*e)*(H+h_tank)	cm		
36						
37	**4. Objective Function**	**Symbol**	**Equation**	**Units**		
38	Minimize mass of support column	f	=(gamma_s/g)*(2*PI()*R_*t)*H	kg		
39						
40	**5. Constraints**	**Value**	**</>/=**	**RS**		
41	g1 (combined stress constraint)	=(fa/sigma_a)+(fb/sigma_b)	<	1		
42	g2 (inner diameter constraint)	=70/di	<	=di/di		
43	g3 (diameter/thickness ratio constraint)	=(do/t)/92	<	=92/92		
44	g4 (deflection constraint)	=delta_u/delta_l	<	=delta_l/delta_l		

1



2



3

	A	B	C	D	E
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5					
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9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		D3	t	10	2.835444427
14		D4	R_	100	129.0127134

6.15

Solve the following NLP problem using the Excel Solver:

Exercise 3.54

Design of a tripod. Design a minimum mass tripod of height *H* to support a vertical load *W* = 60 kN. The tripod bas is an equilateral triangle with sides *B* = 1200 mm. The struts have a solid circular cross section of diameter *D* (Fig. E3.54).

The axial stress in the struts must not exceed the allowable stress in compression, and axial load in the strut *P* must not exceed the critical buckling load P_{cr} divided by a safety factor FS = 2. Use consistent units of Newtons and centimeters. The minimum and maximum values for design variables are $0.5 \le H \le 5$m and $0.5 \le D \le 50$ cm. Material properties and other relationship are given below:

Material: aluminum alloy 2014-T6

Allowable compressive stress,	σ_a = 150MPa
Young's modulus,	E = 75 GPa
Mass density,	ρ = 2800 kg/m^3
Strut length,	$l = (H^2 + \frac{1}{3}B^2)^{0.5}$
Critical buckling load,	$P_{cr} = \frac{\pi^2 EI}{l^2}$
Moment of inertia,	$I = \frac{\pi}{64}D^4$
Strut load,	$P = \frac{Wl}{3H}$

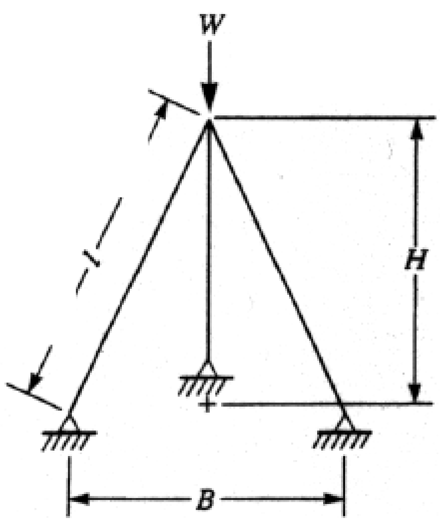


FIGURE E3.54 A tripod.

Solution

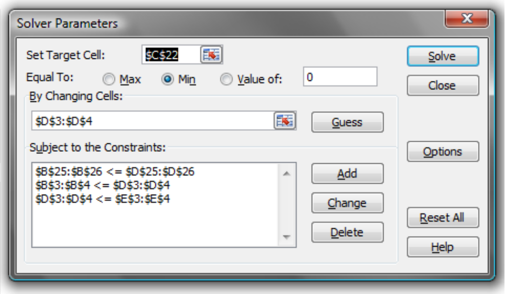
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of H=100 and D=20 a solution of H=50 and D=3.42 which gives an objective function value of 6.61, is obtained.

1

	A	B	C	D	E	F
1	**Exercise 6.15**					
2	1. Design Variables	Lower limit	Symbol	Value	Upper limit	Units
3	Tripod height	=0.5*(100)	H	50	=5*(100)	cm
4	Strut cross-section diameter	0.5	D	3.42322785703146	50	cm
5						
6	2. Parameters	Symbol	Value	Units		
7	Vertical load	W	=60*1000	N		
8	Tripod sides	B	=1200*(100/1000)	cm		
9	Factor of Safety	FS	2			
10	Allowable compressive stress	σ_a, sigma_a	=1.5*10^4	N/cm^2		
11	Young's modulus	E	=7.5*10^6	N/cm^2		
12	Mass density	ρ, ro	=2.8*10^-3	kg/cm^3		
13						
14	3. Analysis variables	Symbol	Equation	Units		
15	Cross-sectional area	A	=PI()*D^2/4	cm^2		
16	Strut length	l	=(H^2+(1/3)*B^2)^(0.5)	cm^2		
17	Moment of inertia	I	=PI()*D^4/64	cm^4		
18	Critical buckling load	Pcr	=(PI()^2*E*I)/(l^2)	N		
19	Strut load	P	=(W*l)/(3*H)	N		
20						
21	4. Objective function	Symbol	Equation	Units		
22	Minimize mass	f	=3*(ro*A*l)	kg		
23						
24	5. Constraints	Value	</>/=	RS		
25	g1 (allowable compressive stress constraint)	=(P/A)/sigma_a	<	=sigma_a/sigma_a		
26	g2 (buckling load constraint)	=(2*P)/Pcr	<	=Pcr/Pcr		



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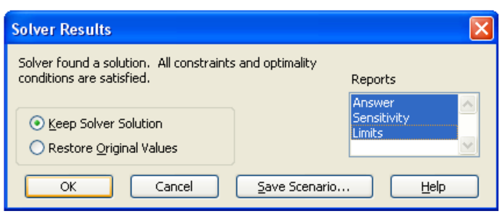

3

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5					
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7		**Cell**	**Name**	**Original Value**	**Final Value**
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9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		D3	H	100	50
14		D4	D	20	3.423227857
15					

6.16

Solve the following NLP problem using the Excel Solver:

Solve the spring design problem for the following data: Applied load (P) = 20 lb.

TABLE 2.2 Information to design a coil spring

Notation	*Data*
Deflection along spring axis	δ, in
Mean Coil Diameter	D, in
Wire Diameter	d, in
Number of active coils	N
Gravitational constant	$g = 386$ in/s^2
Frequency of surge waves	ω, Hz
Weight Density of spring material	$\gamma = 0.285$ lb/in^3
Shear Modulus	$G = (1.15*10^7)$ lb/in^2
Mass density of material ($\rho = \gamma/g$)	$\rho = (7.38342*10^{-4})$ lb-s^2/in^4
Allowable shear stress	$\tau_a = 80{,}000$ lb/in^2
Number of inactive coils	Q = 2
Applied load	P = 20 lb
Minimum spring deflection	$\Delta = 0.5$ in
Lower limit on surge wave frequency	$\omega_o = 100$ Hz
Limit on outer diameter of coil	$D_o = 1.5$ in

Load deflection equation: $P = K\delta$

Spring Constant: $K = \dfrac{d^4 G}{8D^3 N}$

Shear Stress: $\tau = \dfrac{8kPD}{\pi d^3}$

Wahl stress concentration factor: $k = \dfrac{(4D-d)}{4(D-d)} + \dfrac{0.615d}{D}$

Frequency of surge waves: $\varpi = \dfrac{d}{2\pi ND^2}\sqrt{\dfrac{G}{2\rho}}$

Design variables for the problem are defined as below:

d = wire diameter, in
D = mean coil diameter, in
N = number of active coils, integer

The problem to minimize the mass of the spring, given as volume*mass density is:

$$Mass = (\frac{\pi d^2}{4})[(\mathrm{N}+\mathrm{Q})\pi\,\mathrm{D}]\rho = \frac{\pi d^2(\mathrm{N}+\mathrm{Q})\pi\,\mathrm{D}\,\rho}{4}$$

The constraints for the spring design problem are formulated as

$$\frac{P}{K} \geq \Delta\ ,\ \tau \leq \tau_a\ ,\ \omega \geq \omega_0\ ,\ D + d \leq D_0\ ,\ d_{\min} \leq d \leq d_{\max}\ ,\ D_{\min} \leq D \leq D_{\max}\ ,\ N_{\min} \leq N \leq N_{\max}$$

Solution

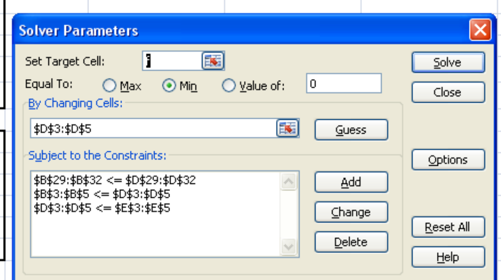
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

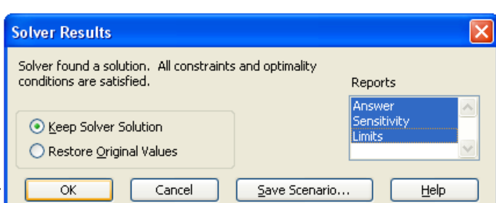
(3) The answer report shows that for initial design variable values of d=0.2, D_coil=1.3, and N=2 a solution of d=0.0705, D_coil=0.444, and N=10.16 which gives an objective function value of 0.0268, is obtained.

	A	B	C	D	E
1	**Design of Coil Springs**				
2	**1. Design Variables**	**Lower limit**	**Symbol**	**Value**	**Upper limit**
3	Wire diameter	=0.05	d, in	0.2	=0.2
4	Mean coil diameter	=0.25	D, in	1.3	=1.3
5	Number of active coils	=2	N	2	20
6					
7	**2. Parameters**	**Symbol**	**Value**	**Units**	
8	Shear modulus	G	=11500000	lb/in^2	
9	Mass density	ρ, ro	=7.38342*10^-4	lb-s^2/in^4	
10	Allowable shear stress	τ_a, tau_a	=80000	lb/in^2	
11	Number of inactive coils	Q	=2		
12	Applied load	P	=20	lb	
13	Minimum spring deflection	Δ, Def_min	=0.5	in	
14	Lower limit on surge wave frequency	ω_o, omega_0	=100	Hz	
15	Limit on outer diameter of the coil	D_o	=1.5	in	
16					
17	**3. Analysis Variables**	**Symbol**	**Equation**	**Units**	
18	Load deflection equation	δ, Def	=P/K	in	
19	Spring Constant	K	=(d^4*G)/(8*D_coil^3*N)	lb/in	
20	Shear Stress	τ, tau	=(8*k_CF*P*D_coil)/(PI()*d^3)	lb/in^2	
21	Wahl stress concentration factor	k, k_CF	=(4*D_coil-d)/(4*(D_coil-d))+(0.615*d)/(D_coil)		
22	Frequency of surge waves	ω, omega	=((d)/(2*PI()*N*D_coil^2))*SQRT(G/(2*ro))	Hz	
23					
24	**4. Objective Function**	**Symbol**	**Equation**	**Units**	
25	Minimize f	f	=(N+2)*D_coil*d^2		
26	Mass	M	=f*ro*PI()^2/4	lb-s^2/in	
27					
28	**5. Constraints**	**Value**	**</>/=**	**RS**	
29	Deflection constraint	=1-P/(K*Def_min)	<	0	
30	Shear stress constraint	=tau/tau_a-1	<	0	
31	Frequency constraint	=1-omega/omega_0	<	0	
32	Outer diameter constraint	=(D_coil+d)/(Do)-1	<	0	
33					
34					

1



2



3

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.16.xlsx]Sheet1**				
3	**Report Created: 6/17/2010 1:22:09 PM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		C25	f	0.208	0.026777982
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		D3	d	0.2	0.07047271
14		D4	D_coil	1.3	0.443594147
15		D5	N	2	10.15486337
16					

6.17

Solve the following NLP problem using the Excel Solver:

Solve the spring design problem for the following data: Number of active coils (N) = 20, limit on outer diameter of the coil (D_0) = 1 in, number of inactive coils (Q) = 4.

TABLE 2.2 Information to design a coil spring

Notation	*Data*
Deflection along spring axis	δ, in
Mean Coil Diameter	D, in
Wire Diameter	d, in
Number of active coils	N
Gravitational constant	$g = 386$ in/s^2
Frequency of surge waves	ω, Hz
Weight Density of spring material	$\gamma = 0.285$ lb/in^3
Shear Modulus	$G = (1.15*10^7)$ lb/in^2
Mass density of material ($\rho = \gamma/g$)	$\rho = (7.38342*10^{-4})$ lb-s^2/in^4
Allowable shear stress	$\tau_a = 80{,}000$ lb/in^2
Number of inactive coils	Q = 4
Applied load	P = 20 lb
Minimum spring deflection	$\Delta = 0.5$ in
Lower limit on surge wave frequency	$\omega_o = 100$ Hz
Limit on outer diameter of coil	$D_o = 1$ in

Load deflection equation: $P = K\delta$

Spring Constant: $K=\dfrac{d^4G}{8D^3N}$

Shear Stress: $\tau=\dfrac{8kPD}{\pi d^3}$

Wahl stress concentration factor: $k=\dfrac{(4D-d)}{4(D-d)}+\dfrac{0.615d}{D}$

Frequency of surge waves: $\varpi = \dfrac{d}{2\pi ND^2}\sqrt{\dfrac{G}{2\rho}}$

Design variables for the problem are defined as below:

d = wire diameter, in
D = mean coil diameter, in
N = number of active coils, integer

The problem to minimize the mass of the spring, given as volume*mass density is:

$$Mass = \left(\frac{\pi d^2}{4}\right)[(N+Q)\pi D]\rho = \frac{\pi d^2 (N+Q)\pi D \rho}{4}$$

The constraints for the spring design problem are formulated as

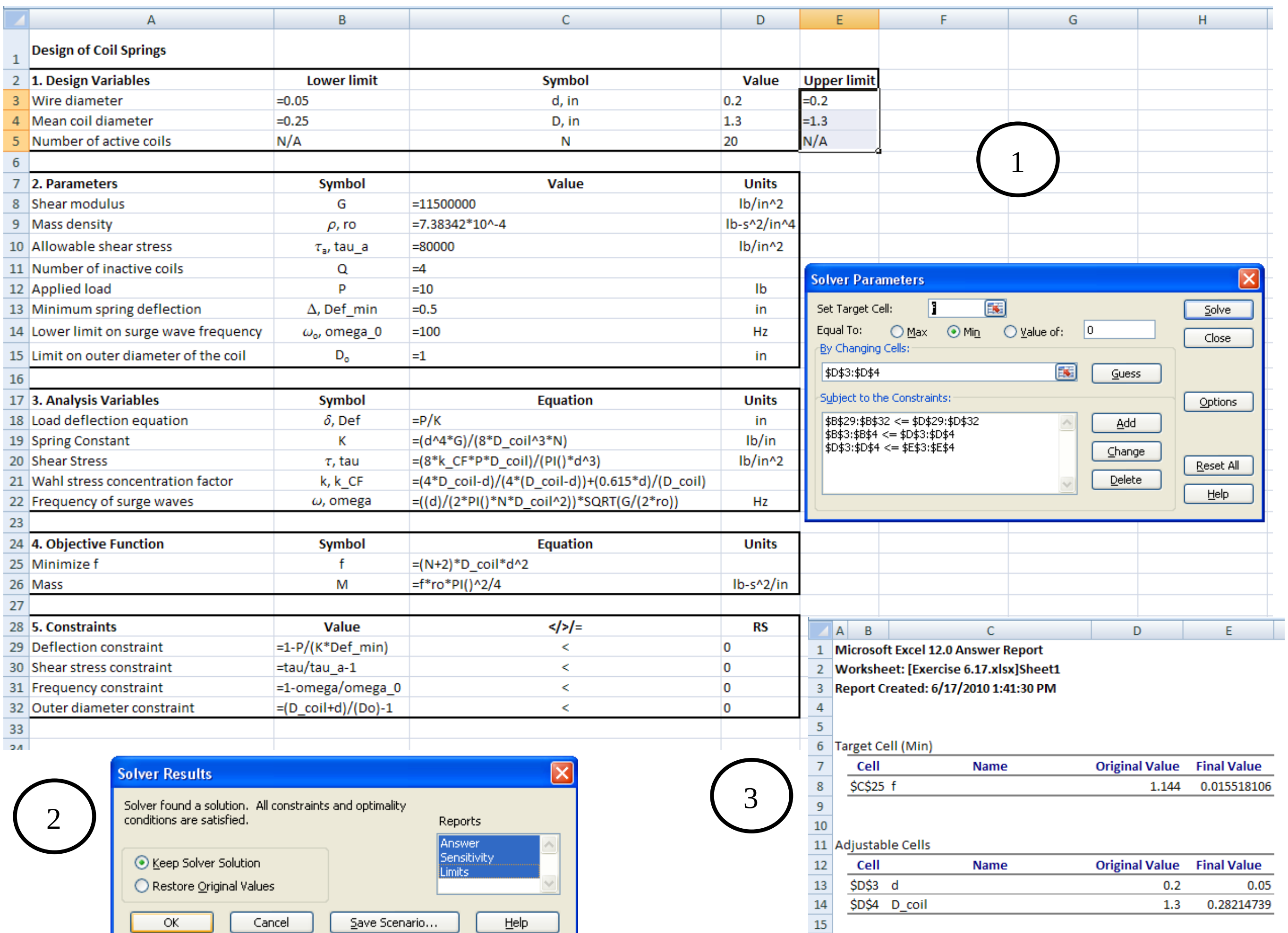
$$\frac{P}{K} \geq \Delta\ ,\ \tau \leq \tau_a\ ,\ \omega \geq \omega_0\ ,\ D + d \leq D_0\ ,\ d_{\min} \leq d \leq d_{\max}\ ,\ D_{\min} \leq D \leq D_{\max}\ ,\ N_{\min} \leq N \leq N_{\max}$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of d=0.2 and D_coil=1.3 a solution of d=0.05 and D_coil=0.282 which gives an objective function value of 0.0155, is obtained.



	A	B	C	D	E
1	**Design of Coil Springs**				
2	**1. Design Variables**	**Lower limit**	**Symbol**	**Value**	**Upper limit**
3	Wire diameter	=0.05	d, in	0.2	=0.2
4	Mean coil diameter	=0.25	D, in	1.3	=1.3
5	Number of active coils	N/A	N	20	N/A
6					
7	**2. Parameters**	**Symbol**	**Value**	**Units**	
8	Shear modulus	G	=11500000	lb/in^2	
9	Mass density	ρ, ro	=7.38342*10^-4	lb-s^2/in^4	
10	Allowable shear stress	τ_a, tau_a	=80000	lb/in^2	
11	Number of inactive coils	Q	=4		
12	Applied load	P	=10	lb	
13	Minimum spring deflection	Δ, Def_min	=0.5	in	
14	Lower limit on surge wave frequency	ω_o, omega_0	=100	Hz	
15	Limit on outer diameter of the coil	D_o	=1	in	
16					
17	**3. Analysis Variables**	**Symbol**	**Equation**	**Units**	
18	Load deflection equation	δ, Def	=P/K	in	
19	Spring Constant	K	=(d^4*G)/(8*D_coil^3*N)	lb/in	
20	Shear Stress	τ, tau	=(8*k_CF*P*D_coil)/(PI()*d^3)	lb/in^2	
21	Wahl stress concentration factor	k, k_CF	=(4*D_coil-d)/(4*(D_coil-d))+(0.615*d)/(D_coil)		
22	Frequency of surge waves	ω, omega	=((d)/(2*PI()*N*D_coil^2))*SQRT(G/(2*ro))	Hz	
23					
24	**4. Objective Function**	**Symbol**	**Equation**	**Units**	
25	Minimize f	f	=(N+2)*D_coil*d^2		
26	Mass	M	=f*ro*PI()^2/4	lb-s^2/in	
27					
28	**5. Constraints**	**Value**	**</>/=**	**RS**	
29	Deflection constraint	=1-P/(K*Def_min)	<	0	
30	Shear stress constraint	=tau/tau_a-1	<	0	
31	Frequency constraint	=1-omega/omega_0	<	0	
32	Outer diameter constraint	=(D_coil+d)/(Do)-1	<	0	
33					

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.17.xlsx]Sheet1**				
3	**Report Created: 6/17/2010 1:41:30 PM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		C25	f	1.144	0.015518106
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		D3	d	0.2	0.05
14		D4	D_coil	1.3	0.28214739
15					

6.18

Solve the following NLP problem using the Excel Solver:

Solve the spring design problem for the following data: Aluminum coil with shear modulus (G) = 4,000,000 psi, mass density (ρ) = 2.58920x10^{-4} lb-s^2/in^4, and allowable shear stress (τ_a) = 50,000 lb/in^2.

TABLE 2.2 Information to design a coil spring

Notation	*Data*
Deflection along spring axis	δ, in
Mean Coil Diameter	D, in
Wire Diameter	d, in
Number of active coils	N
Gravitational constant	g = 386 in/s^2
Frequency of surge waves	ω, Hz
Weight Density of spring material	γ = 0.285 lb/in^3
Shear Modulus	G = (4*10^6) lb/in^2
Mass density of material (ρ = γ/g)	ρ = 2.58920x10^{-4} lb-s^2/in^4
Allowable shear stress	τ_a = 50,000 lb/in^2
Number of inactive coils	Q = 2
Applied load	P = 20 lb
Minimum spring deflection	Δ = 0.5 in
Lower limit on surge wave frequency	ω_o = 100 Hz
Limit on outer diameter of coil	D_o = 1.5 in

Load deflection equation: $$P = K\delta$$

Spring Constant: $$K=\frac{d^4G}{8D^3N}$$

Shear Stress: $$\tau=\frac{8kPD}{\pi d^3}$$

Wahl stress concentration factor: $$k=\frac{(4D-d)}{4(D-d)}+\frac{0.615d}{D}$$

Frequency of surge waves: $$\varpi = \frac{d}{2\pi ND^2}\sqrt{\frac{G}{2\rho}}$$

Design variables for the problem are defined as below:

d = wire diameter, in
D = mean coil diameter, in

N = number of active coils, integer

The problem function to minimize the mass of the spring, given as volume*mass density is:

$$Mass = \left(\frac{\pi d^2}{4}\right)[(\mathrm{N}+\mathrm{Q})\pi\,\mathrm{D}]\rho = \frac{\pi d^2(\mathrm{N}+\mathrm{Q})\pi\,\mathrm{D}\,\rho}{4}$$

The constraints for the spring design problem are formulated as

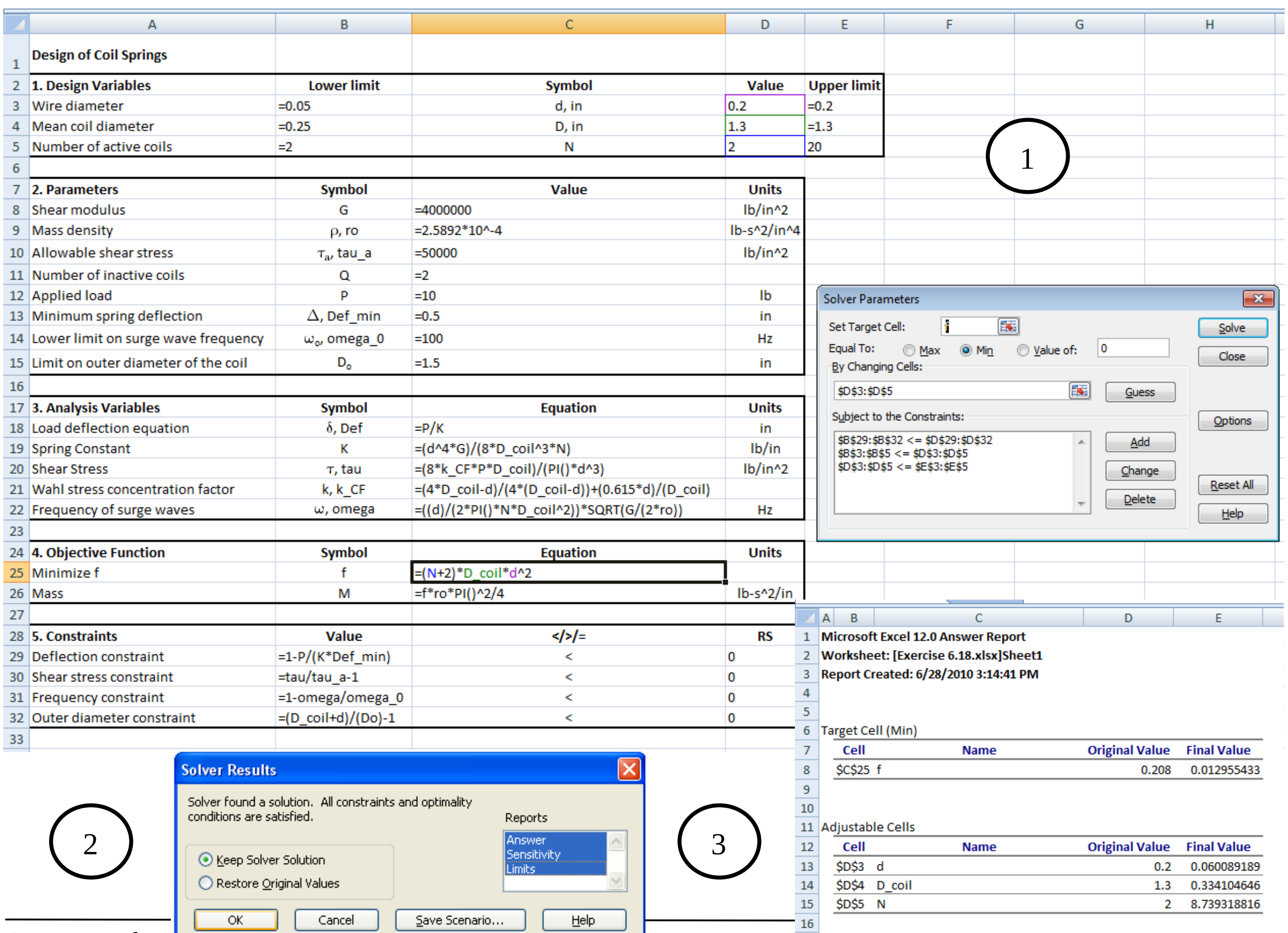
$$\frac{P}{K} \geq \Delta\ ,\ \tau \leq \tau_a\ ,\ \omega \geq \omega_0\ ,\ D + d \leq D_0\ ,\ d_{\min} \leq d \leq d_{\max}\ ,\ D_{\min} \leq D \leq D_{\max}\ ,\ N_{\min} \leq N \leq N_{\max}$$

<u>Solution</u>

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click “Solve” to solve the problem and to bring up the Solver Results dialog box.

(2) Choose “Keep Solver Solution” in the Solver Results dialog box, highlight “Answers, Sensitivity, and Limits” under Reports, and click “OK” to obtain the solution.

(3) The answer report shows that for initial design variable values of d=0.2, D_coil=1.3, and N=2 a solution of d=0.0601, D_coil=0.334, and N=8.74 which gives an objective function value of 0.0130, is obtained.



	A	B	C	D	E
1	**Design of Coil Springs**				
2	**1. Design Variables**	**Lower limit**	**Symbol**	**Value**	**Upper limit**
3	Wire diameter	=0.05	d, in	0.2	=0.2
4	Mean coil diameter	=0.25	D, in	1.3	=1.3
5	Number of active coils	=2	N	2	20
6					
7	**2. Parameters**	**Symbol**	**Value**	**Units**	
8	Shear modulus	G	=4000000	lb/in^2	
9	Mass density	ρ, ro	=2.5892*10^-4	lb-s^2/in^4	
10	Allowable shear stress	τ_a, tau_a	=50000	lb/in^2	
11	Number of inactive coils	Q	=2		
12	Applied load	P	=10	lb	
13	Minimum spring deflection	Δ, Def_min	=0.5	in	
14	Lower limit on surge wave frequency	ω_o, omega_0	=100	Hz	
15	Limit on outer diameter of the coil	D_o	=1.5	in	
16					
17	**3. Analysis Variables**	**Symbol**	**Equation**	**Units**	
18	Load deflection equation	δ, Def	=P/K	in	
19	Spring Constant	K	=(d^4*G)/(8*D_coil^3*N)	lb/in	
20	Shear Stress	τ, tau	=(8*k_CF*P*D_coil)/(PI()*d^3)	lb/in^2	
21	Wahl stress concentration factor	k, k_CF	=(4*D_coil-d)/(4*(D_coil-d))+(0.615*d)/(D_coil)		
22	Frequency of surge waves	ω, omega	=((d)/(2*PI()*N*D_coil^2))*SQRT(G/(2*ro))	Hz	
23					
24	**4. Objective Function**	**Symbol**	**Equation**	**Units**	
25	Minimize f	f	=(N+2)*D_coil*d^2		
26	Mass	M	=f*ro*PI()^2/4	lb-s^2/in	
27					
28	**5. Constraints**	**Value**	**</>/=**	**RS**	
29	Deflection constraint	=1-P/(K*Def_min)	<	0	
30	Shear stress constraint	=tau/tau_a-1	<	0	
31	Frequency constraint	=1-omega/omega_0	<	0	
32	Outer diameter constraint	=(D_coil+d)/(Do)-1	<	0	
33					

	A	B	C	D	E
1	**Microsoft Excel 12.0 Answer Report**				
2	**Worksheet: [Exercise 6.18.xlsx]Sheet1**				
3	**Report Created: 6/28/2010 3:14:41 PM**				
4					
5					
6	Target Cell (Min)				
7		**Cell**	**Name**	**Original Value**	**Final Value**
8		C25	f	0.208	0.012955433
9					
10					
11	Adjustable Cells				
12		**Cell**	**Name**	**Original Value**	**Final Value**
13		D3	d	0.2	0.060089189
14		D4	D_coil	1.3	0.334104646
15		D5	N	2	8.739318816
16					

Section 6.8 Optimum Design of Plate Girders Using Excel Solver

Solve the following problems using Excel Solver:

6.19

Solve the following problem using the Excel Solver:

Solve the plate girder design problem for the following data: Span length (L) = 35 ft.

TABLE E6-19

Notation	*Data*
L	span, 25 m
E	modulus of elasticity, 210 GPa
σ_y	yield stress, 262 MPa
σ_a	allowable bending stress, 0.55 σ_y = 144.1 MPa
τ_a	allowable shear stress, 0.33 σ_y = 86.46 MPa
σ_t	allowable fatigue stress 255 MPa
D_a	allowable deflection, L/800, m
P_m	concentrated load for momemt, 104 kN
P_s	concentrated load for shear, 155 kN
LLIF	live load impact factor, 1+50/(L+125)

Cross-sectional area: $A = (ht_w + 2bt_f)$

Moment of inertia: $I = \dfrac{t_w h^3}{12} + \dfrac{2bt_f^3}{3} + \dfrac{bt_f h(h+2t_f)}{2}$

Uniform load for the girder: $w = (19 + 77A)$

Bending moment: $M = \dfrac{L(2P_m + wL)}{8}$

Bending Stress: $\sigma = \dfrac{M(0.5h + t_f)}{1000I}$

Flange buckling stress limit: $\sigma_f = 72{,}845[\dfrac{t_w}{b}]^2$

Web crippling stress limit: $\sigma_w = 3{,}648{,}276[\dfrac{t_w}{h}]^2$

Shear force: $S = 0.5(P_s + wL)$

Deflection: $D = \dfrac{L^3(8P_m + 5wL)}{384 * 10^6 (EI)}$

Average shear stress: $\tau = \dfrac{S}{1000ht_w}$

Objective function to minimize the material volume of the girder is defined as:

$$Vol = AL = (ht_w + 2bt_f)L$$

Design variables for the plate girder optimization problem are defined as:

h = web height, m

b = flange weight, m

t_f = flange thickness, m
t_w = web thickness, m

The constraints for the spring design problem are formulated as

$$\sigma \le \sigma_a \,, \sigma \le \sigma_f \,, \sigma \le \sigma_w \,, \tau \le \tau_a \,, D \le D_a \,, \sigma \le \frac{\sigma_t}{2} \,,$$
$$0.3 \le h \le 2.5 \,, 0.3 \le b \le 2.5 \,, 0.01 \le t_f \le 0.1 \,, 0.01 \le t_w \le 0.1$$

Solution

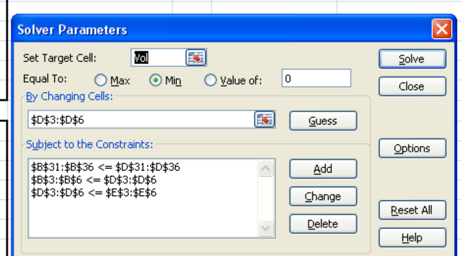
(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of h=1, b=1, tf=0.1, and tw=0.1 a solution of h=2.5, b=0.3, tf=0.045, and tw=0.013, which gives an objective function value of 2.067, is obtained.

	A	B	C	D	E	F
1	Plate Girder Design					
2	**1. Design variable name**	**Lower limit**	**Symbol**	**Value**	**Upper llimit**	**Units**
3	web height	0.3	h	1	2.5	m
4	flange width	0.3	b	1	2.5	m
5	flange thickness	0.01	tf	0.1	0.1	m
6	web thickness	0.01	tw	0.1	0.1	m
7						
8	**2. Parameter name**	**Symbol**	**Value**	**Units**		
9	Span length	L	35	m		
10	Modulus of elasticity	E	210	Gpa		
11	Yield stress	sigma_y	262	Mpa		
12	Allowable fatigue stress	sigma_t	255	MPa		
13	Concentrated load for moment	Pm	104	kN		
14	Concentrated load for shear	Ps	155	kN		
15	Live load impact factor	LLIF	=1+50/(L+125)	none		
16						
17	**3. Dependent variable name**	**Symbol**	**Equation**	**Units**		
18	Cross sectional area	A	=h*tw+2*b*tf	m^2		
19	Moment of inertia	I	=(1/12)*tw*h^3+(2/3)*b*tf^3+(1/2)*b*tf*h*(h+2*tf)	m^4		
20	Uniform load	w	=19+77*A	kN/m		
21	Bending moment	M	=L*(2*Pm+w*L)/8	kN-m		
22	Bending stress	sigma	=M*(h/2+tf)/(1000*I)	Mpa		
23	Shear force	S	=(Ps+w*L)/2	kN		
24	Deflection	D	=L^3*(8*Pm+5*w*L)/(384*E*I*1000000)	m		
25	Average shear stress	tau	=S/(1000*h*tw)	MPa		
26						
27	**4. Objective function name**	**Symbol**	**Equation**	**Units**		
28	Volume of material	Vol	=A*L	m^3		
29						
30	**5. Constraints**	**Value/Eq.**	**</>/=**	**Value/Eq.**	**Name**	
31	Bending stress	=sigma	<	=0.55*sigma_y	Allowable bending stress	
32	Bending stress	=sigma	<	=72845*(tf/b)^2	Flange buckling limit	
33	Bending stress	=sigma	<	=3648276*(tw/h)^2	Web crippling limit	
34	Shear stress	=tau	<	=0.33*sigma_y	Allowable shear stress	
35	Deflection	=D	<	=L/800	Allowable deflection	
36	Bending stress	=sigma	<	=sigma_t/2	Allowable fatigue stress	

1



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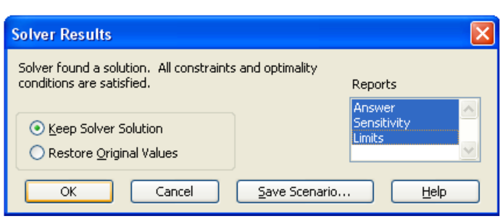


3

6	Target Cell (Min)			
7	**Cell**	**Name**	**Original Value**	**Final Value**
8	C28	Vol	10.5	2.066791187
9				
10				
11	Adjustable Cells			
12	**Cell**	**Name**	**Original Value**	**Final Value**
13	D3	h	1	2.5
14	D4	b	1	0.3
15	D5	tf	0.1	0.044670412
16	D6	tw	0.1	0.012899572

6.20

Solve the following problem using the Excel Solver:

Solve the plate girder design problem for the following data: A36 steel with modulus of elasticity (E) = 200 GPa, yield stress (sigma_y) = 250 MPa, allowable fatigue stress (sigma_t) = 243 Mpa.

TABLE E6-20

Notation	*Data*
L	span, 25 m
E	modulus of elasticity, 200 GPa
σ_y	yield stress, 250 MPa
σ_a	allowable bending stress, 0.55 σ_y = 144.1 MPa
τ_a	allowable shear stress, 0.33 σ_y = 86.46 MPa
σ_t	allowable fatigue stress 243 MPa
D_a	allowable deflection, L/800, m
P_m	concentrated load for momemt, 104 kN
P_s	concentrated load for shear, 155 kN
LLIF	live load impact factor, 1+50/(L+125)

Cross-sectional area: $A = (ht_w + 2bt_f)$

Moment of inertia: $I = \dfrac{t_w h^3}{12} + \dfrac{2bt_f^3}{3} + \dfrac{bt_f h(h+2t_f)}{2}$

Uniform load for the girder: $w = (19 + 77A)$

Bending moment: $M = \dfrac{L(2P_m + wL)}{8}$

Bending Stress: $\sigma = \dfrac{M(0.5h + t_f)}{1000I}$

Flange buckling stress limit: $\sigma_f = 72{,}845[\dfrac{t_w}{b}]^2$

Web crippling stress limit: $\sigma_w = 3{,}648{,}276[\dfrac{t_w}{h}]^2$

Shear force: $S = 0.5(P_s + wL)$

Deflection: $D = \dfrac{L^3(8P_m + 5wL)}{384*10^6(EI)}$

Average shear stress: $\tau = \dfrac{S}{1000ht_w}$

Objective function to minimize the material volume of the girder is defined as:

$$Vol = AL = (ht_w + 2bt_f)L$$

Design variables for the plate girder optimization problem are defined as:
h = web height, m
b = flange weight, m

t_f = flange thickness, m
t_w = web thickness, m

The constraints for the spring design problem are formulated as

$$\sigma \le \sigma_a \text{ , } \sigma \le \sigma_f \text{ , } \sigma \le \sigma_w \text{ , } \tau \le \tau_a \text{ , } D \le D_a \text{ , } \sigma \le \frac{\sigma_t}{2} \text{ ,}$$

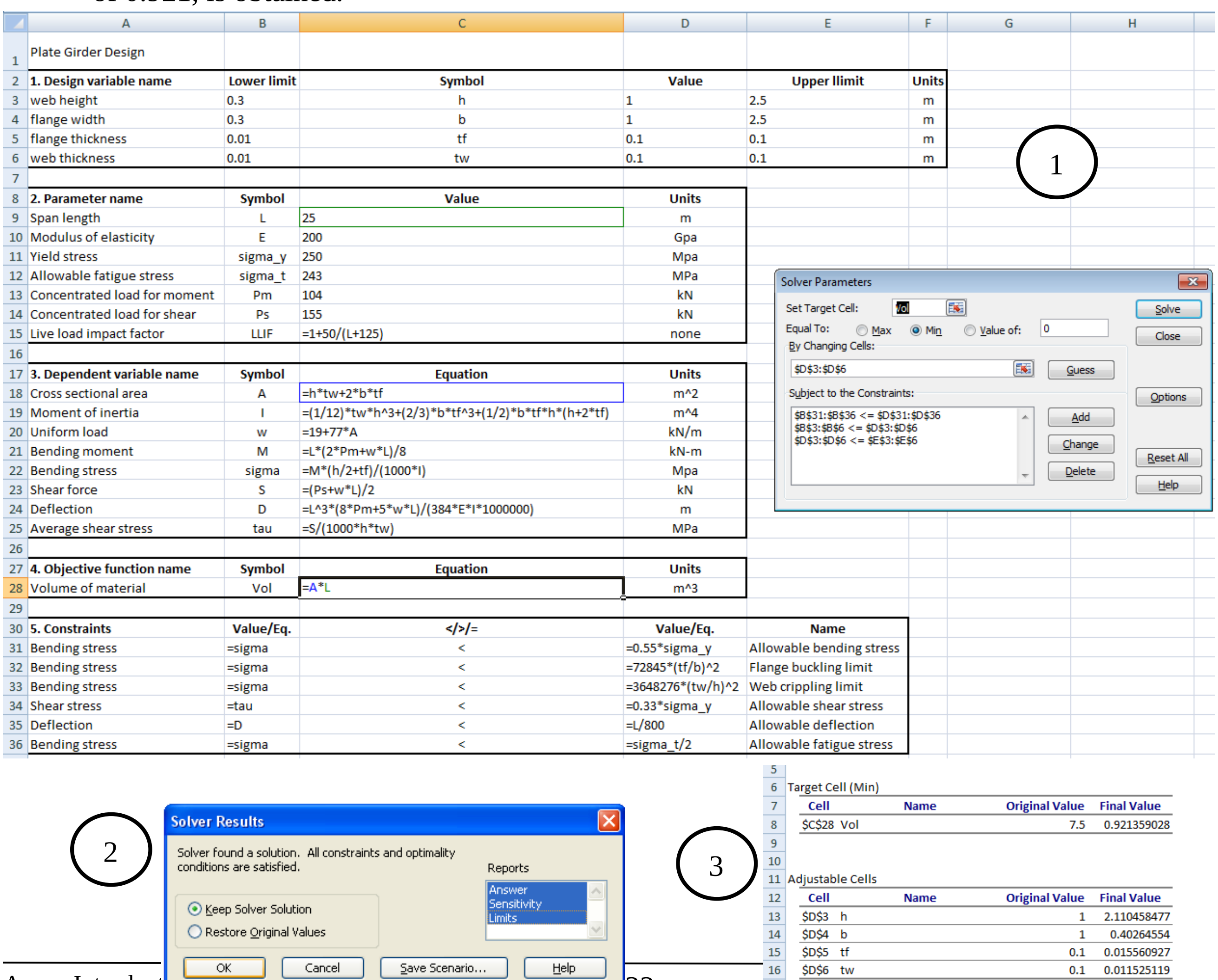
$$0.3 \le h \le 2.5 \text{ , } 0.3 \le b \le 2.5 \text{ , } 0.01 \le t_f \le 0.1 \text{ , } 0.01 \le t_w \le 0.1$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

(3) The answer report shows that for initial design variable values of h=1, b=1, tf=0.1, and tw=0.1 a solution of h=2.11, b=0.403, tf=0.0156, and tw=0.0115, which gives an objective function value of 0.921, is obtained.



	A	B	C	D	E	F
1	Plate Girder Design					
2	1. Design variable name	Lower limit	Symbol	Value	Upper llimit	Units
3	web height	0.3	h	1	2.5	m
4	flange width	0.3	b	1	2.5	m
5	flange thickness	0.01	tf	0.1	0.1	m
6	web thickness	0.01	tw	0.1	0.1	m
7						
8	2. Parameter name	Symbol	Value	Units		
9	Span length	L	25	m		
10	Modulus of elasticity	E	200	Gpa		
11	Yield stress	sigma_y	250	Mpa		
12	Allowable fatigue stress	sigma_t	243	MPa		
13	Concentrated load for moment	Pm	104	kN		
14	Concentrated load for shear	Ps	155	kN		
15	Live load impact factor	LLIF	=1+50/(L+125)	none		
16						
17	3. Dependent variable name	Symbol	Equation	Units		
18	Cross sectional area	A	=h*tw+2*b*tf	m^2		
19	Moment of inertia	I	=(1/12)*tw*h^3+(2/3)*b*tf^3+(1/2)*b*tf*h*(h+2*tf)	m^4		
20	Uniform load	w	=19+77*A	kN/m		
21	Bending moment	M	=L*(2*Pm+w*L)/8	kN-m		
22	Bending stress	sigma	=M*(h/2+tf)/(1000*I)	Mpa		
23	Shear force	S	=(Ps+w*L)/2	kN		
24	Deflection	D	=L^3*(8*Pm+5*w*L)/(384*E*I*1000000)	m		
25	Average shear stress	tau	=S/(1000*h*tw)	MPa		
26						
27	4. Objective function name	Symbol	Equation	Units		
28	Volume of material	Vol	=A*L	m^3		
29						
30	5. Constraints	Value/Eq.	</>/=	Value/Eq.	Name	
31	Bending stress	=sigma	<	=0.55*sigma_y	Allowable bending stress	
32	Bending stress	=sigma	<	=72845*(tf/b)^2	Flange buckling limit	
33	Bending stress	=sigma	<	=3648276*(tw/h)^2	Web crippling limit	
34	Shear stress	=tau	<	=0.33*sigma_y	Allowable shear stress	
35	Deflection	=D	<	=L/800	Allowable deflection	
36	Bending stress	=sigma	<	=sigma_t/2	Allowable fatigue stress	

6	Target Cell (Min)			
7	Cell	Name	Original Value	Final Value
8	C28	Vol	7.5	0.921359028
9				
10				
11	Adjustable Cells			
12	Cell	Name	Original Value	Final Value
13	D3	h	1	2.110458477
14	D4	b	1	0.40264554
15	D5	tf	0.1	0.015560927
16	D6	tw	0.1	0.011525119

6.21

Solve the following problem using the Excel Solver:

Solve the plate girder design problem for the following data: Web height (h) = 1.5 m, flange thickness (tf) = 0.015 m.

TABLE E6-21

Notation	*Data*
L	span, 25 m
E	modulus of elasticity, 200 GPa
σ_y	yield stress, 250 MPa
σ_a	allowable bending stress, 0.55 σ_y = 144.1 MPa
τ_a	allowable shear stress, 0.33 σ_y = 86.46 MPa
σ_t	allowable fatigue stress 243 MPa
D_a	allowable deflection, L/800, m
P_m	concentrated load for momemt, 104 kN
P_s	concentrated load for shear, 155 kN
LLIF	live load impact factor, 1+50/(L+125)

Cross-sectional area: $A = (ht_w + 2bt_f)$

Moment of inertia: $$I = \frac{t_w h^3}{12} + \frac{2bt_f^3}{3} + \frac{bt_f h(h+2t_f)}{2}$$

Uniform load for the girder: $w = (19 + 77A)$

Bending moment: $$M = \frac{L(2P_m + wL)}{8}$$

Bending Stress: $$\sigma = \frac{M(0.5h + t_f)}{1000I}$$

Flange buckling stress limit: $$\sigma_f = 72{,}845\left[\frac{t_w}{b}\right]^2$$

Web crippling stress limit: $$\sigma_w = 3{,}648{,}276\left[\frac{t_w}{h}\right]^2$$

Shear force: $S = 0.5(P_s + wL)$

Deflection: $$D = \frac{L^3(8P_m + 5wL)}{384*10^6(EI)}$$

Average shear stress: $$\tau = \frac{S}{1000ht_w}$$

Objective function to minimize the material volume of the girder is defined as:

$$Vol = AL = (ht_w + 2bt_f)L$$

Design variables for the plate girder optimization problem are defined as:

h = web height, m

b = flange weight, m

t_f = flange thickness, m

t_w = web thickness, m

The constraints for the spring design problem are formulated as

$$\sigma \le \sigma_a \;, \sigma \le \sigma_f \;, \sigma \le \sigma_w \;, \tau \le \tau_a \;, D \le D_a \;, \sigma \le \frac{\sigma_t}{2} \;,$$

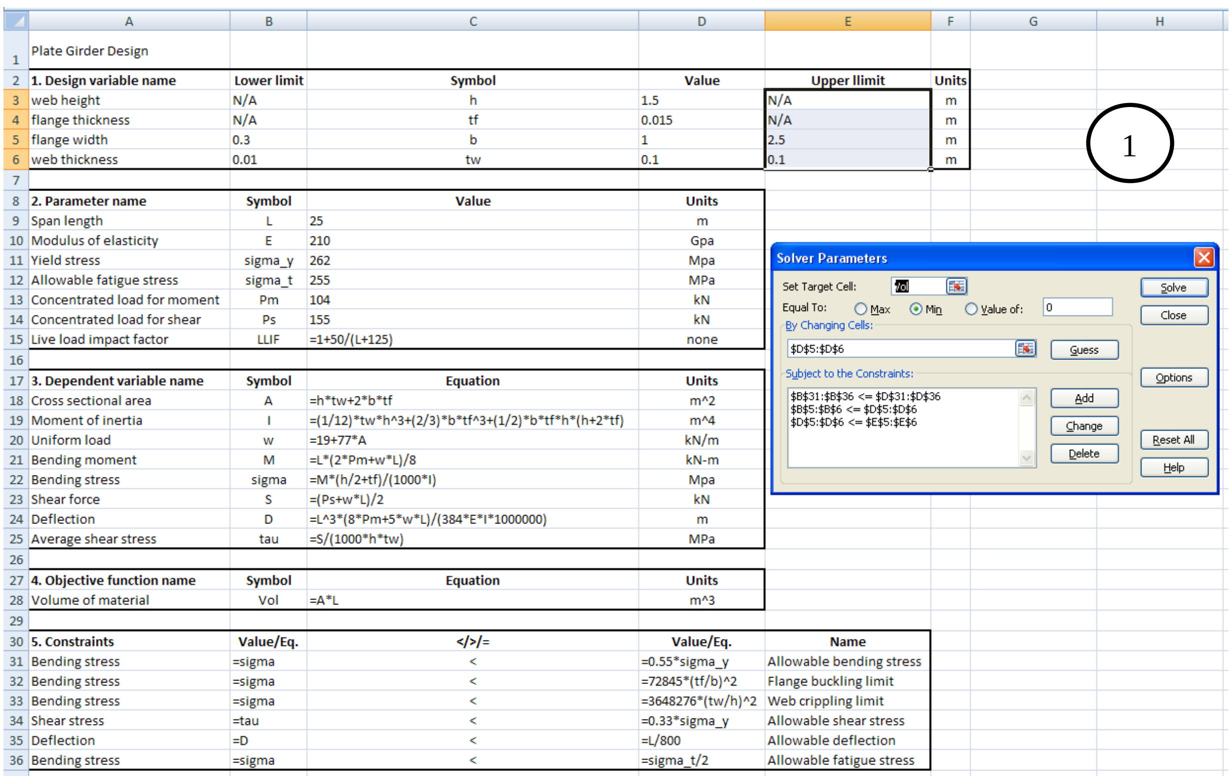
$$0.3 \le h \le 2.5 \;, 0.3 \le b \le 2.5 \;, 0.01 \le t_f \le 0.1 \;, 0.01 \le t_w \le 0.1$$

Solution

(1) One possible format for setting up the Excel worksheet for this problem is shown below. The objective function, variables, and constraints are input into the Solver Parameters dialog box as shown. Once the problem is defined, click "Solve" to solve the problem and to bring up the Solver Results dialog box.

(2) Choose "Keep Solver Solution" in the Solver Results dialog box, highlight "Answers, Sensitivity, and Limits" under Reports, and click "OK" to obtain the solution.

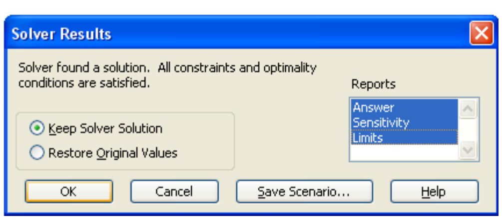
(3) The answer report shows that for initial design variable values of b=1 and tw=0.1 a solution of b=0.4503 and tw=0.0675, which gives an objective function value of 2.87, is obtained.



	A	B	C	D	E	F
1	Plate Girder Design					
2	1. Design variable name	Lower limit	Symbol	Value	Upper llimit	Units
3	web height	N/A	h	1.5	N/A	m
4	flange thickness	N/A	tf	0.015	N/A	m
5	flange width	0.3	b	1	2.5	m
6	web thickness	0.01	tw	0.1	0.1	m
7						
8	2. Parameter name	Symbol	Value	Units		
9	Span length	L	25	m		
10	Modulus of elasticity	E	210	Gpa		
11	Yield stress	sigma_y	262	Mpa		
12	Allowable fatigue stress	sigma_t	255	MPa		
13	Concentrated load for moment	Pm	104	kN		
14	Concentrated load for shear	Ps	155	kN		
15	Live load impact factor	LLIF	=1+50/(L+125)	none		
16						
17	3. Dependent variable name	Symbol	Equation	Units		
18	Cross sectional area	A	=h*tw+2*b*tf	m^2		
19	Moment of inertia	I	=(1/12)*tw*h^3+(2/3)*b*tf^3+(1/2)*b*tf*h*(h+2*tf)	m^4		
20	Uniform load	w	=19+77*A	kN/m		
21	Bending moment	M	=L*(2*Pm+w*L)/8	kN-m		
22	Bending stress	sigma	=M*(h/2+tf)/(1000*I)	Mpa		
23	Shear force	S	=(Ps+w*L)/2	kN		
24	Deflection	D	=L^3*(8*Pm+5*w*L)/(384*E*I*1000000)	m		
25	Average shear stress	tau	=S/(1000*h*tw)	MPa		
26						
27	4. Objective function name	Symbol	Equation	Units		
28	Volume of material	Vol	=A*L	m^3		
29						
30	5. Constraints	Value/Eq.	</>/=	Value/Eq.	Name	
31	Bending stress	=sigma	<	=0.55*sigma_y	Allowable bending stress	
32	Bending stress	=sigma	<	=72845*(tf/b)^2	Flange buckling limit	
33	Bending stress	=sigma	<	=3648276*(tw/h)^2	Web crippling limit	
34	Shear stress	=tau	<	=0.33*sigma_y	Allowable shear stress	
35	Deflection	=D	<	=L/800	Allowable deflection	
36	Bending stress	=sigma	<	=sigma_t/2	Allowable fatigue stress	

1

2



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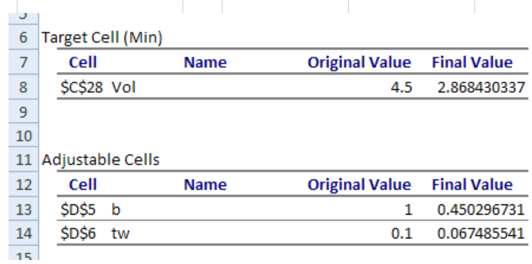

6	Target Cell (Min)			
7	Cell	Name	Original Value	Final Value
8	C28	Vol	4.5	2.868430337
9				
10				
11	Adjustable Cells			
12	Cell	Name	Original Value	Final Value
13	D5	b	1	0.450296731
14	D6	tw	0.1	0.067485541