CHAPTER

3

Graphical Solution Method and Basic Optimization Concepts

Solve the following problems using the graphical method. (3.1-3.10)

3.1

Minimize $f(x_1, x_2) = (x_1 - 3)^2 + (x_2 - 3)^2$
Subject to $x_1 + x_2 \le 4$
$x_1, x_2 \ge 0$

Solution

$f = (x_1 - 3)^2 + (x_2 - 3)^2$;

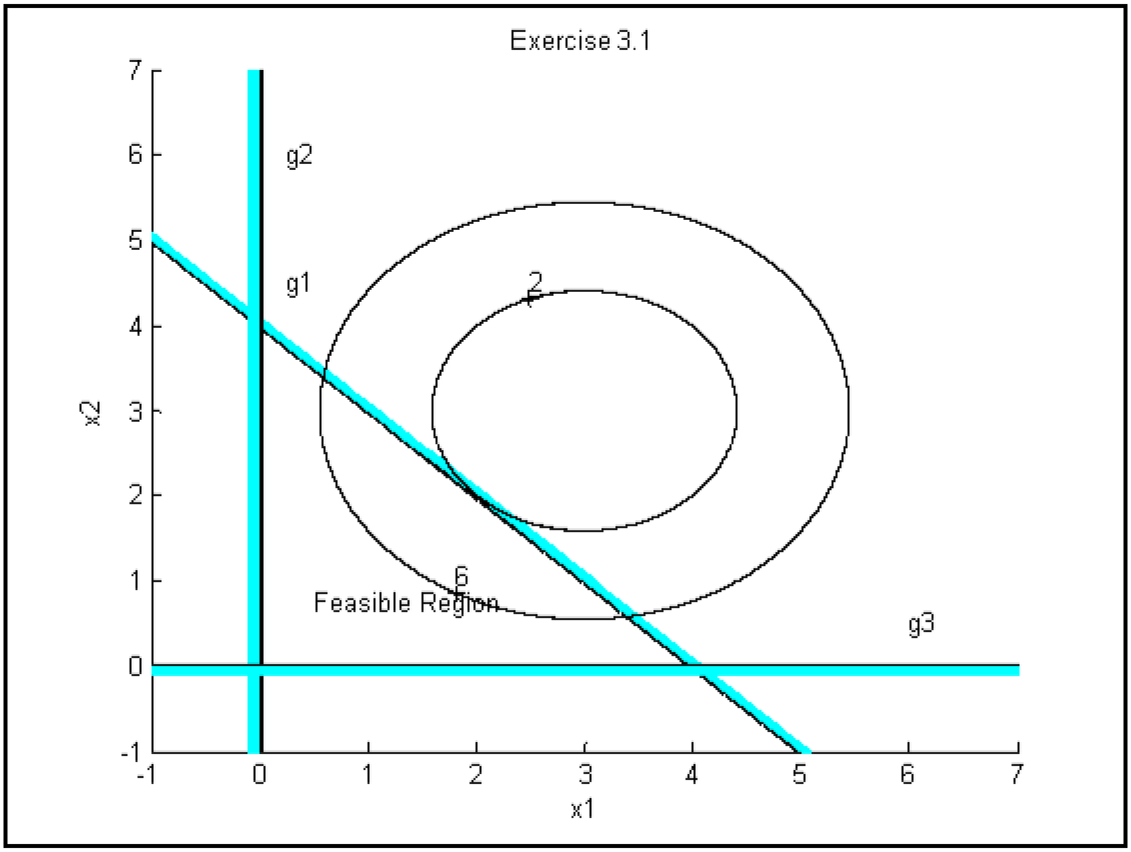
$g_1 = x_1 + x_2 - 4 \le 0$;

$g_2 = -x_1 \le 0$;

$g_3 = -x_2 \le 0$

The optimum solution is: $x_1^* = 2.0$, $x_2^* = 2.0$, $f^* = 2.0$

Active constraint: g_1


Exercise 3.1
g2
g1
2
6
Feasible Region
g3
x1
x2
7
6
5
4
3
2
1
0
-1
-1
0
1
2
3
4
5
6
7

MATLAB Code

```
             %Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-1:0.01:7.0, -1:0.01:7.0);
             %Enter functions for the minimization problem
f=(x1-3).^2+(x2-3).^2;
g1=x1+x2-4;
g2=-x1;
g3=-x2;
cla reset
axis auto                    %Minimum and maximum values for axes are determined automatically
                             %Limits for x- and y-axes may be specified with the command
                             %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')   %Specifies labels for x- and y-axes
hold on                      %retains the current plot and axes properties for all subsequent plots
                             %Use the "contour" command to plot constraint/minimization functions
cv1=[0 0];                   %Specifies two contour values
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);      %Plots two specified contours of g1; k=black color
text(0.25,4.5,'g1')                                  %Writes g1 at the location (0.25, 4.5)
cv11=[0.01:0.01:0.1];
const1=contour(x1,x2,g1,cv11,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv11,'c');
text(.25,6,'g2')
const3=contour(x1,x2,g3,cv1,'k','Linewidth',3);
const3=contour(x1,x2,g3,cv11,'c');
text(6,0.5,'g3')
text(0.5,0.75,'Feasible Region')
fv=[2 6];                    %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');  %'k' specifies black dashed lines for function contours
clabel(fs)                   %Automatically puts the contour value on the graph
hold off                     %Indicates end of this plotting sequence
                             %Subsequent plots will appear in separate windows
```

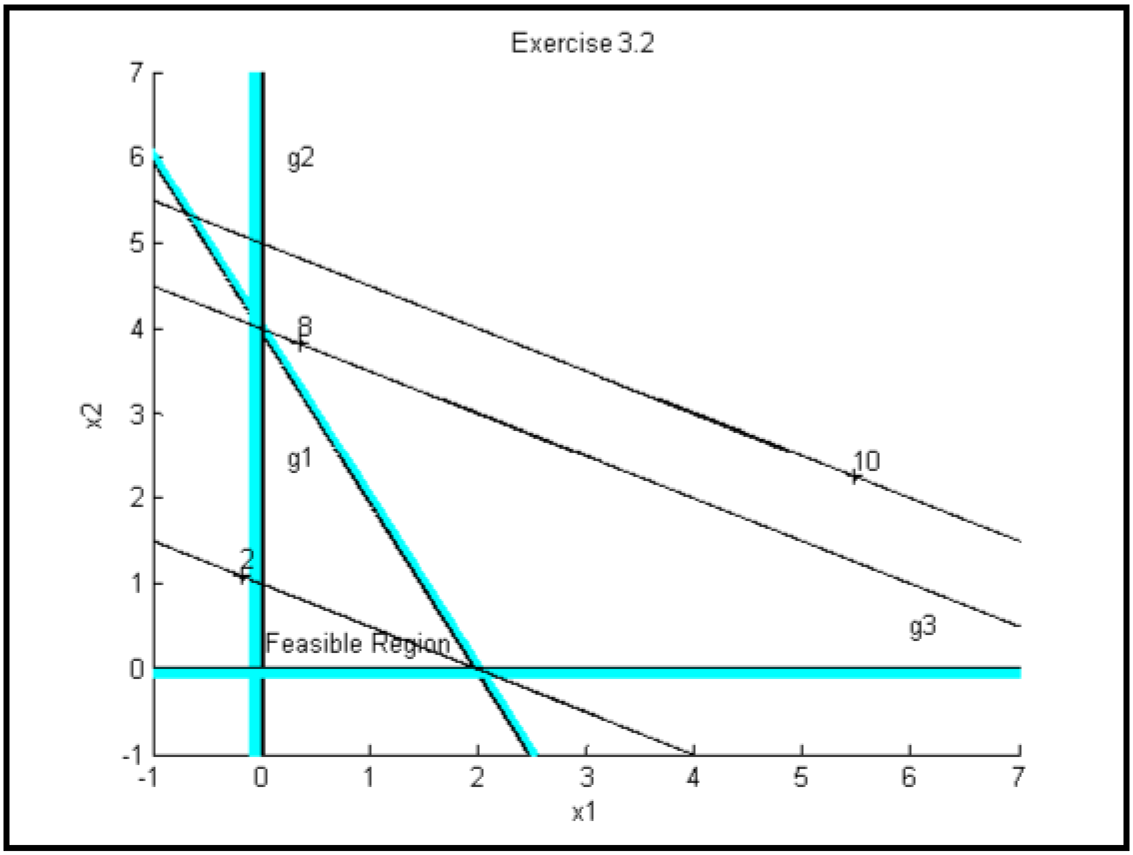
3.2

Maximize $F(x_1, x_2) = x_1 + 2x_2$
Subject to $2x_1 + x_2 \le 4$
$x_1, x_2 \ge 0$

Solution

$F = x_1 + 2x_2;$
$g_1 = 2x_1 + x_2 - 4 \le 0;$
$g_2 = -x_1 \le 0;$
$g_3 = -x_2 \le 0$

The optimum solution is: $x_1^* = 0$, $x_2^* = 4$, $F^* = 8$. Active constraints: g_1 and g_2.



MATLAB Code for Exercise 3.2

```
        %Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-1:0.01:7.0, -1:0.01:7.0);
        %Enter functions for the minimization problem
f=x1+2*x2;
g1=2*x1+x2-4;
g2=-x1;
g3=-x2;
cla reset
axis auto                  %Minimum and maximum values for axes are determined automatically
xlabel('x1'),ylabel('x2')  %Specifies labels for x- and y-axes
hold on                    %retains the current plot and axes properties for all subsequent plots
                           %Use the "contour" command to plot constraint/minimization functions
cv1=[0 0];                 %Specifies two contour values
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);
text(0.25,2.5,'g1')
cv11=[0.01:0.01:0.1];
const1=contour(x1,x2,g1,cv11,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv11,'c');
text(.25,6,'g2')
const3=contour(x1,x2,g3,cv1,'k','Linewidth',3);
const3=contour(x1,x2,g3,cv11,'c');
text(6,0.5,'g3')
text(0.05,0.3,'Feasible Region')
fv=[2 8 10];               %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k'); %'k' specifies black dashed lines for function contours
clabel(fs)                 %Automatically puts the contour value on the graph
hold off                   %Indicates end of this plotting sequence
                           %Subsequent plots will appear in separate windows
```

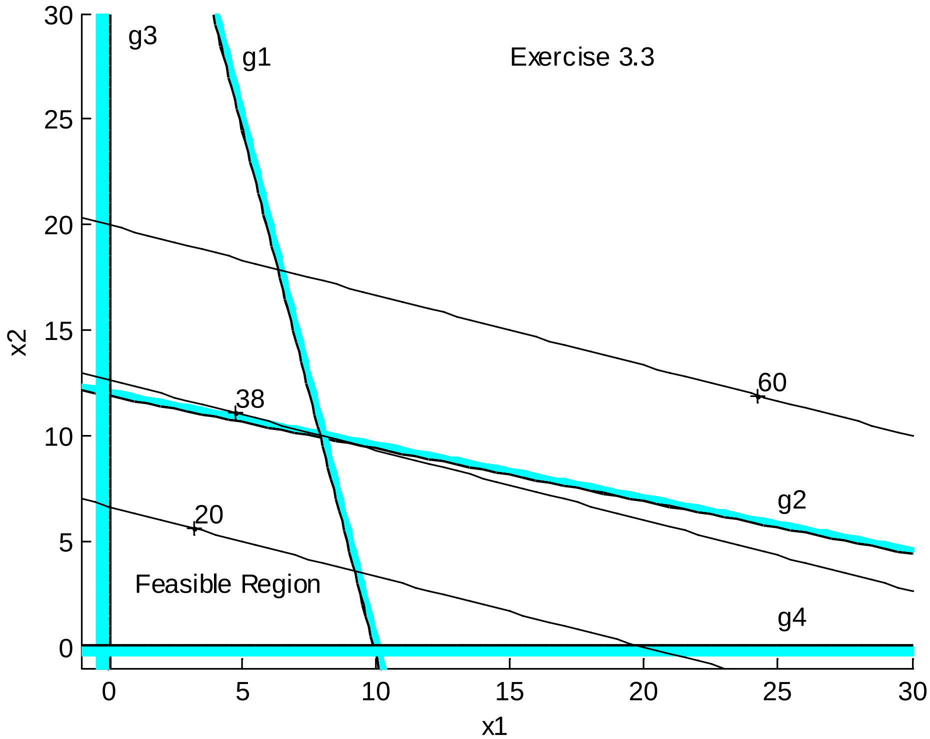
3.3

Minimize $f(x_1, x_2) = x_1 + 3x_2$
Subject to $x_1 + 4x_2 \geq 48$
$5x_1 + x_2 \geq 50$
$x_1, x_2 \geq 0$

Solution

$f = x_1 + 3x_2;$
$g_1 = -x_1 - 4x_2 + 48 \leq 0;$
$g_2 = -5x_1 - x_2 + 50 \leq 0;$
$g_3 = -x_1 \leq 0;$
$g_4 = -x_2 \leq 0$

The optimum solution is: $x_1^* = 8,\ x_2^* = 10,\ f^* = 38$
Active constraints: g_1 and g_2.



MATLAB Code for Exercise 3.3

```
%Exercise 3.3
%Create a grid from -1 to 30 with an increment of 0.5 for the variables x1 and x2
[x1,x2]=meshgrid(-1:0.5:30.0, -1:0.5:30.0);
%Enter functions for the minimization problem
f=x1+3*x2;
g1=x1+4*x2-48;
g2=5*x1+x2-50;
g3=-x1;
g4=-x2;
cla reset
axis auto                %Minimum and maximum values for axes are determined
automatically
                         %Limits for x- and y-axes may be specified with the command
                         %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')        %Specifies labels for x- and y-axes
hold on                  %retains the current plot and axes properties for all
subsequent plots
text(15,28,'Exercise 3.3')
cv1=[0 0];
cv11=[0.01:0.01:0.4];
cv22=[0.01:0.01:0.8];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);
text(5,28,'g1')
const1=contour(x1,x2,g1,cv22,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
text(25,7,'g2')
const2=contour(x1,x2,g2,cv22,'c');
const3=contour(x1,x2,g3,cv1,'k','Linewidth',3);
text(25,1.5,'g4')
const3=contour(x1,x2,g3,cv11,'c');
const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
text(.75,29,'g3')
const4=contour(x1,x2,g4,cv11,'c');
text(1,3,'Feasible Region')
fv=[20 38 60];           %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');      %'k' specifies black dashed lines for function
contours
clabel(fs)                   %Automatically puts the contour value on the graph
hold off                 %Indicates end of this plotting sequence
                         %Subsequent plots will appear in separate windows
```

3.4

Maximize $F(x_1, x_2) = x_1 + x_2 + 2x_3$
Subject to $1 \le x_1 \le 4$
$3x_2 - 2x_3 = 6$
$-1 \le x_3 \le 2$
$x_2 \ge 0$

Solution

Maximize; subject to $1 \le x_1 \le 4$; $3x_2 - 2x_3 = 6$; $-1 \le x_3 \le 2$; $x_2 \ge 0$

Eliminate the design variable x_3 from the problem using the equality constraint:
$x_3 = 1.5x_2 - 3$ Substituting into the objective function, we get

$x_1 + x_2 + 2x_3 = x_1 + x_2 + 2(1.5x_2 - 3) = x_1 + 4x_2 - 6$

Substituting into the third constraint, we get

$-1 \le 1.5x_2 - 3 \le 2$; or $2 \le 1.5x_2 \le 5$; or $4/3 \le x_2 \le 10/3$

Rewrite the problem in the standard form with the remaining 2 design variables:

$f = -x_1 - 4x_2 + 6$;

$g_1 = -x_1 + 1 \le 0$;

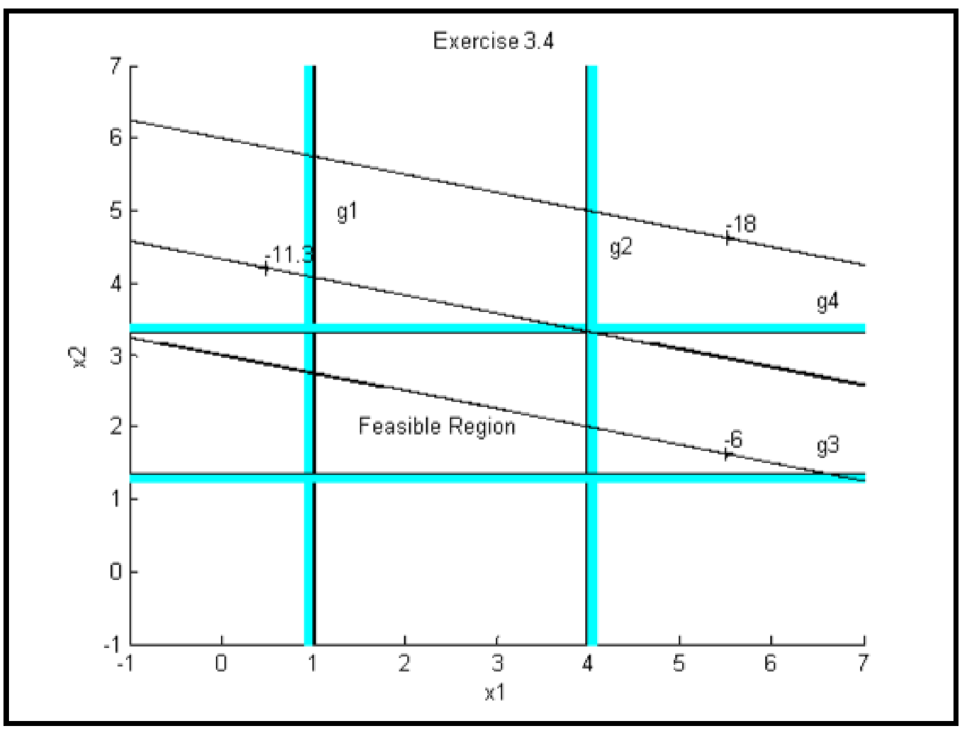
$g_2 = x_1 - 4 \le 0$;

$g_3 = -x_2 + 4/3 \le 0$;

$g_4 = x_2 - 10/3 \le 0$

The optimum solution is: $x_1^* = 4, x_2^* = 3.333, x_3^* = 2, F^* = 11.333$

Active constraints: g_2 and g_4.



MATLAB Code for Exercise 3.4

```
        %Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-1:0.01:7.0, -1:0.01:7.0);
        %Enter functions for the minimization problem
f=-x1-4*x2+6;
g1=-x1+1;
g2=x1-4;
g3=-x2+4/3;
g4=x2-10/3;
cla reset
axis auto                   %Minimum and maximum values for axes are determined automatically
xlabel('x1'),ylabel('x2')   %Specifies labels for x- and y-axes
hold on                     %retains the current plot and axes properties for all subsequent plots
                            %Use the "contour" command to plot constraint/minimization functions
cv1=[0 0];                  %Specifies two contour values
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);
text(1.25,5,'g1')
cv11=[0.01:0.01:0.1];
cv22=[0.01:0.01:0.1];
const1=contour(x1,x2,g1,cv22,'c');
const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
text(6.5,3.75,'g4')
const4=contour(x1,x2,g4,cv22,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv11,'c');
text(4.25,4.5,'g2')
const3=contour(x1,x2,g3,cv1,'k','Linewidth',3);
const3=contour(x1,x2,g3,cv11,'c');
text(6.5,1.75,'g3')
text(1.5,2,'Feasible Region')
fv=[-6 -11.333 -18];        %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k'); %'k' specifies black dashed lines for function contours
clabel(fs)                  %Automatically puts the contour value on the graph
hold off                    %Indicates end of this plotting sequence
                            %Subsequent plots will appear in separate windows
```

3.5

Maximize $F(x_1, x_2) = 4x_1x_2$

Subject to $x_1 + x_2 \leq 20$

$x_2 - x_1 \leq 10$

$x_1, x_2 \geq 0$

Solution

$F = 4x_1x_2;$

$g_1 = x_1 + x_2 - 20 \leq 0;$

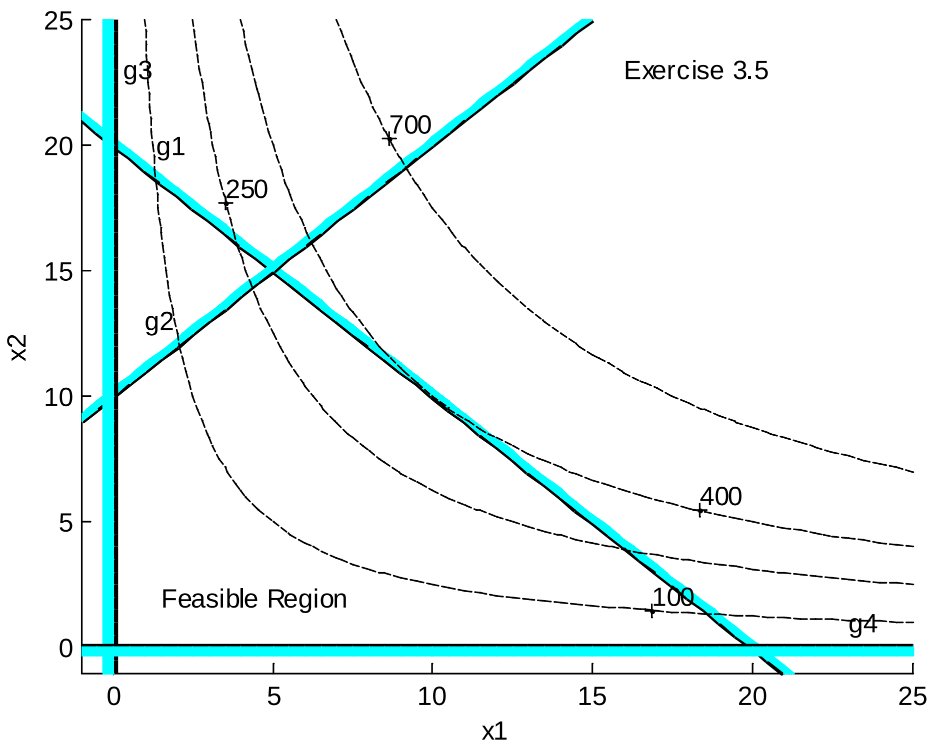
$g_2 = x_2 - x_1 - 10 \leq 0;$

$g_3 = -x_1 \leq 0;$

$g_4 = -x_2 \leq 0$

The optimum solution is: $x_1^* = 10, x_2^* = 10, F^* = 400$

Active constraint: g_1.



MATLAB Code for Exercise 3.5

```
%Exercise 3.5
%Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-1:0.5:25.0, -1:0.5:25.0);
        %Enter functions for the minimization problem
f=4*x1.*x2;
g1=x1+x2-20;
g2=x2-x1-10;
g3=-x1;
g4=-x2;
cla reset
axis auto           %Minimum and maximum values for axes are determined
automatically
                        %Limits for x- and y-axes may be specified with the command
                        %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')       %Specifies labels for x- and y-axes
hold on
text(16,23,'Exercise 3.5')
cv1=[0 0];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);
text(1.35,20,'g1')
cv11=[0.01:0.01:0.3];
cv22=[0.01:0.01:0.3];
const1=contour(x1,x2,g1,cv22,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv11,'c');
text(1,13,'g2')
const3=contour(x1,x2,g3,cv1,'k','Linewidth',4);
const3=contour(x1,x2,g3,cv11,'c');
text(23,1,'g4')
const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
text(0.3,23,'g3')
const4=contour(x1,x2,g4,cv22,'c');
text(1.5,2,'Feasible Region')
fv=[100 250 400 700];       %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k--');       %'k' specifies black dashed lines for function
contours
clabel(fs)              %Automatically puts the contour value on the graph
hold off                %Indicates end of this plotting sequence
                        %Subsequent plots will appear in separate windows
```

3.6

Minimize $f(x_1, x_2) = 5x_1 + 10x_2$

Subject to $10x_1 + 5x_2 \le 50$

$5x_1 - 5x_2 \ge -20$

$x_1, x_2 \ge 0$

Solution

$f = 5x_1 + 10x_2;$

$g_1 = 10x_1 + 5x_2 - 50 \le 0;$

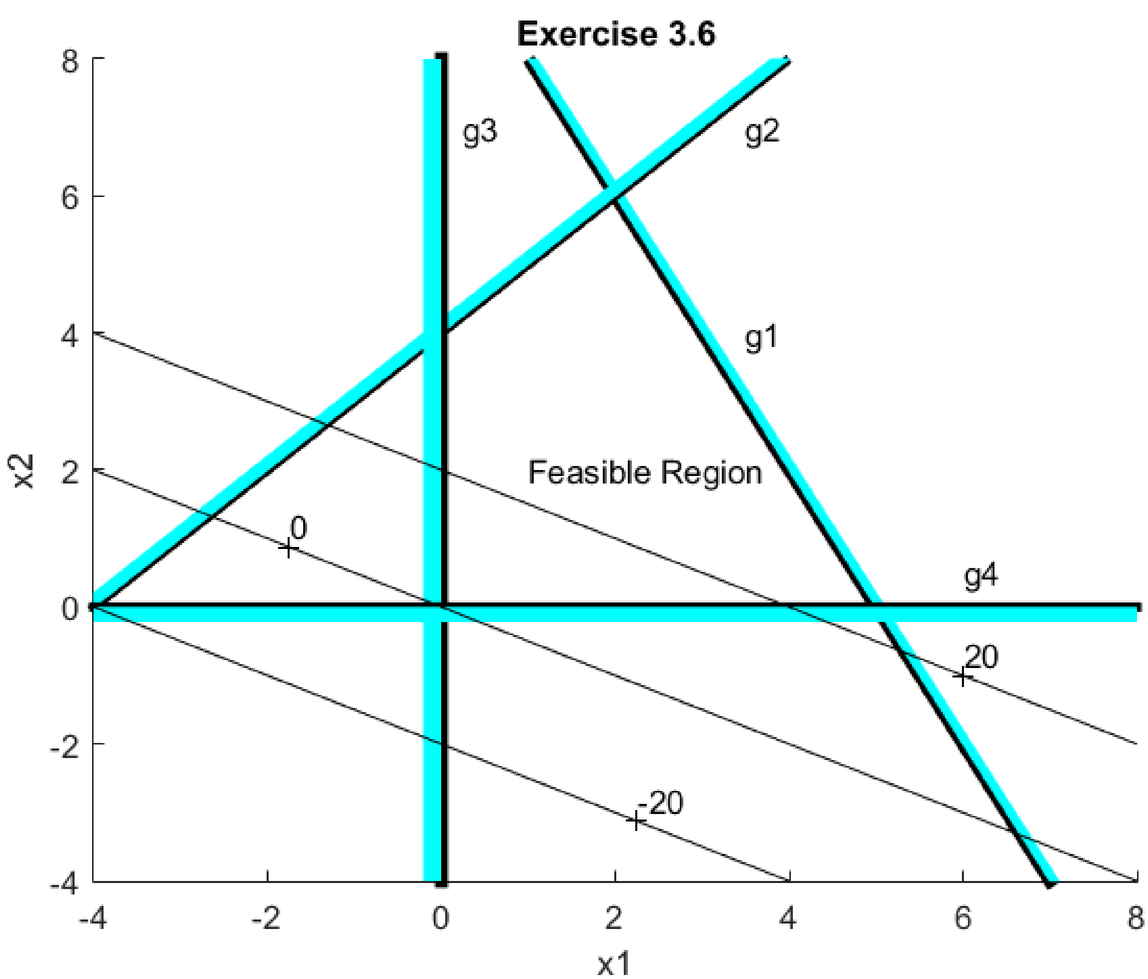
$g_2 = -5x_1 + 5x_2 - 20 \le 0;$

$g_3 = -x_1 \le 0;$

$g_4 = -x_2 \le 0$

The optimum solution is: $x_1^* = 0$, $x_2^* = 0$, $f^* = 0$

Active constraints: g_3 and g_4.



MATLAB Code

```
%Exercise 3.6
            %Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-4:0.5:8.0, -4:0.5:8.0);
            %Enter functions for the minimization problem
f=5*x1+10*x2;
g1=10*x1+5*x2-50;
g2=-5*x1+5*x2-20;
g3=-x1;
g4=-x2;
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
                              %Limits for x- and y-axes may be specified with the command
                              %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')     %Specifies labels for x- and y-axes
Title (‘Exercise 3.6’)
hold on                       %retains the current plot and axes properties for all subsequent plots
                              %Use the "contour" command to plot constraint/minimization functions
cv1=[0 0];                    %Specifies two contour values
cv12=[0.01:0.01:1];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',4);
text(3.5,4,'g1')
const1=contour(x1,x2,g1,cv12,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv12,'c');
text(3.5,7,'g2')
cv34=[0.01:0.01:0.2];
const3=contour(x1,x2,g3,cv1,'k','Linewidth',4);
const3=contour(x1,x2,g3,cv34,'c');
text(0.25,6,'g3')
const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
text(7,0.25,'g4')
const4=contour(x1,x2,g4,cv34,'c');
text(1,2,'Feasible Region')
fv=[-20 0 20];                %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
                              %Subsequent plots will appear in separate windows
```

3.7

Minimize $f(x_1, x_2) = 3x_1 + x_2$
Subject to $2x_1 + 4x_2 \le 21$
$5x_1 + 3x_2 \le 18$
$x_1, x_2 \ge 0$

Solution

$f = 3x_1 + x_2;$

$g_1 = 2x_1 + 4x_2 - 21 \le 0;$

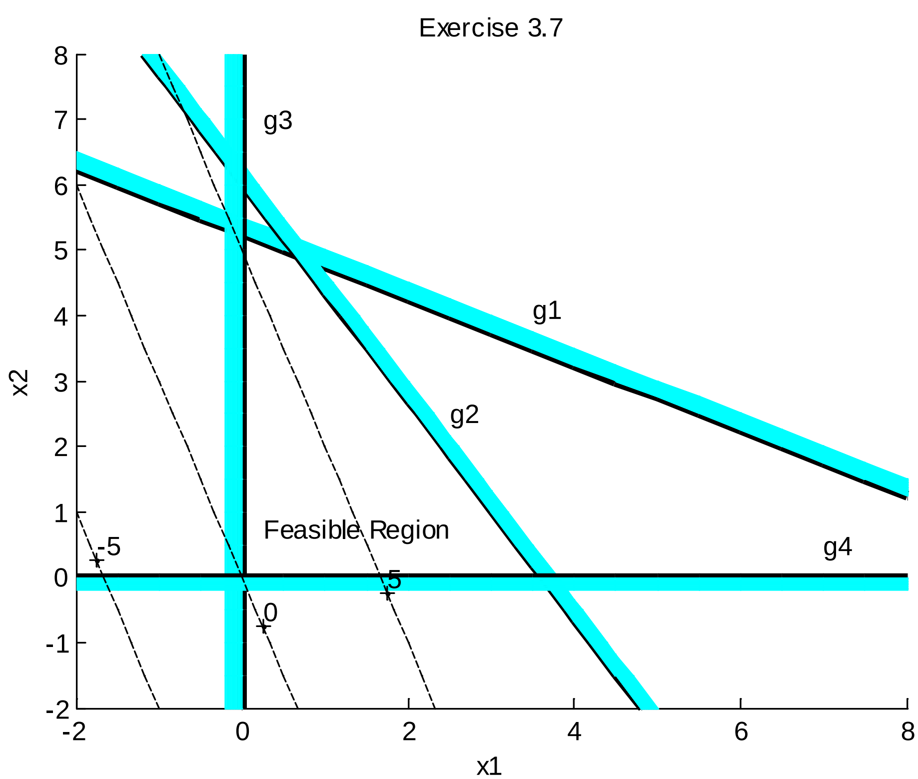
$g_2 = 5x_1 + 3x_2 - 18 \le 0;$

$g_3 = -x_1 \le 0;$

$g_4 = -x_2 \le 0$

The optimum solution is: $x_1^* = 0,\ \ x_2^* = 0,\ \ f^* = 0$

Active constraints: g_3 and g_4.



MATLAB Code

```
%Exercise 3.7
%Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-2:0.5:8.0, -2:0.5:8.0);
%Enter functions for the minimization problem
f=3*x1+x2;
g1=2*x1+4*x2-21;
g2=5*x1+3*x2-18;
g3=-x1;
g4=-x2;
cla reset
axis auto                %Minimum and maximum values for axes are determined
automatically
                         %Limits for x- and y-axes may be specified with the command
                         %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')       %Specifies labels for x- and y-axes
title ('Exercise 3.7')
hold on
cv1=[0 0];
cv12=[0.01:0.01:1];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',4);
text(3.5,4.1,'g1')
const1=contour(x1,x2,g1,cv12,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv12,'c');
text(2.5,2.5,'g2')
cv34=[0.01:0.01:0.2];
const3=contour(x1,x2,g3,cv1,'k','Linewidth',4);
const3=contour(x1,x2,g3,cv34,'c');
text(7,0.5,'g4')
const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
text(0.25,7,'g3')
const4=contour(x1,x2,g4,cv34,'c');
text(0.25,0.75,'Feasible Region')
fv=[-5 0 5];                 %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k--');       %'k' specifies black dashed lines for function
contours
clabel(fs)               %Automatically puts the contour value on the graph
hold off                 %Indicates end of this plotting sequence
                         %Subsequent plots will appear in separate windows
```

3.8

Minimize $f(x_1, x_2) = x_1^2 - 2x_2^2 - 4x_1$
Subject to $x_1 + x_2 \le 6$
$x_2 \le 3$
$x_1, x_2 \ge 0$

Solution

$f = x_1^2 - 2x_2^2 - 4x_1$ (hyperbola);

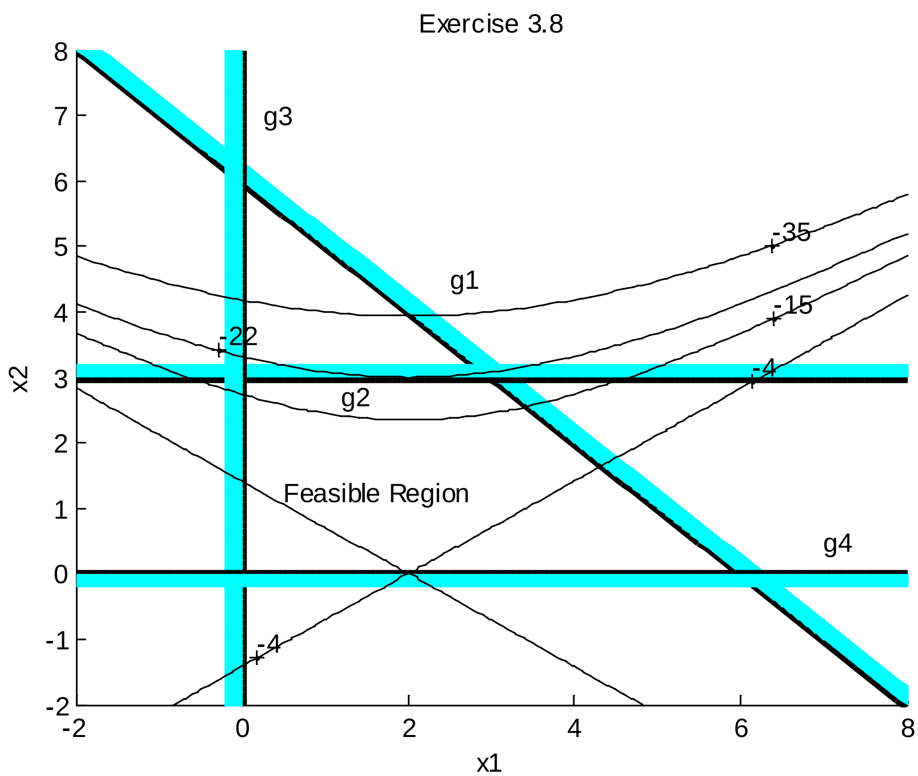
$g_1 = x_1 + x_2 - 6 \le 0$;

$g_2 = x_2 - 3 \le 0$;

$g_3 = -x_1 \le 0$;

$g_4 = -x_2 \le 0$

The optimum solution is: $x_1^* = 2,\ x_2^* = 3,\ f^* = -22$
Active constraint: g_1.



MATLAB Code

```
%Exercise 3.8
%Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-2:0.1:8.0, -2:0.1:8.0);
%Enter functions for the minimization problem
f=(x1.^2)-2*(x2.^2)-4*x1;
g1=x1+x2-6;
g2=x2-3;
g3=-x1;
g4=-x2;
cla reset
axis auto                %Minimum and maximum values for axes are determined
automatically
                         %Limits for x- and y-axes may be specified with the command
                         %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')       %Specifies labels for x- and y-axes
title('Exercise 3.8')
hold on
cv1=[0 0];
cv12=[0.01:0.01:0.3];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',4);
text(2.5,4.5,'g1')
const1=contour(x1,x2,g1,cv12,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',4);
const2=contour(x1,x2,g2,cv34,'c');
text(1.2,2.7,'g2')
cv34=[0.01:0.01:0.2];
const3=contour(x1,x2,g3,cv1,'k','Linewidth',4);
const3=contour(x1,x2,g3,cv34,'c');
text(7,0.5,'g4')
const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
text(0.25,7,'g3')
const4=contour(x1,x2,g4,cv34,'c');
text(0.5,1.25,'Feasible Region')
fv=[-4 -15 -22 -35];            %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');     %'k' specifies black dashed lines for function
contours
clabel(fs)               %Automatically puts the contour value on the graph
hold off                 %Indicates end of this plotting sequence
                         %Subsequent plots will appear in separate windows
```

3.9

Minimize $f(x_1, x_2) = x_1 x_2$
Subject to $x_1 + x_2^2 \le 0$
$x_1^2 + x_2^2 \le 9$

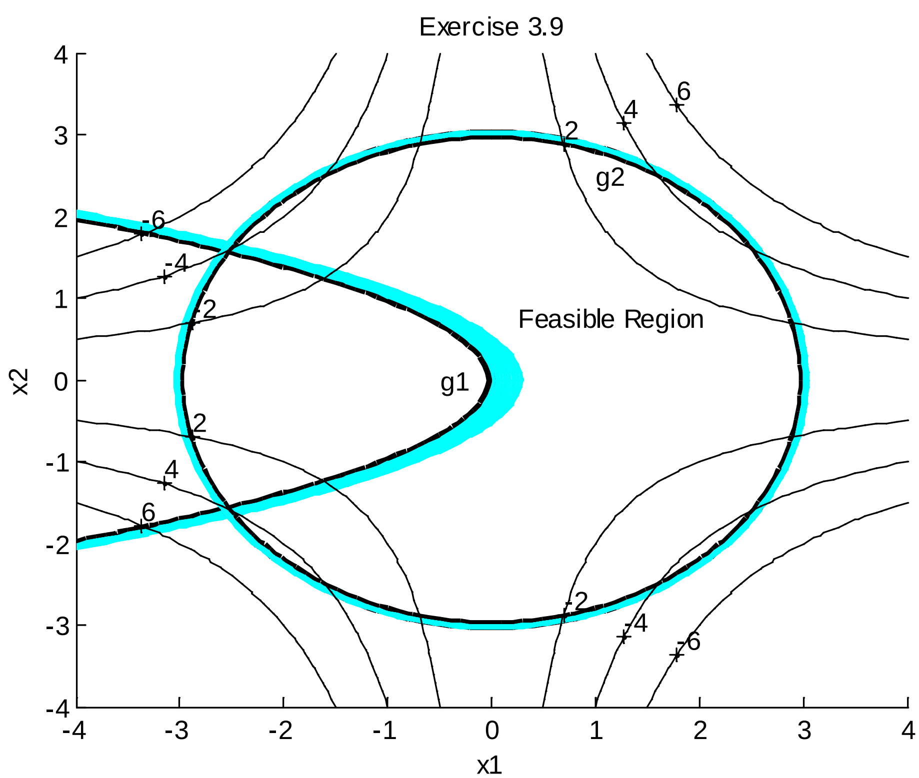
Solution

$f = x_1 x_2;$

$g_1 = x_1 + x_2^2 \le 0;$

$g_2 = x_1^2 + x_2^2 - 9 \le 0$

The optimum solution is: $x_1^* \doteq -2.5,\ x_2^* \doteq 1.58,\ f^* \doteq -3.95$
Active constraints: g_1 and g_2.



MATLAB Code

```
%Exercise 3.9
%Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-4:0.1:4.0, -4:0.1:4.0);
%Enter functions for the minimization problem
f=x1.*x2;
g1=x1+x2.^2;
g2=x1.^2+x2.^2-9;
cla reset
axis auto                %Minimum and maximum values for axes are determined
automatically
                         %Limits for x- and y-axes may be specified with the command
                         %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')        %Specifies labels for x- and y-axes
title('Exercise 3.9')
hold on
cv1=[0 0];
cv12=[0.01:0.01:0.3];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',4);
text(-0.5,0,'g1')
const1=contour(x1,x2,g1,cv12,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',4);
const2=contour(x1,x2,g2,cv12,'c');
text(1,2.5,'g2')
text(0.25,0.75,'Feasible Region')
fv=[2 -2 4 -4 6 -6];             %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');      %'k' specifies black dashed lines for function
contours
clabel(fs)               %Automatically puts the contour value on the graph
hold off                 %Indicates end of this plotting sequence
                         %Subsequent plots will appear in separate windows
```

3.10

Minimize $f(x_1, x_2) = 3x_1 + 6x_2$
Subject to $-3x_1 + 3x_2 \le 2$
$4x_1 + 2x_2 \le 4$
$-x_1 + 3x_2 \ge 1$

Solution

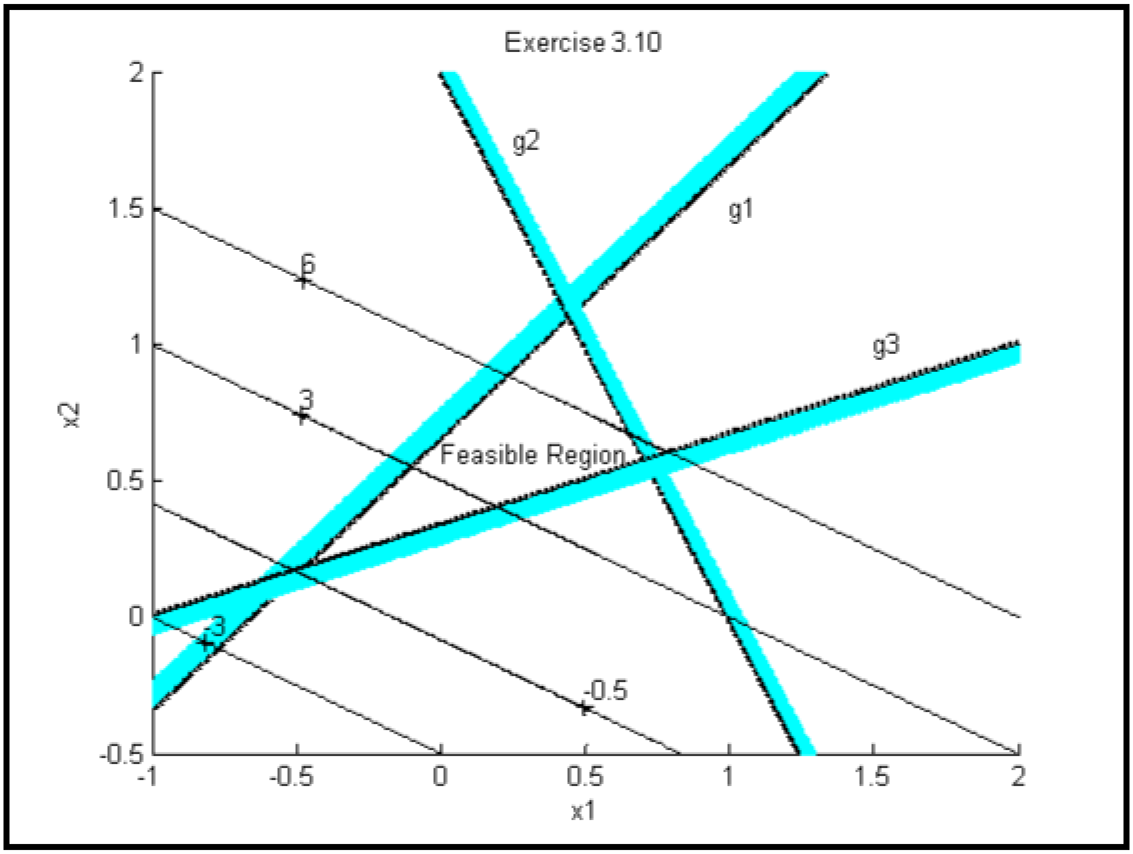
$f = 3x_1 + 6x_2;$

$g_1 = -3x_1 + 3x_2 - 2 \le 0;$

$g_2 = 4x_1 + 2x_2 - 4 \le 0;$

$g_3 = x_1 - 3x_2 + 1 \le 0$

The optimum solution is: $x_1^* = -0.5$, $x_2^* \doteq 0.167$, $f^* \doteq -0.5$
Active constraints: g_1 and g_3.



MATLAB Code

```
%Exercise 3.10
%Create a grid from -1 to 7 with an increment of 0.01 for the variables x1 and x2
[x1,x2]=meshgrid(-1:0.01:2.0, -0.5:0.01:2.0);
%Enter functions for the minimization problem
f=3*x1+6*x2;
g1=-3*x1+3*x2-2;
g2=4*x1+2*x2-4;
g3=x1-3*x2+1;
cla reset
axis auto                %Minimum and maximum values for axes are determined
automatically
                         %Limits for x- and y-axes may be specified with the command
                         %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')        %Specifies labels for x- and y-axes
title('Exercise 3.10')
hold on
cv1=[0 0];
cv12=[0.01:0.01:0.3];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',4);
text(1,1.5,'g1')
const1=contour(x1,x2,g1,cv12,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',4);
const2=contour(x1,x2,g2,cv34,'c');
text(0.25,1.75,'g2')
cv34=[0.01:0.01:0.2];
const3=contour(x1,x2,g3,cv1,'k','Linewidth',4);
const3=contour(x1,x2,g3,cv34,'c');
text(1.5,1.,'g3')
text(0,0.6,'Feasible Region')
fv=[-3 -0.5 3 6];            %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');      %'k' specifies black dashed lines for function
contours
clabel(fs)               %Automatically puts the contour value on the graph
hold off                 %Indicates end of this plotting sequence
                         %Subsequent plots will appear in separate windows
```

Develop an appropriate graphical representation for the following problems and determine all the local minimum and local maximum points.

3.11

$f(x,y) = 2x^2 + y^2 - 2xy - 3x - 2y$

subject to $y - x \le 0$

$x^2 + y^2 - 1 = 0$

Solution

$f(x,y) = 2x^2 + y^2 - 2xy - 3x - 2y$

g1: $y - x \le 0$

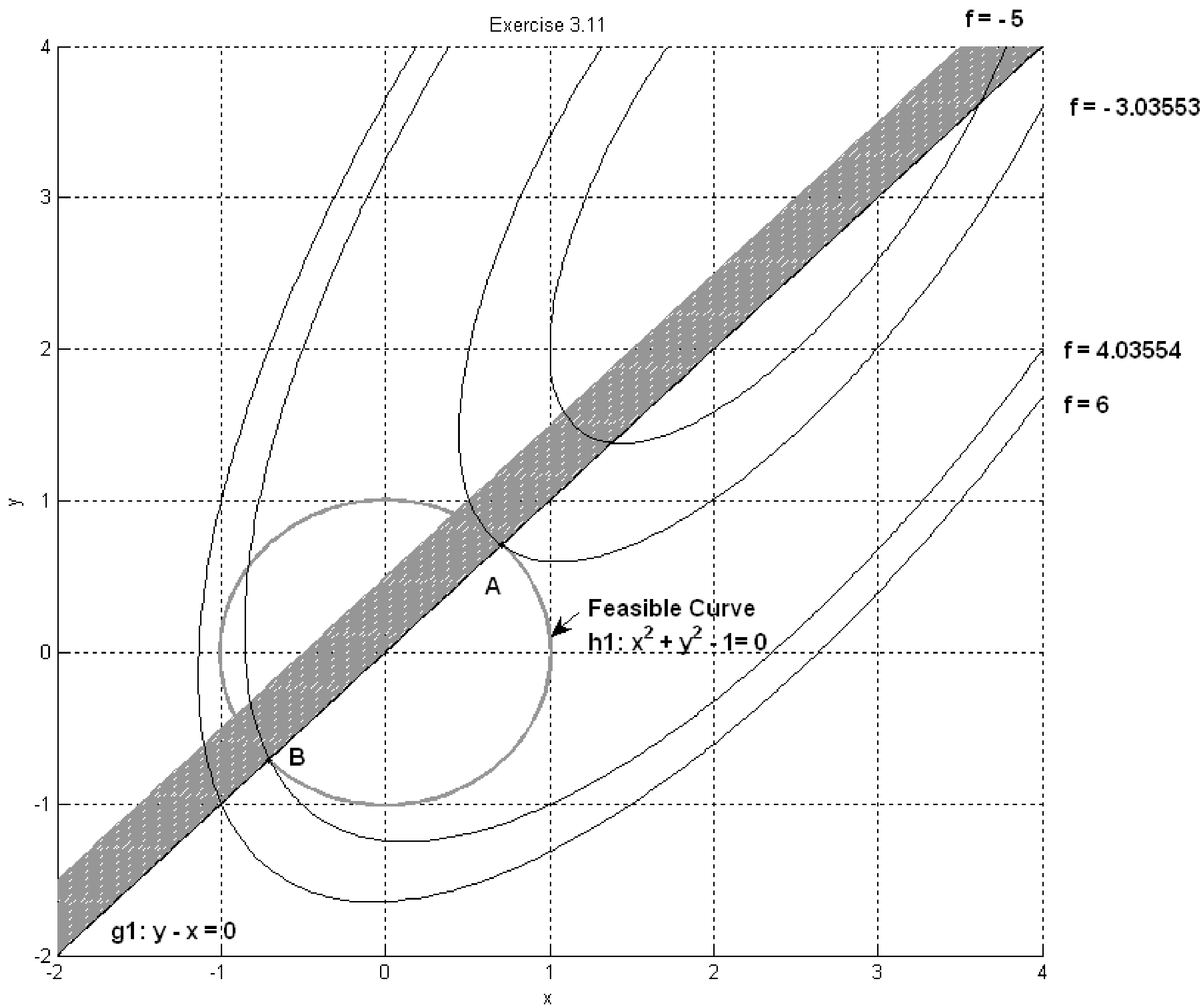
$h1$: $x^2 + y^2 - 1 = 0$

Local and global minimum at $A(0.71, 0.71)$ with $\mathrm{f}^* = -3.04$

Active constraint: g1

Local and global maximum at $\mathrm{B}(-0.71, -0.71)$ with $\mathrm{f}^* = 4.04$

Active constraint: g1



MATLAB Code

```
%Exercise 3.11
%Create a grid from -2 to 4 with an increment of 0.05 for the variables x and y
[x,y]=meshgrid(-2:0.05:4, -2:0.05:4);
                    %Optimization and constraint functions
f=2*x.^2+y.^2-2*x.*y-3*x-2*y;
g1=y-x;
h1=x.^2+y.^2-1;
cla reset
axis auto                   %Minimum and maximum values for axes are determined automatically
xlabel('x'),ylabel('y')     %Specifies labels for x- and y-axes
title('Exercise 3.11')      %Specifies graph title
hold on                     %retains the current plot and axes properties for all subsequent plots
                            %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:0.5];           %Specifies contour values
const1=contour(x,y,g1,cv1,'g');
cv1=[0 0.005];
const1=contour(x,y,g1,cv1,'k');
cv2=[0:0.001:0.03];
const2=contour(x,y,h1,cv2,'g');
fv=[-5 -3.03553 4.03554 6]; %Defines contours for the minimization function
fs=contour(x,y,f,fv,'k');
a=[0.707107,-0.707107];
b=[0.707107,-0.707107];
plot(a,b,'.k');             %Plots points a and b in black
grid
hold off                    %Indicates end of this plotting sequence
                            %Subsequent plots will appear in separate windows
```

3.12

$f(x, y) = 4x^2 + 3y^2 - 5xy - 8x$

subject to $x + y = 4$

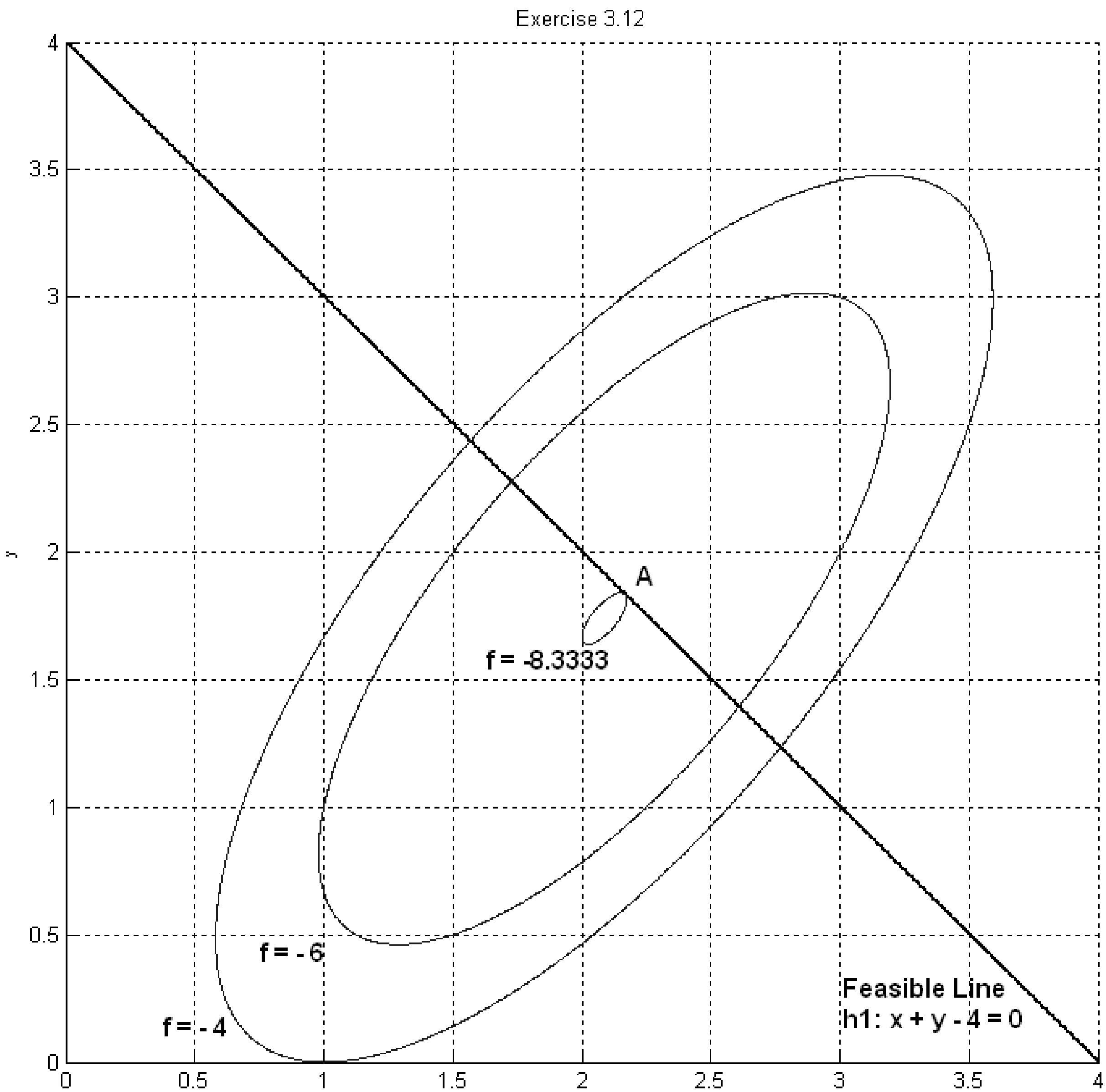
Solution

$f(x, y) = 4x^2 + 3y^2 - 5xy - 8x$

$h1\text{:}\ x + y - 4 = 0$

Local and global minimum at $A(2.17, 1.83)$ with $f^* = -8.33$

There are no local maximum points.



MATLAB Code

```
%Exercise 3.12
%Create a grid from -5 to 5 with an increment of 0.005 for the variables x and y
[x,y]=meshgrid(-5:0.005:5, -5:0.005:5);
                    %Optimization and constraint functions
f=4*x.^2+3*y.^2-5*x.*y-8*x;
h1=x+y-4;
cla reset
axis ([0 4 0 4])            %Minimum and maximum values are defined for plot
xlabel('x'),ylabel('y')     %Specifies labels for x- and y-axes
title('Exercise 3.12')      %Specifies graph title
hold on                     %retains the current plot and axes properties for all subsequent plots
                            %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.001:0.01];         %Specifies contour values
const1=contour(x,y,h1,cv1,'k');
fv=[-25/3 -6 -4];           %Defines contours for the minimization function
fs=contour(x,y,f,fv,'b');
grid
hold off                    %Indicates end of this plotting sequence
                            %Subsequent plots will appear in separate windows
```

3.13

$f(x, y) = 9x^2 + 13y^2 + 18xy - 4$

subject to $x^2 + y^2 + 2x = 16$

Solution

$f(x, y) = 9x^2 + 13y^2 + 18xy - 4$

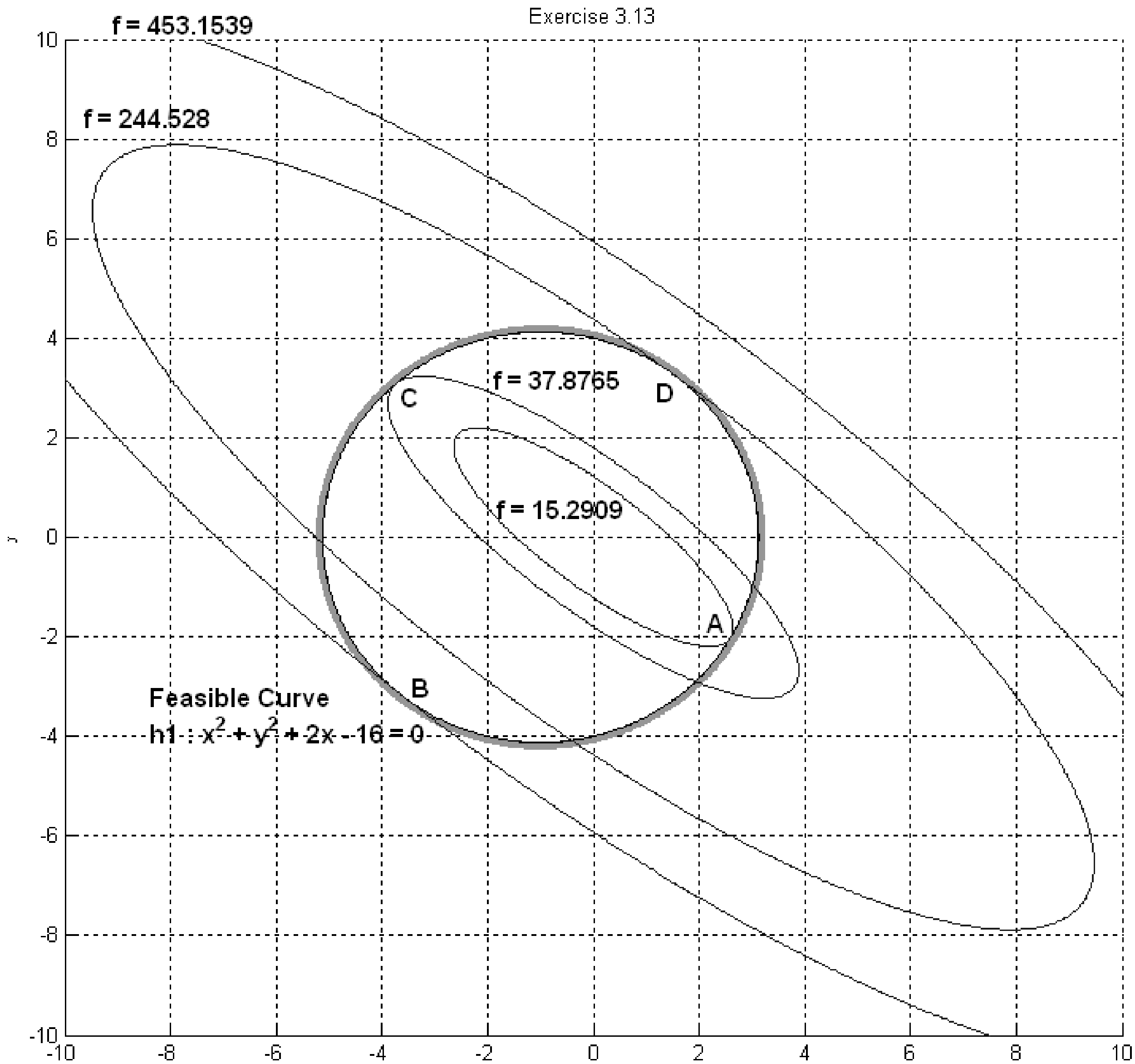
$h1$: $x^2 + y^2 + 2x - 16 = 0$

Local, global minimum at A (2.59, -2.02) with $f^* = 15.3$

Local, global maximum at B (-3.63, -3.18) with $f^* = 453.2$

Local minimum at C (-3.73, 3.09) with $f^* = 37.88$

Local maximum at D (1.51, 3.27) with $f^* = 244.53$



MATLAB Code

```
                    %Create a grid from -10 to 10 with an increment of 0.05 for the variables x and y
[x,y]=meshgrid(-10:0.05:10, -10:0.05:10);
                    %Optimization and constraint functions
f=9*x.^2+13*y.^2.+18*x.*y-4;
g1=x.^2+y.^2+2*x-16;
cla reset
axis ([-10 10 -10 10])        %Minimum and maximum values are defined for plot
xlabel('x'),ylabel('y')       %Specifies labels for x- and y-axes
title('Exercise 3.13')        %Specifies graph title
hold on                       %retains the current plot and axes properties for all subsequent plots
                              %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:1];               %Specifies contour values
const1=contour(x,y,g1,cv1,'g');
cv1=[0 0.01];
const1=contour(x,y,g1,cv1,'k');
fv=[15.2909 37.8765 244.528 453.1539];     %Defines contours for the minimization function
fs=contour(x,y,f,fv,'b');
grid
hold off                      %Indicates end of this plotting sequence
                              %Subsequent plots will appear in separate windows
```

3.14

$f(x, y) = 2x + 3y - x^3 - 2y^2$

subject to $x + 3y \le 6$

$5x + 2y \le 10$

$x, y \ge 0$

Solution

$f(x, y) = 2x + 3y - x^3 - 2y^2$

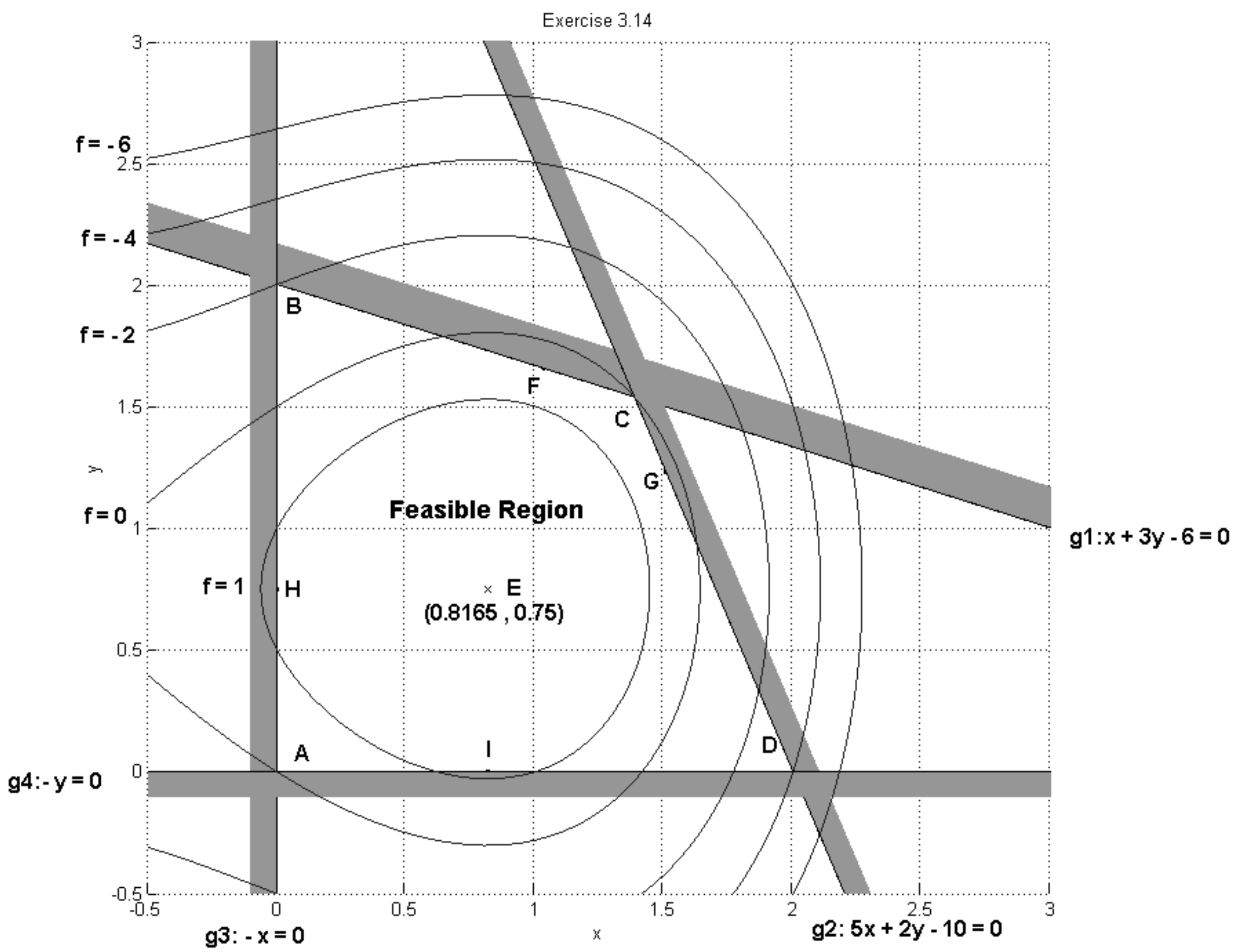
g1: $\mathrm{x} + 3\mathrm{y} - 6 \le 0$

g2: $5\,\mathrm{x} + 2\mathrm{y} - 10 \le 0$

g3: $-x \le 0$

g4: $-y \le 0$

Local minimum at A$(0, 0)$ with $f^* = 0$. Active constraint: g3 and g4
Local minimum at B$(0, 2)$ with $f^* = -2$. Active constraint: g1 and g3
Local minimum at C$(1.39, 1.54)$ with $f^* = 0$. Active constraint: g1 and g2
Local, global minimum at D$(2, 0)$ with $f^* = -4$. Active constraint: g2 and g4
Local, global maximum at E$(0.82, 0.75)$ with $f^* = 2.21$. Active constraint: None.



MATLAB Code

```
                    %Create a grid from -1 to 3 with an increment of 0.05 for the variables x and y
[x,y]=meshgrid(-1:0.05:3, -1:0.05:3);
                    %Optimization and constraint functions
f=2*x+3*y-x.^3-2*y.^2;
g1=x+3*y-6;
g2=5*x+2*y-10;
g3=-x;
g4=-y;
cla reset
axis ([-0.5 3 -0.5 3])          %Minimum and maximum values are defined for plot
xlabel('x'),ylabel('y')         %Specifies labels for x- and y-axes
title('Exercise 3.14')          %Specifies graph title
hold on                         %retains the current plot and axes properties for all subsequent plots
                                %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.0005:0.5];             %Specifies contour values
const1=contour(x,y,g1,cv1,'g');
cv1=[0 0.001];
const1=contour(x,y,g1,cv1,'k');
cv2=[0:0.0005:0.5];
const2=contour(x,y,g2,cv2,'g');
cv2=[0 0.001];
const2=contour(x,y,g2,cv2,'k');
cv3=[0:0.0005:0.1];
const3=contour(x,y,g3,cv3,'g');
cv3=[0 0.001];
const3=contour(x,y,g3,cv3,'k');
cv4=[0:0.0005:0.1];
const4=contour(x,y,g4,cv4,'g');
cv4=[0 0.001];
const4=contour(x,y,g4,cv4,'k');
fv=[-6 -4 -2 0 1];              %Defines contours for the minimization function
fs=contour(x,y,f,fv,'b');
a=[1.03395,1.50735,0,0.8165];
b=[1.65535,1.23163,0.75,0];
plot(a,b,'.k');                 %Plots points a and b in black
c=[0.8165];
d=[0.75];
plot(c,d,'x');                  %Plots points c and d
grid
hold off                        %Indicates end of this plotting sequence
                                %Subsequent plots will appear in separate windows
```

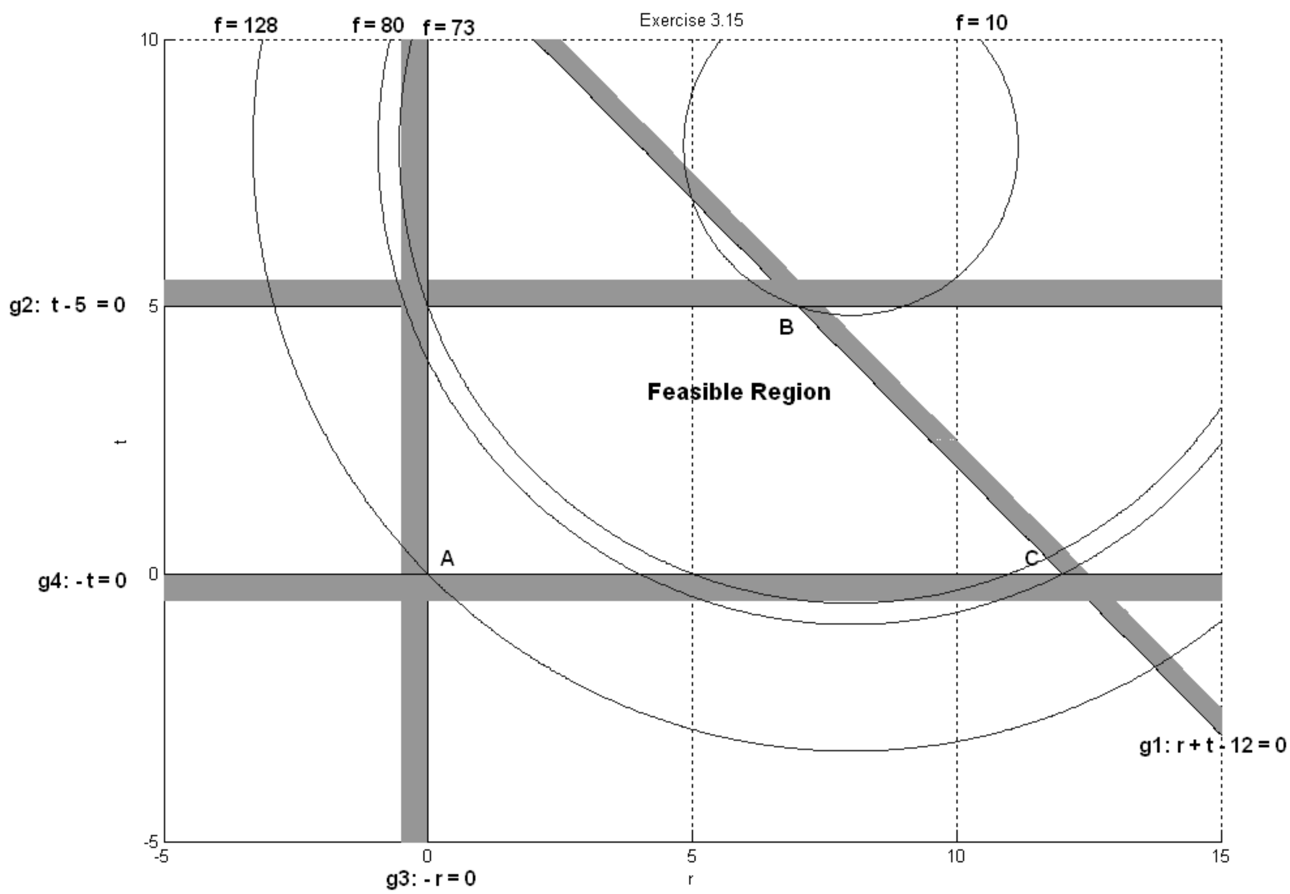
3.15

$f(r,t) = (r-8)^2 + (t-8)^2$
subject to $12 \geq r + t$
$t \leq 5$
$r, t \geq 0$

Solution

$f(r,t) = (r-8)^2 + (t-8)^2$
$\mathrm{g1: r + t - 12 \leq 0}$
$\mathrm{g2: t - 5 \leq 0}$
$\mathrm{g3: -r \leq 0}$
$\mathrm{g4: -t \leq 0}$

Local, global maximum at A (0,0) with $f^* = 128$. Active constraint: g3 and g4
Local, global minimum at B (7, 5) $f^* = 10$. Active constraint: g1 and g2
Local maximum at C (12, 0) $f^* = 80$. Active constraint: g1 and g4



MATLAB Code

```
                    %Create a grid from -5 to 15 with an increment of 0.05 for the variables r and t
 [r,t]=meshgrid(-5:0.05:15, -5:0.05:10);
                    %Optimization and constraint functions
f=(r-8).^2+(t-8).^2;
g1=r+t-12;
g2=t-5;
g3=-r;
g4=-t;
cla reset
axis ([-5 15 -5 10])    %Minimum and maximum values are defined for plot
xlabel('r'),ylabel('t') %Specifies labels for x- and y-axes
title('Exercise 3.15')  %Specifies graph title
hold on                 %retains the current plot and axes properties for all subsequent plots
                        %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:0.5];       %Specifies contour values
const1=contour(r,t,g1,cv1,'g');
cv1=[0 0.001];
const1=contour(r,t,g1,cv1,'k');
cv2=[0:0.01:0.5];
const2=contour(r,t,g2,cv2,'g');
cv2=[0 0.001];
const2=contour(r,t,g2,cv2,'k');
cv3=[0:0.01:0.5];
const3=contour(r,t,g3,cv3,'g');
cv3=[0 0.001];
const3=contour(r,t,g3,cv3,'k');
cv4=[0:0.01:0.5];
const4=contour(r,t,g4,cv4,'g');
cv4=[0 0.001];
const4=contour(r,t,g4,cv4,'k');
fv=[10 73 80 128];      %Defines contours for the minimization function
fs=contour(r,t,f,fv,'b');
grid
hold off                %Indicates end of this plotting sequence
                        %Subsequent plots will appear in separate windows
```

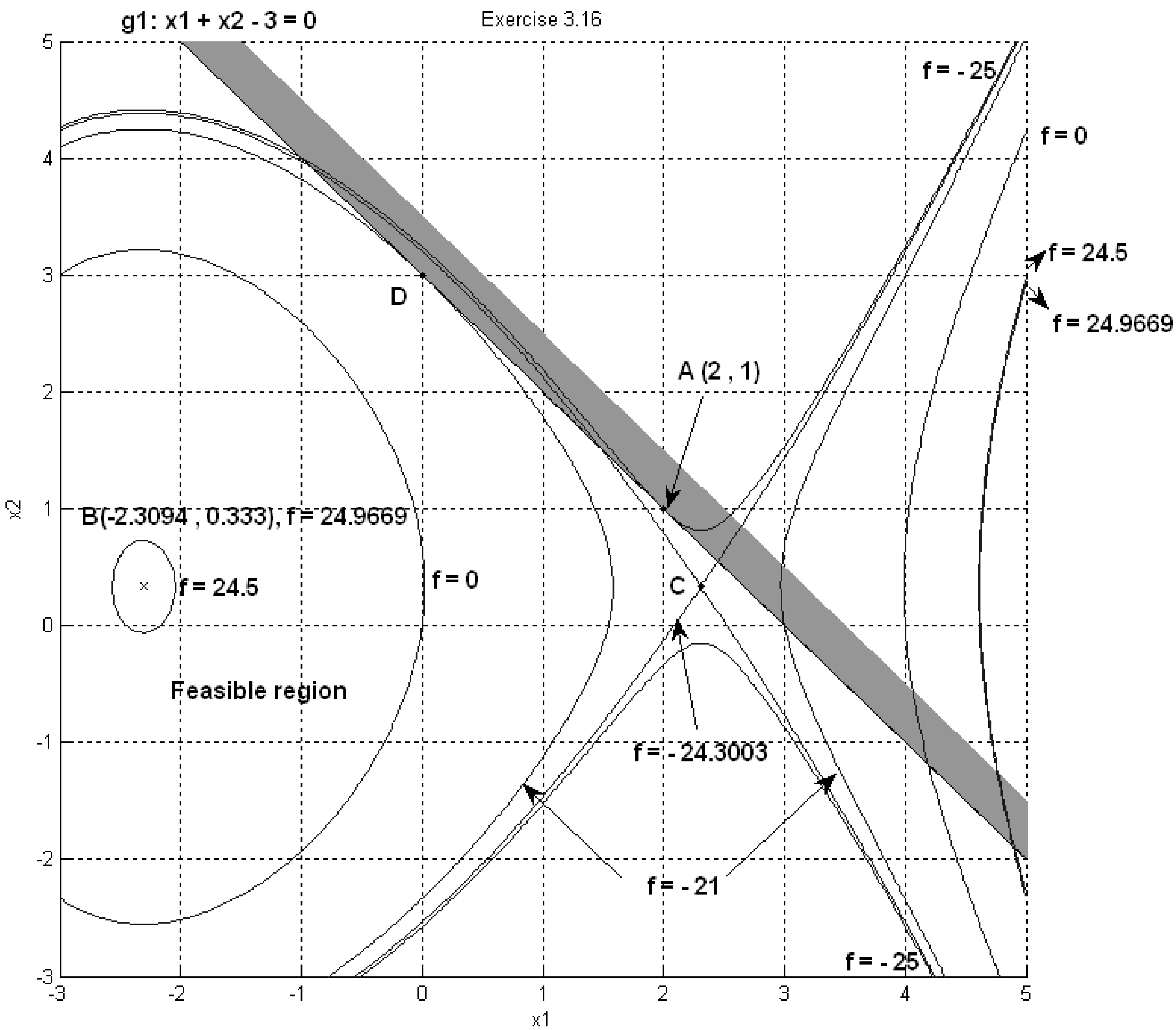
3.16

$f(x_1, x_2) = x_1^3 - 16x_1 + 2x_2 - 3x_2^2$
subect to $x_1 + x_2 \le 3$

Solution

$f(x_1, x_2) = x_1^3 - 16x_1 + 2x_2 - 3x_2^2$
$g1: x_1 + x_2 - 3 \le 0$

Local, global minimum at A (2, 1) with $f^* = -25$. Active constraint: g1
Local, global maximum at B (-2.31 , 0.33) with $f^* = 24.97$. Active constraint: N/A



MATLAB Code

```
                    %Create a grid from -3 to 5 with an increment of 0.05 for the variables x1 and x2
[x1,x2]=meshgrid(-3:0.05:5, -3:0.05:5);
                    %Optimization and constraint functions
f=x1.^3-16*x1+2*x2-3*x2.^2;
g1=x1+x2-3;
cla reset
axis ([-3 5 -3 5])          %Minimum and maximum values are defined for plot
xlabel('x1'),ylabel('x2')   %Specifies labels for x- and y-axes
title('Exercise 3.16')      %Specifies graph title
hold on                     %retains the current plot and axes properties for all subsequent plots
                            %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:0.5];           %Specifies contour values
const1=contour(x1,x2,g1,cv1,'g');
cv1=[0 0.001];
const1=contour(x1,x2,g1,cv1,'k');
fv=[-25 -24.3003 -21 0 24.5 24.9669];     %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'b');
a=[-2.3094];
b=[0.333];
plot(a,b,'x');              %Plots points a and b
c=[2.3094,2,0];
d=[0.333,1,3];
plot(c,d,'.k');             %Plots points c and d
grid
hold off                    %Indicates end of this plotting sequence
                            %Subsequent plots will appear in separate windows
```

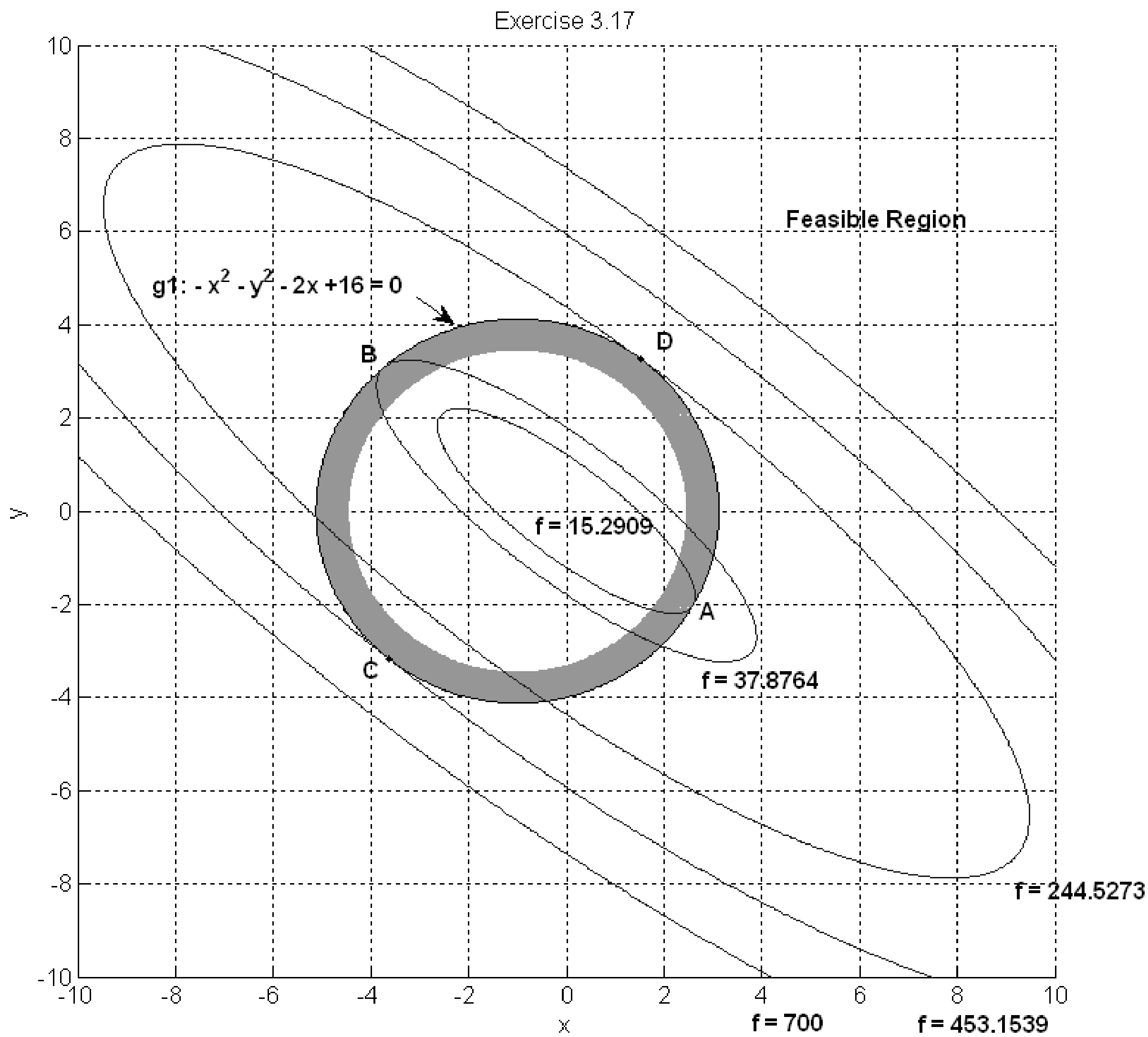
3.17

$f(x, y) = 9x^2 + 13y^2 + 18xy - 4$
subject to $x^2 + y^2 + 2x \geq 16$

Solution

$f(x, y) = 9x^2 + 13y^2 + 18xy - 4$
$g1: -x^2 - y^2 - 2x + 16 \leq 0$

Local, global minimum at A (2.59, -2.01) with $f^* = 15.25$. Active constraint: g1
Local minimum at B (-3.73, 3.09) with $f^* = 37.87$. Active constraint: g1
There are no local maximum points.



MATLAB Code

```
                    %Create a grid from -10 to 10 with an increment of 0.05 for the variables x and y
[x,y]=meshgrid(-10:0.05:10, -10:0.05:10);
                    %Optimization and constraint functions
f=9*x.^2.+13.*y.^2.+18.*x.*y-4;
g1=-x.^2-y.^2-2*x+16;
cla reset
axis ([-10 10 -10 10]) %Minimum and maximum values are defined for plot
xlabel('x'),ylabel('y')  %Specifies labels for x- and y-axes
title('Exercise 3.17')   %Specifies graph title
hold on                  %retains the current plot and axes properties for all subsequent plots
                         %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:5];          %Specifies contour values
const1=contour(x,y,g1,cv1,'g');
cv1=[0 0.001];
const1=contour(x,y,g1,cv1,'k');
fv=[15.2909 37.8764 244.5273 453.1539 700];      %Defines contours for the minimization function
fs=contour(x,y,f,fv,'b');
c=[1.50884,-3.63];
d=[3.27196,-3.1754];
plot(c,d,'.k');          %Plots points c and d in black
grid
hold off                 %Indicates end of this plotting sequence
                         %Subsequent plots will appear in separate windows
```

3.18

$f(r,t) = (r-4)^2 + (t-4)^2$

subject to $10 - r - t \geq 0$

$5 \geq r$

$r, t \geq 0$

Solution

$f(r,t) = (r-4)^2 + (t-4)^2$

$g1: r + t - 10 \leq 0$

$g2: r - 5 \leq 0$

$g3: -r \leq 0$

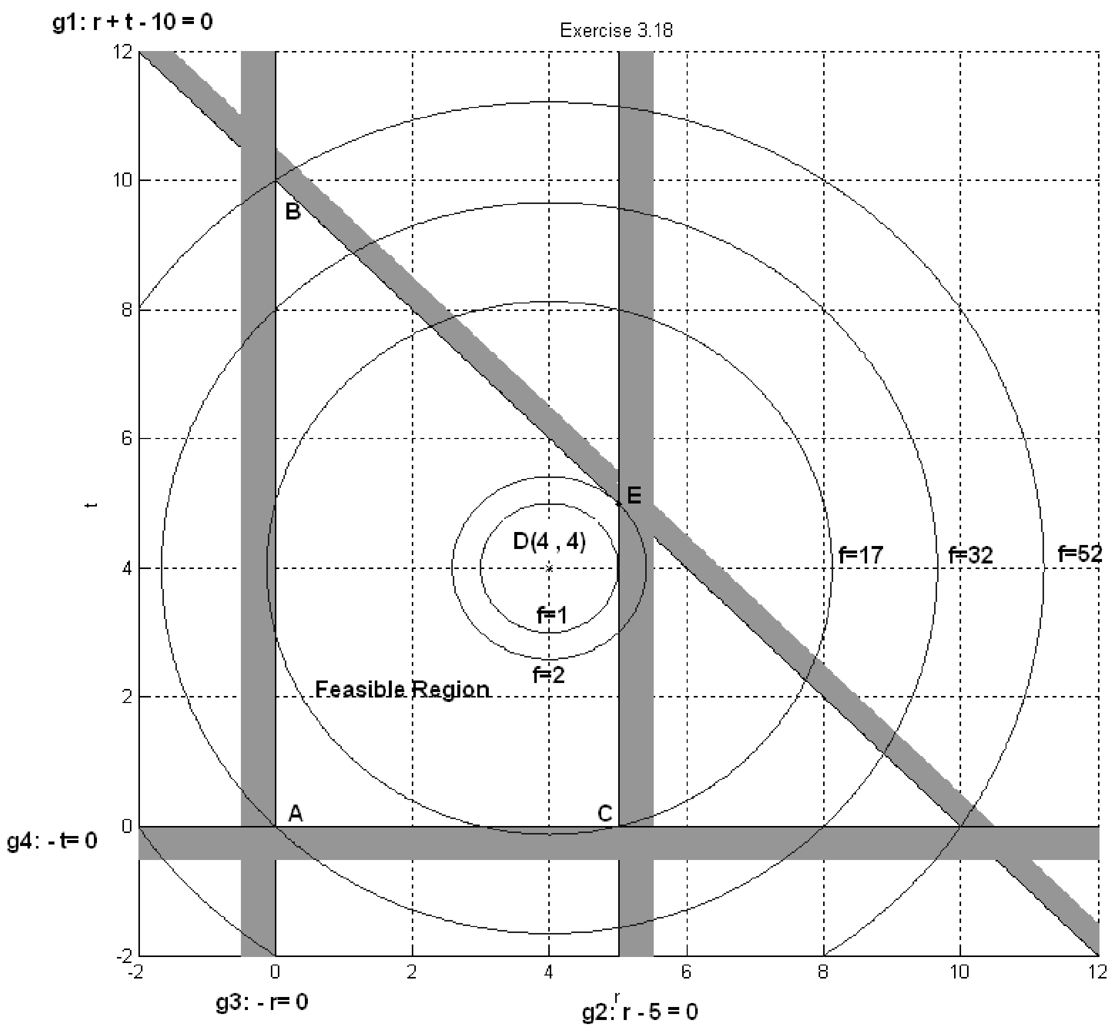
$g4: -t \leq 0$

Local maximum at A (0, 0) with $f^* = 32$. Active constraint: g3 and g4

Local, global maximum at B (0, 10) with $f^* = 52$. Active constraint: g1 and g3

Local maximum at C (5, 0) with $f^* = 17$. Active constraint: g2 and g4

Local, global minimum at D (4, 4) with $f^* = 0$. Active constraint: None.



MATLAB Code

```
                    %Create a grid from -2 to 12 with an increment of 0.05 for the variables r and t
[r,t]=meshgrid(-2:0.05:12, -2:0.05:12);
                    %Optimization and constraint functions
f=(r-4).^2+(t-4).^2;
g1=r+t-10;
g2=r-5;
g3=-r;
g4=-t;
cla reset
axis ([-2 12 -2 12])    %Minimum and maximum values are defined for plot
xlabel('r'),ylabel('t') %Specifies labels for x- and y-axes
title('Exercise 3.18')  %Specifies graph title
hold on                 %retains the current plot and axes properties for all subsequent plots
                        %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:0.5];       %Specifies contour values
const1=contour(r,t,g1,cv1,'g');
cv1=[0 0.001];
const1=contour(r,t,g1,cv1,'k');
cv2=[0:0.01:0.5];
const2=contour(r,t,g2,cv2,'g');
cv2=[0 0.001];
const2=contour(r,t,g2,cv2,'k');
cv3=[0:0.01:0.5];
const3=contour(r,t,g3,cv3,'g');
cv3=[0 0.001];
const3=contour(r,t,g3,cv3,'k');
cv4=[0:0.01:0.5];
const4=contour(r,t,g4,cv4,'g');
cv4=[0 0.001];
const4=contour(r,t,g4,cv4,'k');
fv=[1 2 17 32 52];      %Defines contours for the minimization function
fs=contour(r,t,f,fv,'b');
a=[4];
b=[4];
plot(a,b,'x');          %Plots points a and b
c=[5];
d=[5];
plot(c,d,'.k');         %Plots points c and d
grid
hold off                %Indicates end of this plotting sequence
                        %Subsequent plots will appear in separate windows
```

3.19

$f(x,y) = -x + 2y$

subject to $-x^2 + 6x + 3y \leq 27$

$18x - y^2 \geq 180$

$x, y \geq 0$

Solution

$f(x,y) = -x + 2y$

$g1:\ -x^2 + 6x + 3y - 27\ \leq 0$

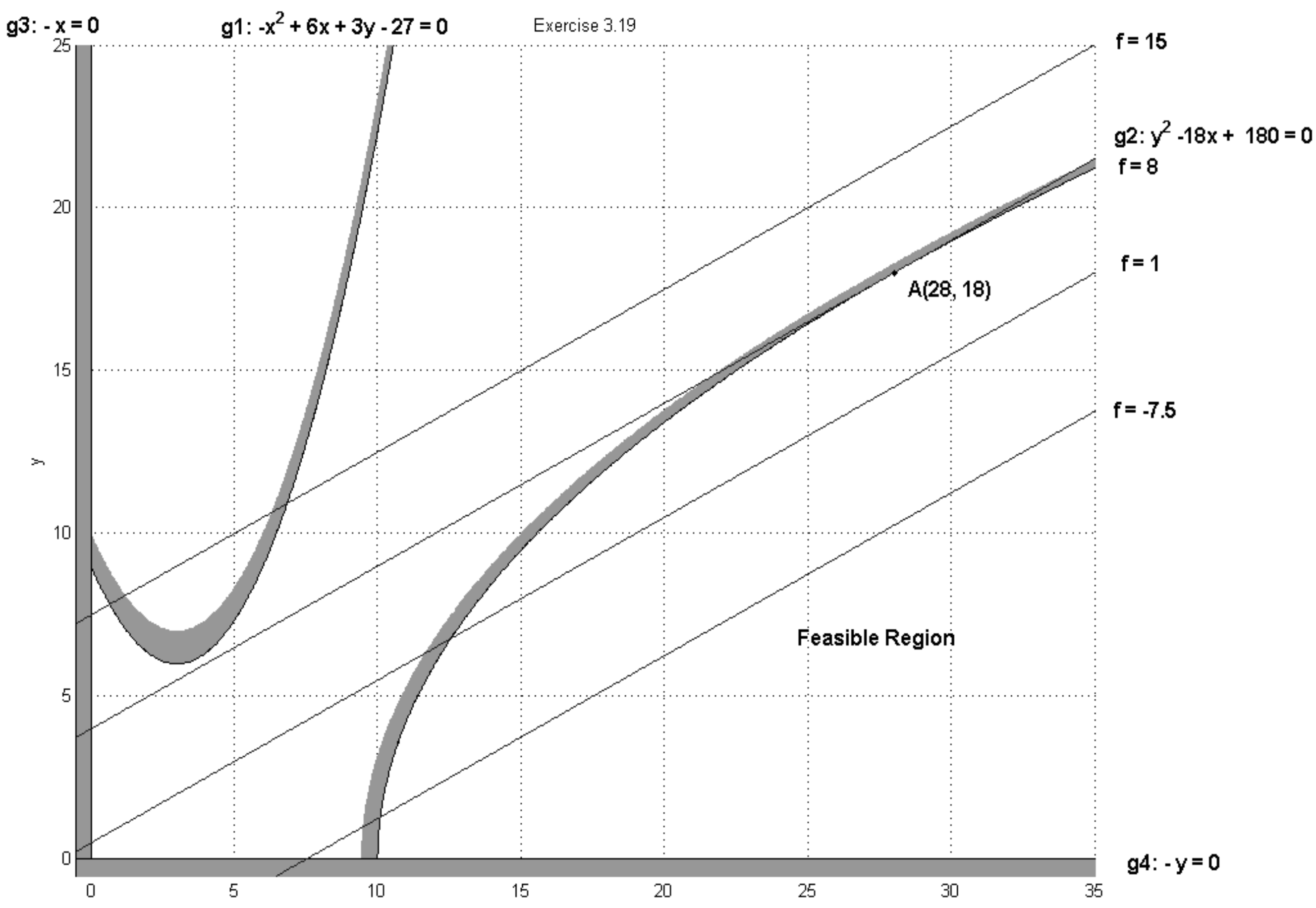
$g2: y^2 - 18x + 180\ \leq 0$

$g3:\ -x \leq 0$

$g4:\ -y\ \leq 0$

There are no local minimum points.

Local, global maximum at A (28, 18) with $f^* = 8$. Active constraint: g2



MATLAB Code

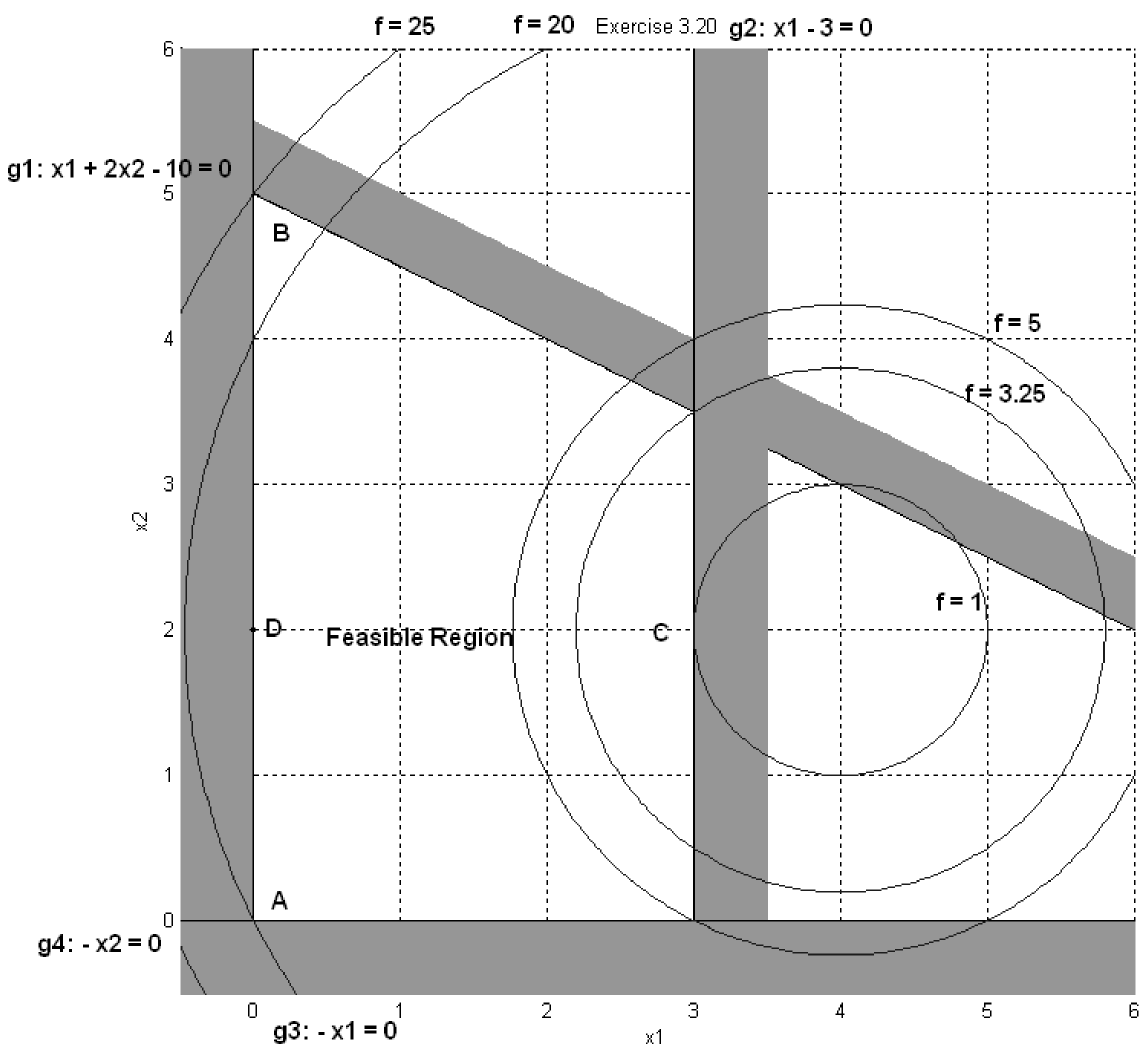
```
                    %Create a grid from -0.5 to 25 with an increment of 0.05 for the variables x and y
 [x,y]=meshgrid(-0.5:0.05:35, -0.5:0.05:25);
                    %Optimization and constraint functions
f=-x+2*y;
g1=-x.^2+6*x+3*y-27;
g2=y.^2-18*x+180;
g3=-x;
g4=-y;
cla reset
axis auto               %Minimum and maximum values are automatically defined for plot
xlabel('x'),ylabel('y') %Specifies labels for x- and y-axes
title('Exercise 3.19')  %Specifies graph title
hold on                 %retains the current plot and axes properties for all subsequent plots
                        %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:3];         %Specifies contour values
const1=contour(x,y,g1,cv1,'g');
cv1=[0 0.001];
const1=contour(x,y,g1,cv1,'k');
cv2=[0:0.01:10];
const2=contour(x,y,g2,cv2,'g');
cv2=[0 0.001];
const2=contour(x,y,g2,cv2,'k');
cv3=[0:0.01:0.5];
const3=contour(x,y,g3,cv3,'g');
cv3=[0 0.001];
const3=contour(x,y,g3,cv3,'k');
cv4=[0:0.01:0.5];
const4=contour(x,y,g4,cv4,'g');
cv4=[0 0.001];
const4=contour(x,y,g4,cv4,'k');
fv=[-7.5 1 8 15];       %Defines contours for the minimization function
fs=contour(x,y,f,fv,'b');
c=[28];
d=[18];
plot(c,d,'.k');         %Plots points c and d in black
grid
hold off                %Indicates end of this plotting sequence
                        %Subsequent plots will appear in separate windows
```

3.20

$f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 2)^2$
subect to $10 \geq x_1 + 2x_2$
$0 \leq x_1 \leq 3$
$x_2 \geq 0$

Solution

$f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 2)^2$
$g1: x_1 + 2x_2 - 10 \leq 0$
$g2: x_1 - 3 \leq 0$
$g3: -\ x_1 \leq 0$
$g4: -x_2\ \leq 0$
Local maximum at A (0, 0) with $f^* = 20$. Active constraint: g3 and g4
Local, global maximum at B(0, 5) with $f^* = 25$. Active constraint: g1 and g3
Local, global minimum at C(3, 2) with $f^* = 1$. Active constraint: g2



MATLAB Code

```
                    %Create a grid from -0.5 to 6 with an increment of 0.05 for the variables x1 and x2
 [x1,x2]=meshgrid(-0.5:0.05:6, -0.5:0.05:6);
                    %Optimization and constraint functions
f=(x1-4).^2+(x2-2).^2;
g1=x1+2*x2-10;
g2=x1-3;
g3=-x1;
g4=-x2;
cla reset
axis ([-0.5 6 -0.5 6])          %Minimum and maximum values are defined for plot
xlabel('x1'),ylabel('x2')       %Specifies labels for x- and y-axes
title('Exercise 3.20')          %Specifies graph title
hold on                         %retains the current plot and axes properties for all subsequent plots
                                %Use the "contour" command to plot constraint/minimization functions
cv1=[0:0.01:1];                 %Specifies contour values
const1=contour(x1,x2,g1,cv1,'g');
cv1=[0 0.001];
const1=contour(x1,x2,g1,cv1,'k');
cv2=[0:0.01:0.5];
const2=contour(x1,x2,g2,cv2,'g');
cv2=[0 0.001];
const2=contour(x1,x2,g2,cv2,'k');
cv3=[0:0.01:0.5];
const3=contour(x1,x2,g3,cv3,'g');
cv3=[0 0.001];
const3=contour(x1,x2,g3,cv3,'k');
cv4=[0:0.01:1];
const4=contour(x1,x2,g4,cv4,'g');
cv4=[0 0.001];
const4=contour(x1,x2,g4,cv4,'k');
fv=[1 3.25 5 20 25];            %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'b');
c=[0];
d=[2];
plot(c,d,'.k');                 %Plots points c and d in black
grid
hold off                        %Indicates end of this plotting sequence
                                %Subsequent plots will appear in separate windows
```