3.21

Solve the rectangular beam problem of Exercise 2.17 graphically for the following data: M = 80 kN·m, V=150 kN, σ_a=8 MPa, and τ_a=3 MPa.

Solution

Rewrite the formulation of Exercise 2.17, and substitute the following data:

M = 80 kN.m = 8.0×10^6 N.cm; V = 150 kN = 1.5×10^5 N; σ_a = 8 MPa = 800 N/cm^2; τ_a = 300 N/cm^2

Using units of Newtons and centimeters, we have: minimize $f = bd$; subject to

$$g_1 : \frac{6M}{bd^2} - \sigma_a \le 0; \quad g_1 = \frac{6(8.0 \text{ x } 10^6)}{bd^2} - 800 \le 0;$$

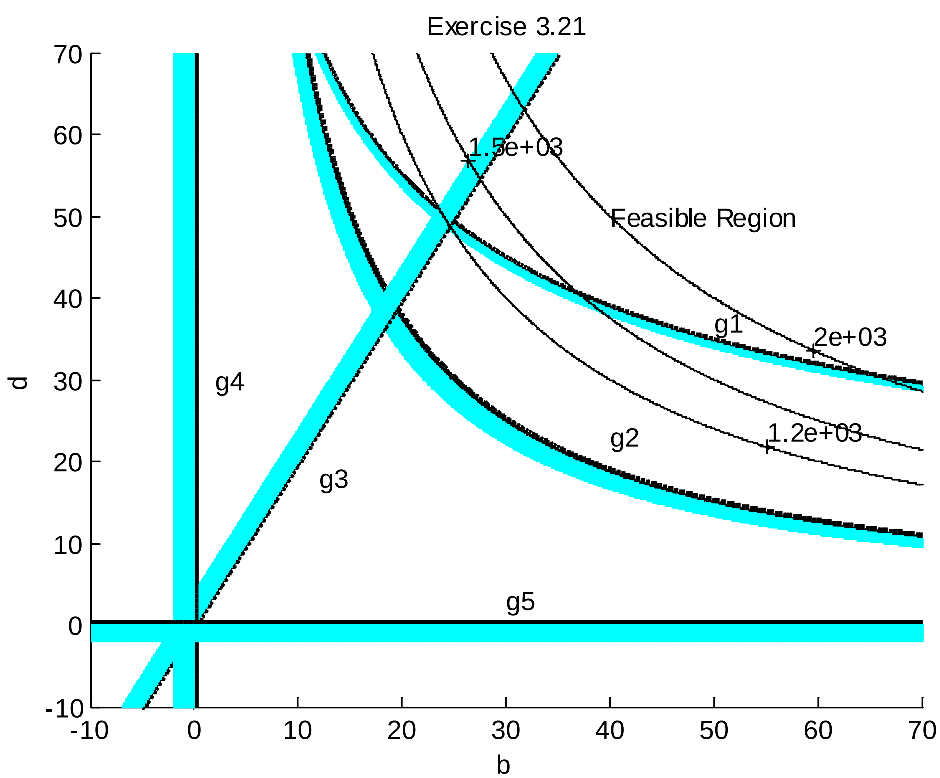
$$g_2 : \frac{3V}{2bd} - \tau_a \le 0; \quad g_2 = \frac{3\left(1.5\times10^5\right)}{2bd} - 300 \le 0;$$

$$g_3 = d - 2b \le 0;$$

$$g_4 = -b \le 0;$$

$$g_5 = -d \le 0$$

Optimum solution: $b^* \doteq 24.66$ cm, $d^* \doteq 49.32$ cm, $f^* \doteq 1216$ cm^2; g_1 (bending stress) and g_3 (depth- ratio) constraints are active.



MATLAB Code

```
%Exercise 3.21
%Create a grid from -10 to 70 with an increment of 0.1 for the variables x1 and x2
[b,d]=meshgrid(-10:0.1:70.0, -10:0.1:70.0);
%Enter functions for the minimization problem
f=b.*d;
g1=(48*10.^6)./(b.*(d.^2))-800;
g2=(2.25*10^5)./(b.*d)-300;
g3=0.5.*d-b;
g4=-b;
g5=-d;
cla reset
axis auto
%Minimum and maximum values for axes are determined automatically
xlabel('b'),ylabel('d')   %Specifies labels for x- and y-axes
title('Exercise 3.21')
hold on     %retains the current plot and axes properties for all subsequent plots
%Use the "contour" command to plot constraint and minimization functions
cv1=[0 0]; %Specifies two contour values
const1=contour(b,d,g1,cv1,'k','LineWidth',4);
text(50,37,'g1')
cv11=[0.01:0.05:2];
cv22=[5:2:40];
const1=contour(b,d,g1,cv22,'c');
const2=contour(b,d,g2,cv1,'k','Linewidth',4);
const2=contour(b,d,g2,cv22,'c');
text(40,23,'g2')
const3=contour(b,d,g3,cv1,'k','Linewidth',4);
const3=contour(b,d,g3,cv11,'c');
text(12,18,'g3')
const4=contour(b,d,g4,cv1,'k','Linewidth',4);
const4=contour(b,d,g4,cv11,'c');
text(30,3,'g5')
const5=contour(b,d,g5,cv1,'k','Linewidth',4);
const5=contour(b,d,g5,cv11,'c');
text(2,30,'g4')
text(40,50,'Feasible Region')
fv=[1200 1500 2000];    %Defines contours for the minimization function
fs=contour(b,d,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)         %Automatically puts the contour value on the graph
hold off    %Indicates end of this plotting sequence
            %Subsequent plots will appear in separate windows
```

3.22

Solve the cantilever beam problem of Exercise 2.23 graphically for the following data:
P = 10 kN; l = 5.0 m; modulus of elasticity, E = 210 GPa; allowable bending stress σ_a= 250 MPa; allowable shear stress, τ_a= 90 MPa; mass density, ρ = 7850 kg/m^3 ; $R_0 \le 20.0$ cm; $R_i \le 20.0$ cm.

Solution

Using kg, N and cm as units

Given Data: (this data will change if different units are used)

P = 10 kN = 10^4 N

L = 5 m = 500 cm

σ_a = 250 MPa = 2.5×10^4 N/cm^2;

τ_a = 90 MPa = 9000 N/cm^2

ρ = 7850 kg/m^3 = 7.85×10^{-3} kg/cm^3;

Cross-sectional area of hollow tubes: $A = \pi(R_o^2 - R_i^2)$

Moment of inertia of a hollow tube is $I = \pi\left(R_o^4 - R_i^4\right)/4$

Maximum bending stress:

$$\sigma = \frac{PL}{I} R_0$$

Maximum shearing stress:

$$\tau = \frac{VQ}{Ib};\ \ V = P;\quad Q = \frac{2}{3}(R_o^3 - R_i^3);\quad b = 2(R_o - R_i)$$

Substituting various quantities and simplifying the expression, we get

$$\tau = \frac{P}{3I}(R_o^2 + R_0 R_i + R_i^2)$$

In addition, it must be ensured that $R_o > R_i$ which can be imposed as a constraint on the wall thickness as $t \ge t_{min}$ with t_{min} as, say 0.1 cm.

Thickness: $t = R_o - R_i$

Referring to Exercise 2.23 and the given data, the problem is formulated in terms of the design variables only as follows:

$$f = (7.85\times10^{-3})(500)\ \pi\left(R_0^2 - R_i^2\right) = 12.331\left(R_0^2 - R_i^2\right)$$

$$g_1 = \frac{4PlR_o}{\pi\left(R_o^4 - R_i^4\right)} \le \sigma_a;\ \text{or}$$

$$g_1 = \frac{R_o\left(4.0\times10^4\right)(500)}{\pi\left(R_o^4 - R_i^4\right)} \le \left(2.5\times10^4\right);\ \text{or}$$

$$g_1 = \frac{R_o 6.3662\times10^6}{\left(R_o^4 - R_i^4\right)} - 2.5\times10^4 \le 0$$

$$g_2 = \frac{4P\left(R_o^2 + R_o R_i + R_i^2\right)}{3\pi\left(R_o^4 - R_i^4\right)} \le \tau_a; \text{ or}$$

$$g_2 = \frac{\left(4.0\times10^4\right)\left(R_o^2 + R_o R_i + R_i^2\right)}{3\pi\left(R_o^4 - R_i^4\right)} \le 9000; \text{ or}$$

$$g_2 = \frac{4244.13\left(R_o^2 + R_o R_i + R_i^2\right)}{\left(R_o^4 - R_i^4\right)} - 9000 \le 0;$$

$g_3 = R_o - 20 \le 0;$

$g_4 = R_i - 20 \le 0;$

$g_5 = -R_o \le 0;$

$g_6 = -R_i \le 0$

FORMULATION 2: Using Intermediate Variables

Step 4: Optimization Criterion

Optimization criterion is to minimize mass of hollow tube, and the cost function is defined as

$f = \rho AL$

Step 5: Formulation of Constraints

g_1 : bending stress should be smaller than the allowable bending stress; $\sigma \le \sigma_a$

$$g_1 = \sigma - \sigma_a \le 0$$

g_2 : shear stress smaller than allowable shear stress: $\tau \le \tau_a$

$$g_2 = \tau - \tau_a \le 0$$

$g_3 = R_o - 20 \le 0$

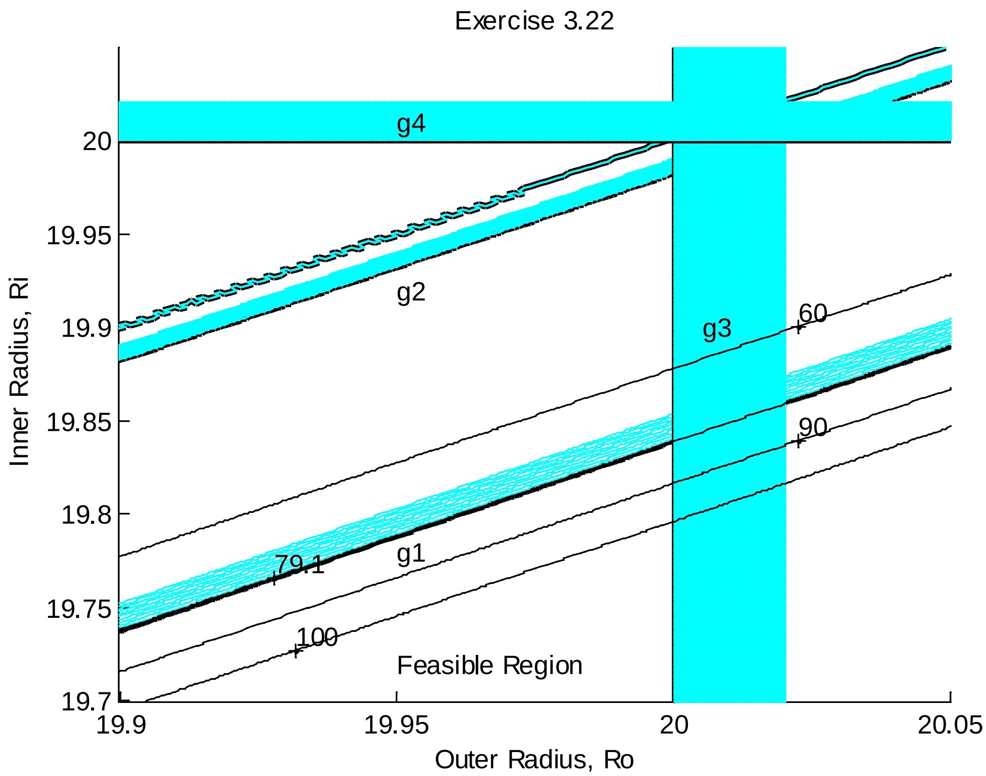
$g_4 = R_i - 20 \le 0$

$g_5 = -R_o \le 0$

$g_6 = -R_i \le 0$

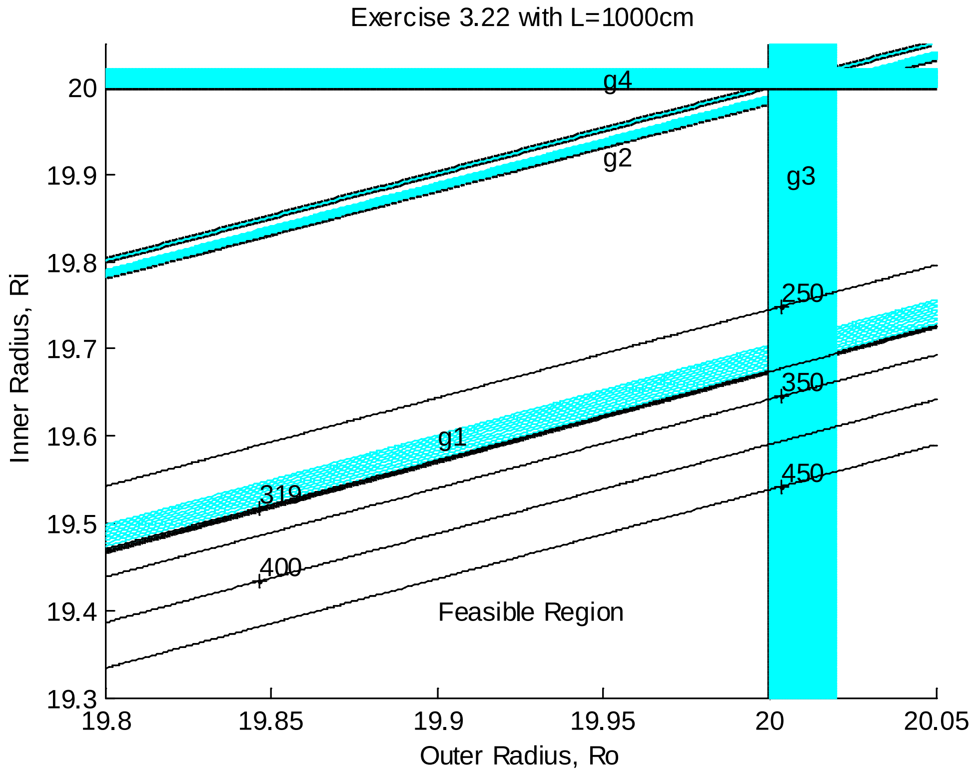
$$g_7 = t_{min} - t \le 0$$

Optimum solution: $R_o^* = 20$ cm, $R_i^* \doteq 19.84$ cm, $f^* \doteq 79.1$ kg, g_1(bending stress) and g_3(max. outer radius) constraints are active.



With L = 1000cm

Optimum solution: $R_o^* = 20$ cm, $R_i^* \doteq 19.674$ cm, $f^* \doteq 319.19$ kg, g_1 (bending stress) and g_3 (max. outer radius) constraints are active.



MATLAB Code

```
%Exercise 3.22
%Create a grid
[Ro,Ri]=meshgrid(19.9:0.001:20.05, 19.70:0.001:20.05);
%Enter functions for the minimization problem; use Newton and cm as units
P=10000;  L=500;  sigma_a=25000;  tau_a=9000;  ro=7.85/1000;
A=pi.*(Ro.*Ro-Ri.*Ri)
I=0.25.*pi.*(Ro.^4-Ri.^4)
sigma=P.*L.*Ro./I
tau=P.*(Ro.*Ro+Ro.*Ri+Ri.*Ri)./(3.*I)
t=Ro-Ri
f=ro.*A.*L
g1=sigma./sigma_a - 1
g2= tau./tau_a - 1
%f=12.331*((Ro.^2)-(Ri.^2));
%g1=Ro*(6.3662*10.^6)-(2.5*10^4)*((Ro.^4)-(Ri.^4));
%g2=(4244.13)*(Ro.*Ro+Ri.*Ro+Ri.*Ri)-9000*(Ro.^4).*(Ri.^4);
g3=Ro./20 - 1;
g4=Ri./20 - 1;
g5=-Ro;
g6=-Ri;
cla reset
axis auto
%Minimum and maximum values for axes are determined automatically
```

```
title('Exercise 3.22')
xlabel('Outer Radius, Ro'),ylabel('Inner Radius, Ri')   %Specifies labels for x-
and y-axes
hold on     %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];  %Specifies two contour values
            %Use the "contour" command to plot constraint and minimization
functions
const1=contour(Ro,Ri,g1,cv1,'k','LineWidth',2);
cv11=[0.0:0.00001:0.001];
cv22=[0.01:0.01:0.1];
cv33=[0.01:0.01:0.8];
const1=contour(Ro,Ri,g1,cv22,'c');
const2=contour(Ro,Ri,g2,cv1,'k','Linewidth',3);
const2=contour(Ro,Ri,g2,cv33,'c');
const3=contour(Ro,Ri,g3,cv1,'k','Linewidth',2);
const3=contour(Ro,Ri,g3,cv11,'c');
const4=contour(Ro,Ri,g4,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g4,cv11,'c');
const4=contour(Ro,Ri,g5,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g5,cv11,'c');
const5=contour(Ro,Ri,g6,cv1,'k','Linewidth',3);
const5=contour(Ro,Ri,g6,cv11,'c');
%Label constraints
text(19.95,19.78,'g1')
text(19.95,19.92,'g2')
text(20.005,19.9,'g3')
text(19.95,20.01,'g4')
text(6,0.5,'g5')
text(19.95,19.72,'Feasible Region')
fv=[60 79.1 90 100];    %Defines contours for the minimization function
fs=contour(Ro,Ri,f,fv,'k--');   %'k' specifies black dashed lines for function
contours
clabel(fs)         %Automatically puts the contour value on the graph
hold off    %Indicates end of this plotting sequence
            %Subsequent plots will appear in separate windows
```

```
%Exercise 3.22; L=10m
%Create a grid
[Ro,Ri]=meshgrid(19.8:0.001:20.05, 19.30:0.001:20.05);
%Enter functions for the minimization problem; use Newton and cm as units
P=10000;  L=1000;  sigma_a=25000;  tau_a=9000;  ro=7.85/1000;
A=pi.*(Ro.*Ro-Ri.*Ri)
I=0.25.*pi.*(Ro.^4-Ri.^4)
sigma=P.*L.*Ro./I
tau=P.*(Ro.*Ro+Ro.*Ri+Ri.*Ri)./(3.*I)
t=Ro-Ri
f=ro.*A.*L
g1=sigma./sigma_a - 1
g2= tau./tau_a - 1
%f=12.331*((Ro.^2)-(Ri.^2));
%g1=Ro*(6.3662*10.^6)-(2.5*10^4)*((Ro.^4)-(Ri.^4));
%g2=(4244.13)*(Ro.*Ro+Ri.*Ro+Ri.*Ri)-9000*(Ro.^4).*(Ri.^4);
g3=Ro./20 - 1;
g4=Ri./20 - 1;
g5=-Ro;
```

```
g6=-Ri;
cla reset
axis auto
%Minimum and maximum values for axes are determined automatically
title('Exercise 3.22 with L=1000cm')
xlabel('Outer Radius, Ro'),ylabel('Inner Radius, Ri')   %Specifies labels for x-
and y-axes
hold on     %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];  %Specifies two contour values
            %Use the "contour" command to plot constraint and minimization
functions
const1=contour(Ro,Ri,g1,cv1,'k','LineWidth',2);
cv11=[0.0:0.00001:0.001];
cv22=[0.01:0.01:0.1];
cv33=[0.01:0.01:0.8];
const1=contour(Ro,Ri,g1,cv22,'c');
const2=contour(Ro,Ri,g2,cv1,'k','Linewidth',3);
const2=contour(Ro,Ri,g2,cv33,'c');
const3=contour(Ro,Ri,g3,cv1,'k','Linewidth',2);
const3=contour(Ro,Ri,g3,cv11,'c');
const4=contour(Ro,Ri,g4,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g4,cv11,'c');
const4=contour(Ro,Ri,g5,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g5,cv11,'c');
const5=contour(Ro,Ri,g6,cv1,'k','Linewidth',3);
const5=contour(Ro,Ri,g6,cv11,'c');
%Label constraints
text(19.9,19.6,'g1')
text(19.95,19.92,'g2')
text(20.005,19.9,'g3')
text(19.95,20.01,'g4')
text(6,0.5,'g5')
text(19.9,19.4,'Feasible Region')
fv=[250 319.185 350 400 450];    %Defines contours for the minimization function
fs=contour(Ro,Ri,f,fv,'k');   %'k' specifies black dashed lines for function
contours
clabel(fs)         %Automatically puts the contour value on the graph
hold off    %Indicates end of this plotting sequence
            %Subsequent plots will appear in separate windows
```

3.23

For the minimum mass tubular column design problem formulated in Section 2.7, consider the following data: P = 50 kN; l = 5.0 m; modulus of elasticity, E= 210 GPa; allowable stress, σ_a= 250 MPa; mass density, ρ = 7850 kg/m^3.

Treating mean radius R and wall thickness t as design variables, solve the design problem graphically imposing an additional constraint $R/t \le 50$. This constraint is needed to avoid local crippling of the column. Also impose the member size constraints as

$$0.01 \le R \le 1.0 \text{ m}; \qquad 5 \le t \le 200 \text{ mm}$$

Solution

Referring to Formulation 1 of Section 2.7.1 and imposing an additional constraint $R/t \le 50$, we summarize the formulation as follows:

$f = 2\rho l \pi R t$, kg;

g_1: $\dfrac{P}{2\pi Rt} \le \sigma_a$;

g_2: $P \le \dfrac{\pi^3 E R^3 t}{4l^2}$;

g_3: $\dfrac{R}{t} \le 50$;

g_4: $R \ge R_{\min}$;

g_5: $R \le R_{\max}$;

g_6: $t \ge t_{\min}$;

g_7: $t \le t_{\max}$

Use Newtons and millimeters as units.

P = 50 kN = 5×10^4 N; l = 5.0 m = 5000 mm; E = 210 GPa = 2.1×10^5 N/mm^2;

σ_a = 250 MPa = 250 N/mm^2; ρ = 7850 kg/m^3 = 7.85×10^{-6} kg/mm^3; $10 \le R \le 1000$ mm;

$5 \le t \le 200$ mm

Substitute the given data into f, g_1 and g_2:

$f = 2(7.85\times10^{-6})(5000)\pi Rt = 0.2466\, Rt$, kg;

g_1: $\dfrac{50000}{2Rt} \le 250$ or

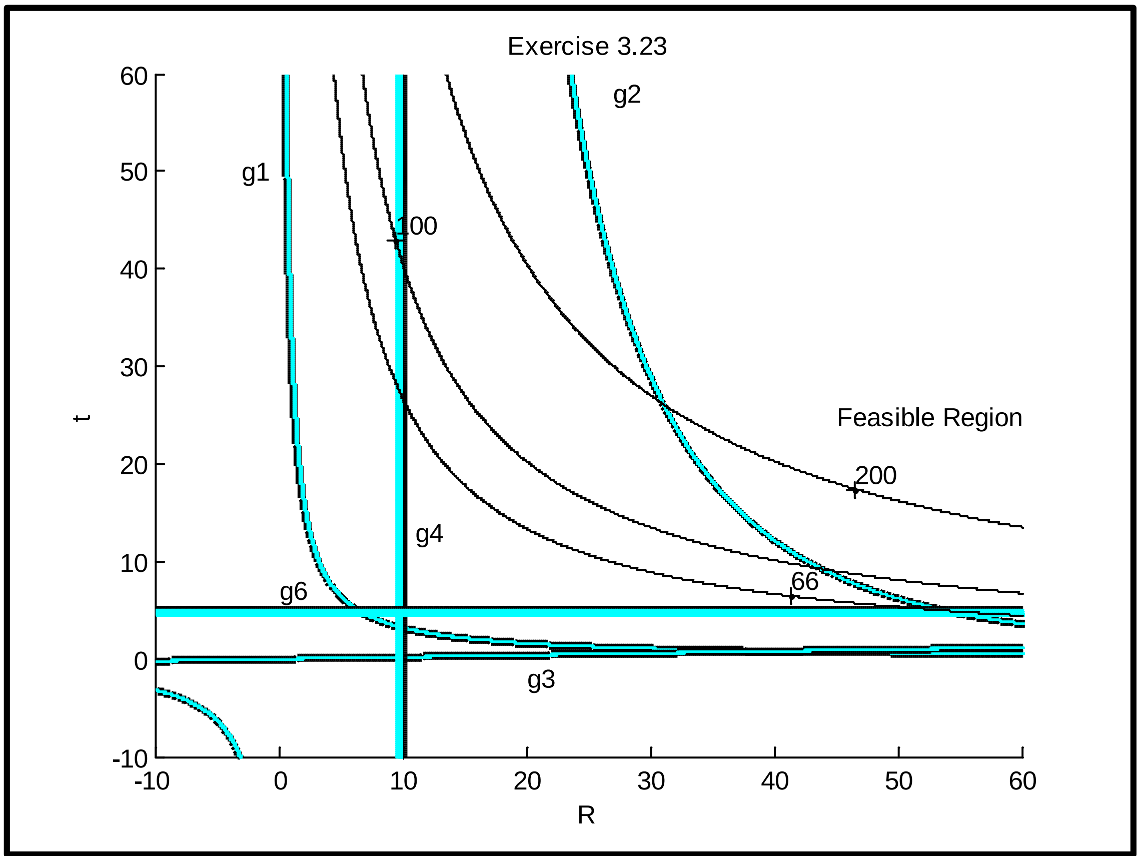
$g_1 = \dfrac{7957.7}{Rt} - 250 \le 0$

g_2: $(5.0\times10^4) \le \pi^3\left(2.1\times10^5\right)R^3\, t/4\,(5000)^2$, or

$g_2 = \left(5\times10^4\right) - 0.06511R^3 t \le 0$

$$g_3 = \frac{R}{t} - 50 \le 0$$
$$g_4 = 10 - R \le 0$$
$$g_5 = R - 1000 \le 0$$
$$g_6 = 5 - t \le 0$$
$$g_7 = t - 200 \le 0$$

Optimum solution: $R^* \doteq 53.6$ mm, $t^* = 5.0$ mm, $f^* \doteq 66$ kg; g_2 (buckling constraint) and g_6 (minimum thickness constraint) are active.



MATLAB Code

```
 [R,t]=meshgrid(-10:0.1:60, -10:0.1:60);
                         %Enter functions for the minimization problem
f=0.2466*R.*t;
g1=7957.7-250*R.*t;
g2=5*10^4-0.06511*(R.^3).*t;
g3=R-50*t;
g4=10-R;
g5=R-1000;
g6=5-t;
g7=t-200;
cla reset
axis auto               %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('t') %Specifies labels for x- and y-axes
hold on                 %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,t,g1,cv1,'k','LineWidth',3);
text(-3,50,'g1')
cv11=[0.01:0.01:0.5];
const1=contour(R,t,g1,cv11,'c');
const2=contour(R,t,g2,cv1,'k','Linewidth',3);
const2=contour(R,t,g2,cv11,'c');
text(27,58,'g2')
const3=contour(R,t,g3,cv1,'k','Linewidth',3);
const3=contour(R,t,g3,cv11,'c');
text(20,-2,'g3')
const4=contour(R,t,g4,cv1,'k','Linewidth',3);
const4=contour(R,t,g4,cv11,'c');
text(11,13,'g4')
const5=contour(R,t,g5,cv1,'k','Linewidth',3);
const5=contour(R,t,g5,cv11,'c');
text(7,100,'g5')
const6=contour(R,t,g6,cv1,'k','Linewidth',3);
const6=contour(R,t,g6,cv11,'c');
text(0,7,'g6')
const7=contour(R,t,g7,cv1,'k','Linewidth',3);
const7=contour(R,t,g7,cv11,'c');
text(6,200,'g7')
text(45,25,'Feasible Region')
fv=[66 100 200];        %Defines contours for the minimization function
fs=contour(R,t,f,fv,'k'); %'k' specifies black dashed lines for function contours
clabel(fs)              %Automatically puts the contour value on the graph
hold off                %Indicates end of this plotting sequence
```

3.24

For Exercise 3.23, treat outer radius R_0 and inner radius R_i as design variables, and solve the design problem graphically. Impose the same constraints as in Exercise 3.23.

Solution

Referring to Formulation 2 of Section 2.7.2 and Exercise 3.23, the problem is formulated as

$$f = \pi\rho l\left(R_0^2 - R_i^2\right);$$

$$g_1: \frac{P}{\pi\left(R_o^2 - R_i^2\right)} \le \sigma_a ;$$

$$g_2: P \le \frac{\pi^3 E\left(R_o^4 - R_i^4\right)}{16 l^2};$$

$$g_3: \frac{\left(R_o + R_i\right)}{2\left(R_o - R_i\right)} - 50 \le 0;$$

$$5 \le \left(R_o - R_i\right) \le 200;$$

$$10 \le 0.5\left(R_o - R_i\right) \le 1000 \text{ mm}$$

Use Newtons and millimeters as units, and the data given in Exercise 3.23:

$$f = \pi\left(7.85\times10^{-6}\right)\left(5000\right)\left(R_o^2 - R_i^2\right) = 0.1233\left(R_o^2 - R_i^2\right)$$

$$g_1 = \frac{15915.5}{\left(R_o^2 - R_i^2\right)} - 250 \le 0 ;$$

$$g_2: 5.0\times10^4 \le \frac{\pi^3\left(2.1\times10^5\right)\left(R_o^4 - R_i^4\right)}{16\left(5000\right)^2}; \quad \text{or} \qquad)$$

$$g_2 = \left(5\times10^4\right) - 0.016278\left(R_o^4 - R_i^4\right) \le 0$$

$$g_3 = \frac{\left(R_o + R_i\right)}{2\left(R_o - R_i\right)} - 50 \le 0;$$

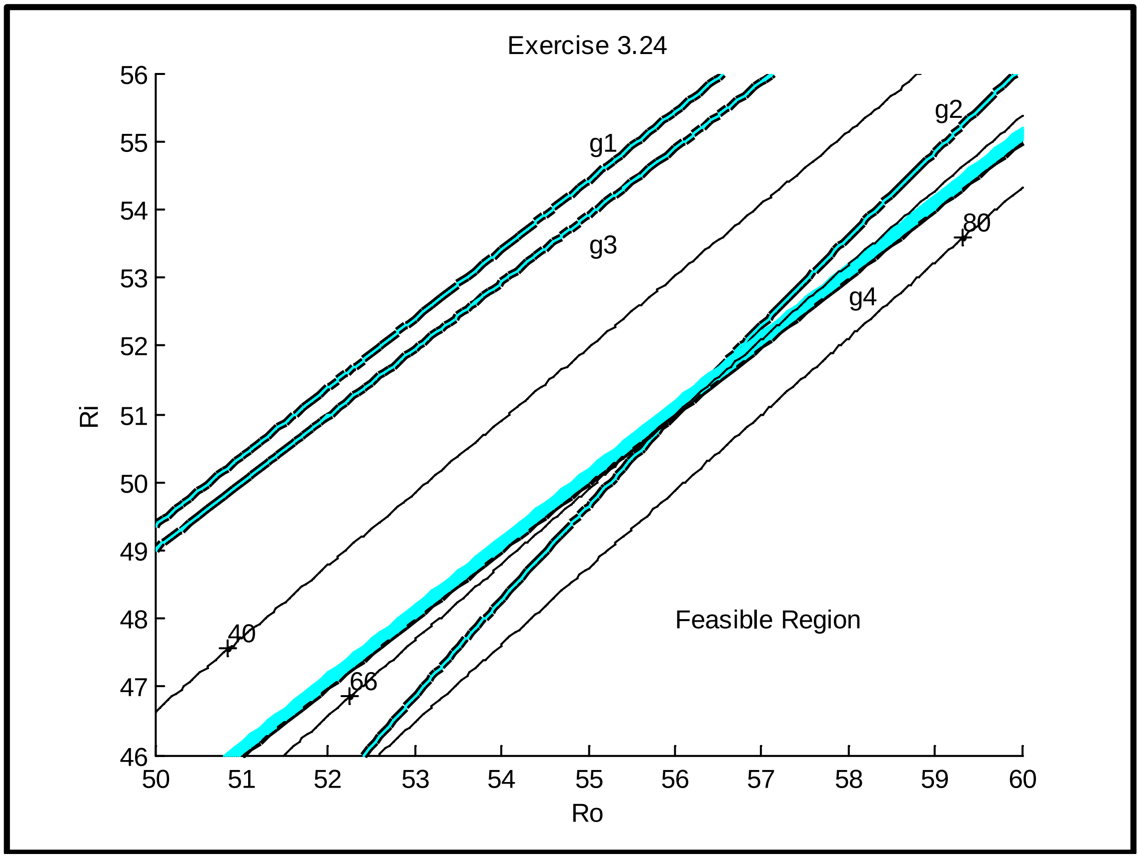
$$g_4 = -R_o + R_i + 5 \le 0;$$

$$g_5 = R_o - R_i - 200 \le 0$$

$$g_6 = -0.5\left(R_o + R_i\right) - 1000 \le 0 ;$$

$$g_7 = 0.5\left(R_o + R_i\right) - 1000 \le 0$$

Optimum solution: $R_0^* \doteq 56$ mm, $R_i^* \doteq 51$ mm, $f^* = 66$ kg; g_2 (buckling constraint) and g_4 (minimum thickness) are active.



MATLAB Code

```
 [Ro,Ri]=meshgrid(50:0.1:60, 46:0.1:56);
                              %Enter functions for the minimization problem
f=0.1233*(Ro.^2-Ri.^2);
g1=15915.5-250*(Ro.^2-Ri.^2);
g2=5*10^4-0.016278*(Ro.^4-Ri.^4);
g3=(Ro+Ri)-100*(Ro-Ri);
g4=5-Ro+Ri;
g5=Ro-Ri-200;
g6=-0.5*(Ro+Ri)-1000;
g7=0.5*(Ro+Ri)-1000;
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
xlabel('Ro'),ylabel('Ri')     %Specifies labels for x- and y-axes
hold on                       %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(Ro,Ri,g1,cv1,'k','LineWidth',3);
text(55,55,'g1')
cv11=[0.01:0.01:0.2];
const1=contour(Ro,Ri,g1,cv11,'c');
const2=contour(Ro,Ri,g2,cv1,'k','Linewidth',3);
const2=contour(Ro,Ri,g2,cv11,'c');
text(59,55.5,'g2')
const3=contour(Ro,Ri,g3,cv1,'k','Linewidth',3);
const3=contour(Ro,Ri,g3,cv11,'c');
text(55,53.5,'g3')
const4=contour(Ro,Ri,g4,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g4,cv11,'c');
text(58,52.75,'g4')
const5=contour(Ro,Ri,g5,cv1,'k','Linewidth',3);
const5=contour(Ro,Ri,g5,cv11,'c');
text(7,100,'g5')
const6=contour(Ro,Ri,g6,cv1,'k','Linewidth',3);
const6=contour(Ro,Ri,g6,cv11,'c');
text(0,7,'g6')
const7=contour(Ro,Ri,g7,cv1,'k','Linewidth',3);
const7=contour(Ro,Ri,g7,cv11,'c');
text(6,200,'g7')
text(56,48,'Feasible Region')
fv=[66 40 80];                %Defines contours for the minimization function
fs=contour(Ro,Ri,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
```

3.25

Formulate the minimum mass column design problem of Section 2.7 using a hollow square cross section with outside dimension *w* and thickness *t* as design variables. Solve the problem graphically using the constraints and the data given in Exercise 3.23.

Solution

For a hollow square cross-section with outside dimension *w* and thickness *t*, the cross-sectional area and moment of inertia are

$$A = w^2 - (w-2t)^2 = 4wt - 4t^2; \quad I = \frac{w^4}{12} - \frac{(w-2t)^4}{12} = \frac{w^4 - (w-2t)^4}{12}$$

Referring to Section 2.7 and Exercise 3.23, the problem is formulated as

$f = \rho l\left(4wt - 4t^2\right)$

g_1: $P/\left(4wt-4t^2\right) \le \sigma_a$;

g_2: $P \le \pi^2 E\left[w^4 - (w-2t)^4\right]/48l^2$;

g_3: $(w-t)/t \le 100$;

$20 \le (w-t) \le 2000$ mm;

$5 \le t \le 200$ mm

Use Newtons and millimeters as units, and the data given in Exercise 3.23:

$f = (7.85\times10^{-6})(5000)(4wt-4t^2) = 0.157(wt - t^2)$

g_1: $(5.0\times10^4)/4(wt-t^2) \le 250$; or

$g_1 = 12500/\left(wt-t^2\right) - 250 \le 0$

g_2: $(5.0\times10^4) \le \dfrac{\pi^2\left(2.1\times10^5\right)\left[w^4-(w-2t)^4\right]}{48(5000)^2}$; or

$g_2 = (5.0\times10^4) - \left(1.7271\times10^{-3}\right)\left[w^4-(w-2t)^4\right] \le 0$

$g_3 = w/t - 151 \le 0$

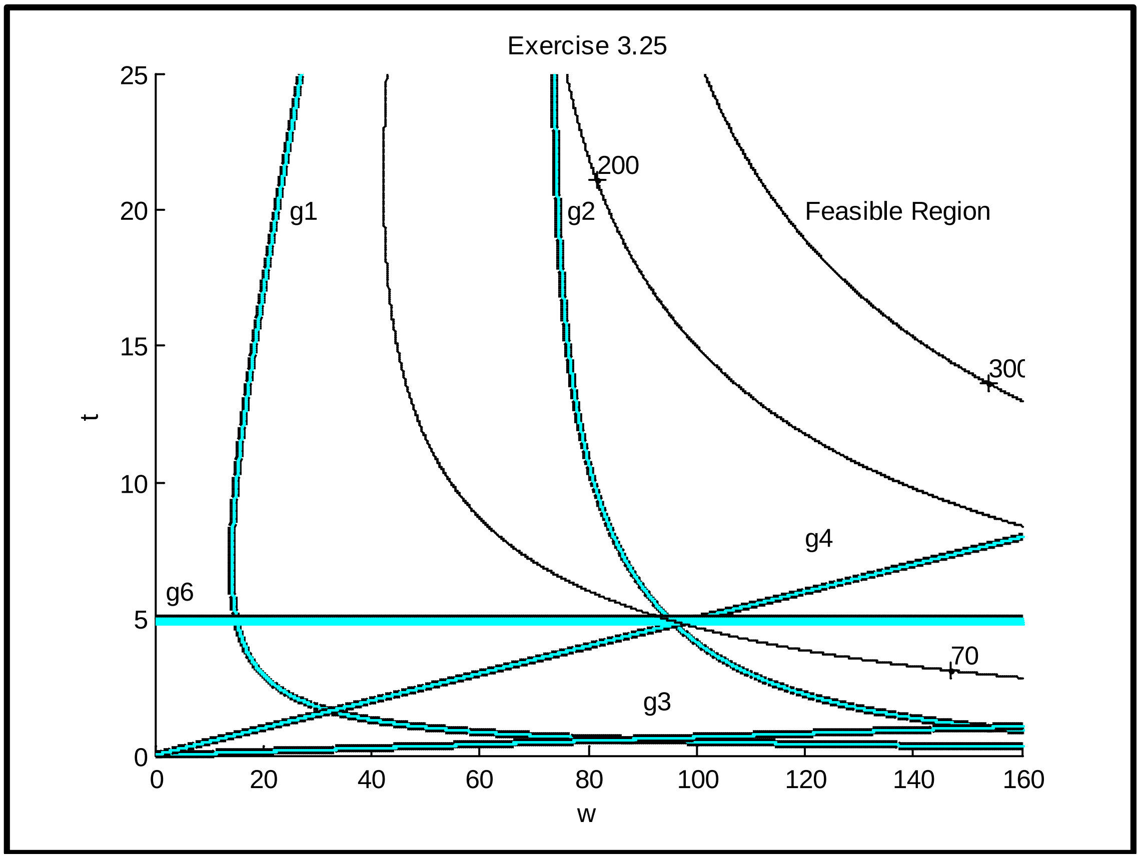
$g_4 = 20 - (w/t) \le 0$

$g_5 = (w-t) - 2000 \le 0$

$g_6 = 5 - t \le 0$

$g_7 = t - 200 \le 0$

Optimum solution: $w^* \doteq 93$ mm, $t^* = 5$ mm, $f^* \doteq 70$ kg; g_2 (buckling constraint) and g_6 (minimum thickness) are active.



MATLAB Code

```
 [w,t]=meshgrid(0:0.1:160, 0:0.1:25);
                    %Enter functions for the minimization problem
f=0.157*(w.*t-t.^2);
g1=12500-250*(w.*t-t.^2);
g2=5*10^4-(1.7271*10^-3)*(w.^4-(w-2*t).^4);
g3=w-151*t;
g4=20*t-w;
g5=w-t-2000;
g6=5-t;
g7=t-200;
cla reset
axis auto               %Minimum and maximum values for axes are determined automatically
xlabel('w'),ylabel('t') %Specifies labels for x- and y-axes
hold on                 %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(w,t,g1,cv1,'k','LineWidth',3);
text(25,20,'g1')
cv11=[0.01:0.01:0.2];
const1=contour(w,t,g1,cv11,'c');
const2=contour(w,t,g2,cv1,'k','Linewidth',3);
const2=contour(w,t,g2,cv11,'c');
text(76,20,'g2')
const3=contour(w,t,g3,cv1,'k','Linewidth',3);
const3=contour(w,t,g3,cv11,'c');
text(90,2,'g3')
const4=contour(w,t,g4,cv1,'k','Linewidth',3);
const4=contour(w,t,g4,cv11,'c');
text(120,8,'g4')
const5=contour(w,t,g5,cv1,'k','Linewidth',3);
const5=contour(w,t,g5,cv11,'c');
text(7,100,'g5')
const6=contour(w,t,g6,cv1,'k','Linewidth',3);
const6=contour(w,t,g6,cv11,'c');
text(2,6,'g6')
const7=contour(w,t,g7,cv1,'k','Linewidth',3);
const7=contour(w,t,g7,cv11,'c');
text(6,200,'g7')
text(120,20,'Feasible Region')
fv=[70 200 300];              %Defines contours for the minimization function
fs=contour(w,t,f,fv,'k');     %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
```

3.26

Consider the symmetric (member are identical) case of the two-bar truss problem discussed in Section 2.5 with the following data: W = 10kN ; $\theta = 30^\circ$; height h = 1.0m; span s = 1.5m; allowable stress σ_a = 250MPa; modulus of elasticity, E = 210GPa.

Formulate the minimum mass design problem with constraints on member stresses and bounds on design variables. Solve the problem graphically using circular tubes as members.

Solution

Design variables: D_0 = outer diameter of the tube; D_i = inner diameter of the tube

Cost function: $f = 2\rho\left[\pi\left(D_o^2 - D_i^2\right)/4\right]l = \rho(\pi l/2)\left(D_o^2 - D_i^2\right)$

Constraints:

g1: $2Wl\,(\sin\theta/h + 2\cos\theta/s)/[\pi\left(D_o^2 - D_i^2\right)] \le \sigma_a$

g2: $2Wl\,(\sin\theta/h - 2\cos\theta/s)/[\pi\left(D_o^2 - D_i^2\right)] \le \sigma_a$

g3: $2Wl\,(2\cos\theta/s - \sin\theta/h)/[\pi\left(D_o^2 - D_i^2\right)] \le \sigma_a$

g4: $D_0 \ge 0$;

g5: $D_i \ge 0$;

g6: $D_0 - D_i \ge 0$;

where W = 10 kN, θ = 30°, ρ = 7850 kg/m^3, h = 1.0 m, s = 1.5 m, σ_a = 250 MPa

$$l = \sqrt{h^2 + (s/2)^2} = \sqrt{1 + (0.75)^2} = 1.25$$

For $0° \le \theta \le 90°$; $\sin\theta \ge 0$, $\cos\theta \ge 0$;

$(\sin\theta/h + 2\cos\theta/s) \ge (\sin\theta/h - 2\cos\theta/s)$;

$(\sin\theta/h + 2\cos\theta/s) \ge (2\cos\theta/s - \sin\theta/h)$

We could drop g2 and g3, since they are redundant.

Use kN and cm as the units; substitute the constants into f and g1,

$$f = (\pi l/2)\left(D_o^2 - D_i^2\right) = 196.3\left(D_o^2 - D_i^2\right)$$

$$g_1 = 2(10)(125)\left[0.005 + \left(2\cos 30°/150\right)\right]\Big/\left[\pi\left(D_o^2 - D_i^2\right)\right] - 25 \le 0; \text{ or}$$

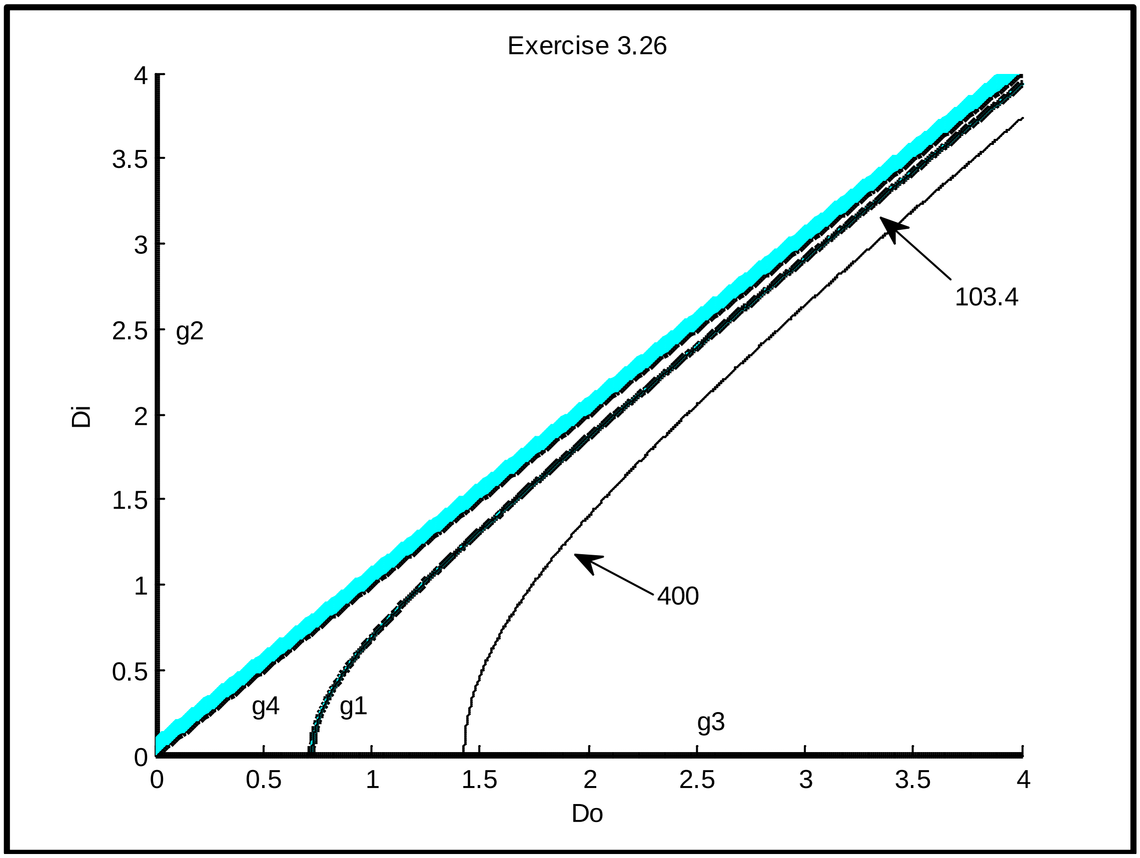
$$g_1 = 13.1675\Big/\left(D_o^2 - D_i^2\right) - 25 \le 0;$$

$$g_2:\ -D_o \le 0;$$

$$g_3:\ -D_i \le 0;$$

$$g_4:\ -D_o + D_i \le 0$$

Since cost function is identical to constraint g_1 (member stress constraint), there are infinite optimum points. The optimum cost is 103.4 cm^3. Thus, the minimum mass is $7.85 \times 10^{-3}(103.4) = 0.812$ kg.



MATLAB Code

```
[Do,Di]=meshgrid(0:0.01:4, 0:0.01:4);
                              %Enter functions for the minimization problem
f=196.3*(Do.^2-Di.^2);
g1=13.1675-25*(Do.^2-Di.^2);
g2=-Do;
g3=-Di;
g4=Di-Do;
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
xlabel('Do'),ylabel('Di')   %Specifies labels for x- and y-axes
hold on                       %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(Do,Di,g1,cv1,'k','LineWidth',3);
text(0.85,0.3,'g1')
cv11=[0.01:0.01:0.1];
const1=contour(Do,Di,g1,cv11,'c');
const2=contour(Do,Di,g2,cv1,'k','Linewidth',3);
const2=contour(Do,Di,g2,cv11,'c');
text(0.1,2.5,'g2')
const3=contour(Do,Di,g3,cv1,'k','Linewidth',3);
const3=contour(Do,Di,g3,cv11,'c');
text(2.5,0.2,'g3')
const4=contour(Do,Di,g4,cv1,'k','Linewidth',3);
const4=contour(Do,Di,g4,cv11,'c');
text(0.45,0.3,'g4')
text(120,20,'Feasible Region')
fv=[103.4 400];                 %Defines contours for the minimization function
fs=contour(Do,Di,f,fv,'k');     %'k' specifies black dashed lines for function contours
%clabel(fs)                     %Automatically puts the contour value on the graph
hold off                        %Indicates end of this plotting sequence
```

3.27

Formulate and solve the problem of Exercise 2.1 graphically.

Solution

$f = 0.6h + 0.001A$

$g_1 = 20000 - hA/3.5 \le 0$ (floor space);

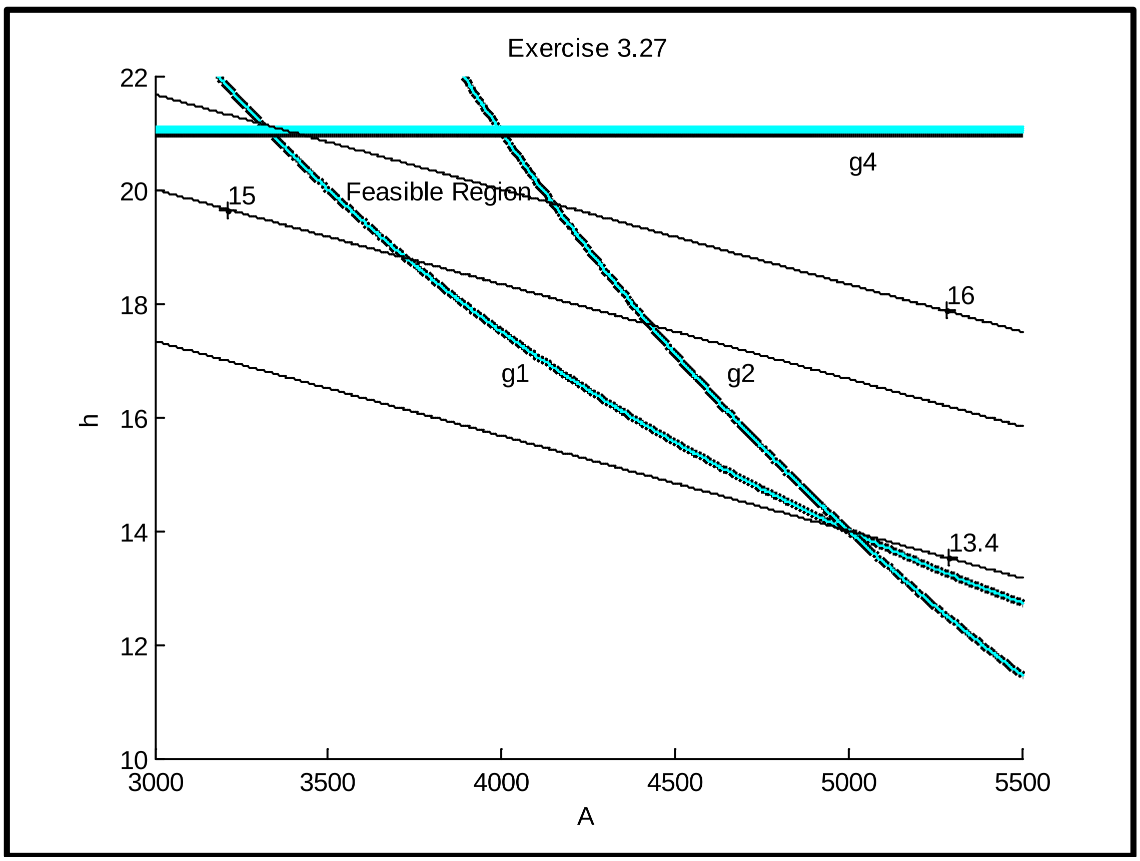
$g_2 = 0.25\,hA/3.5 + A - 10000 \le 0$ (parking space)

$g_3 = 3.5 - h \le 0;$

$g_4 = h - 21 \le 0;$

$g_5 = -A \le 0$

Optimum solution: $A^* = 5000$, $h^* = 14$, $f^* = 13.4$ million dollars; g_1 (floor space) and g_2 (parking space) constraints are active.



MATLAB Code

```
[A,h]=meshgrid(3000:5:5500, 10:0.1:22);
                                %Enter functions for the minimization problem
f=0.6*h+0.001*A;
g1=20000*3.5-h.*A;
g2=0.25*h.*A+A*3.5-10000*3.5;
g3=3.5-h;
g4=h-21;
g5=-A;
cla reset
axis auto                       %Minimum and maximum values for axes are determined automatically
xlabel('A'),ylabel('h')         %Specifies labels for x- and y-axes
hold on                         %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(A,h,g1,cv1,'k','LineWidth',3);
text(4000,16.8,'g1')
cv11=[0.01:0.01:0.1];
const1=contour(A,h,g1,cv11,'c');
const2=contour(A,h,g2,cv1,'k','Linewidth',3);
const2=contour(A,h,g2,cv11,'c');
text(4650,16.8,'g2')
const3=contour(A,h,g3,cv1,'k','Linewidth',3);
const3=contour(A,h,g3,cv11,'c');
text(2.5,0.2,'g3')
const4=contour(A,h,g4,cv1,'k','Linewidth',4);
const4=contour(A,h,g4,cv11,'c');
text(5000,20.5,'g4')
const5=contour(A,h,g5,cv1,'k','Linewidth',3);
const5=contour(A,h,g5,cv11,'c');
text(7,100,'g5')
text(3550,20,'Feasible Region')
fv=[13.4 15 16];                %Defines contours for the minimization function
fs=contour(A,h,f,fv,'k');       %'k' specifies black dashed lines for function contours
clabel(fs)                      %Automatically puts the contour value on the graph
hold off                        %Indicates end of this plotting sequence
```

3.28

In the design of a closed-end, thin-walled cylindrical pressure vessel shown in Fig. E3.28, the design objective is to select the mean radius R and wall thickness t to minimize the total mass. The vessel should contain at least 25.0 m^3 of gas at an internal pressure of 3.5 MPa. It is required that the circumferential stress in the pressure vessel not exceed 210 MPa and the circumferential strain not exceed (1.0E-03). The circumferential stress and strain are calculated from the equations

$$\sigma_c = \frac{PR}{t} \;, \varepsilon_c = \frac{PR(2-\nu)}{2Et}$$

where ρ =mass density (7850 kg/m^3), σ_c-circumferential stress (Pa), ε_c = circumferential strain, P = internal pressure (Pa), E = Young's modulus (210 GPa), and ν =Poisson's ratio(0.3).

(i) Formulate the optimum design problem and (ii) solve the problem graphically.

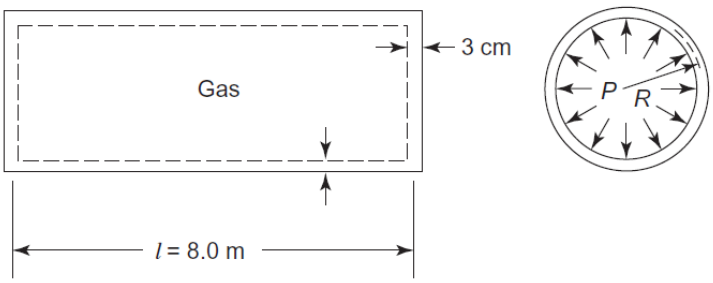


FIGURE E3.28 Cylindrical pressure vessel.

Solution

Design Variables: R = mean radius, m; t = wall thickness, m.

Use MPa and meters as units.

Objective Function: Total mass of the cylindrical pressure vessel;

$$f = \left[2\pi Rt(8.0-0.3) + 2\pi(R+0.5t)^2(0.03)\right]\rho = \left[50.077Rt + 0.1885(R+0.5t)^2\right]\rho$$

Constraints: Volume of the vessel is at least 25.0 m^3:

$g_1: \pi(R-0.5t)^2(7.97) \ge 25.0$, or $g_1 = 25 - 25.038(R-0.5t)^2 \le 0$

Circumferential stress should not exceed 210 MPa at a pressure of 3.5 MPa:

$$g_2 = 3.5R/t - 210 \le 0$$

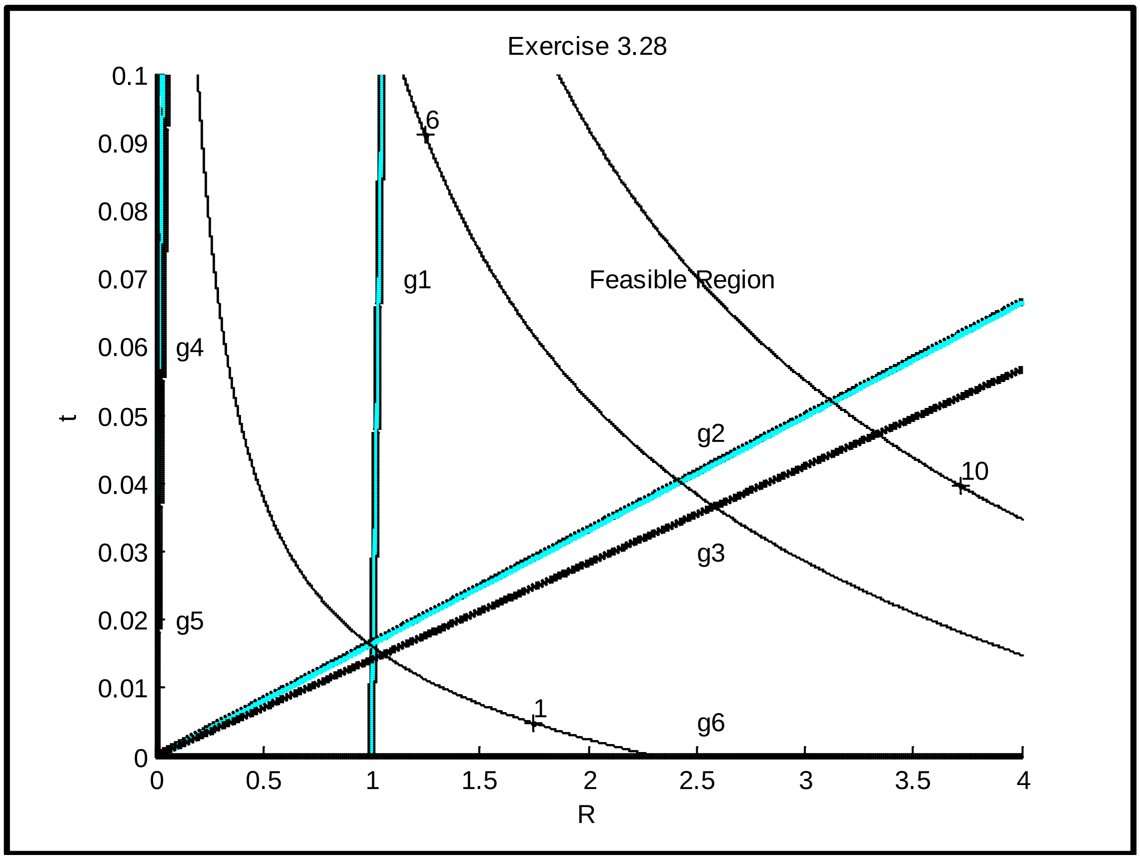
Circumferential strain should not exceed 10^{-3}:

g_3: $\dfrac{3.5\mathrm{R}(2-\nu)}{2Et} \le 0.001$; or $g_3 = (1.41667\times10^{-5})R/t - 0.001 \le 0$

Mean radius should be greater than $t/2$:
$g_4 = -R + 0.5t \le 0$

Both R and t should be greater than zero:
$g_5 = -R \le 0,$
$g_6 = -t \le 0$

Optimum solution: $R^* \doteq 1.0077$ m; $t^* \doteq 0.0168$ m = 1.68 cm; $f^* \doteq 1.0$ m^3; g_1(volume) and g_2 (circumferential stress) constraints are active. (Note: ρ is neglected in f). Thus, the minimum mass is 8182.84 kg if ρ is 7850 kg/m^3.



MATLAB Code.

```
[R,t]=meshgrid(0:0.01:4, 0:0.0001:0.1);
                                %Enter functions for the minimization problem
f=(50.077*R.*t+0.1885*(R+0.5*t).^2);
g1=25-25.038*((R-0.5*t).^2);
g2=3.5*R-210*t;
g3=(1.41667*10^-5)*R-0.001*t;
g4=-R+0.5*t;
g5=-R;
g6=-t;
cla reset
axis auto                       %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('t')         %Specifies labels for x- and y-axes
hold on                         %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,t,g1,cv1,'k','LineWidth',3);
text(1.15,0.07,'g1')
cv11=[0.01:0.01:0.1];
const1=contour(R,t,g1,cv11,'c');
const2=contour(R,t,g2,cv1,'k','Linewidth',3);
const2=contour(R,t,g2,cv11,'c');
text(2.5,0.0475,'g2')
const3=contour(R,t,g3,cv1,'k','Linewidth',3);
const3=contour(R,t,g3,cv11,'c');
text(2.5,0.03,'g3')
const4=contour(R,t,g4,cv1,'k','Linewidth',4);
const4=contour(R,t,g4,cv11,'c');
text(0.1,0.06,'g4')
const5=contour(R,t,g5,cv1,'k','Linewidth',3);
const5=contour(R,t,g5,cv11,'c');
text(0.1,0.02,'g5')
const6=contour(R,t,g6,cv1,'k','Linewidth',3);
const6=contour(R,t,g6,cv11,'c');
text(2.5,0.005,'g6')
text(2,0.07,'Feasible Region')
fv=[1 6 10];                    %Defines contours for the minimization function
fs=contour(R,t,f,fv,'k');       %'k' specifies black dashed lines for function contours
clabel(fs)                      %Automatically puts the contour value on the graph
hold off                        %Indicates end of this plotting sequence
```

3.29

Consider the symmetric three-bar truss design problem formulated in Section 2.10. Formulate and solve the problem graphically for the following data:
l = 1.0 m; P = 100 kN; θ = 30°; mass density, ρ = 2800 kg/m^3; modulus of elasticity, E = 70 GPa; allowable stress, σ_a = 140 MPa; Δ_u = 0.5 cm; Δ_v = 0.5 cm; ω_o = 50 Hz; β = 1.0; A_1, $A_2 \geq 2$ cm^2.

Solution

As described in Section 2.10, the problem is formulated as

$$f = \rho l\left(2\sqrt{2}A_1 + A_2\right)$$
$$g_1:\ \left(P\cos\theta / A_1 + P\sin\theta / \left(A_1 + \sqrt{2}A_2\right)\right) / \sqrt{2} \leq \sigma_a;$$
$$g_2:\ \sqrt{2}P\sin\theta / \left(A_1 + \sqrt{2}A_2\right) \leq \sigma_a$$
$$g_3:\ \sqrt{2}\, l\, P\cos\theta / A_1 E \leq \Delta_u;$$
$$g_4:\ \sqrt{2} l\, P\sin\theta / E\left(A_1 + \sqrt{2}A_2\right) \leq \Delta_v$$
$$g_5:\ 3EA_1 / \left(\rho l^2\left(4A_1 + \sqrt{2}A_2\right)\right) \geq \left(2\pi\omega_0\right)^2;$$
$$g_6:\ -\left(P\sin\theta / \left(A_1 + \sqrt{2}A_2\right) - P\cos\theta / A_1\right) / \sqrt{2} \leq \pi^2 E\beta A_1 / 2l^2$$

Note that the buckling load constraint for only member 3 is needed.

$$g_7:\ A_1 \geq 2;$$
$$g_8:\ A_2 \geq 2$$

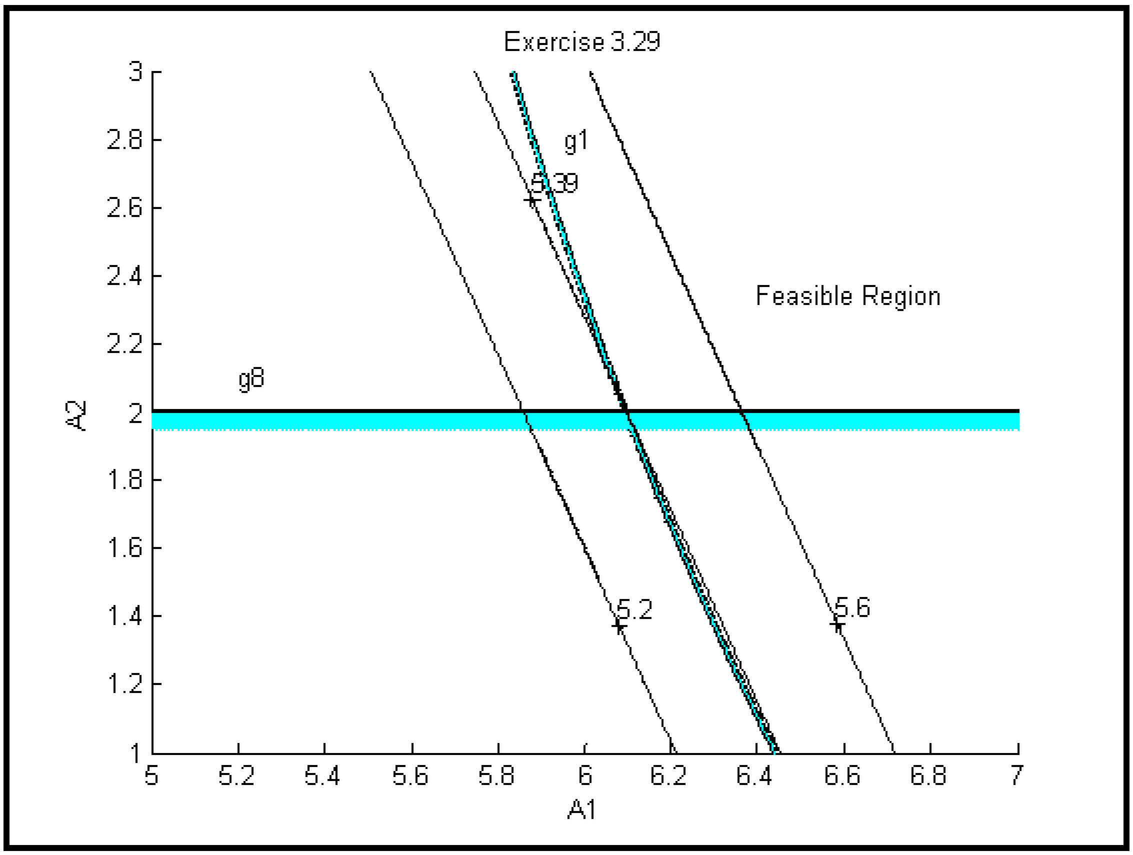
Data: l = 1.0 m, P = 100 kN, θ= 30 degrees, ρ = 2800 kg/m^3, E = 70 GPa, σ_a = 140 MPa,
Δ_u = 0.5 cm, Δ_v = 0.5 cm, ω_0 = 50 Hz, β= 1.0

Using kg, N and cm as unit of mass, force and length respectively, the foregoing equations can be rewritten as follows:

$$f = (2.8 \times 10^{-3})(100)(2\sqrt{2}A_1 + A_2) = 0.28(2\sqrt{2}A_1 + A_2) = 0.791960 A_1 + 0.28 A_2$$
$$g_1 = 6.11926 \times 10^4 / A_1 + 3.53553 \times 10^4 / \left(A_1 + \sqrt{2}A_2\right) - 1.4 \times 10^4 \leq 0;$$
$$g_2:\ \sqrt{2}\left(10^5\right)\sin 30° / \left(A_1 + \sqrt{2}A_2\right) \leq 1.4 \times 10^4;\quad g_2 = 7.07106 \times 10^4 / \left(A_1 + \sqrt{2}A_2\right) - 1.4 \times 10^4 \leq 0$$
$$g_3:\ \sqrt{2}(100)\left(10^5\right)\cos 30° / A_1\left(7.0 \times 10^6\right) \leq 0.5;\quad g_3 = 1.7496 / A_1 - 0.5 \leq 0$$
$$g_4:\ \sqrt{2}(100)\left(10^5\right)\sin 30° / \left(7.0 \times 10^6\right)\left(A_1 + \sqrt{2}A_2\right) \leq 0.5; \text{ or}$$
$$g_4 = 1.01016 / \left(A_1 + \sqrt{2}A_2\right) - 0.5 \leq 0$$
$$g_5:\ 3\left(7.0 \times 10^6\right)A_1 / \left(\left(2.8 \times 10^{-3}\right)\left(10^4\right)\left(4A_1 + \sqrt{2}A_2\right)\right) \geq \left(2\pi \times 50\right)^2; \text{or}$$

$$g_5 = 9.86904\times10^4 - 7.5\times10^5 A_1 \Big/ \left(4A_1 + \sqrt{2}\,A_2\right) \le 0$$
$$g_6 = -3.53554\times10^4 \Big/ \left(A_1 + \sqrt{2}\,A_2\right) + 6.12372\times10^4 / A_1 - 3.45436\times10^3 A_1 \le 0$$
$$g_7 = 2 - A_1 \le 0;$$
$$g_8 = 2 - A_2 \le 0$$

Optimum solution: $A_1^* \doteq 6.1$ cm^2, $A_2^* \doteq 2.0$ cm^2, $f^* \doteq 5.39$ kg; g_1 (member stress constraint) and g_8 (lower limit on A_2) are active.



MATLAB Code

```
[A1,A2]=meshgrid(5:0.01:7, 1:0.01:3);
f=0.79196*A1+0.28*A2;
g1=(6.11926*10^4)*(A1+(2^(1/2))*A2)+(3.53553*10^4)*A1-(1.4*10^4)*(A1+(2^(1/2))*A2).*A1;
g2=7.07106*10^4-(1.4*10^4).*(A1+(2^(1/2))*A2);
g3=1.7496-0.5*A1;
g4=1.01016-0.5.*(A1+(2^(1/2))*A2);
g5=(9.86904*10^4)*(4*A1+(2^(1/2))*A2)-(7.5*10^5)*A1;
g6=(6.12372*10^4)*(A1+(2^(1/2))*A2)-(3.53553*10^4)*A1-
(3.45436*10^3)*(A1+(2^(1/2))*A2).*(A1.^2);
g7=2-A1;
g8=2-A2;
cla reset
axis auto                   %Minimum and maximum values for axes are determined automatically
xlabel('A1'),ylabel('A2')   %Specifies labels for x- and y-axes
hold on                     %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(A1,A2,g1,cv1,'k','LineWidth',3);
text(5.95,2.8,'g1')
cv11=[0.01:0.001:0.05];
const1=contour(A1,A2,g1,cv11,'c');
const2=contour(A1,A2,g2,cv1,'k','Linewidth',3);
const2=contour(A1,A2,g2,cv11,'c');
text(2.5,0.0475,'g2')
const3=contour(A1,A2,g3,cv1,'k','Linewidth',3);
const3=contour(A1,A2,g3,cv11,'c');
const4=contour(A1,A2,g4,cv1,'k','Linewidth',4);
const4=contour(A1,A2,g4,cv11,'c');
text(0.1,0.06,'g4')
const5=contour(A1,A2,g5,cv1,'k','Linewidth',3);
const5=contour(A1,A2,g5,cv11,'c');
text(0.1,0.02,'g5')
const6=contour(A1,A2,g6,cv1,'k','Linewidth',3);
const6=contour(A1,A2,g6,cv11,'c');
text(2.5,0.005,'g6')
const7=contour(A1,A2,g7,cv1,'k','Linewidth',3);
const7=contour(A1,A2,g7,cv11,'c');
text(6,200,'g7')
const8=contour(A1,A2,g8,cv1,'k','Linewidth',3);
const8=contour(A1,A2,g8,cv11,'c');
text(5.2,2.1,'g8')
text(6.4,2.34,'Feasible Region')
fv=[5.2 5.39 5.6];          %Defines contours for the minimization function
fs=contour(A1,A2,f,fv,'k'); %'k' specifies black dashed lines for function contours
clabel(fs)                  %Automatically puts the contour value on the graph
hold off                    %Indicates end of this plotting sequence
```

3.30

Consider the cabinet design problem given in Section 2.6. Use the equality constraints to eliminate three design variables from the problem. Restate the problem in terms of the remaining three variables, transcribing it into the standard form.

Solution

$f = 3.5x_1 + 3.0x_2 + 6.0x_3 + 4.8x_4 + 1.8x_5 + 3.0x_6$

$h_1 = x_1 + x_2 - 800 = 0$
$h_2 = x_3 + x_4 - 500 = 0$
$h_3 = x_5 + x_6 - 1500 = 0$
$g_1 = 5x_1 + 6x_3 + 3x_5 - 6000 \le 0$
$g_2 = 5x_2 + 6x_4 + 3x_6 - 8000 \le 0$;
$g_{2+i} = -x_i \le 0$, i=1 to 6

Using the three equality constraints to eliminate x_2, x_4 and x_6 from all the other equations, we get

$f = 0.5x_1 + 1.2x_3 - 1.2x_5 + 9300$
$g_1 = 5x_1 + 6x_3 + 3x_5 - 6000 \le 0$
$g_2 = 3500 - (5x_1 + 6x_3 + 3x_5) \le 0;$
$g_3 = -x_1 \le 0;$
$g_4 = x_1 - 800 \le 0;$
$g_5 = -x_3 \le 0;$
$g_6 = x_3 - 500 \le 0;$
$g_7 = -x_5 \le 0;$
$g_8 = x_5 - 1500 \le 0$

3.31

Solve the insulated spherical tank design problem formulate in Section 2.3 graphically for the following data:
$r = 3.0$ m, $c_1 = 10000$, $c_2 = \$1000$, $c_3 = \$1$, $c_4 = \$0.1$, $\Delta T = 5$

Solution

Referring to the formulation in Section 2.3, we rewrite it as follows:

minimize $f = at + b/t$ subject to $g_1 = -t \le 0$

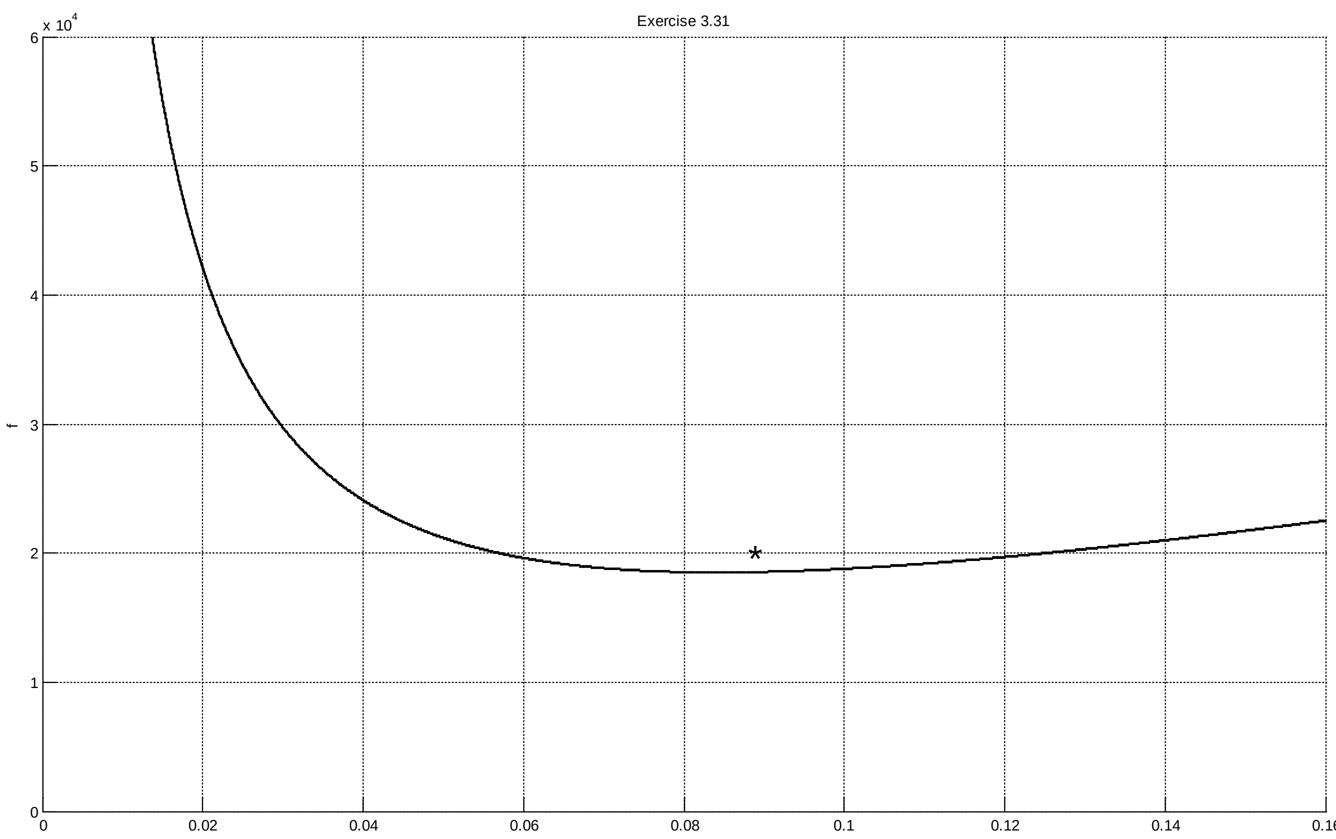
$a = c_2(4\pi r^2)$; $b = [c_3 + f(0.1, 10)c_4](365)(24)(\Delta T)(4\pi r^2)/c_1$
$r = 3.0$, $c_1 = 10000$, $c_2 = 1000$, $c_3 = 1$, $c_4 = 0.1$, $\Delta T = 5$

$a = 1000(4\pi)(3.0)^2 = 1.13097\times10^6$;

$b = [1+\left(\dfrac{1-(1+0.1)^{-10}}{0.1}\right)(0.1)](365)(24)(5)(4\pi)(3)^2/10000 = 800$

Substitute into f: $f = (1.13097\times10^6)t + (800)/t$

Optimum solution: $t^* = 0.084$ *m*, $f^* = \$19024$



3.32

Solve the cylindrical tank design problem given in Section 2.8 graphically for the following data: c=\$1500/m^2, V=3000 m^3.

Solution

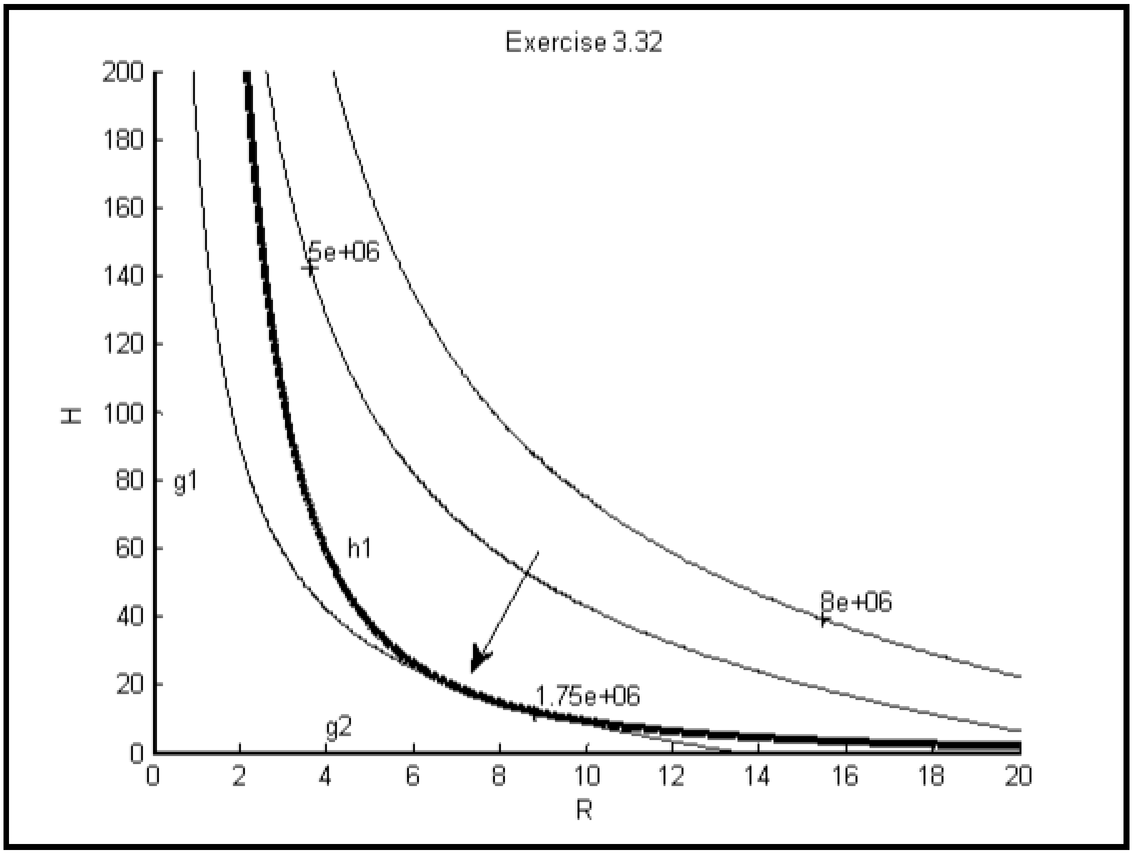
$f = 1500(2\pi R^2 + 2\pi RH)$;

$h_1 = \pi R^2 H - 3000 = 0$

$g_1 = -R \le 0$;

$g_2 = -H \le 0$

Optimum solution: $R^* \doteq 7.8$, $H^* \doteq 15.6$, $f^* \doteq 1.75 \times 10^6$



MATLAB Code

```
[R,H]=meshgrid(0:0.1:20, 0:1:200);
                            %Enter functions for the minimization problem
f=1500*(2*pi*(R.^2)+2*pi*(R).*H);
h1=pi*(R.^2).*H-3000;
g1=-R;
g2=-H;
cla reset
axis auto                   %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('H')     %Specifies labels for x- and y-axes
hold on                     %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,H,h1,cv1,'k','LineWidth',3);
text(4.5,60,'h1')
cv11=[0.01:0:0.01];
const1=contour(R,H,h1,cv11,'c');
const2=contour(R,H,g1,cv1,'k','Linewidth',3);
const2=contour(R,H,g1,cv11,'c');
text(0.5,80,'g1')
const3=contour(R,H,g2,cv1,'k','Linewidth',3);
const3=contour(R,H,g2,cv11,'c');
text(4,8,'g2')
text(25,2.34,'Feasible Region')
fv=[1750000 5000000 8000000];         %Defines contours for the minimization function
fs=contour(R,H,f,fv,'k');             %'k' specifies black dashed lines for function contours
clabel(fs)                            %Automatically puts the contour value on the graph
hold off                              %Indicates end of this plotting sequence
```

3.33

Consider the minimum mass tubular column problem formulated in Section 2.7. Find the optimum solution for the problem using the graphical method for the data:
load, P = 100 kN; length, l = 5.0 m; Young's modulus, E = 210GPa; allowable stress, σ_a = 250 MPa; mass density, ρ =7850 kg/m^3, R ≤ 0.4 m; t ≤ 0.1 m; R, t ≥ 0.

Solution

Formulation 1: Design Variables: R = mean radius of the column; t = wall thickness

$$f = 2\rho l \pi R t = 2(7850)(5)(\pi) R t = \left(2.46615\times 10^5\right) R t$$

g1: P/2$\pi R t \le \sigma_a$; $g_1 = 15915.5/RT - 2.50\times 10^8 \le 0$;

g2: $P \le \pi^3 E R^3 t/4l^2$; $g_2 = 10^5 - \left(6.51132\times 10^{10}\right) R^3 t \le 0$

$$g_3 = R - 0.4 \le 0;$$
$$g_4 = -R \le 0;$$
$$g_5 = t - 0.1 \le 0;$$
$$g_6 = -t \le 0$$
;

Optimum solution: multiple optima on constraint $g_1 = 0$ between points (0.157, 0.000405) and (0.40, 0.000159); $Rt = 6.3662\times 10^{-5}$, $f^* \doteq 15.7$ kg; g_1 (axial stress constraint) is active. (Figure Ex. 3.33a).

Formulation 2: Design Variables**:** R_o= outer radius of the column; R_i = inner radius of the column

$$f = 2\rho l \pi R t = 2(7850)(5)(\pi)(R t) = \left(2.46615\times 10^5\right) R t$$

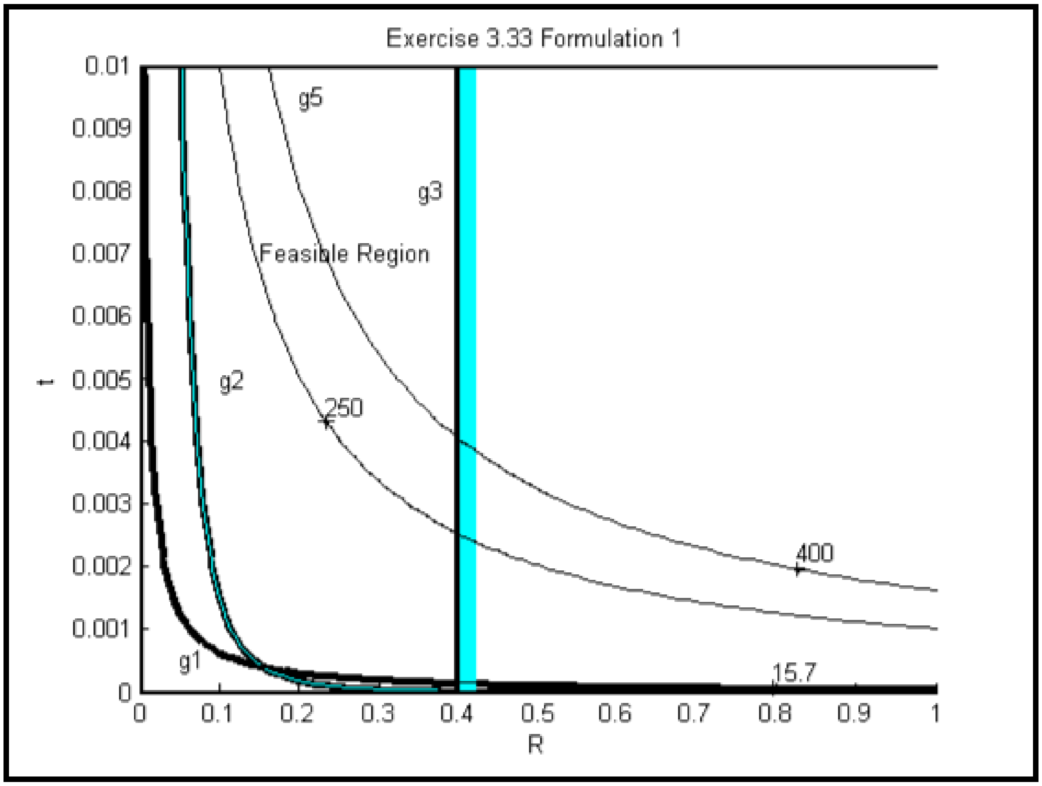
g1: $P/2\pi R t \le \sigma_a$; $g_1 = 15915.5/Rt - 2.50\times 10^8 \le 0$;

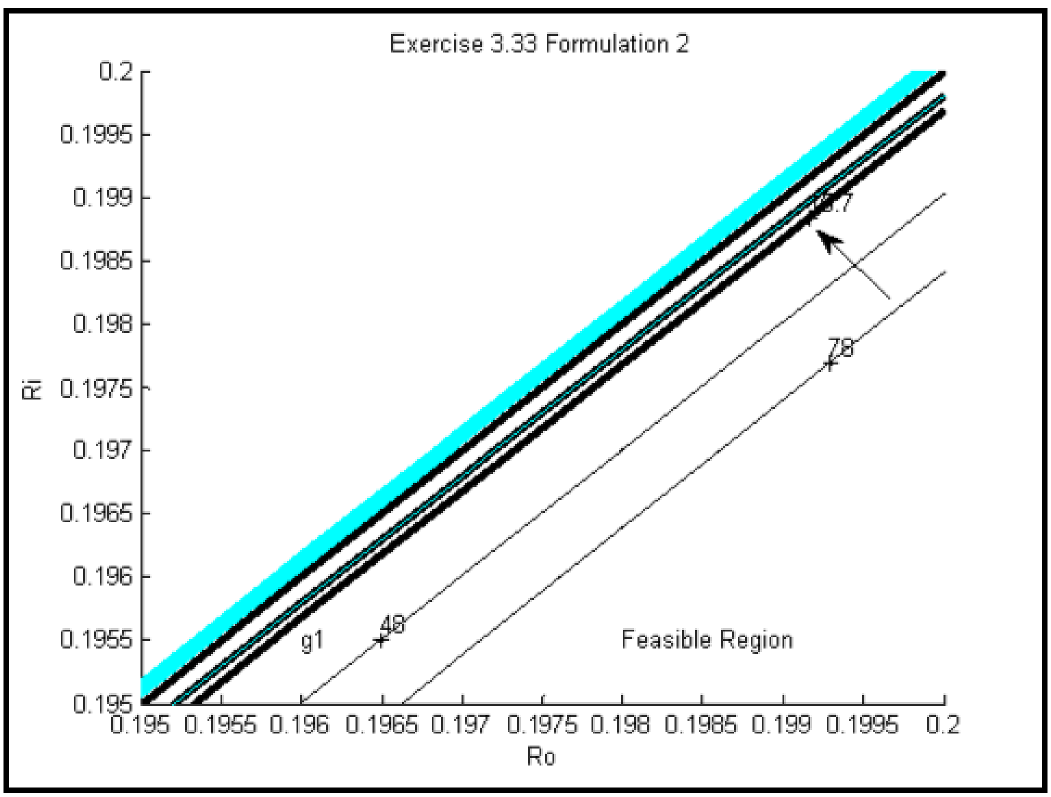
g2: $P \le \pi^3 E\left(R_o^4 - R_i^4\right)/16l^2$; $g_2 = 10^5 - \left(1.62783\times 10^{10}\right)\left(R_o^4 - R_i^4\right) \le 0$

Since mean radius and thickness are given as 0.5($R_0 + R_i$) and ($R_0 - R_i$), the explicit design variable bound constraints become:

$$g_3 = 0.5\left(R_o + R_i\right) - 0.4 \le 0;$$
$$g_4 = -0.5\left(R_o + R_i\right) \le 0;$$
$$g_5 = \left(R_o - R_i\right) - 0.1 \le 0;$$
$$g_6 = -\left(R_o - R_i\right) \le 0$$

Optimum solution: multiple optima on constraint $g_1 = 0$, $f^* \doteq 15.7$ kg; g_1 is active. (Figure Ex. 3.33b).


Exercise 3.33 Formulation 1
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
t
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
R
g5
g3
Feasible Region
g2
250
400
g1
15.7


Exercise 3.33 Formulation 2
0.2
0.1995
0.199
0.1985
0.198
0.1975
0.197
0.1965
0.196
0.1955
0.195
Ri
0.195
0.1955
0.196
0.1965
0.197
0.1975
0.198
0.1985
0.199
0.1995
0.2
Ro
78
48
g1
Feasible Region

MATLAB Code
Formulation 1:

```
[R,t]=meshgrid(0:0.01:1, 0:0.001:0.01);
%Enter functions for the minimization problem
f=(2.46615*10^5)*R.*t;
g1=15915.5-(2.50*10^8)*R.*t;
g2=10^5-(6.51132*10^10)*(R.^3).*t;
g3=R-0.4;
g4=-R;
g5=t-0.01;
g6=-t;
cla reset
axis auto   %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('t')   %Specifies labels for x- and y-axes
hold on     %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,t,g1,cv1,'k','LineWidth',3);
text(0.05,0.0005,'g1')
cv11=[0.001:0.001:0.02];
const1=contour(R,t,g1,cv11,'c');
const2=contour(R,t,g2,cv1,'k','Linewidth',3);
const2=contour(R,t,g2,cv11,'c');
text(0.1,0.005,'g2')
const3=contour(R,t,g3,cv1,'k','Linewidth',3);
const3=contour(R,t,g3,cv11,'c');
text(0.35,0.008,'g3')
const4=contour(R,t,g4,cv1,'k','Linewidth',3);
const4=contour(R,t,g4,cv11,'c');
text(11,13,'g4')
const5=contour(R,t,g5,cv1,'k','Linewidth',3);
const5=contour(R,t,g5,cv11,'c');
text(0.2,0.0095,'g5')
const6=contour(R,t,g6,cv1,'k','Linewidth',3);
const6=contour(R,t,g6,cv11,'c');
text(0,7,'g6')
text(0.15,0.007,'Feasible Region')
fv=[15.7 250 400];    %Defines contours for the minimization function
fs=contour(R,t,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)        %Automatically puts the contour value on the graph
hold off    %Indicates end of this plotting sequence
```

MATLAB Code
Formulation 2:

```
[Ro,Ri]=meshgrid(0.195:0.001:0.2, 0.195:0.001:0.2);
%Enter functions for the minimization problem
f=(2.46615*10^5)*0.5*(Ro+Ri).*(Ro-Ri);
g1=15915.5-(2.50*10^8)*0.5*(Ro+Ri).*(Ro-Ri);
g2=10^5-(1.62783*10^10)*((Ro.^4)-(Ri.^4));
g3=0.5*(Ro+Ri)-0.4;
g4=-0.5*(Ro+Ri);
g5=(Ro-Ri)-0.01;
g6=-(Ro-Ri);
cla reset
axis auto   %Minimum and maximum values for axes are determined automatically
xlabel('Ro'),ylabel('Ri')   %Specifies labels for x- and y-axes
hold on     %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(Ro,Ri,g1,cv1,'k','LineWidth',3);
text(0.196,0.1955,'g1')
cv11=[0.00005:0.00001:0.0002];
const1=contour(Ro,Ri,g1,cv11,'c');
const2=contour(Ro,Ri,g2,cv1,'k','Linewidth',3);
const2=contour(Ro,Ri,g2,cv11,'c');
text(0.1,0.005,'g2')
const3=contour(Ro,Ri,g3,cv1,'k','Linewidth',3);
const3=contour(Ro,Ri,g3,cv11,'c');
text(0.35,0.008,'g3')
const4=contour(Ro,Ri,g4,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g4,cv11,'c');
text(11,13,'g4')
const5=contour(Ro,Ri,g5,cv1,'k','Linewidth',3);
const5=contour(Ro,Ri,g5,cv11,'c');
text(0.2,0.0095,'g5')
const6=contour(Ro,Ri,g6,cv1,'k','Linewidth',3);
const6=contour(Ro,Ri,g6,cv11,'c');
text(0,7,'g6')
text(0.198,0.1955,'Feasible Region')
fv=[15.7 48 78];    %Defines contours for the minimization function
fs=contour(Ro,Ri,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)         %Automatically puts the contour value on the graph
hold off    %Indicates end of this plotting sequence
```

3.34*

Design a hollow torsion rod shown in Fig.E3.34 to satisfy the following requirements (created by J.M. Trummel):

1. The calculated shear stress, τ , shall not exceed the allowable shear stress τ_a under the normal operation torque T_o (N·m).
2. The calculated angle of twist, θ, shall not exceed the allowable twist, θ_a (radians).
3. The member shall not buckle under a short duration torque of T_{max} (N·m).

Requirements for the rod and material properties are given in Table E3.34(A) and E3.34(B) (select a material for one rod). Use the following design variables:
x_1 = outside diameter of the shaft; x_2 = ratio of inside/outside diameter, d_i/d_o.

Using graphical optimization, determine the inside and outside diameters for a minimum mass rod to meet the above design requirements. Compare the hollow rod with an equivalent solid rod (d_i/d_o = 0). Use consistent set of units (e.g. Newtons and millimeters) and let the minimum and maximum values for design variables be given as

$$0.02 \le d_o \le 0.5 \text{ m}, \quad 0.60 \le \frac{d_i}{d_o} \le 0.999$$

Useful expressions for the rod are:

Mass of rod: $$M = \frac{\pi}{4}\rho l(d_o^2 - d_i^2), kg$$

Calculated shear stress: $$\tau = \frac{c}{J}T_o, Pa$$

Calculated angle of twist: $$\theta = \frac{l}{GJ}T_o, radians$$

Critical buckling torque: $$T_{cr} = \frac{\pi d_o^3 E}{12\sqrt{2}(1-\nu^2)^{0.75}}\left(1-\frac{d_i}{d_o}\right)^{2.5}, \text{N.m}$$

Notation

M = mass of the rod (kg),
d_o = outside diameter of the rod (m),
d_i = inside diameter of the rod (m),
ρ = mass density of material (kg/m^3),
l = length of the rod (m),
T_0 = Normal operation torque (N · m),
c = Distance from rod axis to extreme fiber (m),
J = Polar moment of inertia (m^4),
θ = Angle of twist (radians),
G = Modulus of rigidity (Pa),
T_{cr} = Critical buckling torque (N · m),
E = Modulus of elasticity (Pa), and
ν = Poisson's ratio.

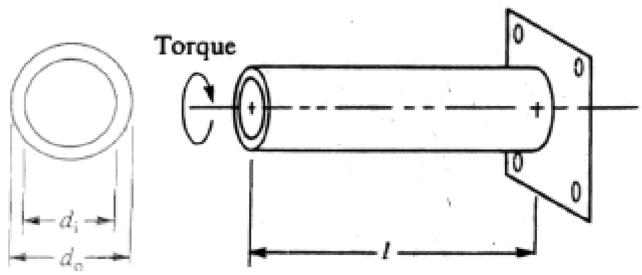


FIGURE E3-34 Hollow torsion rod.

TABLE E3-34(A) Rod Requirements

Torsion rod number	*Length, l (m)*	*Normal torque, T_0 (kN·m)*	*Max. torque, T_{max} (kN·m)*	*Allowable twist, θ_a (degrees)*
1	0.50	10.0	20.0	2
2	0.75	15.0	25.0	2
3	1.00	20.0	30.0	2

TABLE E3-34(B) Materials and Properties for the Torsion Rod

Material	*Density, ρ (kg/m^3),*	*Allowable Shear stress, τ_a (MPa)*	*Elastic modulus, E (GPa)*	*Shear modulus, G (GPa)*	*Poisson's ratio (ν)*
1. 4140 alloy steel	7850	275	210	80	0.30
2. Aluminum alloy 24 ST4	2750	165	75	28	0.32
3. Magnesium alloy A261	1800	90	45	16	0.35
4. Berylium	1850	110	300	147	0.02
5. Titanium	4500	165	110	42	0.30

Solution

Design Variables: x_1 = outside diameter of the shaft; x_2 = ratio of inside/outside diameter, d_i/d_o
Units of mass, force and length are kg, N and mm respectively.

Cost Function: minimize mass of hollow shaft

$$f = \rho A l = \rho(\pi/4)\left(d_o^2 - d_i^2\right)l$$

$$= \rho(\pi/4)d_o^2\left(1-\left(d_i/d_o\right)^2\right)l = \left(\pi\rho l/4\right)x_1^2\left(1-x_2^2\right)$$

Constraints:

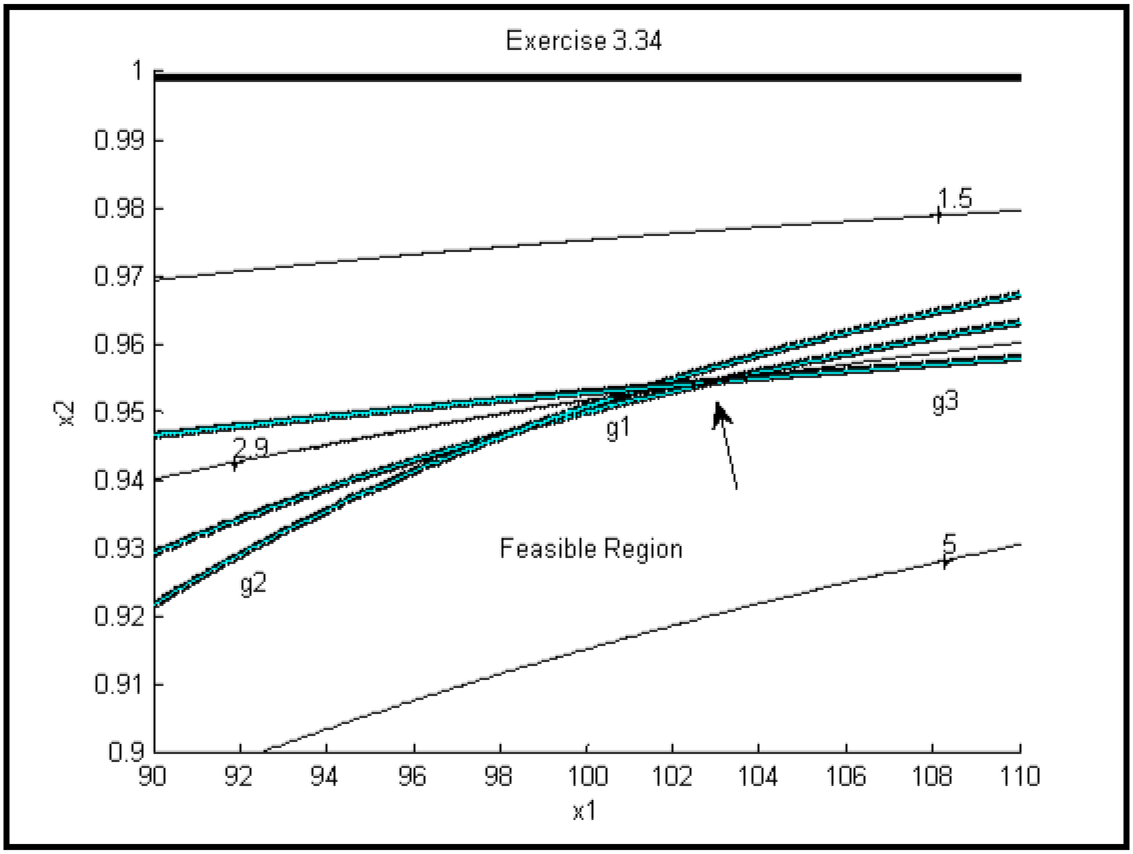
$$g_1 = \tau - \tau_a \le 0$$
$$= T_o c/J - \tau_a = T_o\left(0.5x_1\right)\Big/\left(\pi x_1^4(1-x_2)/32\right) - \tau_a$$
$$= \left(16T_o/\pi\right)\Big/x_1^3\left(1-x_2^4\right) - \tau_a \le 0$$
$$g_2 = \theta - \theta_a \le 0$$
$$= T_0 l/GJ - \theta_a = T_0 l\Big/\left(G\pi\left(d_o^4 - d_i^4\right)/32\right) - \theta_a$$
$$= \left(32T_0 l/G\pi\right)/x_1^4\left(1-x_2^4\right) - \theta_a \le 0$$
$$g_3 = -T_{cr} + T_{max} \le 0$$
$$= -\left(\frac{\pi E}{12\sqrt{2}\left(1-\upsilon^2\right)^{0.75}}\right)\left(x_1^3(1-x_2)2.5\right) + T_{max} \le 0$$

Transform the parameters used in the foregoing equations to have consistent units. Note the first case of requirements and first case of material properties in Tables E3.34(A) and E3.34(B) are used.

l = 0.5 m = 500 mm;
T_o = 10.0 kN.m = 10.0(10^3)(10^3) = 10^7 N.mm;
T_{max} = 20.0 kN.m = 2.0×10^7 N.mm;
$\theta_a = 2° = 2\left(\pi/180\right) = \pi/90$ rad ;
ρ = 7850 kg/m^3 = 7850(10^{-9}) = (7.85×10^{-6}) kg/mm^3;
τ_a = 275 MPa = 275 N/mm^2
E = 210 GPa = (2.1×10^5) N/mm2;
G = 80 GPa = (8.0×10^4) N/mm^2;
υ= 0.3

$$f = \left(7.85\times10^{-6}\right)\left(\pi/4\right)\left(500\right)x_1^2\left(1-x_2^2\right) = \left(3.08269\times10^{-3}\right)x_1^2\left(1-x_2^2\right)$$
$$g_1 = \left(16\times10^7/\pi\right)\Big/x_1^3\left(1-x_2^4\right) - 275 = 5.093\times10^7\Big/x_1^3\left(1-x_2^4\right) - 275 \le 0$$
$$g_2 = \frac{32\left(10^7\right)\left(500\right)}{\left(8.0\times10^4\right)\pi}\Bigg/x_1^4\left(1-x_2^4\right) - \pi/90 = 6.36619\times10^5/x_1^4\left(1-x_2^4\right) - 3.49066\times10^{-2} \le 0$$
$$g_3 = \frac{-\pi\left(2.1\times10^5\right)}{12\sqrt{2}\left(1-0.3^2\right)^{0.75}}\left(x_1^3(1-x_2)2.5\right) + \left(2.0\times10^7\right)$$
$$= 2.0\times10^7 - \left(4.17246\times10^4\right)\left(x_1^3\right)\left(1-x_2\right)^{2.5} \le 0$$
$$g_4 = 20 - x_1 \le 0;$$
$$g_5 = x_1 - 500 \le 0;$$
$$g_6 = 0.6 - x_2 \le 0;$$
$$g_7 = x_2 - 0.999 \le 0$$

Optimum solution: $x_1^* \doteq 103.0$ mm, $x_2^* \doteq 0.955$, $f^* \doteq 2.9$ kg; g_1 (shear stress constraint) and g_3 (buckling constraint) are active.



MATLAB Code Exercise 3.34

```
[x1,x2]=meshgrid(90:0.1:110, 0.9:0.001:1);
                              %Enter functions for the minimization problem
f=(3.08269*10^-3)*(x1.^2).*(1-x2.^2);
g1=(5.093*10^7)-(275)*(x1.^3).*(1-x2.^4);
g2=(6.36619*10^5)-(3.49066*10^-2)*(x1.^4).*(1-x2.^4);
g3=(2*10^7)-(4.17246*10^4)*(x1.^3).*(1-x2).^2.5;
g4=20-x1;
g5=x1-500;
g6=0.6-x2;
g7=x2-0.999;
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
xlabel('x1'),ylabel('x2')     %Specifies labels for x- and y-axes
hold on                       %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);
text(100.5,0.948,'g1')
cv11=[0.01:0.001:0.1];
const1=contour(x1,x2,g1,cv11,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv11,'c');
text(92,0.925,'g2')
const3=contour(x1,x2,g3,cv1,'k','Linewidth',3);
const3=contour(x1,x2,g3,cv11,'c');
text(108,0.952,'g3')
const4=contour(x1,x2,g4,cv1,'k','Linewidth',4);
const4=contour(x1,x2,g4,cv11,'c');
text(0.1,0.06,'g4')
const5=contour(x1,x2,g5,cv1,'k','Linewidth',3);
const5=contour(x1,x2,g5,cv11,'c');
text(0.1,0.02,'g5')
const6=contour(x1,x2,g6,cv1,'k','Linewidth',3);
const6=contour(x1,x2,g6,cv11,'c');
text(2.5,0.005,'g6')
const7=contour(x1,x2,g7,cv1,'k','Linewidth',3);
const7=contour(x1,x2,g7,cv11,'c');
text(6,200,'g7')
text(98,0.93,'Feasible Region')
fv=[1.5 2.9 5];               %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
```

MATLAB Code for Alternate Formulation in Terms of Intermediate Variables

```
%Exercise 3.34
% Axis increments of x1 and x2
[x1,x2]=meshgrid(-.1:0.001:0.6, 0:0.01:1.25);

%Variables in the problem
density=4500;
length=0.75;
modrigidity=42000000000;
modelasticity=110000000000;
poissons=0.3;
theta_a=0.035;
Tau_a=165000000;
T_max=25000;
T_o=15000;
xtremefiber=x1./2;


%Equation Information
Area=(pi/4).*((x1.^2)-((x1.*x2).^2));
J_inertia=(pi/64).*((x1.^4)-((x1.*x2).^4));
Mass=density.*Area.*length;
Tau=(xtremefiber./J_inertia).*T_o;
theta=(length./(modrigidity.*J_inertia)).*T_o;
T_cr=((pi.*(x1.^3).*modelasticity)./(16.97.*((1-(poissons.^2)).^0.75))).*((1-
x2).^2.5);

%Function of the constraints
f=Mass;
g1=0.02-x1;
g2=x1-0.5;
g3=x2-0.999;
g4=0.6-x2;
g5=theta-theta_a;
g6=Tau-Tau_a;
g7=T_cr-T_max;

%Initialization statements
cla reset
axis auto
xlabel('x1 - Outer Diameter of Shaft'),ylabel('x2 - Ratio of Inside/Outside
Diameter')
title('Exercise 3.34')
hold on

%Shading out of infeasible region
cv1=[0 0]; %Specifies two contour values
cv11=[0.005:0.001:0.02]; %Defines a series of closely spaced contours
cv12=[0.005:0.001:.05];
cv13=[0.005:0.001:0.015];
cv14=[300:100000:50000000];
cv15=[300:100:30000];
```

```
%Plotting and shading of constaint 1
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);
const1=contour(x1,x2,g1,cv11,'r');
text(0.03,1.1,'g1')

const2=contour(x1,x2,g2,cv1,'k','LineWidth',3);
const2=contour(x1,x2,g2,cv11,'r');
text(0.53, 1.1, 'g2')

const3=contour(x1,x2,g3,cv1,'k','LineWidth',3);
const3=contour(x1,x2,g3,cv12,'r');
text(0.25,1.09,'g3')

const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
const4=contour(x1,x2,g4,cv11,'r');
text(0.25,0.55,'g4')

const5=contour(x1,x2,g5,cv1,'k','LineWidth',3);
const5=contour(x1,x2,g5,cv13,'r');
text(0.152,0.9,'g5')

const6=contour(x1,x2,g6,cv1,'k','LineWidth',5);
const6=contour(x1,x2,g6,cv14,'r');
text(0.065,0.7,'g6')

const7=contour(x1,x2,g7,cv1,'k','LineWidth',3);
const7=contour(x1,x2,g7,cv15,'r');
text(0.03,0.9,'g7')

text(0.3, 0.75, 'Feasible Region')
fv=[18, 25, 30, 40];    %Defines 4 contours for the profit function
fs=contour(x1,x2,f,fv,'k--');   %'k--'specifies black dashed lines for contours
clabel(fs)
```

3.35*

Formulate and solve Exercise 3.34 using the outside diameter d_o and the inside diameter d_i as design variables.

Solution

Design Variables: d_o = outside diameter of the shaft; d_i = inside diameter of the shaft
Units of mass, force and length are kg, N and mm respectively.

Objective Function: Minimize $f = \rho(\pi/4)\left(d_o^2 - d_i^2\right)l = \left(\pi\rho l/4\right)\left(d_o^2 - d_i^2\right)$

Constraints:

$$g_1 = T_o c/J - \tau_a = T_o\left(d_o/2\right)\Big/\left(\frac{\pi}{32}\left(d_o^4 - d_i^4\right)\right) - \tau_a = \left(16T_o/\pi\right)d_o\Big/\left(d_o^4 - d_i^4\right) - \tau_a \le 0$$

$$g_2 = T_o l/GJ - \theta_a = T_o l\Big/G\left(\frac{\pi}{32}\left(d_o^4 - d_i^4\right)\right) - \theta_a = \left(32T_o l/G\pi\right)\Big/\left(d_o^4 - d_i^4\right) - \theta_a \le 0$$

$$g_3 = -T_{cr} + T_{max} = -\left(\frac{\pi E}{12\sqrt{2}\left(1-\upsilon^2\right)^{0.75}}\right)d_o^3\left(1 - d_i/d_o\right)^{2.5} + T_{max} \le 0$$

$$g_4 = -d_o + 20 \le 0;$$

$$g_5 = d_o - 500 \le 0;$$

$$g_6 = -d_i/d_o + 0.6 \le 0;$$

$$g_7 = d_i/d_o - 0.999 \le 0$$

Using the parameters given in the previous problem, cost and constraint functions become

$$f = (3.08269 \times 10^{-3})\left(d_o^2 - d_i^2\right)$$

$$g_1 = \left(5.093\times10^7\right)d_o\Big/\left(d_o^4 - d_i^4\right) - 275 \le 0$$

$$g_2 = \left(6.3662\times10^5\right)\Big/\left(d_o^4 - d_i^4\right) - \pi/90 \le 0$$

$$g_3 = 2.0\times10^7 - \left(4.17246\times10^4\right)d_o^3\left(1 - d_i/d_o\right)^{2.5} \le 0$$

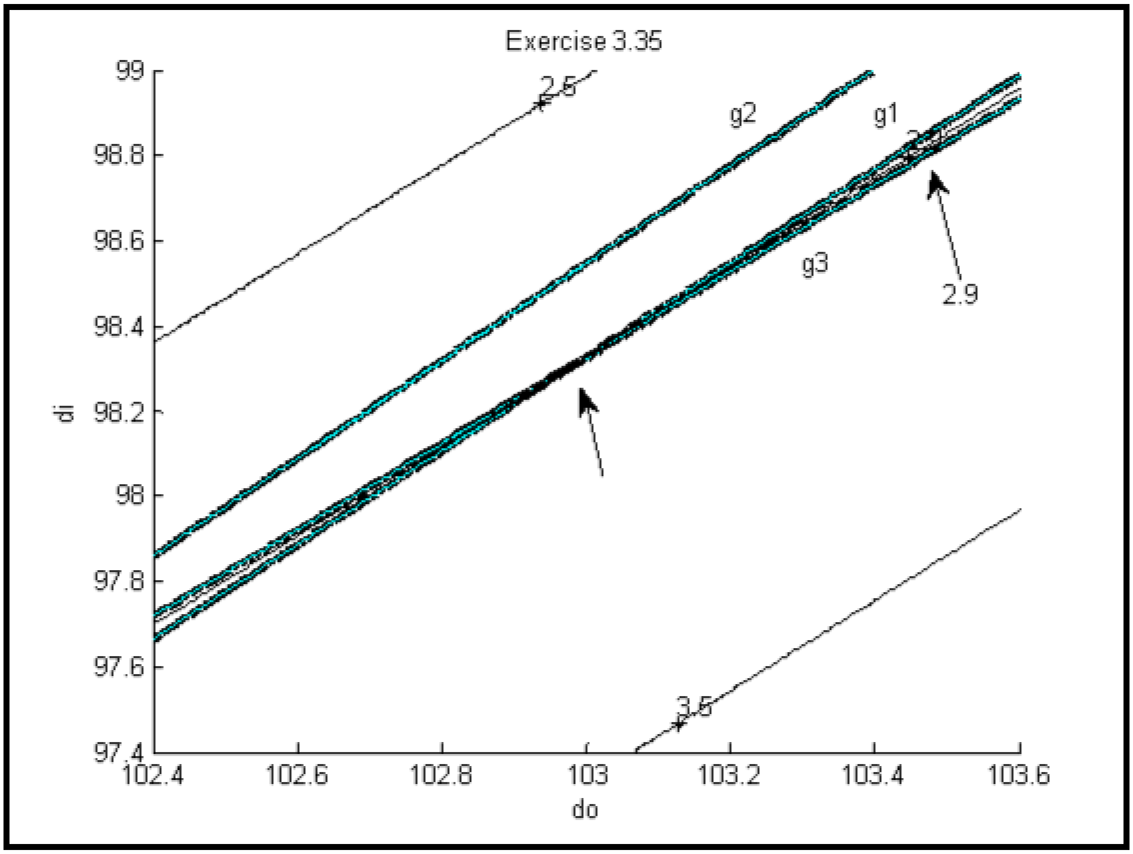
$$g_4 = 20 - d_o \le 0$$

$$g_5 = d_o \le 0$$

$$g_6 = 0.6 - d_i/d_o \le 0$$

$$g_7 = d_i/d_o - 0.999 \le 0$$

Optimum solution: $d_0^* \doteq 103.0$ mm, $d_i^* \doteq 98.36$ mm, $f^* \doteq 2.9$ kg; g_1 (shear stress constraint) and g_3 (buckling constraint) are active.



MATLAB Code

```
[do,di]=meshgrid(102.4:0.01:103.6, 97.4:0.01:99);
                                    %Enter functions for the minimization problem
f=(3.08269*10^-3)*((do.^2)-(di.^2));
g1=(5.093*10^7)*do-(275)*((do.^4)-(di.^4));
g2=(6.36619*10^5)-(3.49066*10^-2)*((do.^4)-(di.^4));
g3=(2*10^7)-(4.17246*10^4)*(do.^3).*(1-(di./do)).^2.5;
g4=20-do;
g5=-do;
g6=0.6-(di./do);
g7=(di./do)-0.999;
cla reset
axis auto                           %Minimum and maximum values for axes are determined automatically
xlabel('do'),ylabel('di')           %Specifies labels for x- and y-axes
hold on                             %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(do,di,g1,cv1,'k','LineWidth',3);
text(103.4,98.9,'g1')
cv11=[0.01:0.001:0.1];
const1=contour(do,di,g1,cv11,'c');
const2=contour(do,di,g2,cv1,'k','Linewidth',3);
const2=contour(do,di,g2,cv11,'c');
text(103.2,98.9,'g2')
const3=contour(do,di,g3,cv1,'k','Linewidth',3);
const3=contour(do,di,g3,cv11,'c');
text(103.3,98.55,'g3')
const4=contour(do,di,g4,cv1,'k','Linewidth',4);
const4=contour(do,di,g4,cv11,'c');
text(0.1,0.06,'g4')
const5=contour(do,di,g5,cv1,'k','Linewidth',3);
const5=contour(do,di,g5,cv11,'c');
text(0.1,0.02,'g5')
const6=contour(do,di,g6,cv1,'k','Linewidth',3);
const6=contour(do,di,g6,cv11,'c');
text(2.5,0.005,'g6')
const7=contour(do,di,g7,cv1,'k','Linewidth',3);
const7=contour(do,di,g7,cv11,'c');
text(6,200,'g7')
text(98,0.93,'Feasible Region')
fv=[2.5 2.9 3.5];                   %Defines contours for the minimization function
fs=contour(do,di,f,fv,'k');         %'k' specifies black dashed lines for function contours
clabel(fs)                          %Automatically puts the contour value on the graph
hold off                            %Indicates end of this plotting sequence
```

3.36*

Formulate and solve Exercise 3.34 using the mean radius R and wall thickness t as design variables.

Let the bounds on design variables be given as $5 \le R \le 20\ cm$ and $0.2 \le t \le 4\ cm$.

Solution

Design Variables: R = mean radius of the shaft; t = wall thickness of the shaft
Units of mass, force and length are kg, N and mm respectively.

Objective Function: minimize $f = \rho(2\pi Rt)l = (2\pi\rho l)Rt$

Constraints:

$$g_1 = T_o c/J - \tau_a = T_o(R+0.5t)\Big/\left(0.5\pi\left((R+0.5t)^4-(R-0.5t)^4\right)\right)-\tau_a$$

$$=(T_o/\pi)(2R+t)\Big/\left(4R^3t+Rt^3\right)-\tau_a \le 0$$

$$g_2 = T_o l/GJ - \theta_a = T_o l/G\left(0.5\pi(R+0.5t)^4-(R-0.5t)^4\right)-\theta_a$$

$$=(2T_o l/\pi G)\Big/\left(4R^3t+Rt^3\right)-\theta_a \le 0$$

$$g_3 = -T_{cr}+T_{max} = -\left(\frac{\pi E}{12\sqrt{2}\left(1-\upsilon^2\right)^{0.75}}\right)(2R+t)^3\left(1-\frac{R-0.5t}{R+0.5t}\right)^{2.5}+T_{max} \le 0$$

$$g_4 = -R+50 \le 0;$$

$$g_5 = R-200 \le 0;$$

$$g_6 = -\ t+2 \le 0;$$

$$g_7 = t-40 \le 0$$

Referring the data given in Exercise 3.34, cost and constraint functions become

$$f = 2\pi\left(7.85\times10^{-6}\right)(500)Rt = \left(2.46615\times10^{-2}\right)Rt$$

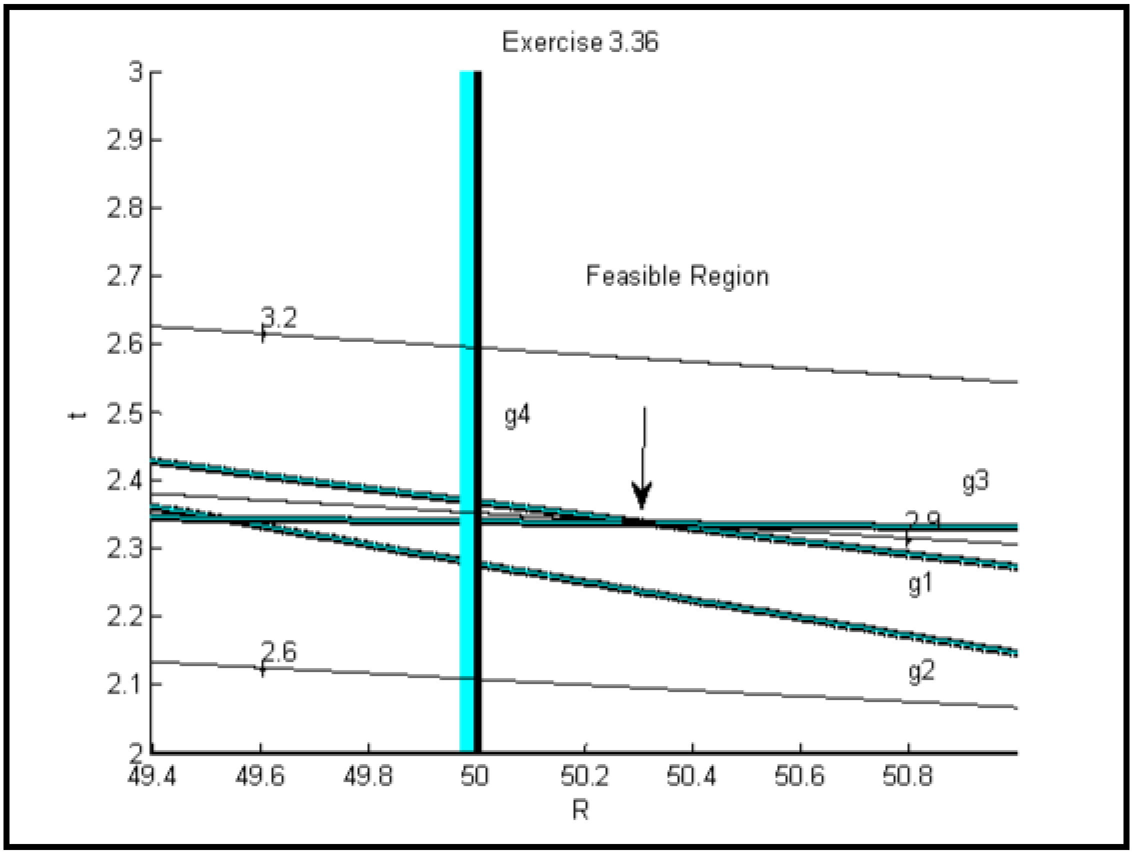
$$g_1 = \left(3.1831\times10^6\right)(2R+t)\Big/\left(4R^3t+Rt^3\right)-275 \le 0$$

$$g_2 = \left[\frac{2\left(10^7\right)(500)}{\pi\left(8.0\times10^4\right)}\right]\Big/\left(4R^3t+Rt^3\right)-\pi/90 = \left(3.97886\times10^4\right)\Big/\left(4R^3t+Rt^3\right)-3.49066\times10^{-2} \le 0$$

$$g_3 = \frac{-\left(2.1\times10^5\right)}{12\sqrt{2}\left(1-0.3^2\right)^{0.75}}(2R+t)^3\left(1-\frac{R-0.5t}{R+0.5t}\right)^{2.5}+\left(2.0\times10^7\right)$$

$$= 2.0\times10^7-\left(4.17246\times10^4\right)(2R+t)^3\left(1-\frac{R-0.5t}{R+0.5t}\right)^{2.5} \le 0$$

Optimum solution: $R^* = 50.3$ mm, $t^* \doteq 2.35$ mm, $f^* \doteq 2.9$ kg; g_1 (shearing stress constraint) and g_3 (buckling constraint) are active.



MATLAB Code

```
[R,t]=meshgrid(49.4:0.01:51, 2:0.01:3);
                                 %Enter functions for the minimization problem
f=(2.46615*10^-2)*(R.*t);
g1=(3.1831*10^6)*(2*R+t)-(275)*((4*t.*R.^3)+(R.*t.^3));
g2=(3.97886*10^4)-(3.49066*10^-2)*((4*t.*R.^3)+(R.*t.^3));
g3=2*10^7-(4.17246*10^4)*((2*R+t).^3).*(1-((R-0.5*t)./(R+0.5*t))).^2.5;
g4=50-R;
g5=-200+R;
g6=-t+2;
g7=t-40;
cla reset
axis auto                        %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('t')          %Specifies labels for x- and y-axes
hold on                          %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,t,g1,cv1,'k','LineWidth',3);
text(50.8,2.25,'g1')
cv11=[0.01:0.001:0.03];
const1=contour(R,t,g1,cv11,'c');
const2=contour(R,t,g2,cv1,'k','Linewidth',3);
const2=contour(R,t,g2,cv11,'c');
text(50.8,2.12,'g2')
const3=contour(R,t,g3,cv1,'k','Linewidth',3);
const3=contour(R,t,g3,cv11,'c');
text(50.9,2.4,'g3')
const4=contour(R,t,g4,cv1,'k','Linewidth',4);
const4=contour(R,t,g4,cv11,'c');
text(50.05,2.5,'g4')
const5=contour(R,t,g5,cv1,'k','Linewidth',3);
const5=contour(R,t,g5,cv11,'c');
text(0.1,0.02,'g5')
const6=contour(R,t,g6,cv1,'k','Linewidth',3);
const6=contour(R,t,g6,cv11,'c');
text(2.5,0.005,'g6')
const7=contour(R,t,g7,cv1,'k','Linewidth',3);
const7=contour(R,t,g7,cv11,'c');
text(6,200,'g7')
text(50.2,2.7,'Feasible Region')
fv=[2.6 2.9 3.2];                %Defines contours for the minimization function
fs=contour(R,t,f,fv,'k');        %'k' specifies black dashed lines for function contours
clabel(fs)                       %Automatically puts the contour value on the graph
hold off                         %Indicates end of this plotting sequence
```

3.37

Formulate the problem of Exercise 2.3 and solve it using the graphical method.

Solution

Writing the Exercise 2.3 in the standard form, we get

$f = -\pi R^2 H$;

$g_1 = 2\pi RH - 900 \le 0;$

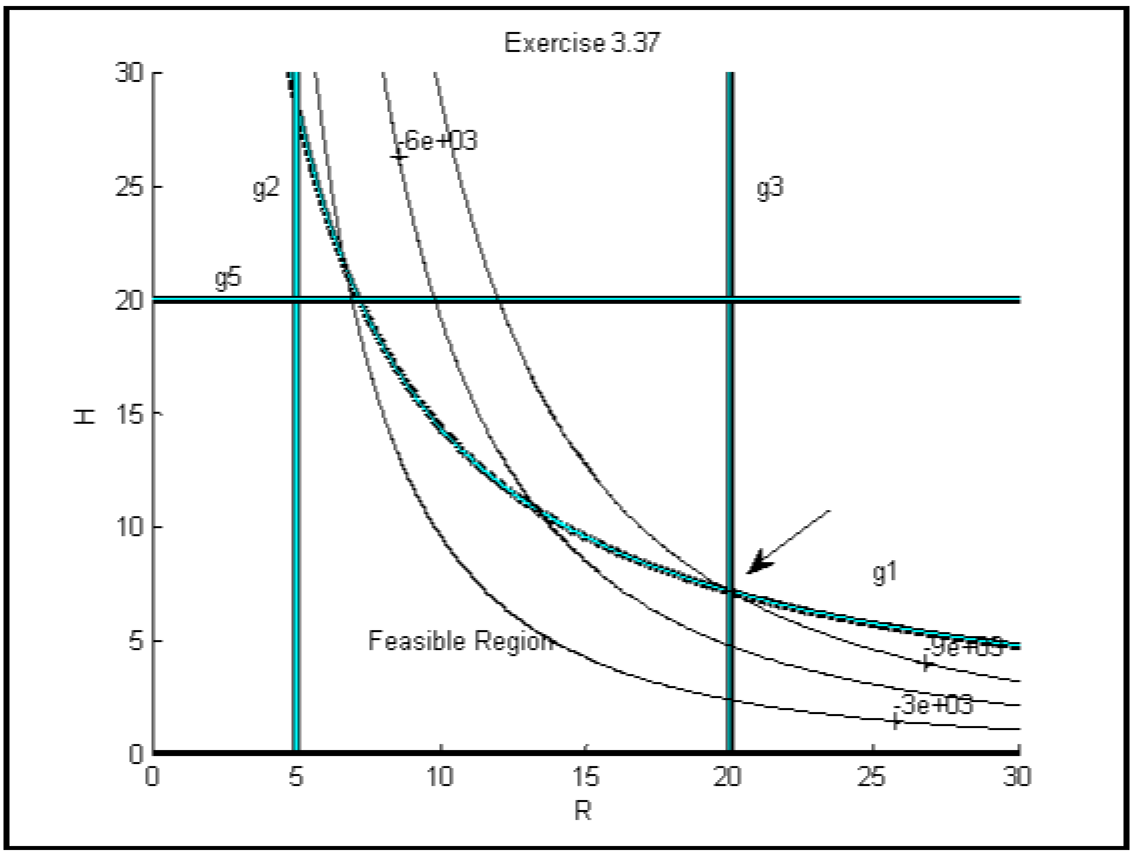
$g_2 = 5 - R \le 0;$

$g_3 = R - 20 \le 0;$

$g_4 = -\ H \le 0;$

$g_5 = H - 20 \le 0$

Optimum solution: R^* = 20 cm, H^* = 7.2 cm, $f^* = -9000$ cm^3; g_1 (surface area constraint) and g_3 (max. radius constraint) are active.



MATLAB Code

```
[R,H]=meshgrid(0:0.1:30, 0:0.1:30);
                                %Enter functions for the minimization problem
f=-pi*(R.*R.*H);
g1=2*pi*R.*H-900;
g2=5-R;
g3=R-20;
g4=-H;
g5=H-20;
cla reset
axis auto                       %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('H')         %Specifies labels for x- and y-axes
hold on                         %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,H,g1,cv1,'k','LineWidth',3);
text(25,8,'g1')
cv11=[0.01:0.001:0.03];
const1=contour(R,H,g1,cv11,'c');
const2=contour(R,H,g2,cv1,'k','Linewidth',3);
const2=contour(R,H,g2,cv11,'c');
text(3.5,25,'g2')
const3=contour(R,H,g3,cv1,'k','Linewidth',3);
const3=contour(R,H,g3,cv11,'c');
text(21,25,'g3')
const4=contour(R,H,g4,cv1,'k','Linewidth',4);
const4=contour(R,H,g4,cv11,'c');
text(50.05,2.5,'g4')
const5=contour(R,H,g5,cv1,'k','Linewidth',3);
const5=contour(R,H,g5,cv11,'c');
text(2.2,21,'g5')
text(7.5,5,'Feasible Region')
fv=[-3000 -6000 -9000];         %Defines contours for the minimization function
fs=contour(R,H,f,fv,'k');       %'k' specifies black dashed lines for function contours
clabel(fs)                      %Automatically puts the contour value on the graph
hold off                        %Indicates end of this plotting sequence
```

3.38

Formulate the problem of Exercise 2.4 and solve it using the graphical method.

Solution

Writing the Exercise 2.4 in the standard form, we get

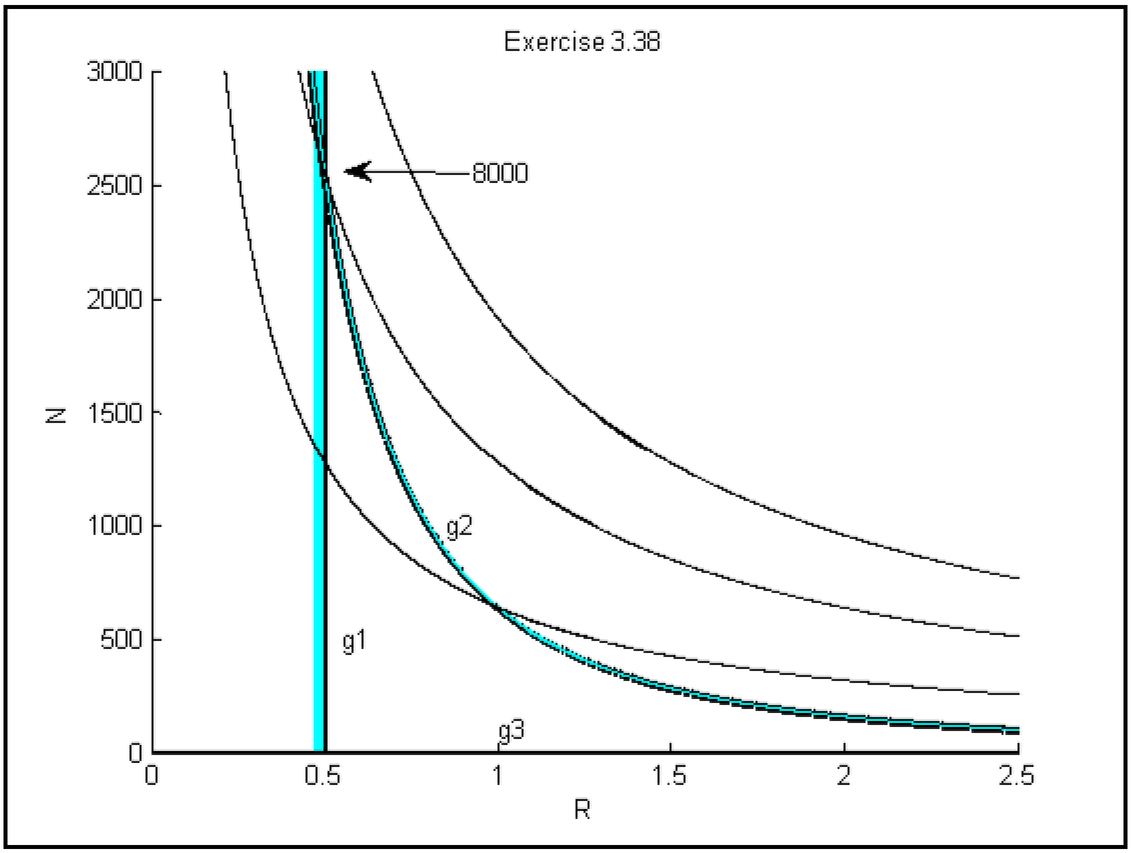
$f = -2\pi l N R$;

$g_1 = 0.5 - R \le 0$;

$g_2 = N\pi R^2 - 2000 \le 0$;

$g_3 = -N \le 0$

Optimum solution: $R^* = 0.5$ cm, $N^* = 2550$, $f^* = -8000\,(l = 10)$; g_1 (min. radius constraint) and g_2 (cross-sectional area constraint) are active.



MATLAB Code

```
[R,N]=meshgrid(0:0.01:2.5, 0:1:3000);
                              %Enter functions for the minimization problem
f=-20*pi*(R.*N);
g1=0.5-R;
g2=pi*N.*R.*R-2000;
g3=-N;
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('N')       %Specifies labels for x- and y-axes
hold on                       %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,N,g1,cv1,'k','LineWidth',3);
text(0.55,500,'g1')
cv11=[0.01:0.001:0.03];
const1=contour(R,N,g1,cv11,'c');
const2=contour(R,N,g2,cv1,'k','Linewidth',3);
const2=contour(R,N,g2,cv11,'c');
text(0.85,1000,'g2')
const3=contour(R,N,g3,cv1,'k','Linewidth',3);
const3=contour(R,N,g3,cv11,'c');
text(1,100,'g3')
text(7.5,5,'Feasible Region')
fv=[-40000 -80000 -120000];           %Defines contours for the minimization function
fs=contour(R,N,f,fv,'k');             %'k' specifies black dashed lines for function contours
hold off                              %Indicates end of this plotting sequence
```

3.39

Formulate Execise 3.23 for a column pinned at both ends. The buckling load for such a column is given as $\pi^2 EI/l^2$. Use graphical method.

Solution

Referring to Exercise 3.23, the only difference between the two problems is that the buckling load is now increased by a factor of four. Therefore, the new formulation is:

$f = 0.2466Rt$

$g_1 = 7957.7/Rt - 250 \le 0;$

$g_2 = \left(5\times10^4\right) - 0.26045R^3t \le 0;$

$g_3 = R/t - 50 \le 0$

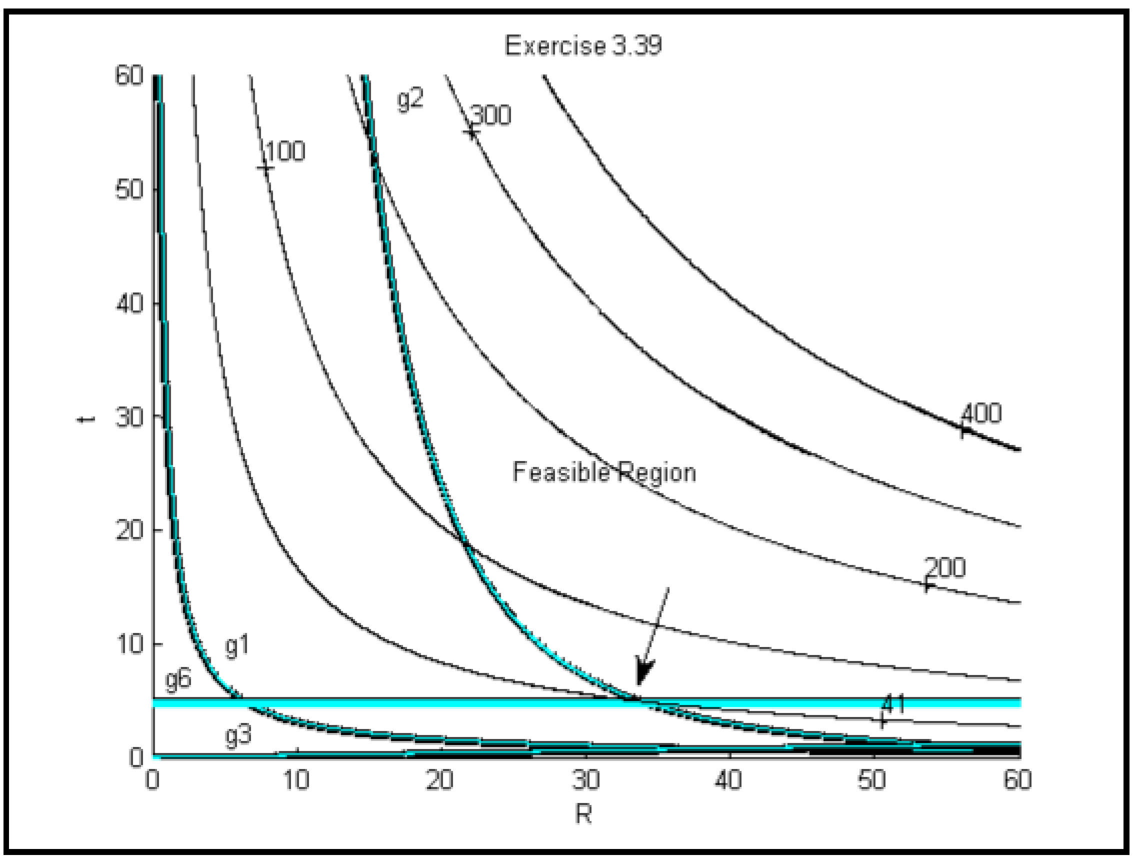
$g_4 = 10 - R \le 0;$

$g_5 = R - 1000 \le 0;$

$g_6 = 5 - t \le 0;$

$g_7 = t - 200 \le 0$

Optimum solution: $R^* \doteq 33.7$ mm, $t^* = 5.0$ mm, $f^* = 41$ kg; g_2 (buckling constraint) and g_6 (minimum thickness) are active.



MATLAB Code

```
[R,t]=meshgrid(0:0.1:60, 0:0.1:60);
%Enter functions for the minimization problem
f=0.2466*R.*t;
g1=7957.7-250*R.*t;
g2=5*10^4-0.26045*(R.^3).*t;
g3=R-50*t;
g4=10-R;
g5=R-1000;
g6=5-t;
g7=t-200;
cla reset
axis auto                %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('t')  %Specifies labels for x- and y-axes
hold on                  %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,t,g1,cv1,'k','LineWidth',3);
text(5,10,'g1')
cv11=[0.01:0.01:0.5];
const1=contour(R,t,g1,cv11,'c');
const2=contour(R,t,g2,cv1,'k','Linewidth',3);
const2=contour(R,t,g2,cv11,'c');
text(17,58,'g2')
const3=contour(R,t,g3,cv1,'k','Linewidth',3);
const3=contour(R,t,g3,cv11,'c');
text(5,2,'g3')
const4=contour(R,t,g4,cv1,'k','Linewidth',3);
const4=contour(R,t,g4,cv11,'c');
text(11,78,'g4')
const5=contour(R,t,g5,cv1,'k','Linewidth',3);
const5=contour(R,t,g5,cv11,'c');
text(7,100,'g5')
const6=contour(R,t,g6,cv1,'k','Linewidth',3);
const6=contour(R,t,g6,cv11,'c');
text(0.9,7,'g6')
const7=contour(R,t,g7,cv1,'k','Linewidth',3);
const7=contour(R,t,g7,cv11,'c');
text(6,200,'g7')
text(25,25,'Feasible Region')
fv=[41 100 200 300 400];        %Defines contours for the minimization function
fs=contour(R,t,f,fv,'k');        %'k' specifies black dashed lines for function contours
clabel(fs)                      %Automatically puts the contour value on the graph
hold off                        %Indicates end of this plotting sequence
```

3.40

Solve Exercise 3.23 for a column fixed at both ends. The buckling load for such a column is given as $4\pi^2 EI/l^2$. Use graphical method.

Solution

Referring to Exercise 3.23, the problem is formulated as follows: (only the buckling load constraint is changed; buckling load is 16 times of that in Exercise 3.23):

$f = 0.2466Rt$

$g_1 = 7957.7/Rt - 250 \le 0;$

$g_2 = 5 \times 10^4 - 1.04181R^3t \le 0;$

$g_3 = R/t - 50 \le 0$

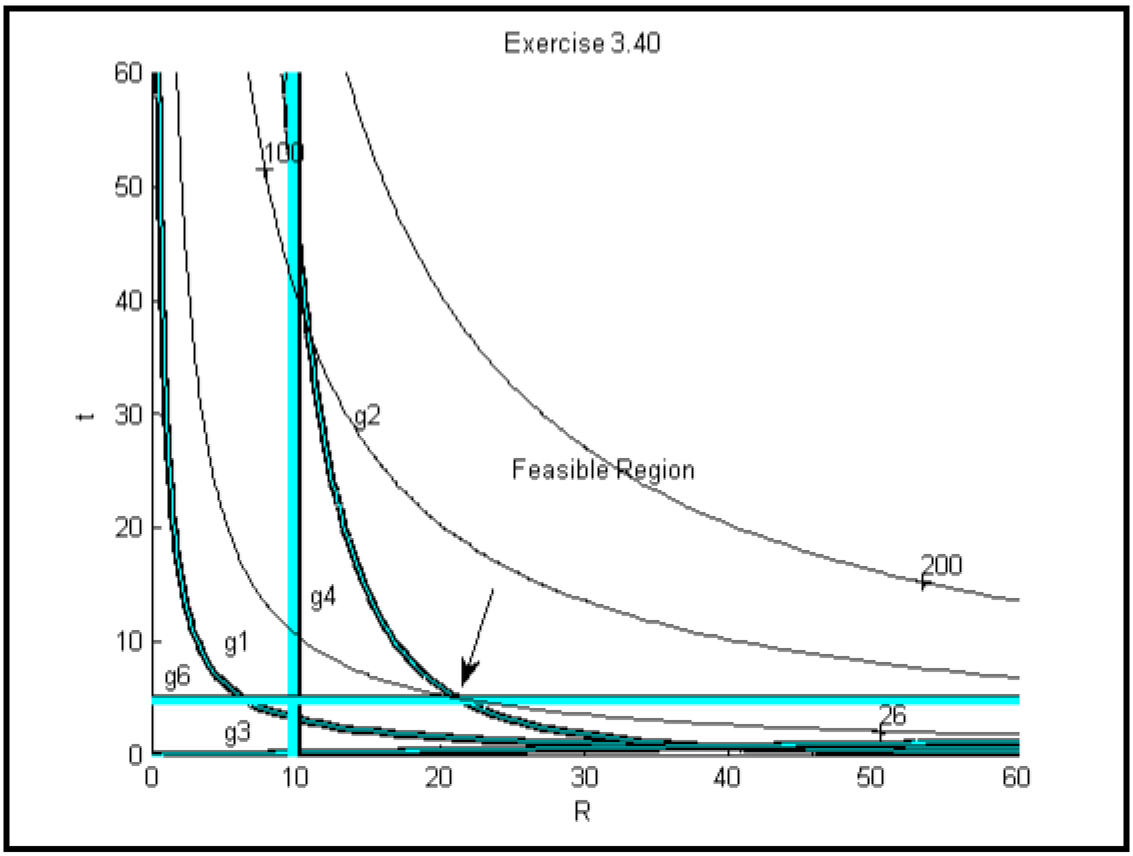
$g_4 = 10 - R \le 0;$

$g_5 = R - 1000 \le 0;$

$g_6 = 5 - t \le 0;$

$g_7 = t - 200 \le 0$

Optimum solution: $R^* \doteq 21.5$ mm, $t^* \doteq 5.0$ mm, $f^* \doteq 26.0$ kg; g_2 (buckling constraint) and g_6 (minimum thickness constraint) are active.



MATLAB Code

```
[R,t]=meshgrid(0:1:60, 0:1:60);
                              %Enter functions for the minimization problem
f=0.2466*R.*t;
g1=7957.7-250*R.*t;
g2=5*10^4-1.04181*(R.^3).*t;
g3=R-50*t;
g4=10-R;
g5=R-1000;
g6=5-t;
g7=t-200;
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('t')       %Specifies labels for x- and y-axes
hold on                       %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,t,g1,cv1,'k','LineWidth',3);
text(5,10,'g1')
cv11=[0.01:0.01:0.5];
const1=contour(R,t,g1,cv11,'c');
const2=contour(R,t,g2,cv1,'k','Linewidth',3);
const2=contour(R,t,g2,cv11,'c');
text(14,30,'g2')
const3=contour(R,t,g3,cv1,'k','Linewidth',3);
const3=contour(R,t,g3,cv11,'c');
text(5,2,'g3')
const4=contour(R,t,g4,cv1,'k','Linewidth',4);
const4=contour(R,t,g4,cv11,'c');
text(11,14,'g4')
const5=contour(R,t,g5,cv1,'k','Linewidth',3);
const5=contour(R,t,g5,cv11,'c');
text(7,100,'g5')
const6=contour(R,t,g6,cv1,'k','Linewidth',3);
const6=contour(R,t,g6,cv11,'c');
text(0.9,7,'g6')
const7=contour(R,t,g7,cv1,'k','Linewidth',3);
const7=contour(R,t,g7,cv11,'c');
text(6,200,'g7')
text(25,25,'Feasible Region')
fv=[26 100 200];              %Defines contours for the minimization function
fs=contour(R,t,f,fv,'k');     %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
```

3.41

Solve Exercise 3.23 for a column fixed at one end and pinned at the other. The buckling load for a column is given as $2\pi^2 EI/l^2$. Use graphical method.

Solution

Referring to Exercise 3.23, this problem is formulated as follows: (only the buckling load constraint is changed; buckling load is 8 times of that in Exercise 3.23):

$f = 0.2466Rt$;

$g_1 = 7957.7/Rt - 250 \le 0$;

$g_2 = 5\times10^4 - 0.52088R^3 t \le 0$;

$g_3 = R/t - 50 \le 0$

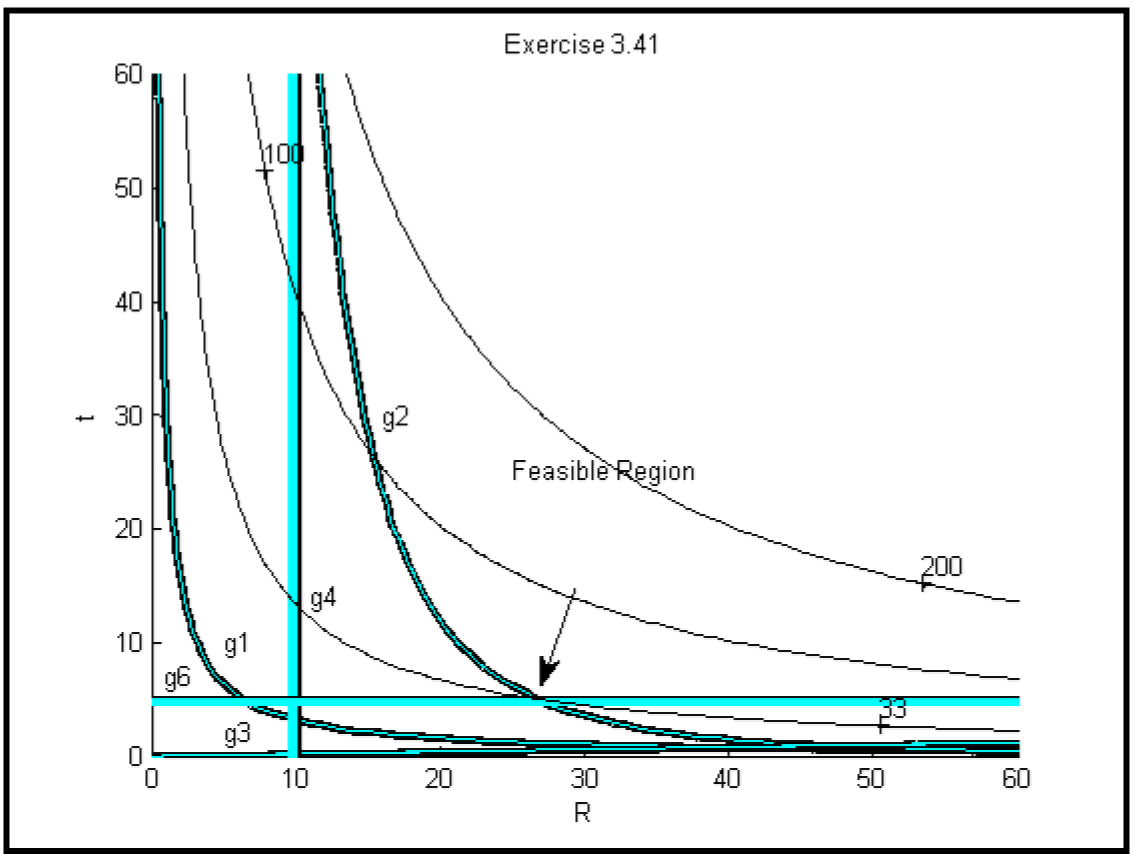
$g_4 = 10 - R \le 0$;

$g_5 = R - 1000 \le 0$;

$g_6 = 5 - t \le 0$;

$g_7 = t - 200 \le 0$

Optimum solution: $R^* \doteq 27.0$ mm, $t^* \doteq 5.0$ mm, $f^* \doteq 33.0$ kg; g_2 (buckling constraint) and g_6 (min. thickness constraint) are active.



MATLAB Code

```
[R,t]=meshgrid(0:1:60, 0:1:60);
                          %Enter functions for the minimization problem
f=0.2466*R.*t;
g1=7957.7-250*R.*t;
g2=5*10^4-0.52088*(R.^3).*t;
g3=R-50*t;
g4=10-R;
g5=R-1000;
g6=5-t;
g7=t-200;
cla reset
axis auto                 %Minimum and maximum values for axes are determined automatically
xlabel('R'),ylabel('t')   %Specifies labels for x- and y-axes
hold on                   %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(R,t,g1,cv1,'k','LineWidth',3);
text(5,10,'g1')
cv11=[0.01:0.01:0.5];
const1=contour(R,t,g1,cv11,'c');
const2=contour(R,t,g2,cv1,'k','Linewidth',3);
const2=contour(R,t,g2,cv11,'c');
text(16,30,'g2')
const3=contour(R,t,g3,cv1,'k','Linewidth',3);
const3=contour(R,t,g3,cv11,'c');
text(5,2,'g3')
const4=contour(R,t,g4,cv1,'k','Linewidth',4);
const4=contour(R,t,g4,cv11,'c');
text(11,14,'g4')
const5=contour(R,t,g5,cv1,'k','Linewidth',3);
const5=contour(R,t,g5,cv11,'c');
text(7,100,'g5')
const6=contour(R,t,g6,cv1,'k','Linewidth',3);
const6=contour(R,t,g6,cv11,'c');
text(0.9,7,'g6')
const7=contour(R,t,g7,cv1,'k','Linewidth',3);
const7=contour(R,t,g7,cv11,'c');
text(6,200,'g7')
text(25,25,'Feasible Region')
fv=[33 100 200];              %Defines contours for the minimization function
fs=contour(R,t,f,fv,'k');     %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
```

3.42

Solve Exercise 3.24 for a column pinned at both ends. The buckling load for such a column is given as $\pi^2 EI/l^2$. Use the graphical method.

Solution

Referring to Exercise 3.24, the problem is formulated as follows:

$f = 0.1233\left(R_o^2 - R_i^2\right);$

$g_1 = 15915.5/\left(R_o^2 - R_i^2\right) - 250 \le 0;$

$g_2 = \left(5\times10^5\right) - 0.06511\left(R_o^4 - R_i^4\right) \le 0;$

$g_3 = \left(R_o + R_i\right)/2\left(R_o - R_i\right) - 50 \le 0;$

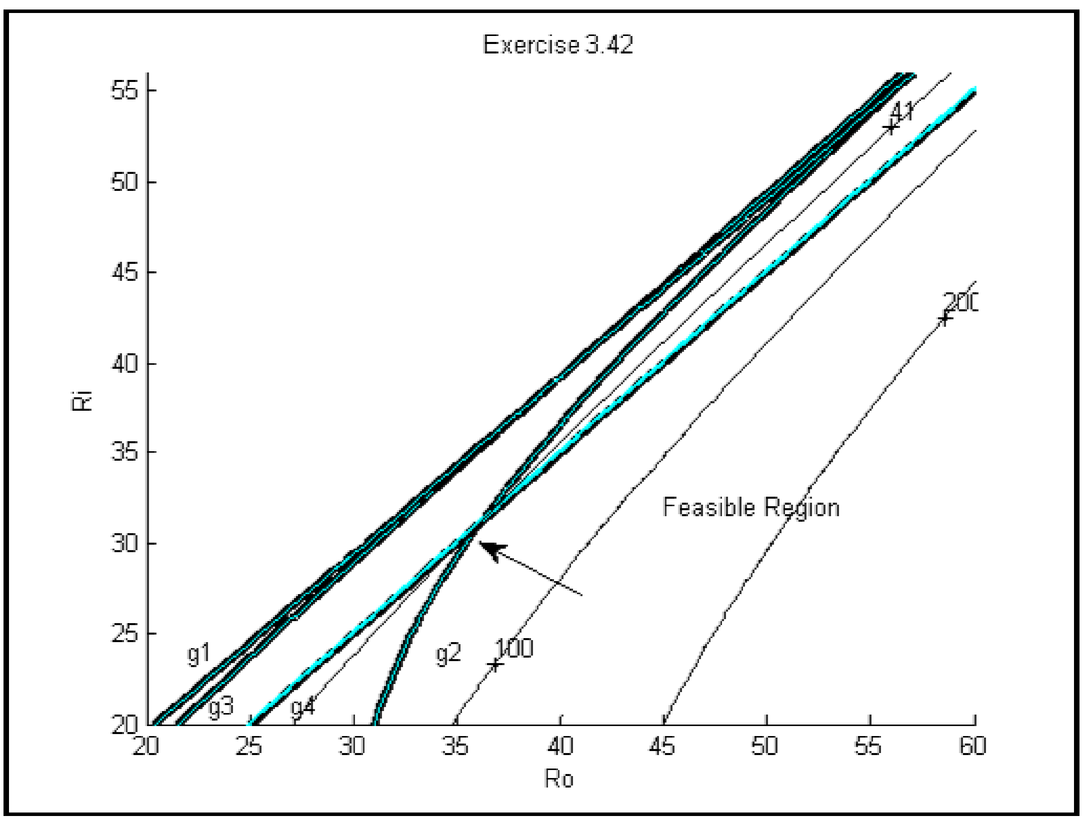
$g_4 = -R_o + R_i + 5 \le 0;$

$g_5 = R_o - R_i - 200 \le 0$

$g_6 = -0.5\left(R_o + R_i\right) + 10 \le 0;$

$g_7 = 0.5\left(R_o + R_i\right) - 1000 \le 0$

Optimum solution: $R_0^* \doteq 36$ mm, $R_i^* \doteq 31$ mm, $f^* \doteq 41$ kg; g_2 (buckling constraint) and g_4 (minimum thickness) are active.



MATLAB Code

```
[Ro,Ri]=meshgrid(20:1:60, 20:1:56);
                                %Enter functions for the minimization problem
f=0.1233*(Ro.^2-Ri.^2);
g1=15915.5-250*(Ro.^2-Ri.^2);
g2=5*10^4-0.06511*(Ro.^4-Ri.^4);
g3=(Ro+Ri)-100*(Ro-Ri);
g4=5-Ro+Ri;
g5=Ro-Ri-200;
g6=-0.5*(Ro+Ri)+10;
g7=0.5*(Ro+Ri)-1000;
cla reset
axis auto                       %Minimum and maximum values for axes are determined automatically
xlabel('Ro'),ylabel('Ri')       %Specifies labels for x- and y-axes
hold on                         %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(Ro,Ri,g1,cv1,'k','LineWidth',3);
text(22,24,'g1')
cv11=[0.01:0.01:0.2];
const1=contour(Ro,Ri,g1,cv11,'c');
const2=contour(Ro,Ri,g2,cv1,'k','Linewidth',3);
const2=contour(Ro,Ri,g2,cv11,'c');
text(34,24,'g2')
const3=contour(Ro,Ri,g3,cv1,'k','Linewidth',3);
const3=contour(Ro,Ri,g3,cv11,'c');
text(23,21,'g3')
const4=contour(Ro,Ri,g4,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g4,cv11,'c');
text(27,21,'g4')
const5=contour(Ro,Ri,g5,cv1,'k','Linewidth',3);
const5=contour(Ro,Ri,g5,cv11,'c');
text(7,100,'g5')
const6=contour(Ro,Ri,g6,cv1,'k','Linewidth',3);
const6=contour(Ro,Ri,g6,cv11,'c');
text(0,7,'g6')
const7=contour(Ro,Ri,g7,cv1,'k','Linewidth',3);
const7=contour(Ro,Ri,g7,cv11,'c');
text(6,200,'g7')
text(45,32,'Feasible Region')
fv=[41 100 200];                %Defines contours for the minimization function
fs=contour(Ro,Ri,f,fv,'k');     %'k' specifies black dashed lines for function contours
clabel(fs)                      %Automatically puts the contour value on the graph
hold off                        %Indicates end of this plotting sequence
```

3.43

Solve Exercise 3.24 for a column pinned at both ends. The buckling load for such a column is given as $4\pi^2 EI/l^2$. Use the graphical method.

Solution

Referring to Exercise 3.24, this problem is formulated as follows:

$$f = 0.1233\left(R_o^2 - R_i^2\right)$$

$$g_1 = 15915.5\Big/\left(R_o^2 - R_i^2\right) - 250 \le 0;$$

$$g_2 = 5\times10^4 - 0.26045\left(R_o^4 - R_i^4\right) \le 0;$$

$$g_3 = \left(R_o + R_i\right)\Big/2\left(R_o - R_i\right) - 50 \le 0;$$

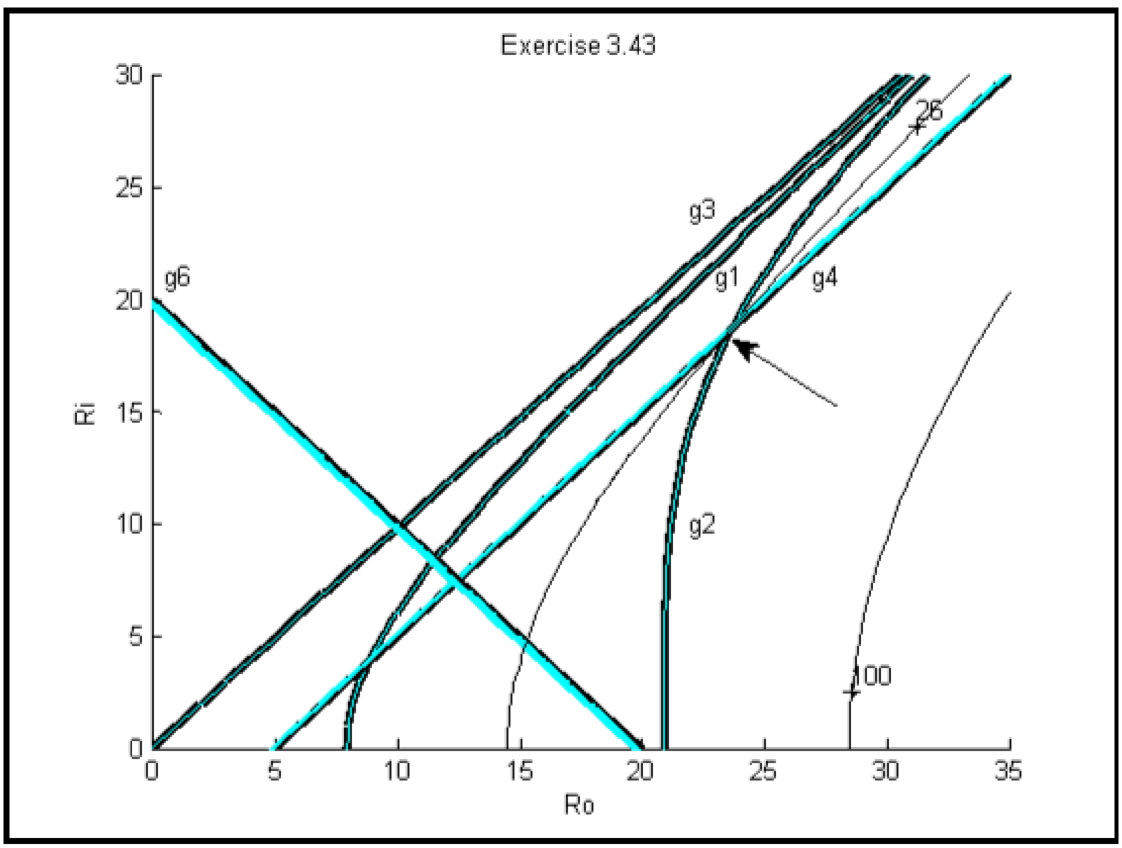
$$g_4 = -R_o + R_i + 5 \le 0;$$

$$g_5 = R_o - R_i - 200 \le 0$$

$$g_6 = -0.5\left(R_o + R_i\right) + 10 \le 0;$$

$$g_7 = 0.5\left(R_o + R_i\right) - 1000 \le 0$$

Optimum solution: $R^* \doteq 24.0$ mm, $R^* \doteq 19.0$ mm, $f^* \doteq 26.0$ kg; g_2 (buckling constraint) and g_4 (min. thickness constraint) are active.



MATLAB Code

```
[Ro,Ri]=meshgrid(0:1:35, 0:1:30);
                                    %Enter functions for the minimization problem
f=0.1233*(Ro.^2-Ri.^2);
g1=15915.5-250*(Ro.^2-Ri.^2);
g2=5*10^4-0.26045*(Ro.^4-Ri.^4);
g3=(Ro+Ri)-100*(Ro-Ri);
g4=5-Ro+Ri;
g5=Ro-Ri-200;
g6=-0.5*(Ro+Ri)+10;
g7=0.5*(Ro+Ri)-1000;
cla reset
axis auto                           %Minimum and maximum values for axes are determined automatically
xlabel('Ro'),ylabel('Ri')           %Specifies labels for x- and y-axes
hold on                             %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(Ro,Ri,g1,cv1,'k','LineWidth',3);
text(23,21,'g1')
cv11=[0.01:0.01:0.2];
const1=contour(Ro,Ri,g1,cv11,'c');
const2=contour(Ro,Ri,g2,cv1,'k','Linewidth',3);
const2=contour(Ro,Ri,g2,cv11,'c');
text(22,10,'g2')
const3=contour(Ro,Ri,g3,cv1,'k','Linewidth',3);
const3=contour(Ro,Ri,g3,cv11,'c');
text(22,24,'g3')
const4=contour(Ro,Ri,g4,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g4,cv11,'c');
text(27,21,'g4')
const5=contour(Ro,Ri,g5,cv1,'k','Linewidth',3);
const5=contour(Ro,Ri,g5,cv11,'c');
text(7,100,'g5')
const6=contour(Ro,Ri,g6,cv1,'k','Linewidth',3);
const6=contour(Ro,Ri,g6,cv11,'c');
text(0.5,21,'g6')
const7=contour(Ro,Ri,g7,cv1,'k','Linewidth',3);
const7=contour(Ro,Ri,g7,cv11,'c');
text(6,200,'g7')
text(45,32,'Feasible Region')
fv=[26 100 200];                    %Defines contours for the minimization function
fs=contour(Ro,Ri,f,fv,'k');         %'k' specifies black dashed lines for function contours
clabel(fs)                          %Automatically puts the contour value on the graph
hold off                            %Indicates end of this plotting sequence
```

3.44

Solve Exercise 3.24 for a column fixed at one end and pinned at the other. The buckling load for such a column is given as $2\pi^2 EI/l^2$. Use the graphical method.

Solution

Referring to Exercise 3.24, this problem is formulated as follows:

$$f = 0.1233\left(R_o^2 - R_i^2\right)$$

$$g_1 = \frac{15915.5}{(R_o^2 - R_i^2)} - 250 \le 0;$$

$$g_2 = 5\times10^4 - 0.13022\left(R_o^4 - R_i^4\right) \le 0;$$

$$g_3 = \left(R_o + R_i\right)/2\left(R_o - R_i\right) - 50 \le 0;$$

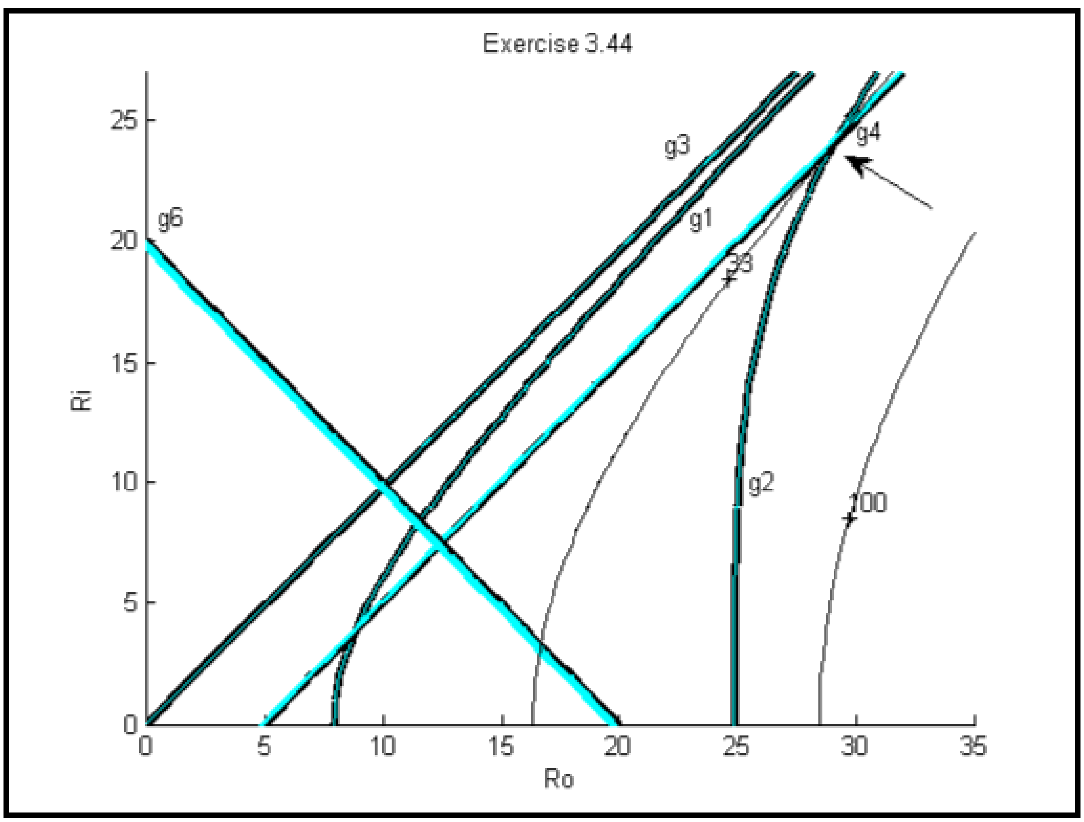
$$g_4 = -R_o + R_i + 5 \le 0;$$

$$g_5 = R_o - R_i - 200 \le 0$$

$$g_6 = -0.5\left(R_o + R_i\right) + 10 \le 0;$$

$$g_7 = 0.5\left(R_o + R_i\right) - 1000 \le 0$$

Optimum solution: $R_o^* \doteq 29.5$ mm, $R_i^* \doteq 24.5$ mm, $f^* \doteq 33.0$ kg; g_2 (buckling constraint) and g_4 (min. thickness constraint) are active.



MATLAB Code: 3.44

```
[Ro,Ri]=meshgrid(0:1:35, 0:1:27);
                              %Enter functions for the minimization problem
f=0.1233*(Ro.^2-Ri.^2);
g1=15915.5-250*(Ro.^2-Ri.^2);
g2=5*10^4-0.13022*(Ro.^4-Ri.^4);
g3=(Ro+Ri)-100*(Ro-Ri);
g4=5-Ro+Ri;
g5=Ro-Ri-200;
g6=-0.5*(Ro+Ri)+10;
g7=0.5*(Ro+Ri)-1000;
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
xlabel('Ro'),ylabel('Ri')     %Specifies labels for x- and y-axes
hold on                       %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(Ro,Ri,g1,cv1,'k','LineWidth',3);
text(23,21,'g1')
cv11=[0.01:0.01:0.2];
const1=contour(Ro,Ri,g1,cv11,'c');
const2=contour(Ro,Ri,g2,cv1,'k','Linewidth',3);
const2=contour(Ro,Ri,g2,cv11,'c');
text(25.5,10,'g2')
const3=contour(Ro,Ri,g3,cv1,'k','Linewidth',3);
const3=contour(Ro,Ri,g3,cv11,'c');
text(22,24,'g3')
const4=contour(Ro,Ri,g4,cv1,'k','Linewidth',3);
const4=contour(Ro,Ri,g4,cv11,'c');
text(30,24.5,'g4')
const5=contour(Ro,Ri,g5,cv1,'k','Linewidth',3);
const5=contour(Ro,Ri,g5,cv11,'c');
text(7,100,'g5')
const6=contour(Ro,Ri,g6,cv1,'k','Linewidth',3);
const6=contour(Ro,Ri,g6,cv11,'c');
text(0.5,21,'g6')
const7=contour(Ro,Ri,g7,cv1,'k','Linewidth',3);
const7=contour(Ro,Ri,g7,cv11,'c');
text(6,200,'g7')
text(45,32,'Feasible Region')
fv=[33 100 200];              %Defines contours for the minimization function
fs=contour(Ro,Ri,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
```

3.45

Solve the can design problem formulated in Section 2.2 using the graphical approach.

Solution

Referring to Section 2.2, the problem is formulated as follows:

Design Variables: D = diameter of can in cm; H = height of can in cm

Cost Function: minimize surface area of sheet metal,

$$f = \pi DH + \pi D^2/2$$

Constraints:

$$g_1 = 400 - \pi D^2 H/4 \le 0;$$

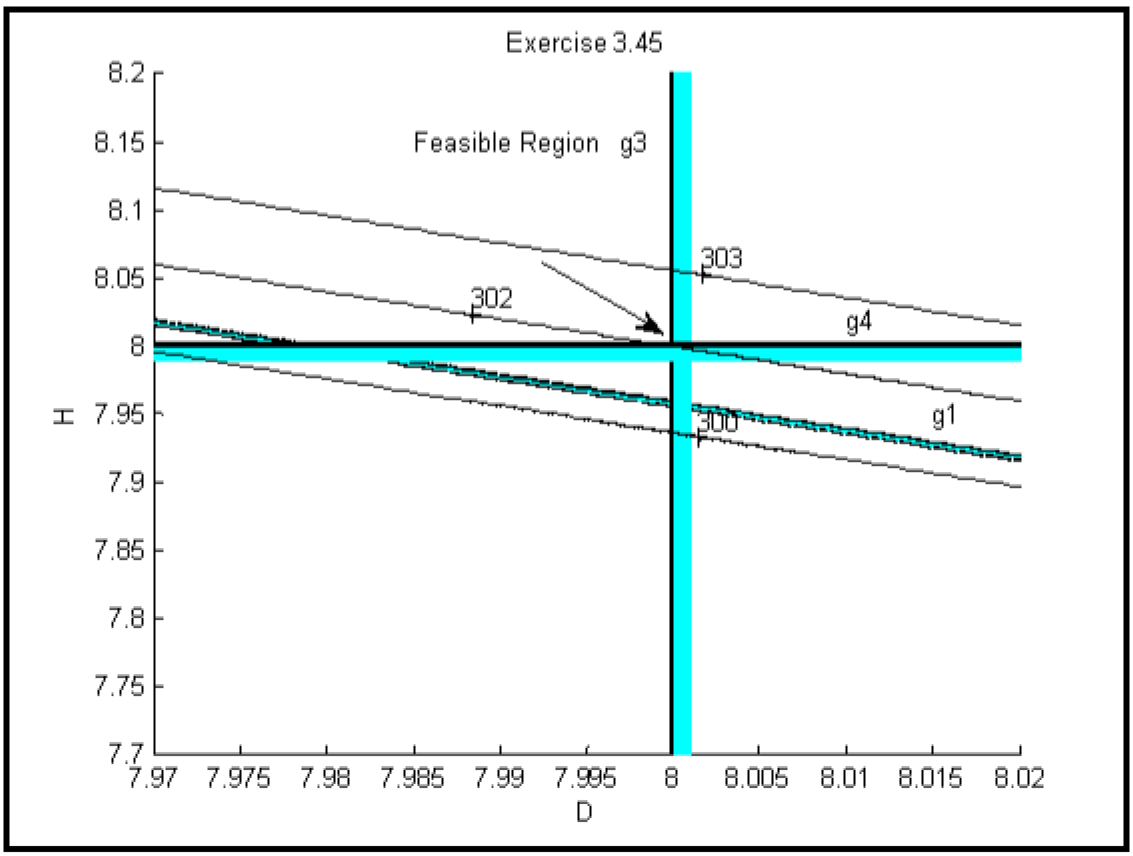
$$g_2 = 3.5 - D \le 0;$$

$$g_3 = D - 8.0 \le 0;$$

$$g_4 = 8.0 - H \le 0;$$

$$g_5 = H - 18.0 \le 0$$

Optimum solution: $D^* \doteq 8.0$ cm, $H^* \doteq 8.0$ cm, $f^* \doteq 301.6$ cm_2; g_3 (max. diameter constraint) and g_4 (min. height constraint) are active.



MATLAB Code: 3.45

```
[D,H]=meshgrid(7.97:0.001:8.02, 7.7:0.001:8.2);
                                %Enter functions for the minimization problem
f=pi*D.*H+0.5*pi*(D.^2);
g1=400-0.25*pi*(D.^2).*H;
g2=3.5-D;
g3=D-8;
g4=8-H;
g5=H-18;
cla reset
axis auto                       %Minimum and maximum values for axes are determined automatically
xlabel('D'),ylabel('H')         %Specifies labels for x- and y-axes
hold on                         %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(D,H,g1,cv1,'k','LineWidth',3);
text(8.015,7.95,'g1')
cv11=[0.001:0.001:0.01];
cv22=[0.0001:0.0001:0.001];
const1=contour(D,H,g1,cv11,'c');
const2=contour(D,H,g2,cv1,'k','Linewidth',3);
const2=contour(D,H,g2,cv11,'c');
text(0.85,1000,'g2')
const3=contour(D,H,g3,cv1,'k','Linewidth',3);
const3=contour(D,H,g3,cv22,'c');
text(7.997,8.15,'g3')
const4=contour(D,H,g4,cv1,'k','Linewidth',4);
const4=contour(D,H,g4,cv11,'c');
text(8.01,8.02,'g4')
const5=contour(D,H,g5,cv1,'k','Linewidth',3);
const5=contour(D,H,g5,cv11,'c');
text(2.2,21,'g5')
text(7.985,8.15,'Feasible Region')
fv=[300 301.6 303];             %Defines contours for the minimization function
fs=contour(D,H,f,fv,'k');       %'k' specifies black dashed lines for function contours
clabel(fs)                      %Automatically puts the contour value on the graph
hold off                        %Indicates end of this plotting sequence
```

3.46

Consider the two-bar truss shown in Figure 2.5. Using the given data, design a minimum mass structure where W = 100 kN; θ = 30°; h = 1 m; s = 1.5 m; modulus of elasticity, E = 210 GPa; allowable stress, σ_a = 250 MPa, mass density, ρ = 7850 kg/m^3. Use Newtons and millimeters as units. The members should not fail on stress and their buckling should be avoided. Deflection at the top in either direction should not be more than 5 cm.

Use cross –sectional area A_1 and A_2 of the two members as design variables and let the moment of inertia of the members be given as $I = A^2$. Area must also satisfy the constraint $1 \le A_i \le 50\text{cm}^2$.

Solution

Consider the two bar truss shown in Figure 2.5 and refer to the formulation in Section 2.5. Using the cross-sectional areas A_1 and A_2 of the two members as design variables, this problem can be formulated as follows:

$$f = \rho l (A_1 + A_2)$$

$$g_1 = -F_1/A_1 - \sigma_a = 0.5Wl\left(\sin\theta/h + 2\cos\theta/s\right)/A_1 - \sigma_a \le 0$$

$$g_2 = -F_2/A_2 - \sigma_a = 0.5Wl\left(\sin\theta/h - 2\cos\theta/s\right)/A_2 - \sigma_a \le 0$$

$$g_3 = F_2/A_2 - \sigma_a = -0.5Wl\left(\sin\theta/h - 2\cos\theta/s\right)/A_2 - \sigma_a \le 0$$

$$g_4 = -F_1 - P_{cr} = 0.5Wl\left(\sin\theta/h + 2\cos\theta/s\right) - \pi^2 E\left(\beta_1 A_1^2\right)/l^2 \le 0$$

$$g_5 = -F_2 - P_{cr} = 0.5Wl\left(\sin\theta/h - 2\cos\theta/s\right) - \pi^2 E\left(\beta_2 A_2^2\right)/l^2 \le 0$$

Horizontal Deflection, $u \le \Delta_u$;

$$g_6 = Wl\left[\left(A_1 + A_2\right)\cos^2\alpha\cos\theta - \left(A_1 - A_2\right)\sin\alpha\cos\alpha\sin\theta\right]/\left[4EA_1A_2\sin^2\alpha\cos^2\alpha\right] - \Delta_u \le 0$$

Vertical Deflection, $v \le \Delta_v$; or

$$g_7 = Wl\left[\left(A_1 + A_2\right)\sin^2\alpha\sin\theta - \left(A_1 - A_2\right)\sin\alpha\cos\alpha\cos\theta\right]/\left[4EA_1A_2\sin^2\alpha\cos^2\alpha\right] - \Delta_v \le 0$$

$$g_8 = -A_1 + A_{1\min} \le 0;$$

$$g_9 = A_1 - A_{1\max} \le 0;$$

$$g_{10} = -A_2 + A_{2\min} \le 0;$$

$$g_{11} = A_2 - A_{2\max} \le 0$$

Use kilograms, Newtons and millimeters as units for the given data:
W = 100 kN = 10^5 N; h = 1 m = 1000 mm; s = 1.5 m = 1500 mm; E = 210 GPa = 2.1×10^5 N/mm^2; σ_a = 250 MPa = 250 N/mm^2; ρ = 7850 kg/m^3 = 7.85×10^{-6} kg/mm^3; $\Delta_u = \Delta_v$ = 5 cm = 50 mm; $A_{i\min}$ = 1 cm^2 = 100 mm^2; $A_{i\max}$ = 50 cm^2 = 5000 mm^2; $l = (h^2+s^2/4)^{1/2}$ = 1250 mm; $\sin\alpha = s/2l$ = 0.6; $\cos\alpha = h/l$ = 0.8; also, we have θ = 30° and $\beta_1 = \beta_2 = 1$. Substituting these data, we get

$$f = \rho l\left(A_1 + A_2\right) = \left(7.85\times10^{-6}\right)(1250)\left(A_1 + A_2\right) = 9.8125\times10^{-3}\left(A_1 + A_2\right)$$

$g_1 = 103420/A_1 - 250 \le 0;$

$g_2 = -40925/A_2 - 250 \le 0;$

$g_3 = 40925/A_2 - 250 \le 0;$

$g_4 = 1.034188 \times 10^5 - 1.32647 A_1^2 \le 0$

Constraint g_5 is neglected, since the corresponding bar is subjected to tensile force.

$g_6 = \left(202.97 A_1 + 512.95 A_2\right) / \left(A_1 A_2\right);$

$g_7 = \left(-150.22275 A_1 + 383.474 A_2\right) / \left(A_1 A_2\right) - 50 \le 0$

$g_8 = 100 - A_1 \le 0;$

$g_9 = A_1 - 5000 \le 0;$

$g_{10} = 100 - A_2 \le 0;$

$g_{11} = A_2 - 5000 \le 0$

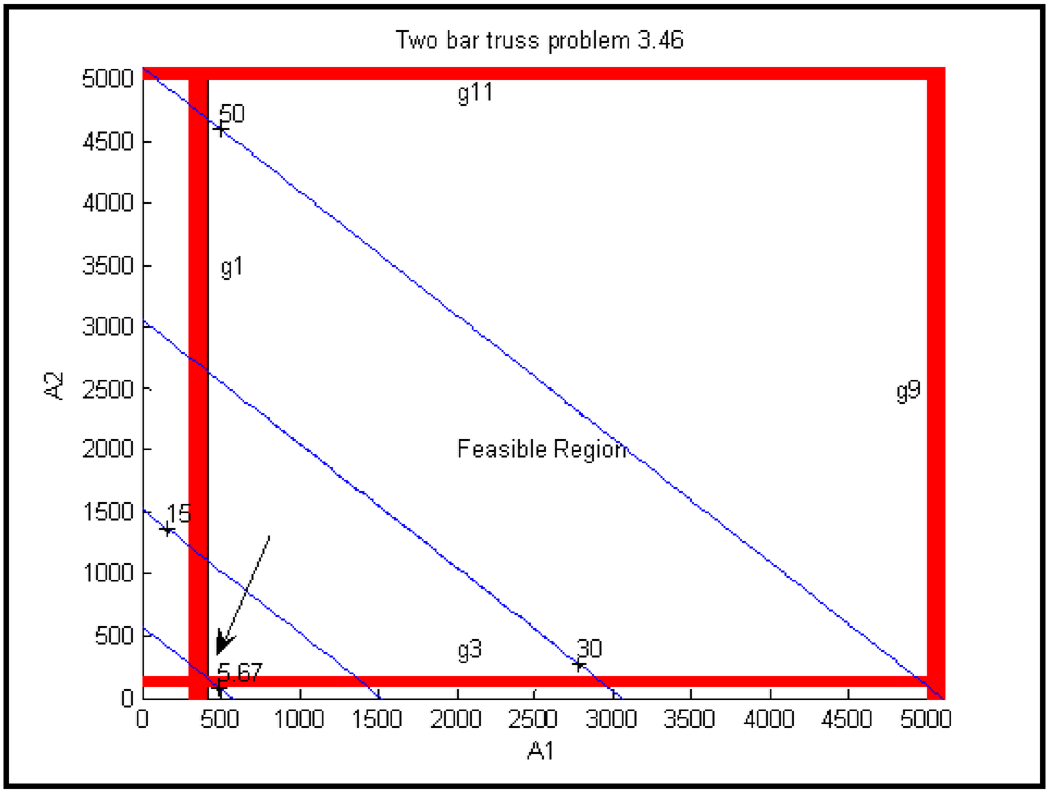
Neglecting the redundant constraints, i.e., g_2, g_4, g_5, g_7, g_8 and g_{10}, and rearranging the constraints we get the formulation as follows:

$f = (9.8125 \times 10^{-3})(A_1 + A_2);$

$g_1 = 103420/A_1 - 250 \le 0;$

$g_2 = 40925/A_2 - 250 \le 0$

Optimum solution: $A_1^* \doteq 413.68$ mm, $A_2^* \doteq 163.7$ mm, $f^* = 5.7$ kg; g_1(stress constraint of member 1) and g_2(horizontal deflection constraint) are active.



MATLAB Code: 3.46

```
% Problem 3.46
% Hyeongjin Song
clear all;
% define mesh for A1 and A2
[A1, A2]=meshgrid(0:100:5100,0:100:5100);
% given parameters
rho=7850e-9;
W=100e3;
theta=30*pi/180;
h=1000;
s=1500;
l=sqrt(h^2+s^2/4);
E1=210e3;
sigmaa=250;
sina=0.6;
cosa=0.8;
delta=50;
% objective function
f=rho*l*(A1+A2);
% constraints
g1=0.5*W*l*(sin(theta)/h+2*cos(theta)/s)./A1-sigmaa;
g2=0.5*W*l*(sin(theta)/h-2*cos(theta)/s)./A2-sigmaa;
g3=-0.5*W*l*(sin(theta)/h-2*cos(theta)/s)./A2-sigmaa;
```

```
g4=0.5*W*l*(sin(theta)/h+2*cos(theta)/s)-pi^2*E1*A1^2/l^2;
g5=0.5*W*l*(sin(theta)/h-2*cos(theta)/s)-pi^2*E1*A2^2/l^2;
g6=W*l*((A1+A2)*cosa^2*cos(theta)-(A1-A2)*sina*cosa*sin(theta))/(4*E1*A1.*A2*sina^2*cosa^2)-
delta;
g7=W*l*((A1+A2)*sina^2*sin(theta)-(A1-A2)*sina*cosa*cos(theta))/(4*E1*A1.*A2*sina^2*cosa^2)-
delta;
g8=-A1+100;
g9=A1-5000;
g10=-A2+100;
g11=A2-5000;
% Since g2 include g3, g2 is neglected.
% Since g4 include g1, g4 is neglected.
% Since the bar is subjected to tensile force, g5 is neglected.
% Since g6 and g7 include union of g8,g9,g10 and g11, both are neglected.
% Therefore, only g1,g3,g8,g9,g10 and g11 are necessary.
cla reset;
axis auto;
xlabel('A1'),ylabel('A2');
title('Two bar truss problem 3.46');
hold on;
cv1=[0 0];
const1=contour(A1,A2,g1,cv1,'k');
text(500,3500,'g1');
cv11=100*(0.05:0.05:1);
contour(A1,A2,g1,cv11,'r');
cv3=[0 0];
const3=contour(A1,A2,g3,cv3,'k');
text(2000,400,'g3');
cv31=20*(0.05:0.25:25);
contour(A1,A2,g3,cv31,'r');
cv9=[0 0];
const9=contour(A1,A2,g9,cv3,'k');
text(4800,2500,'g9');
cv91=20*(0.05:0.05:5);
contour(A1,A2,g9,cv91,'r');
cv11=[0 0];
const11=contour(A1,A2,g11,cv3,'k');
text(2000,4900,'g11');
cv111=20*(0.05:0.05:5);
contour(A1,A2,g11,cv111,'r');
text(2000,2000,'Feasible Region');
opt=rho*l*(413.68+163.7);
fv=[opt, 15, 30, 50];
% fv=[10];
fs=contour(A1,A2,f,fv,'b--');
clabel(fs);
```

hold off

3.47

For Exercise 3.46, use the hollow circular tubes as members with mean radius *R* and wall thickness *t* as design variables. Make sure that R/t ≤ 50. Design the structure so that member 1 is symmetric with member 2. The radius and thickness must also satisfy the constraints 2≤t≤40 mm and 2≤R≤40 cm.

Solution

Consider the two bar truss shown in Fig. 2.2 and refer to the formulation in Section 2.5. Using the hollow circular tubes as members with mean radius *R* and wall thickness *t* as design variables, the problem is formulated as follows:

$$f = 2\rho A l = 2\rho\left(2\pi Rt\right)l = 4\pi\rho l Rt$$

$$g_1 = -F_1/A - \sigma_a = 0.5Wl\left(\sin\theta/h + 2\cos\theta/s\right)/2\pi Rt - \sigma_a \le 0$$

$$g_2 = -F_1 - P_{cr} = 0.5Wl\left(\sin\theta/h + 2\cos\theta/s\right) - \pi^2 E\left(\pi R^3 t\right)/l^2 \le 0$$

Horizontal Deflection, $u \le \Delta_u$;

$$g_3 = Wl^3\cos\theta/\left(\pi RtEs^2\right) - \Delta_u \le 0$$

Vertical Deflection, $v \le \Delta_v$;

$$g_4 = Wl^3\sin\theta/\left(4\pi RtEh^2\right) - \Delta_v \le 0;$$

$$g_5 = R/t - 50 \le 0$$

$$g_6 = -R + R_{min} \le 0;$$

$$g_7 = R - R_{max} \le 0;$$

$$g_8 = -t + t_{min} \le 0;$$

$$g_9 = t - t_{max} \le 0$$

Substituting the data given in Exercise 3.46 and the following simple bounds, R_{min} = 20 mm,

R_{max} = 400 mm, t_{min} = 2 mm, t_{max} = 40 mm, we get

$$f = 4\pi\left(7.85\times10^{-6}\right)\left(1250\right)Rt = 0.123308Rt;$$

$$g_1 = 16459.5/Rt - 250 \le 0;$$

$$g_2 = \left(1.034188\times10^5\right) - 4.16726R^3t \le 0;$$

$$g_3 = 113.95/Rt - 50 \le 0;$$

$$g_4 = 37.01/Rt - 50 \le 0;$$

$$g_5 = R/t - 50 \le 0;$$

$$g_6 = 20 - R \le 0;$$

$$g_7 = R - 400 \le 0;$$

$g_8 = 2 - t \le 0;$

$g_9 = t - 40 \le 0$

Neglecting the redundant constraints, i.e., g_3 and g_4, and rearranging them we get

$f = 0.123308Rt$;

$g_1 = 16459.5/Rt - 250 \le 0;$

$g_2 = 1.034188 \times 10^5 - 4.16726R^3t \le 0;$

$g_3 = R/t - 50 \le 0;$

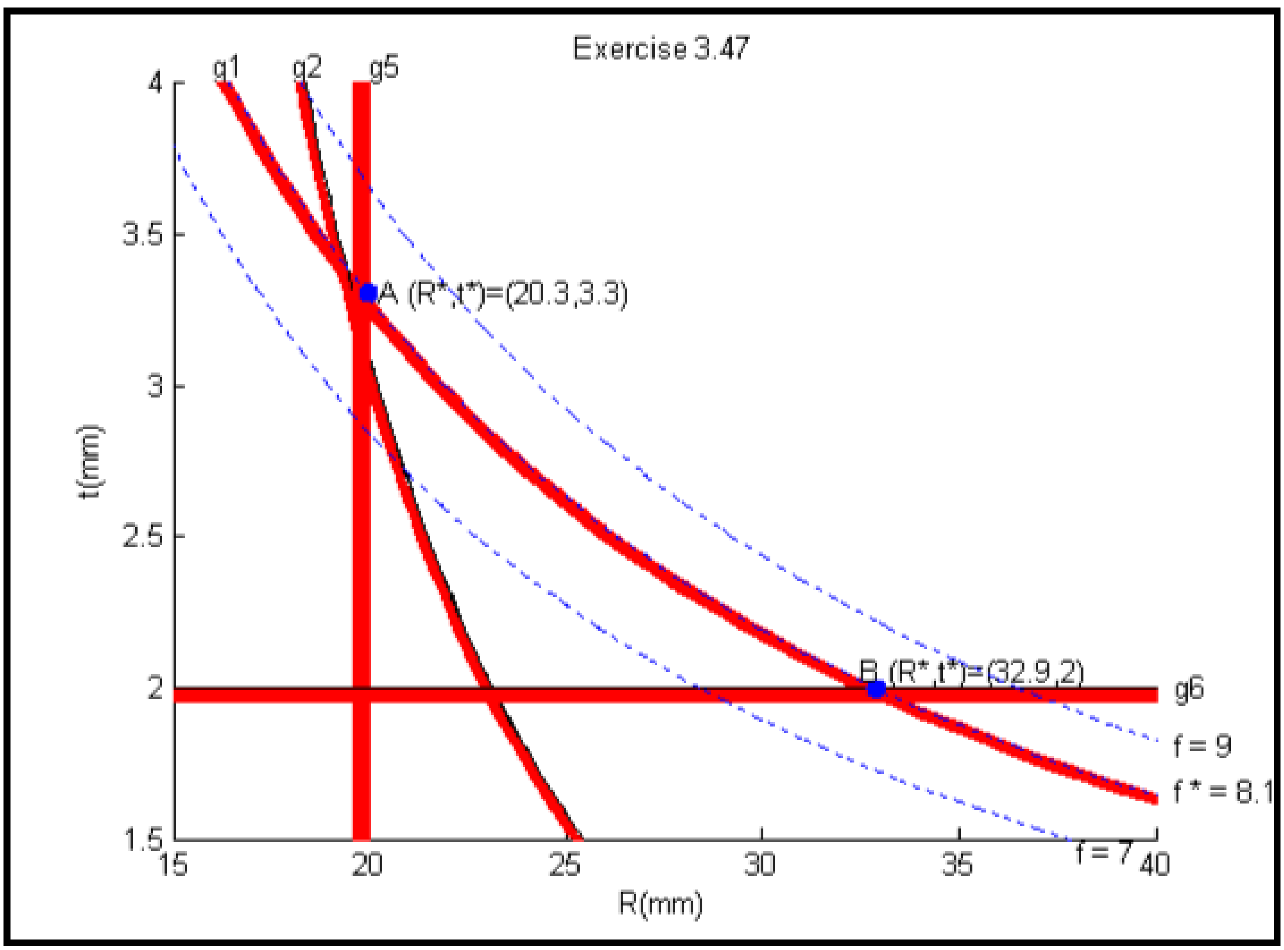
$g_4 = 20 - R \le 0;$

$g_5 = R - 400 \le 0;$

$g_6 = 2 - t \le 0;$

$g_7 = t - 40 \le 0$

Optimum solution: multiple optimum points on the g_1 (member stress) constraint between the points (20, 3.3) and (32.9, 2); $f^* = 8.1$ kg.



MATLAB Code: 3.47

```
% 3.47 graphical solution by Jun Choi
[R,t] = meshgrid (15:1:40 , 1.5:0.1:4);
f = 0.123308*R.*t;
g1 = (16459.5./(R.*t))-250;
g2 = 1.034188*10^5 - (4.16726*R.^3).*t;
g3 = R./t-50;
g4 = 20 - R;
g5 = R - 400;
g6 = 2 - t;
g7 = t - 40;
cla reset
axis auto
xlabel ('R(mm)'),ylabel ('t(mm)')
title('Exercise 3.47','FontSize',10)
hold on
cv1 = [0: 0.1:0.3];
const1 = contour(R,t,g1,cv1,'k');
text(16,4.05,'g1')
cv11 = [0:0.1:5];
const1 = contour(R,t,g1,cv11,'r');
cv2 = [0:100:300];
const2 = contour(R,t,g2,cv2,'k');
text(18,4.05,'g2')
cv22 = [1000:10:4000];
const2 = contour(R,t,g2,cv22,'r');
%Not Shown in Figure
cv3 = [0:0.1:2];
const3 = contour(R,t,g3,cv3,'k');
cv33 = [0:1:10];
const3 = contour(R,t,g3,cv33,'r');
cv4 = [0:0.01:0.04];
const4 = contour(R,t,g4,cv4,'k');
text(20,4.05,'g5')
cv44 = [0:0.01:0.4];
const4 = contour(R,t,g4,cv44,'r');
%Not Shown in Figure
cv5 = [0 0];
const5 = contour(R,t,g5,cv5,'k');
cv55 = [5:0.1:6];
const5 = contour(R,t,g5,cv55,'r');
cv6 = [0.0:0.001: 0.01];
const6 = contour(R,t,g6,cv6,'k');
text(40.5,2,'g6')
cv66 = [0.01:0.001:0.05];
```

```
const6 = contour(R,t,g6,cv66,'r');
%Not Shown in Figure
cv7 = [0 0];
const7 = contour(R,t,g7,cv7,'k');
cv77 = [5:1:10];
const7 = contour(R,t,g7,cv77,'r');
plot(20,3.3,'o','LineWidth',4,'MarkerSize',3)
text(20.2,3.3,'A (R*,t*)=(20.3,3.3)');
plot(32.9,2,'o','LineWidth',4,'MarkerSize',3)
text(32.5,2.05,'B (R*,t*)=(32.9,2)');
fv=[7,8.1,9];
fs = contour (R,t,f,fv,'b:');
text(38,1.45,'f = 7');text(40.5,1.65,'f * = 8.1');text(40.5,1.8,'f = 9');
% Additional Comment
hold off
```

3.48

Design a symmetric structure defined in Exercise 3.46 treating cross-sectional area A and height h as design variables. The design variables must also satisfy the constraints $1 \le A \le 50$ cm^2 and $0.5 \le h \le 3$ m.

Solution

Refer to Fig. 2.1 and the formulation in Section 2.5. Treating cross-sectional area A (for each member) and h as design variables, the problem is formulated as follows:

$$f = 2\rho A l = 2\rho A\left[h^2 + (s/2)^2\right]^{0.5}$$

$$g_1 = -F/A - \sigma_a = 0.5W\left[h^2 + (s/2)^2\right]^{0.5}(\sin\theta/h + 2\cos\theta/s)/A - \sigma_a \le 0$$

$$g_2 = -F_1 - P_{cr} = 0.5W\left[h^2 + (s/2)^2\right]^{0.5}(\sin\theta/h + 2\cos\theta/s) - \pi^2 E\left(\beta A^2\right)\Big/\left[h^2 + (s/2)^2\right] \le 0$$

Horizontal Deflection, $u \le \Delta_u$;

$$g_3 = 2W\left[h^2 + (s/2)^2\right]^{1.5}\cos\theta\Big/\left(AEs^2\right) - \Delta_u \le 0$$

Vertical Deflection, $v \le \Delta_v$;

$$g_4 = W\left[h^2 + (s/2)^2\right]^{1.5}\sin\theta\Big/\left(2AEh^2\right) - \Delta_v \le 0$$

$$g_5 = -A + A_{\min} \le 0;$$

$$g_6 = A - A_{\max} \le 0;$$

$$g_7 = -h + h_{\min} \le 0;$$

$$g_8 = h - h_{\max}$$

Substituting the data given in Exercise 3.46 and the data, $A_{\min}$= 1 cm^2 = 100 mm^2, $A_{\max}$ = 50 cm^2 = 5000 mm^2, $h_{\min}$= 0.5 m = 500 mm, $h_{\max}$= 3 m = 3000 mm, and β = 1, the cost and constraints are:

$$f = 2\left(7.5\times10^{-6}\right)A\left[h^2 + (750)^2\right]^{\frac{1}{2}} = \left(1.57\times10^{-5}\right)A\left[h^2 + 5.625\times10^5\right]^{\frac{1}{2}}$$

$$g_1 = 0.5\left(10^5\right)\left(h^2 + 5.625\times10^5\right)^{\frac{1}{2}}\left[0.5/h + 0.866025/750\right]/A - 250 \le 0$$

$$g_2 = 0.5\left(10^5\right)\left(h^2 + 5.625\times10^5\right)^{\frac{1}{2}}\left[0.5/h + 0.866025/750\right]$$

$$-2.072617\times10^6\, A^2\Big/\left(h^2 + 5.625\times10^5\right) \le 0$$

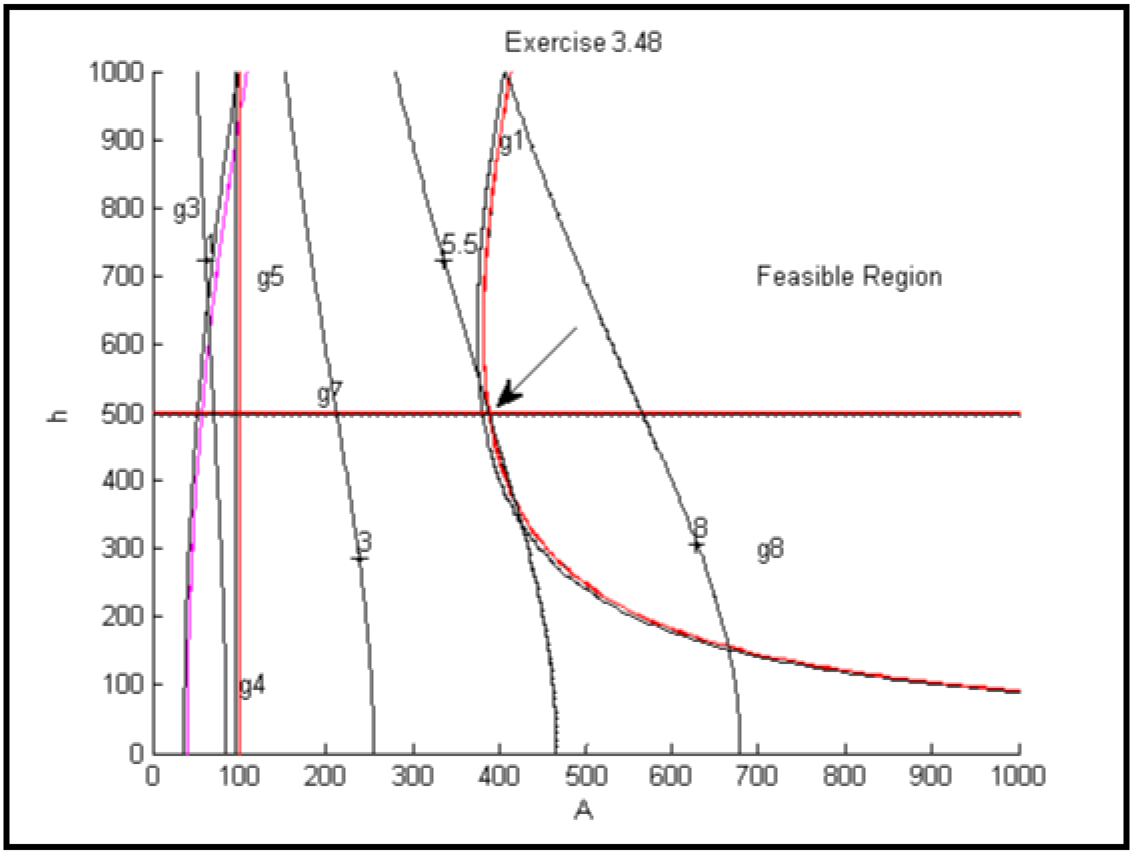
$$g_3 = \left(1.46628\times10^{-6}\right)\left(h^2 + 5.625\times10^5\right)^{1.5}/A - 50 \le 0$$

$$g_4 = 0.119048\left(h^2 + 5.625\times10^5\right)^{1.5}/Ah^2 - 50 \le 0$$

$$g_5 = 100 - A \le 0;$$

$g_6 = A - 5000 \le 0;$
$g_7 = 500 - h \le 0;$
$g_8 = h - 3000 \le 0$

Optimum solution: $A^* \doteq 390$ mm^2, $h^* = 500$ mm, $f^* \doteq 5.5$ kg; g_1 (member stress constraint) and g_7 (min. height constraint) are active.



<u>MATLAB Code: 3.48</u>

```
[A,h]=meshgrid(0:10:1000.0, 0:10:1000.0);
ro=7850*1e-9;s=1500;w=10^5;theta=30*pi/180;sa=250;
du=50;dv=50;Es=210000;Amin=100;Amax=5000;hmin=500;hmax=3000;
l=sqrt(h.^2+(s/2)^2);
F= -.5*w*l.*(sin(theta)./h+2*cos(theta)/s);
Pcr=Es*A.^2*pi^2./l^2;
f=2*ro*A.*l;
g1=-F./A-sa;
g2=-F-Pcr;
g3=2*w*(h.^2+(s/2)^2)^1.5*cos(theta)./(A*Es*s^2)-du;
g4=w*(h.^2+(s/2)^2)^1.5*sin(theta)./(2*A*Es*h.^2)-dv;
g5=-A+Amin;
g6=A-Amax;
g7=-h+hmin;
g8=h-hmax;
cla reset
axis auto   %Minimum and maximum values for axes are determined automatically
xlabel('A'),ylabel('h')   %Specifies labels for x- and y-axes
title('Exercise 3.48')
hold on
cv1=[0 5];
const1=contour(A,h,g1,cv1,'k');
text(400,900,'g1')
cv11=[0.05:0.01: .5];
const11=contour(A,h,g1,cv11,'r');
cv2=[0 5];
const2=contour(A,h,g2,cv2,'k');
text(300,1200,'g2')
cv22=[0.05:0.01: .5];
const21=contour(A,h,g2,cv22,'r');
cv3=[0 5];
const3=contour(A,h,g3,cv3,'k');
text(25,800,'g3')
cv31=[0.0025:0.0025:.03];
const31=contour(A,h,g3,cv31,'m');
cv4=[0 5];
const4=contour(A,h,g4,cv1,'k');
text(100,100,'g4')
cv41=[0.0025:0.0025:.03];
const41=contour(A,h,g4,cv41,'r');
cv5=[0 5];
const5=contour(A,h,g5,cv1,'k');
text(120,700,'g5')
cv51=[0.0025:0.0025:.03];
```

```
const51=contour(A,h,g5,cv51,'r');
cv6=[0 5];
const6=contour(A,h,g6,cv1,'k');
text(8000,400,'g6')
cv61=[0.0025:0.0025:.03];
const61=contour(A,h,g6,cv61,'r');
cv7=[0 5];
const7=contour(A,h,g7,cv1,'k');
text(190,530,'g7')
cv71=[0.0025:0.0025:.03];
const71=contour(A,h,g7,cv71,'r');
cv8=[0 5];
const8=contour(A,h,g8,cv8,'k');
text(700,300,'g8')
cv81=[0.0025:0.0025:.03];
const81=contour(A,h,g8,cv81,'r');
text(700,700,'Feasible Region')
fv=[1, 3, 5.5, 8];
fs=contour(A,h,f,fv,'k--');
clabel(fs)
hold off
```

3.49

Design a symmetric structure defined in Exercise 3.46 treating cross-sectional area A and span s as design variables. The design variables must also satisfy the constraints $1 \le A \le 50$ cm^2 and $0.5 \le s \le 4$ m.

Solution

Refer to Fig. 2.2 and the formulation of the two bar truss problem in Section 2.5. Treating cross-sectional area A and the span s as design variables, the problem is formulated as follows:

$$f = 2\rho A l = 2\rho A\left[h^2+(s/2)^2\right]^{0.5}$$

$$g_1 = -F_1/A - \sigma_a = 0.5W\left[h^2+(s/2)^2\right]^{0.5}(\sin\theta/h + 2\cos\theta/s)/A - \sigma_a \le 0$$

$$g_2 = -F_1 - P_{cr} = 0.5W\left[h^2+(s/2)^2\right]^{0.5}(\sin\theta/h + 2\cos\theta/s) - \frac{\pi^2 E\left(\beta A^2\right)}{h^2+(s/2)^2} \le 0$$

Horizontal Deflection, $u \le \Delta_u$;

$$g_3 = 2W\left[h^2+(s/2)^2\right]^{0.5}\cos\theta/\left(AEs^2\right) - \Delta_u \le 0$$

Vertical Deflection, $v \le \Delta_v$;

$$g_4 = W\left[h^2+(s/2)^2\right]^{1.5}\sin\theta/\left(2AEh^2\right) - \Delta_v \le 0$$

$$g_5 = -A + A_{\min} \le 0;$$

$$g_6 = A - A_{\max} \le 0;$$

$$g_7 = -s + s_{\min} \le 0;$$

$$g_8 = s - s_{\max} \le 0$$

Substituting the data given in Exercise 3.46 and the data, A_{min} = 100 mm^2, $A_{\max}$ = 5000 mm^2, $s_{\min}$ = 500 mm, $s_{\max}$ = 4000 mm, and β = 1, the cost and constraints become

$$f = \left(1.57\times10^{-5}\right)A\left(10^6+0.25s^2\right)^{0.5}; \quad g_1 = \left(0.25s^2+10^6\right)^{0.5}(25+86602.5/s)/A - 250 \le 0;$$

$$g_2 = 0.5\left(10^5\right)\left(10^6+0.25s^2\right)^{0.5}(0.5/1000 + 2(0.866025)/s) - \pi^2\left(2.1\times10^6\right)A^2/\left(10^6+0.25s^2\right)$$

$$= \left(10^6+0.25s^2\right)^{0.5}(25+86602.5/s) - \left(2.072617\times10^7\right)A^2/\left(10^6+0.25s^2\right) \le 0$$

$$g_3 = 0.82479\left(10^6+0.25s^2\right)^{1.5}/\left(As^2\right) - 50 \le 0;$$

$$g_4 = \left(1.19048\times10^{-7}\right)\left(10^6+0.25s^2\right)^{1.5}/A - 50 \le 0$$

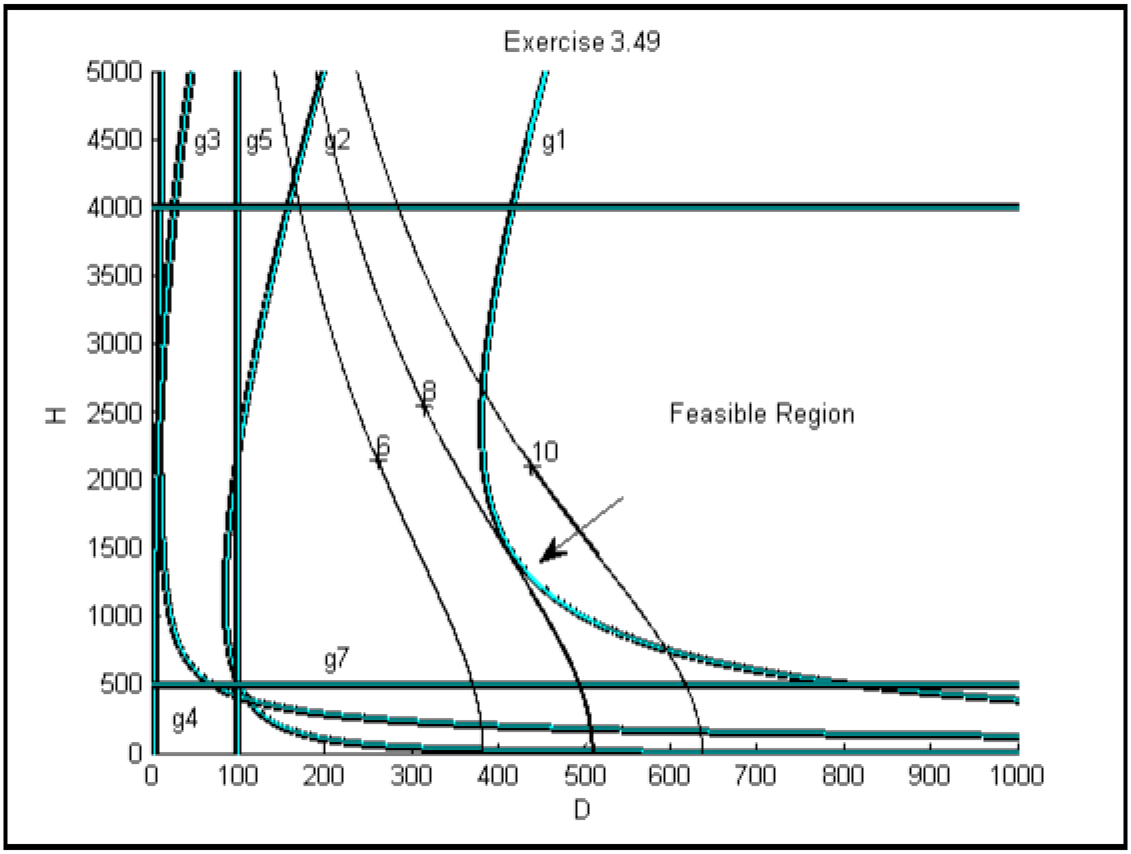
$$g_5 = 100 - A \le 0;$$

$$g_6 = A - 5000 \le 0;$$

$$g_7 = 500 - s \le 0;$$

$g_8 = s - 4000 \le 0$

Optimum solution: $A^* \doteq 410$ mm^2, $s^* = 1500$ mm, $f^* \doteq 8.0$ kg; g_1 (member stress constraint) is active.



MATLAB Code: 3.49

```
[A,s]=meshgrid(0:1:1000, 0:10:5000);
                                %Enter functions for the minimization problem
f=(1.57*10^-5)*A.*((10^6+0.25*s.*s).^0.5);
g1=-250+((10^6+0.25*s.*s).^0.5).*(25+86602.5./s)./A;
g2=(25+86602.5./s).*((10^6+0.25*s.*s).^0.5)-(2.072617*10^7)*A.*A./((10^6+0.25*s.*s));
g3=-50*(A.*s.*s)+0.82479*(10^6+0.25*s.*s).^1.5;
g4=-50*A+(1.19048*10^-7)*(10^6+0.25*s.*s).^1.5;
g5=100-A;
g6=A-5000;
g7=500-s;
g8=s-4000;
cla reset
axis auto                       %Minimum and maximum values for axes are determined automatically
xlabel('D'),ylabel('H')         %Specifies labels for x- and y-axes
hold on                         %retains the current plot and axes properties for all subsequent plots
cv1=[0 0];
const1=contour(A,s,g1,cv1,'k','LineWidth',3);
text(450,4500,'g1')
cv11=[0.001:0.001:0.01];
cv22=[0.0001:0.0001:0.001];
const1=contour(A,s,g1,cv11,'c');
const2=contour(A,s,g2,cv1,'k','Linewidth',3);
const2=contour(A,s,g2,cv11,'c');
text(200,4500,'g2')
const3=contour(A,s,g3,cv1,'k','Linewidth',3);
const3=contour(A,s,g3,cv22,'c');
text(50,4500,'g3')
const4=contour(A,s,g4,cv1,'k','Linewidth',4);
const4=contour(A,s,g4,cv11,'c');
text(25,250,'g4')
const5=contour(A,s,g5,cv1,'k','Linewidth',3);
const5=contour(A,s,g5,cv11,'c');
text(110,4500,'g5')
const6=contour(A,s,g6,cv1,'k','Linewidth',3);
const6=contour(A,s,g6,cv11,'c');
text(6000,0.005,'g6')
const7=contour(A,s,g7,cv1,'k','Linewidth',3);
const7=contour(A,s,g7,cv11,'c');
text(200,690,'g7')
const8=contour(A,s,g8,cv1,'k','Linewidth',3);
const8=contour(A,s,g8,cv11,'c');
text(8000,2.1,'g8')
text(600,2500,'Feasible Region')
fv=[6 8 10];                    %Defines contours for the minimization function
```

```
fs=contour(A,s,f,fv,'k');      %'k' specifies black dashed lines for function contours
clabel(fs)                     %Automatically puts the contour value on the graph
hold off                       %Indicates end of this plotting sequence
```

3.50

A minimum mass structure (area of member 1 is the same as member 3) three-bar truss is to be designed to support a load P as shown in Fig. 2.9. The following notation may be used: $P_u = P\cos\theta$, $P_v = P\sin\theta$, A_1 = cross-sectional area of members 1 and 3, A_2 = cross-sectional area of member 2.

The members must not fail under the stress, and deflection at node 4 must not exceed 2cm in either direction. Use Newtons and millimeters as units. The data is given as P = 50 kN; θ = 30°; mass density, ρ = 7850 kg/m^3; modulus of elasticity, E = 210 GPa; allowable stress, σ_a = 150 MPa. The design variables must also satisfy the constraints $50 \le A_i \le 5000$ mm^2.

Solution

Referring to Section 2.10, the problem is formulated as follows:

$$f = \rho l\left(2\sqrt{2}A_1 + A_2\right)$$

$$g_1 = \left[P\cos\theta/A_1 + P\sin\theta\Big/\left(A_1 + \sqrt{2}A_2\right)\right]\Big/\sqrt{2} - \sigma_a \le 0;$$

$$g_2 = \sqrt{2}\ P\sin\theta\Big/\left(A_1 + \sqrt{2}A_2\right) - \sigma_a \le 0;$$

$$g_3 = \sqrt{2}Pl\cos\theta/A_1E - \Delta_u \le 0;$$

$$g_4 = \sqrt{2}Pl\sin\theta\Big/\left[\left(A_1 + \sqrt{2}A_2\right)E\right] - \Delta_v \le 0;$$

$$g_5 = -A_1 + A_{1\min} \le 0;$$

$$g_6 = A_1 - A_{1\max} \le 0;$$

$$g_7 = -A_2 + A_{2\min} \le 0;$$

$$g_8 = A_2 - A_{2\min} \le 0$$

Use kilogram, Newton and millimeter as units for the given data:

P = 50 kN = 5.0×10^4 N, l = 1 m = 1000 mm, θ= 30°, ρ = 7850 kg/m^3 = 7.85×10^6 kg/mm^3,

σ_a= 150 MPa = 150 N/mm^2, $A_{i\min}$ = 50 mm^2, $A_{i\max}$ = 5000 mm^2, E = 210 GPa = 2.1×10^5 N/mm^2

Substituting these data into cost and constraints, we get

$$f = 7.85\times10^{-6}(1000)\left(2\sqrt{2}A_1 + A_2\right) = \left(7.85\times10^{-3}\right)\left(2\sqrt{2}A_1 + A_2\right);$$

$$g_1 = 30618.6/A_1 + 17677.65\Big/\left(A_1 + \sqrt{2}A_2\right) - 150 \le 0;$$

$$g_2 = 35355.3\Big/\left(A_1 + \sqrt{2}A_2\right) - 150 \le 0;$$

$$g_3 = 291.6/A_1 - 20 \le 0;$$

$$g_4 = 168.4\Big/\left(A_1 + \sqrt{2}A_2\right) - 20 \le 0;$$

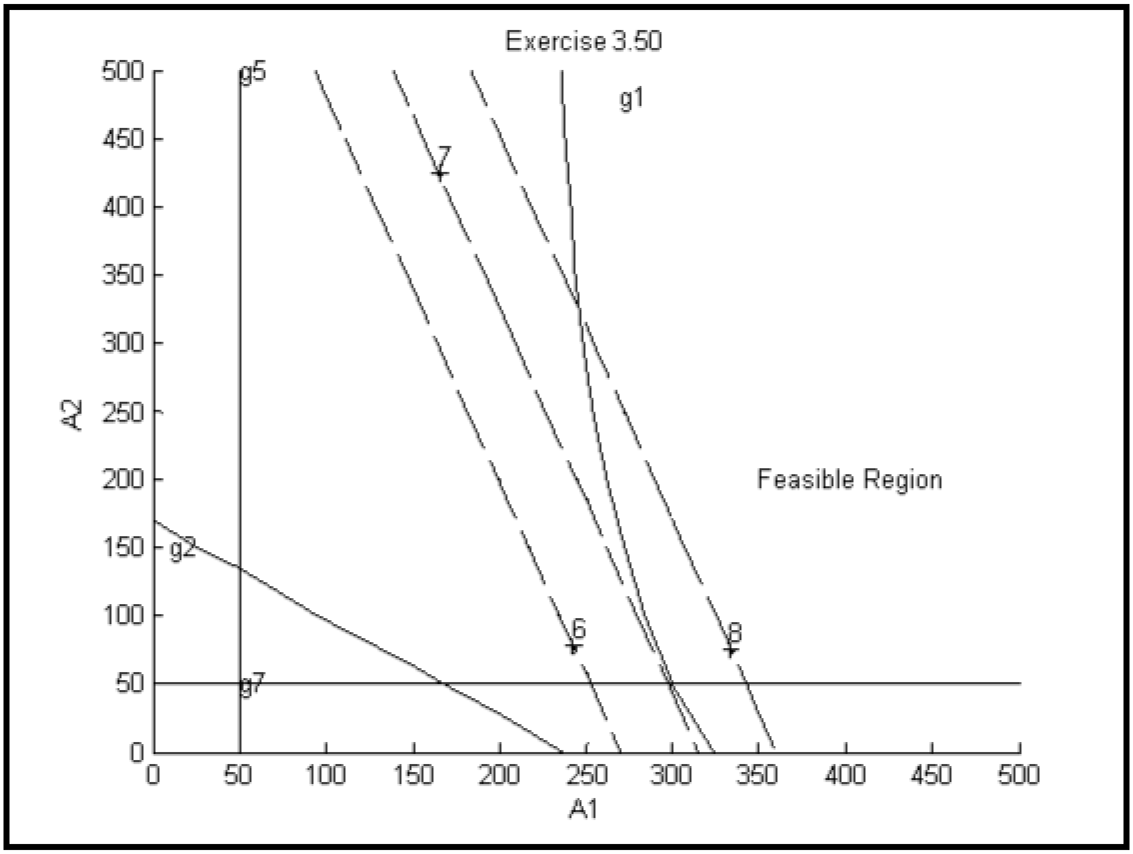
$$g_5 = 50 - A_1 \le 0;$$

$$g_6 = A_1 - 5000 \le 0;$$

$g_7 = 50 - A_2 \leq 0;$
$g_8 = A_2 - 5000 \leq 0$

The constraints g_3 and g_4, are redundant and can be neglected.

Optimum solution: $A_1^* \doteq 300$ mm^2, $A_2^* \doteq 50.0$ mm^2, $f^* \doteq 7.0$ kg; g_1 (member 1 stress constraint) is active.



MATLAB Code: 3.50

```
                        %Create a grid from 0 to 500 with an increment of 1 for the variables A1 and A2
[A1,A2]=meshgrid(0:50:500.0, 0:50:500.0);
                        %Enter functions for the minimum mass structure problem
f=0.00785*(2*sqrt(2)*A1+A2);
g1=30618.6./A1+17677.65./(A1+sqrt(2)*A2)-150;
g2=35355.3./(A1+sqrt(2)*A2)-150;
g5=50-A1;
g6=A1-5000;
g7=50-A2;
g8=A2-5000;
                              %Initialization statements; these need not end with a semicolon
cla reset
axis auto                     %Minimum and maximum values for axes are determined automatically
                              %Limits for x- and y-axes may be specified with the command
                              %axis ([xmin xmax ymin ymax])
xlabel('A1'),ylabel('A2')     %Specifies labels for x- and y-axes
title('Exercise 3.50')
hold on
cv1=[0 0];
const1=contour(A1,A2,g1,cv1,'k');
text(270,480,'g1')
cv2=[0 0];
const2=contour(A1,A2,g2,cv2,'k');
text(10,150,'g2')
cv3=[0 0];
const3=contour(A1,A2,g5,cv3,'k');
text(50,500,'g5')
cv4=[0 0];
const4=contour(A1,A2,g7,cv4,'k');
text(50,50,'g7')
text(350,200,'Feasible Region')
fv=[6.0, 7.0, 8.0];
fs=contour(A1,A2,f,fv,'k--');
clabel(fs)
hold off
```

3.51*———————————————————————————————————

Design of a water tower support column. As a member of the ABC consulting Engineers you have been asked to design a cantilever cylindrical support column of minimum mass for a new water tank. The tank itself has already been designed in the tear-drop shape shown in Fig. E3.51. The height of the base of the tank (H), the diameter of the tank (D), and wind pressure on the tank (w) are given as H = 30 m, D = 10 m, and w = 700 N/m^2. Formulate the design optimization problem and solve it graphically. (created by G.Baenziger).

In addition to designing for combined axial and bending stresses and buckling, several limitations have been placed on the design. The support column must have an inside diameter of at least 0.70 m (d_i) to allow for piping and ladder access to the interior of the tank. To prevent local buckling of the column walls the diameter/thickness ratio (d_0/t) shall not be greater than 92. The large mass of water and steel makes deflections critical as they add to the bending moment. The deflection effects as well as an assumed construction eccentricity (e) of 10 cm must be accounted for in the design process. Deflection at C.G. of the tank should not be greater than Δ.

Limits on the inner radius and wall thickness are $0.35 \le R \le 2.0$ m and $1.0 \le t \le 20$ cm.

Pertinent constraints and formulas

Height of water tank,	h = 10 m
Allowable deflection,	Δ = 20 cm
Unit weight of water,	γ_w= 10 kN/m^3
Unit weight of steel,	γ_s= 80 kN/m^3
Modulus of elasticity,	E = 210 GPa
Moment of inertia of the column,	$I = \frac{\pi}{64}[d_o^4 - (d_o - 2t)^4]$
Cross-sectional area of column material,	$A = \pi t(d_o - t)$
Allowable bending stress,	σ_b = 165 MPa
Allowable axial stress,	$\sigma_a = \frac{12\pi^2 E}{92(H/r)^2}$ (calculated using the critical buckling load with factor of safety of $\frac{23}{12}$
Radius of gyration,	$r = \sqrt{\frac{I}{A}}$
Average thickness of tank wall,	t_t = 1.5 cm
Volume of tank,	$V = 1.2\pi D^2 h$
Surface area of tank,	$A_s = 1.25\pi D^2$
Projected area of tank, for wind loading,	$A_p = \frac{2Dh}{3}$
Load on the column due to weight of water and steel tank,	$P = V\gamma_w + A_s t_t \gamma_s$
Lateral load at the tank C.G due to wind pressure,	$W = wA_p$
Deflection at C.G. of tank,	$\delta = \delta_1 + \delta_2$,where

$$\delta_1 = \frac{WH^2}{12EI}(4H + 3h)$$
$$\delta_2 = \frac{H}{2EI}(0.5Wh + Pe)(H + h)$$

Moment at base, $M = W(H + 0.5h) + (\delta + e)P$

Bending stress, $f_b = \frac{M}{2I}d_o$

Axial stress, $f_a = \left(P/A\right) = \frac{V\gamma_w + A_s t_t \gamma_s}{\pi t(d_o - t)}$

Combined stress constraint, $\frac{f_a}{\sigma_a} + \frac{f_b}{\sigma_b} \le 1$

Gravitational acceleration, g = 9.81 m/s^2

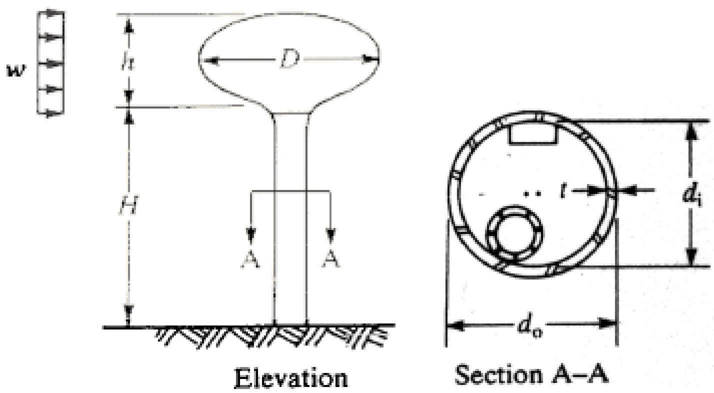


FIGURE E3.51 Water Tower support column.

Solution

Design of a water tower support column-

Design Variables: R = mean radius; t = wall thickness

Cost Function: Minimize the mass of the support column; Constraints:

$\left(f_a/\sigma_a\right)+\left(f_b/\sigma_b\right)\le 1;\;\; d_i \ge 0.7\text{ m};\;\; d_o/t \le 92;\;\; \delta \le \Delta;\;\; 0.35 \le \left(R-0.5t\right) \le 2.5\text{ m};\; 1.0 \le t \le 40\text{ cm}$

Use Newtons and centimeters as units. The pertinent constants are transformed into the adopted units: D = 10 m = 1000 cm; H = 30 m = 3000 cm; w = 700 N/m^2 = 0.07 N/cm^2; h = 10 m = 1000 cm; e = 10 cm; Δ = 20 cm; γ_w = 10 kN/m^3 = 0.01 N/cm^3; γ_s = 80 kN/m^3 = 0.08 N/cm^3; E = 210 GPa = 2.1×10^7 N/cm^2; σ_b = 165 MPa = 1.65×10^4 N/cm^2; t_t = 1.5 cm, g = 9.81 m/s^2 = 981 cm/s^2

Simplify some quantities in terms of design variables:

$d_o = (2R+t)$; $d_i = (2R-t)$; $I = \pi\left(d_o^4 - d_i^4\right)/64 = \pi\left(R^3 t + Rt^3/4\right)$;

$A = \pi\left(d_o^2 - d_i^2\right)/4 = 2\pi Rt$; $\quad r = (I/A)^{\frac{1}{2}} = \left[0.5\left(R^2 + t^2/4\right)\right]^{\frac{1}{2}}$;

$V = 1.2\pi D^2 h_2 = 1.2\pi(1000)^2(1000) = 3.7699\times10^9\ \text{cm}^3$;

$A_s = 1.25\pi(1000)^2 = 3.92699\times10^6\ \text{cm}^2$;

$A_p = 2Dh/3 = 2(1000)(1000)/3\left(2.0\times10^6\right)/3\ \text{cm}^2$;

$P = V\gamma_w + A_s t_t \gamma_s = \left(3.7699\times10^9\right)(0.01) + \left(3.92699\times10^6\right)(1.5)(0.08) = 3.8170\times10^7\ \text{N}$

$W = wA_p = 0.07\left(2.0\times10^6\right)/3 = (1.4\times10^5)/3\ \text{N}$

$$\delta_1 = \frac{WH^2}{12EI}(4H+3H) = \frac{1.4\times10^5}{3(12)\left(2.1\times10^7\right)I}(4\times3000 + 3\times1000) = 2.5\times10^7/I\ \ \text{cm}$$

$$\delta_2 = (H/2EI)(Wh/2 + Pe)(H+h) = \left(3000/2\left(2.1\times10^7\right)I\right)$$

$$\left(\left(1.4\times10^5\right)1000/3(2) + \left(3.8170\times10^7\right)10\right)(3000+1000) = 1.15724\times10^8/I\ \ \text{cm}$$

$$\delta = \delta_1 + \delta_2 = 1.40724\times10^8/I\ \ \text{cm}$$

$$M = W(H + h/2) + P(\delta + e)$$

$$= \left(1.4\times10^5\right)(3000+500)/3 + \left(3.8170\times10^7\right)\left(1.40724\times10^8/I + 10\right)$$

$$= 5.45\times10^8 + 5.3715\times10^{15}/I\quad \text{N}\cdot\text{cm}$$

$$f_a = P/A = 3.8170\times10^7/2\pi Rt = 6.075\times10^6/Rt\ \ \text{N/cm}^2$$

$$f_b = Md_o/2I = \left(\left(5.4503\times10^8 + 5.3715\times10^{15}/I\right)(2R+t)\right)/2I$$

$$= \left(2.725\times10^8/I + 2.72515\times10^{15}/I^2\right)(2R+t) = \left(\frac{2.72515\times10^8}{\pi\left(R^3t + Rt^3/4\right)} + \frac{2.68575\times10^{15}}{\left[\pi\left(R^3t+Rt^3/4\right)\right]^2}\right)(2R+t)$$

$$= \left[\frac{8.6744\times10^7}{\left(R^3t + Rt^3/4\right)} + \frac{2.7212\times10^{14}}{\left(R^3t + Rt^3/4\right)^2}\right](2R+t)\ \ \text{N/cm}^2$$

$$\sigma_a = \frac{12\pi^2 E}{92(H/r)^2} = \frac{12\pi^2\left(2.1\times10^{14}\right)}{92\left[3000/\left(\frac{1}{2}\left(R^2+t^2/4\right)\right)^{\frac{1}{2}}\right]^2} = 1.50190\left(R^2 + t^2/4\right)$$

Writing the cost and constraints in terms of design variables, we get

$$f = (\gamma_s/g)(2\pi Rt)H = (0.08/9.81)(2\pi Rt)(3000) = 153.71Rt$$

$$f_a/\sigma_a + f_b/\sigma_b - 1 \le 0, \quad \text{or} \quad \frac{\left(6.075\times10^6\right)/Rt}{1.50190\left(R^2+t^2/4\right)}$$

$$+ \frac{\left\{\left[8.6744\times10^7/\left(R^3t+Rt^3/4\right)\right]+\left[2.7212\times10^{14}/\left(R^3t+Rt^3/4\right)^2\right]\right\}\left(2R+t\right)}{1.65\times10^4} - 1 \le 0;$$

$$g_1 = \frac{4.04488\times10^6}{\left(R^3t+Rt^3/4\right)} + \frac{5257.21\left(2R+t\right)}{\left(R^3t+Rt^3/4\right)} + \frac{\left(1.6492\times10^{10}\right)\left(2R+t\right)}{\left(R^3t+Rt^3/4\right)^2} - 1 \le 0;$$

$$g_2 = 70 - d_i = 70 - 2R + t \le 0;$$

$$g_3 = d_o/t - 92 = \left(2R+t\right)/t - 92 = 2R/t - 91 \le 0;$$

$$g_4 = \delta - \Delta = 1.4072\times10^8/\left[\pi\left(R^3t+Rt^3/4\right)\right] - 20 = 4.479\times10^7/\left(R^3t+Rt^3/4\right) - 20 \le 0;$$

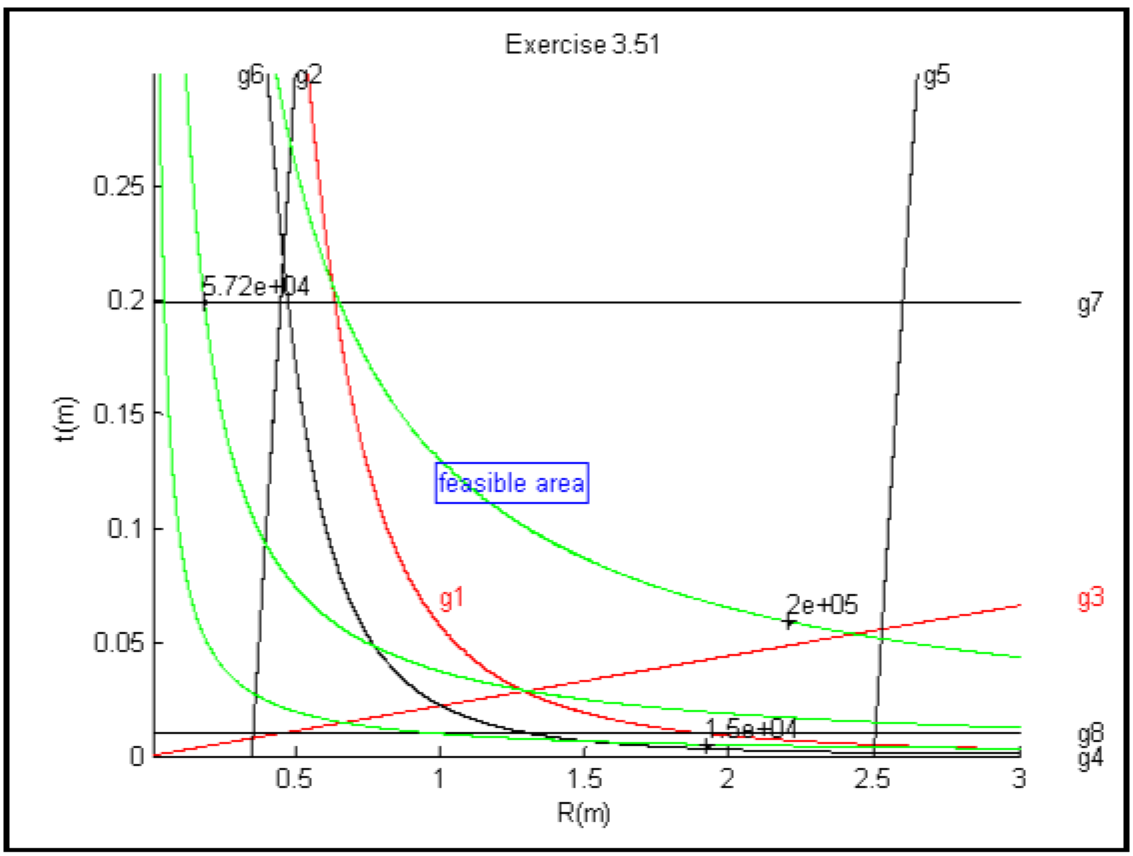
$$g_5 = R - 0.5t - 250 \le 0;$$

$$g_6 = 35 - R + 0.5t \le 0;$$

$$g_7 = t - 40 \le 0;$$

$$g_8 = 1 - t \le 0$$

Optimum solution: $R^* \doteq 130$ cm, $t^* \doteq 2.86$ cm, $f^* \doteq 57000$ kg; g_1 (combined stress constraint) and g_3 (diameter/thickness ratio constraint) are active.



MATLAB Code: 3.51

```
                    %Create a grid from -1 to 25 with an increment of 0.5 for the variables R and t
[R,t]=meshgrid(0.01:0.005:3,0.0:0.0005:0.3);
w=700;D=10;h=10;rw=10000;rs=80000;tt=0.015;E=210*10^9;H=30;e=0.1;
fai_b=165*10^6;g=9.81;
                                %some calculated parameters
Ap=2*D*h/3;                     %area of tank, for wind loading;
W=w*Ap;                         %lateral load at the tank C.G due to wind pressure
V=1.2*pi*D^2*h;
As=1.25*pi*D^2;
P=V*rw+As*tt*rs;
do=2*R+t;                       % diameter of outside column
di=2*R-t;                       % diameter of inside column
I=pi*( (R.^3).*t+R.*(t.^3)/4 ); %I=pi*(do.^4-di.^4)/64;% interia of the column
A=2*pi*R.*t;              %pi*(do.^2-di.^2)/4;    %
r=(0.5*(R.^2+t.^2/4)).^0.5; %(I./A).^0.5;
deta1=W*H^2/12/E*(4*H+3*h)./I;%/I
deta2=(H/2/E)*(W*h/2+P*e)*(H+h)./I;
deta=deta1+deta2;
M=W*(H+0.5*h)+(deta+e)*P;
fa=(P./A);
fb=M.*do/2./I;
fai_a=12*pi^2*E./((92*H^2)./(r.^2));%12*pi^2*E/(92*H^2)./(r.^2)
Mass=(do.*do-di.*di)*pi/4*H*rs/g;
f=Mass;
g1=fa./fai_a+fb./fai_b-1;
g2=0.7-di;
g3=do./t-92;
g4=deta-0.2;
g5=R-0.5*t-2.5;
g6=0.5*t-R+0.35;
g7=t-0.2;
g8=0.01-t;
cla reset
axis auto               %Minimum and maximum values for axes are determined automatically
                        %Limits for x- and y-axes may be specified with the command
                        %axis ([xmin xmax ymin ymax])
xlabel('R(m)'),ylabel('t(m)')
title('Exercise 3.51')
hold on
cv1=[-0.0 0.0];
const1=contour(R,t,g1,cv1,'r');
text(1.0,0.07,'g1','color','r')
cv2=[0.0 0.0];
const2=contour(R,t,g2,cv2,'k');
text(0.5,0.3,'g2')
```

```
cv3=[-0 0];
const3=contour(R,t,g3,cv3,'r');
text(3.2,0.07,'g3','color','r')
cv4=[-0.0 0.0];
const4=contour(R,t,g4,cv4,'k');
text(3.2,0.000,'g4')
cv5=[0 0];
const5=contour(R,t,g5,cv5,'--k');
text(2.67,.3,'g5')
cv6=[0.000 0.00];
const6=contour(R,t,g6,cv6,'--k');
text(0.3,0.3,'g6','color','k')
cv7=[-0.0 0.0];
const7=contour(R,t,g7,cv7,'--k');
text(3.2,0.200,'g7')
cv8=[0.0 0.1];
const8=contour(R,t,g8,cv8,'--k');
text(3.2,0.01,'g8')
fv=[5.7152*10^4, 2*10^5, 3.5^10^5, 1.5*10^4];    %Defines 4 contours for the profit function
fs=contour(R,t,f,fv,':g');
text(1.0,0.12,'feasible area','color','b','edgecolor','b')
clabel(fs)
hold off
```

3.52*

Design of a flag pole. Your consulting firm has been design a minimum mass flag pole of height *H*. The pole will be made of uniform hollow circular tubing with d_o and d_i as outer and inner diameters, respectively. The pole must not fail under the action of high winds.

For design purpose, the pole will be treated as a cantilever that is subjected to a uniform lateral wind load of *w* (kN/m). In addition to the uniform load, the wind introduces a concentrated load *P* (kN) at the top of the pole, as shown in Fig. E3.52. The flag pole must not fail in bending or shear. The deflection at the top should not exceed 10 cm. The ratio of mean diameter to thickness must not exceed 60. The pertinent data are given below. Assume any other data if needed. The minimum and maximum values of design variables are $5 \le d_o \le 50$ cm and $4 \le d_i \le 45$ cm.

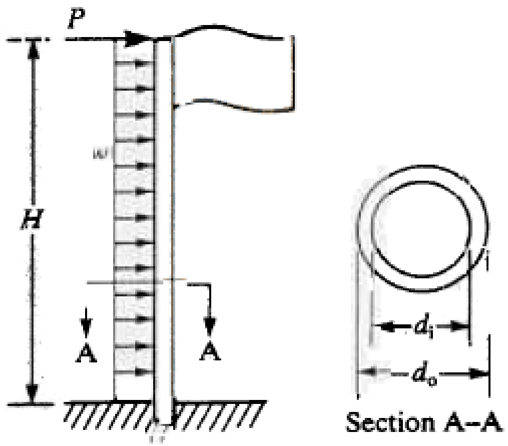


FIGURE E3-52 Flag pole.

Pertinent constraints and equations

Cross-sectional area,	$A = \frac{\pi}{4}(d_o^2 - d_i^2)$
Moment of inertia,	$I = \frac{\pi}{64}(d_o^4 - d_i^4)$
Modulus of elasticity,	E = 210 GPa
Allowable bending stress,	σ_b = 165 MPa
Allowable shear stress,	τ_s = 50 MPa
Mass density,	ρ = 7800 kg/m^3
Wind density,	*w* = 2.0 kN/m
Height of flag pole,	*H* = 10 m
Concentrated load at top,	*P* = 4.0 kN
Moment at the base,	$M = (PH+0.5wH^2)$, kN·m
Bending stress,	$\sigma = \frac{M}{2I}d_o, kPa$

Shear stress,	$S = (P + wH)$, kN
Deflection at the top,	$\delta = \dfrac{PH^3}{3EI} + \dfrac{wH^4}{8EI}$
Minimum and maximum thickness,	0.5 and 2 cm

Formulate the design problem and solve it using the graphical optimization technique.

<u>Solution:</u> Design of a flag pole-

Design Variables: d_o = outer diameter of the flag pole; d_i = inner diameter of the flag pole

Cost Function: $f = \rho AH = (\pi\rho H/4)(d_o^2 - d_i^2)$

Constraints : $\sigma \le \sigma_b$; $\tau \le \tau_s$; $\delta \le 10$ cm; $\dfrac{\text{mean diameter}}{\text{thickness}} \le 60$; $0.5 \le$ thickness ≤ 2 cm;

$5 \le d_o \le 50$; $4 \le d_i \le 45$ cm

Use units of Newtons and centimeters. The pertinent data are given as : P = 4.0 kN = 4000 N

E = 210 GPa = 2.1×10^7 N/cm^2; σ_b = 165 MPa = 1.65×10^4 N/cm^2; τ_s = 50 MPa = 5000 N/cm^2;

ρ = 7800 kg/m3 = 7.8×10^{-3} kg/cm^3; w = 2.0 kN/m = 20 N/cm; H = 10 m = 1000 cm;

$A = \pi(d_o^2 - d_i^2)/4$;

$I = \pi(d_o^4 - d_i^4)/64$;

$S = P + wH = 4000 + 20(1000) = 2.4\times10^4$ N

$$M = (PH + wH^2/2) = 4\times10^3(1000) + 20(1000)^2/2 = 1.4\times10^7 \text{ N.cm}$$

$$\sigma = Md_o/2I = \frac{1.4\times10^7 d_o}{2(\pi(d_o^4 - d_i^4)/64)} = \frac{1.42603\times10^8 d_o}{(d_o^4 - d_i^4)}$$

$$\tau = \frac{S}{12I}(d_o^2 + d_o d_i + d_i^2) = 40743.7(d_o^2 + d_o d_i + d_i^2)/(d_o^4 - d_i^4)$$

$$\delta = PH^3/3EI + wH^4/8EI = (PH^3/3 + wH^4/8)(1/EI)$$

$$= \left(\frac{4\times1000(1000)^3}{3} + \frac{20(1000)^4}{8}\right)\left(\frac{1}{2.1\times10^7(\pi(d_o^4 - d_i^4)/64)}\right) = \frac{3.71867\times10^6}{(d_o^4 - d_i^4)}$$

Substituting the preceding data into cost and constraints, we get:

$f = \left((\pi/4)\left(7.8\times10^{-3}\right)(1000)\right)\left(d_o^2 - d_i^2\right) = 6.12611\left(d_o^2 - d_i^2\right)$, kg

$g_1 = \sigma - \sigma_b = 1.42603\times10^8\, d_o \big/ \left(d_o^4 - d_i^4\right) - 1.65\times10^4 \le 0$ (bending stress)

$g_2 = \tau - \tau_s = 40743.7\left(d_o^2 + d_o d_i + d_i^2\right)\big/\left(d_o^4 - d_i^4\right) - 5000 \le 0$ (sheer stress)

$g_3 = \delta - 10 = 3.71867\times10^6 \big/ \left(d_o^4 - d_i^4\right) - 10 \le 0$ (top deflection)

$g_4 = \left(d_o + d_i\right)\big/\left(d_o - d_i\right) - 60 \le 0$ (diameter-thickness ratio)

$g_5 = \left(d_o - d_i\right)\big/2 - 2 \le 0$ (maximum thickness)

$g_6 = 0.5 - \left(d_o - d_i\right)\big/2 \le 0$ (minimum thickness)

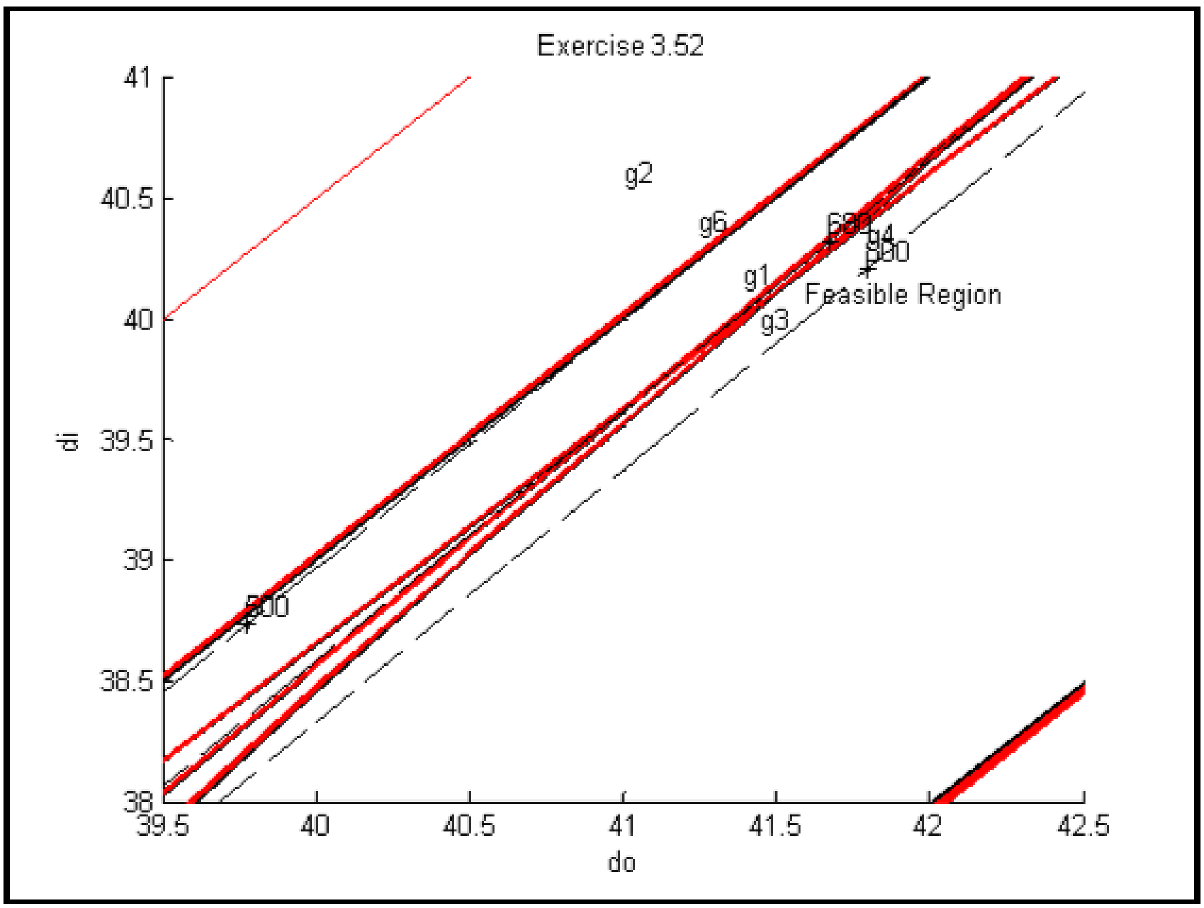
$g_7 = d_o - 50 \le 0;$

$g_8 = 5 - d_o \le 0;$

$g_9 = d_i - 45 \le 0;$

$g_{10} = 4 - d_i \le 0$

Optimum solution: $d_0^* \doteq 41.56$ cm, $d_i^* \doteq 40.19$ cm, $f^* = 680.0$ kg; g_3 (top deflection constraint) and g_4 (diameter/thickness ratio constraint) are active.



MATLAB Code: 3.52

```
[do,di]=meshgrid(39.5:0.5:42.5,38:0.5:41);
f=6.12611*(do.^2-di.^2);
g1=1.42603*10^8*do./(do.^4-di.^4)-1.65*10^4;
g2=40743.7*(do.^2+do.*di+di.^2)./(do.^4-di.^4)-5000;
g3=3.71867*10^6./(do.^4-di.^4)-10;
g4=(do+di)./(do-di)-60;
g5=(do-di)/2-2;
g6=0.5-(do-di)/2;
g7=do-50;
g8=5-do;
g9=di-45;
g10=4-di;
cla reset
axis auto
xlabel('do'),ylabel('di')
title('Exercise 3.52')
hold on
cv1=(0:1:80);
cv11=(80:1:300);
const1=contour(do,di,g1,cv1,'k');
const11=contour(do,di,g1,cv11,'r');
text(41.4,40.18,'g1')
cv2=(0:0.1:20);
cv22=(20:1:500);
const2=contour(do,di,g2,cv2,'k');
const22=contour(do,di,g2,cv22,'r');
text(41,40.6,'g2')
cv3=(0:0.001:0.075);
cv33=(0.075:0.001:0.2);
const3=contour(do,di,g3,cv3,'k');
const33=contour(do,di,g3,cv33,'r');
text(41.45,40,'g3')
cv4=(0.0:0.01:0.3);
cv44=(0.3:0.01:1.0);
const4=contour(do,di,g4,cv4,'k');
const44=contour(do,di,g4,cv44,'r');
text(41.8,40.35,'g4')
cv5=(0:0.001:0.01);
cv55=(0.01:.001:.03);
const5=contour(do,di,g5,cv5,'k');
const55=contour(do,di,g5,cv55,'r');
text(43,39.1,'g5')
cv6=(0:0.001:0.006);
cv66=(0.006:.001:.015);
```

```
const6=contour(do,di,g6,cv6,'k');
const66=contour(do,di,g6,cv66,'r');
text(41.25,40.4,'g6')
cv7=(0:0.001:0.012);
cv77=(0.012:.001:.04);
const7=contour(do,di,g7,cv7,'k');
const77=contour(do,di,g7,cv77,'r');
text(49.8,40.37,'g7')
cv8=(0:0.001:0.012);
cv88=(0.012:.001:.04);
const8=contour(do,di,g8,cv8,'k');
const88=contour(do,di,g8,cv88,'r');
text(5.2,40.37,'g8')
cv9=(0:0.001:0.012);
cv99=(0.012:.001:.04);
const9=contour(do,di,g9,cv9,'k');
const99=contour(do,di,g9,cv99,'r');
text(40,44.8,'g9')
cv10=(0:0.001:0.012);
cv1010=(0.012:.001:.04);
const10=contour(do,di,g10,cv10,'k');
const1010=contour(do,di,g10,cv1010,'r');
text(40,5.1,'g10')
text(41.6,40.1,'Feasible Region')
fv=[500, 680, 800];
fs=contour(do,di,f,fv,'k--');
clabel(fs)
hold off
```

3.53*

Design of a sign support column. The design department of a company has been asked to design a support column of minimum weight for the sign shown. The height to the bottom of the sign H, the width of the sign b, and the wind pressure p, on the sign are as follows: H = 20 m, b = 8 m, p = 800 N/m^2 (Figure E3.53).

The sign itself weights 2.5 kN/m^2 (w). The column must be safe with respect to combined axial and bending stresses. The allowable axial stress includes a factor of safety with respect to buckling. To prevent local buckling of the plate the diameter/thickness ratio d_o/t must not exceed 92. Note that the bending stress in the column will increase as a result of the deflection of the sign under the wind load. The maximum deflection at the center of gravity of the sign should not exceed 0.1 m. The minimum and maximum values of design variables are $25 \le d_o \le 150$ cm and $0.5 \le t \le 10$ cm (created by H. Kane).

Pertinent constraints and equations

Height of the sign,	$h = 4.0$ m
Area,	$A = \frac{\pi}{4}[d_o^2 - (d_o - 2t)^2]$
Moment of inertia,	$I = \frac{\pi}{64}[d_o^4 - (d_o - 2t)^4]$
Radius of gyration,	$r = \sqrt{I/A}$
Young's modulus (aluminum alloy),	E = 75 GPa
Unit weight of material,	γ = 80 kN/m^3
Allowable bending stress,	σ_b = 140 MPa
Allowable axial stress,	$\sigma_a = \frac{12\pi^2 E}{92(H/r)^2}$
Wind force,	$F = pbh$
Weight of sign,	$W = wbh$
Deflection at center of gravity of sign,	$\delta = \frac{F}{EI}\left(\frac{H^3}{3} + \frac{H^2 h}{2} + \frac{H h^2}{4}\right)$
Bending stress in column,	$f_b = \frac{M}{2I} d_o$
Axial stress,	$f_a = \left({}^{W}/_{A}\right)$
Moment at the base,	$M = F\left(H + \frac{h}{2}\right) + W\delta$
Combined stress requirement,	$\frac{f_a}{\sigma_a} + \frac{f_b}{\sigma_b} \le 1$

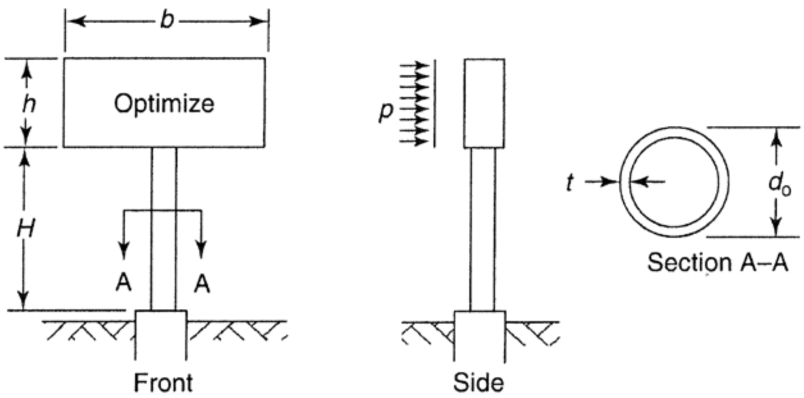


FIGURE E3.53 Sign support column.

Solution: Design of a sign support column-

Design Variables: d_o = outer diameter of the column; t = wall thickness of the column

Cost Function: the weight of the column

Constraints: $(f_a/\sigma_a)+(f_b/\sigma_b)\le 1$; $d_o/t \le 92$; $\delta \le 0.1$ m; $25 \le d_o \le 150$ cm; $0.5 \le t \le 10$ cm

Using Newtons and millimeters as units, the pertinent constants are as follows:
b = 8 m = 8000 mm; H = 20 m = 2.0×10^4 mm; p = 800 N/m^2 = 8.0×10^{-4} N/mm^2; h = 4.0 m = 4000 mm; w = 2.5 kN/m^2 = 2.5×10^{-3} N/mm^2; E = 75 GPa = 7.5×10^4 N/mm^2;

σ_b = 140 MPa = 140 N/mm^2; γ = 80 kN/m^3 = 8.0×10^{-5} N/mm^3

Writing some quantities in terms of the design variables, we get

$$r=(I/A)^{\frac{1}{2}}=\left[\frac{\pi\left(d_o^4-(d_o-2t)^4\right)/64}{\pi\left(d_o^2-(d_o-2t)^2\right)/4}\right]^{\frac{1}{2}}=\left[\frac{d_o^2(d_o-2t)^2}{16}\right]^{\frac{1}{2}}$$

$$\delta=\frac{F}{EI}\left(H^3/3+H^2h/2+Hh^2/4\right)=\frac{pbh\left(H^3/3+H^2h/2+Hh^2/4\right)}{E\left[(\pi/64)\left(d_o^4-(d_o-2t)^4\right)\right]}$$

$$f_a=\frac{W}{A}=\frac{wbh}{\frac{\pi}{4}\left(d_o^2-(d_o-2t)^2\right)}=\frac{wbh}{\pi(d_o-t)};$$

$$f_b = \frac{M\,d_{\mathrm{o}}}{2I} = \frac{\left[pbh\left(H + h/2\right) + wbh\delta\right]d_{\mathrm{o}}}{\left(\pi/32\right)\left(d_{\mathrm{o}}^{\,4} - \left(d_{\mathrm{o}} - 2t\right)^4\right)}$$

$$\sigma_a = \frac{12\pi^2 E}{92\left(H/r\right)^2} = \frac{12\pi^2 E\left[d_{\mathrm{o}}^{\,2} - \left(d_{\mathrm{o}} - 2t\right)^2\right]}{92H^2\,(16)} = \frac{3\pi^2 E\left[d_{\mathrm{o}}^{\,2} - \left(d_{\mathrm{o}} - 2t\right)^2\right]}{368H^2}$$

Writing the cost and constraints in terms of design variables, we get

$$f = \gamma AH = \left(\pi\gamma/4\right)\left(d_0^{\,2} - \left(d_{\mathrm{o}} - 2t\right)^2\right)H = \left(\pi\gamma H/4\right)\left(d_0^{\,2} - \left(d_{\mathrm{o}} - 2t\right)^2\right) = \pi\gamma H\left(d_{\mathrm{o}} - t\right)\mathrm{t}$$

$$\mathrm{g}_1 = \left(f_a/\sigma_a\right) + \left(f_b/\sigma_b\right) - 1 \le 0;\ \ or$$

$$= \frac{wbh/\pi\left(d_o - t\right)t}{3\pi^2 E\left[d_o^{\,2} + \left(d_o - 2t\right)^2\right]\Big/368H^2} + \frac{\left[pbh\left(H + h/2\right) + wbh\delta\right]d_o}{\left(\pi/32\right)\left[d_o^{\,4} - \left(d_o - 2t\right)^4\right]\sigma_b} - 1 \le 0;$$

$$\mathrm{g}_2 = d_{\mathrm{o}}/\mathrm{t} - 92 \le 0;$$

$$\mathrm{g}_3 = \frac{pbh\left(H^3/3 + H^2\,h/2 + H\,h^2/4\right)}{E\left(\pi/64\right)\left[d_{\mathrm{o}}^{\,4} - \left(d_{\mathrm{o}} - 2t\right)^4\right]} - 100 \le 0;$$

$$\mathrm{g}_4 = -d_o + 250 \le 0;$$

$$\mathrm{g}_5 = d_o - 1500 \le 0;$$

$$\mathrm{g}_6 = -t + 5 \le 0;$$

$$\mathrm{g}_7 = t - 100 \le 0$$

Substituting the given data into cost and constraints, we get

$$f = \pi\gamma H\left(d_{\mathrm{o}} - t\right)\mathrm{t} = \pi\left(8.0\times10^{-5}\right)\left(2.0\times10^4\right)\left(d_{\mathrm{o}} - t\right)t = 5.02655t\left(d_{\mathrm{o}} - t\right)$$

$$\delta = \frac{\left(8.0\times10^{-4}\right)\ (8000)\ (4000)\ \left(8\times10^{12}/3 + 8\times10^{11} + 8\times10^{10}\right)}{\left(7.5\times10^4\right)\ \left(\pi/64\right)\ \left(d_0^{\,4} - \left(d_{\mathrm{o}} - 2t\right)^4\right)}$$

$$= \left(7.747812\times10^{13}\right)\Big/\left[\pi\left(d_0^{\,4} - \left(d_{\mathrm{o}} - 2t\right)^4\right)\right]$$

$$f_{\mathrm{a}} = \frac{\left(2.5\times10^{-3}\right)(8000)(4000)}{\pi\left(d_{\mathrm{o}} - t\right)t} = \left(80000/\pi\right)\Big/\left[t\left(d_{\mathrm{o}} - t\right)\right]$$

$$f_b = \frac{(8000)(4000)d_o\left[\left(8.0\times10^{-4}\right)\left(2.0\times10^4+2000\right)+\dfrac{\left(2.5\times10^{-3}\right)\left(7.74781\times10^{13}\right)}{\pi\left[d_o^4-(d_o-2t)^4\right]}\right]}{(\pi/32)\left[d_o^4-(d_o-2t)^4\right]}$$

$$= \frac{\left(1.80224\times10^{10}\right)d_o}{\pi\left[d_o^4-(d_o-2t)^4\right]}+\frac{\left(1.98344\times10^{20}\right)d_o}{\pi^2\left[d_o^4-(d_o-2t)^4\right]^2}$$

$$\sigma_a = \frac{3\pi^2\left(7.5\times10^4\right)\left[d_o^2+(d_o-2t)^2\right]}{368\left(2.0\times10^4\right)^2} = \left(\pi^2/6.542222\times10^5\right)\left[d_o^2+(d_o-2t)^2\right]$$

$$g_1 = \frac{1.687973\times10^9}{t(d_o-t)\left[d_o^2+(d_o-2t)^2\right]}+\frac{\left(4.0976487\times10^7\right)d_o}{\left[d_o^4-(d_o-2t)^4\right]}+\frac{\left(1.435461\times10^{17}\right)d_o}{\left[d_o^4-(d_o-2t)^4\right]^2}-1.0\le 0$$

$$g_2 = d_o/t-92\le 0;$$

$$g_3 = \frac{2.4662052\times10^{13}}{d_o^4-(d_o-2t)^4}-100\le 0$$

$$g_4 = 250-d_o\le 0;$$

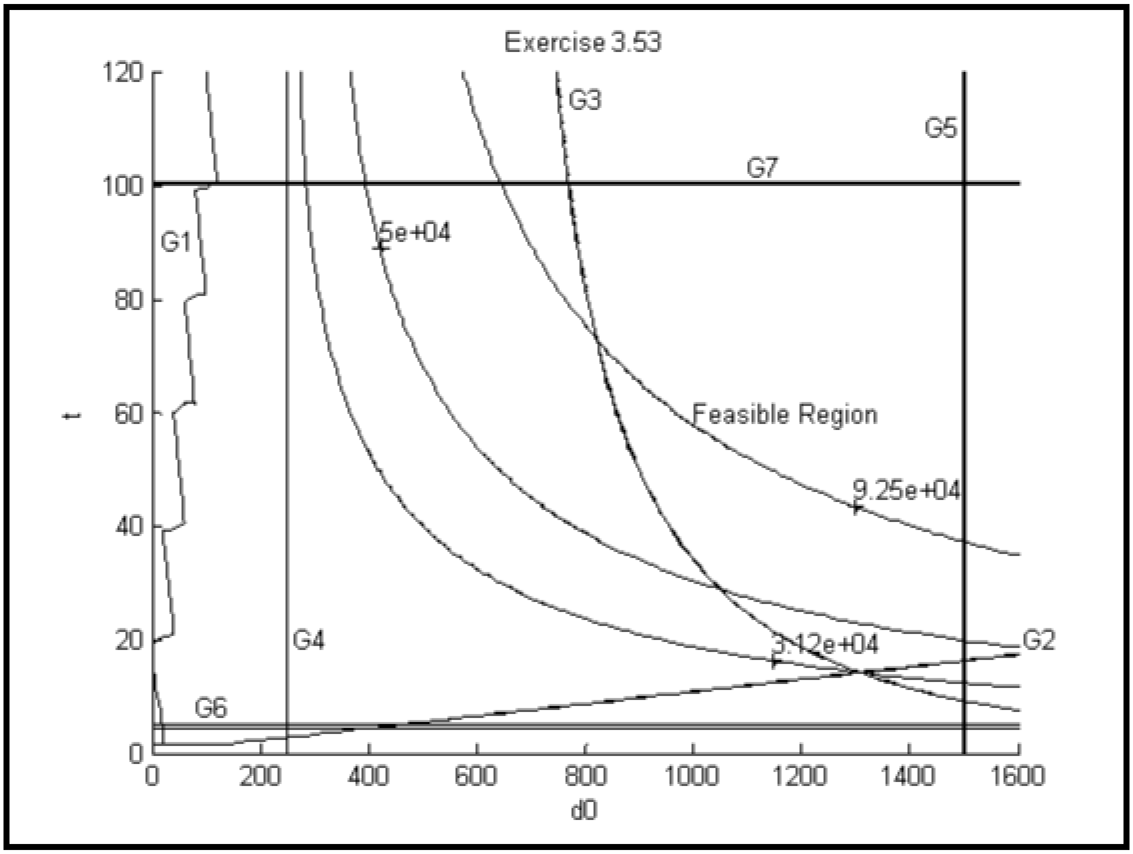
$$g_5 = d_o-1500\le 0;$$

$$g_6 = 5-t\le 0;$$

$$g_7 = t-100\le 0$$

Optimum solution: $d_o^* \doteq 1310$ mm, $t^* \doteq 14.2$ mm, $f^* \doteq 92{,}500$ N; g_3 (maximum deflection constraint) and g_2 (diameter/thickness ratio constraint) are active.

If the unit weight of the material is taken as 27 kN/m^3, then $f^* = 31{,}219$ N.



MATLAB Code: 3.53

```
b=8000;H=20000;p=8*10^(-4);h=4000;w=2.5*10^(-3);E=7.5*10^4;
sigma_b=140;gamma=2.7*10^(-5);
[d0,t]=meshgrid(0:20:1600,0:1.5:120);
r=sqrt((d0.^2.*(d0-2*t).^2)/16);
delta=p*b*h*(H^3/3+H^2*h/2+H*h^2/4)./(E*((pi/64)*(d0.^4-(d0-2*t).^4)));
f_a=w*b*h./(pi*(d0-t));
f_b=(p*b*h*(H+h/2).*d0+w*b*h.*delta.*d0)./((pi/32)*(d0.^4-(d0-2*t).^4));
sigma_a=(3*pi^2*E*(d0.^2-(d0-2*t).^2))/(368*H^2);
f=pi*gamma*H.*(d0-t).*t;
g1=(f_a./sigma_a)+(f_b./sigma_b)-1.0;
g2=d0./t-92;
g3=delta-100;
g4=250-d0;
g5=d0-1500;
g6=5-t;
g7=t-100;
cla reset
axis auto
xlabel('d0');ylabel('t');
title('Exercise 3.53');
hold on
cv1=[0,0.5];
const1=contour(d0,t,g1,cv1,'k');
text(15,90,'G1');
const2=contour(d0,t,g2,cv1,'k');
text(1610,20,'G2');
const3=contour(d0,t,g3,cv1,'k');
text(770,115,'G3');
const4=contour(d0,t,g4,cv1,'k');
text(260,20,'G4');
const5=contour(d0,t,g5,cv1,'k');
text(1430,110,'G5');
const6=contour(d0,t,g6,cv1,'k');
text(80,8,'G6');
const7=contour(d0,t,g7,cv1,'k');
text(1100,103,'G7');
text(1000,60,'Feasible Region');
fv=[3.1215e+004,50000,92500];
fs=contour(d0,t,f,fv,'k-');
clabel(fs)
hold off
```

3.54*

Design of a tripod. Design a minimum mass tripod of height *H* to support a vertical load *W* = 60 kN. The tripod base is an equilateral triangle with sides *B* = 1200 mm. The struts have a solid circular cross section of diameter *D* (Fig. E3.54).

The axial stress in the struts must not exceed the allowable stress in compression, and axial load in the strut *P* must not exceed the critical buckling load P_{cr} divided by a safety factor FS = 2. Use consistent units of Newtons and centimeters. The minimum and maximum values for design variables are $0.5 \le H \le 5$m and $0.5 \le D \le 50$ cm. Material properties and other relationship are given below:

Material: aluminum alloy 2014-T6

Allowable compressive stress,	σ_a = 150MPa
Young's modulus,	E = 75 GPa
Mass density,	ρ = 2800 kg/m^3
Strut length,	$l = (H^2 + \frac{1}{3}B^2)^{0.5}$
Critical buckling load,	$P_{cr} = \frac{\pi^2 EI}{l^2}$
Moment of inertia,	$I = \frac{\pi}{64}D^4$
Strut load,	$P = \frac{Wl}{3H}$

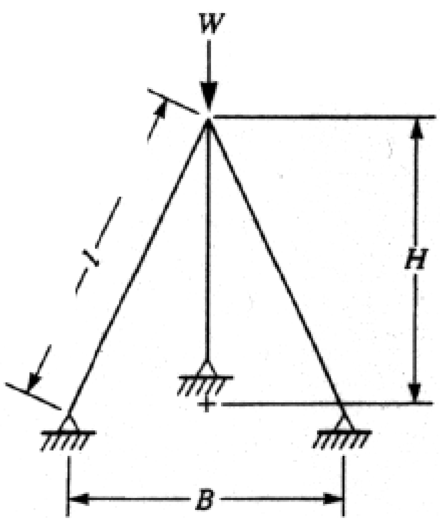


FIGURE E3.54 A tripod.

<u>Solution:</u> Design of a tripod-

Design Variables: *H* = height of the tripod; *D* = diameter of cross-section of the struts

Cost Function: minimize mass; $f = 3(\rho A l) = 3\rho(\pi D^2/4)(H^2 + B^2/3)^{\frac{1}{2}}$

Constraints:

$$g_1 = \frac{P}{A} - \sigma_a = \frac{Wl/3H}{\pi D^2/4} - \sigma_a = \frac{4W\left(H^2 + B^2/3\right)^{\frac{1}{2}}}{3\pi D^2 H} - \sigma_a \le 0$$

$$g_2 = P - \frac{P_{cr}}{2} = \frac{Wl}{3H} - \frac{\pi^2 EI}{2l^2} = \frac{W\left(H^2 + B^2/3\right)^{\frac{1}{2}}}{3H} - \frac{\pi^2 E\left(\pi D^4/64\right)}{2\left(H^2 + B^2/3\right)} \le 0$$

$g_3 = H - 500 \le 0;$

$g_4 = -H + 50 \le 0;$

$g_5 = D - 50 \le 0;$

$g_6 = -D + 0.5 \le 0$

Using units of Newtons and centimeters, the data are calculated as follows:

$\sigma_a = 150 \text{ MPa} = 1.5\times10^4 \text{ N/cm}^2;\ E = 75 \text{ GPa} = 7.5\times10^6 \text{ N/cm}^2;$

$\rho = 2800 \text{ kg/m}^3 = 2.8\times10^{-3}\text{ kg/cm}^3;\ B = 1200 \text{ mm} = 120 \text{ cm};\ W = 60 \text{ kN} = 6.0\times10^4 \text{ N}$

Introducing these constants into the cost and constraints, we get

$$f = 3\left(2.8\times10^{-3}\right)\left(\pi D^2/4\right)\left(H^2 + 120^2/3\right)^{\frac{1}{2}} = \left(6.59734\times10^{-3}\right)D^2\left(H^2 + 4800\right)^{\frac{1}{2}}$$

$$g_1 = \left(2.546475\times10^4\right)\left(H^2 + 4800\right)^{\frac{1}{2}} \Big/ D^2 H - 1.5\times10^4 \le 0;$$

$$g_2 = \frac{\left(2.0\times10^4\right)\left(H^2 + 4800\right)^{\frac{1}{2}}}{H} - \left(1.816774\times10^6\right)D^4 \Big/ \left(H^2 + 4800\right) \le 0;$$

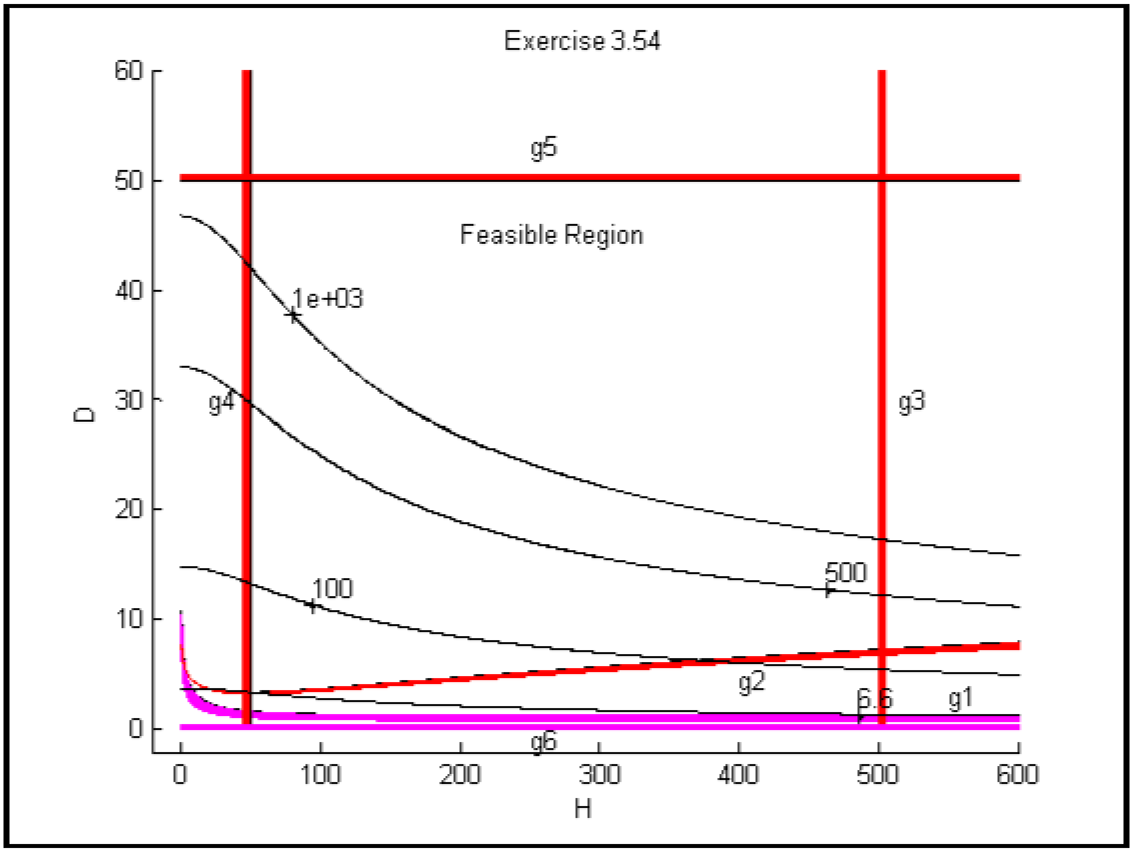
$g_3 = H - 500 \le 0;$

$g_4 = 50 - H \le 0;$

$g_5 = D - 50 \le 0;$

$g_6 = 0.5 - D \le 0$

Optimum solution: $H^* = 50.0$ cm, $D^* \doteq 3.42$ cm, $f^* \doteq 6.6$ kg; g_2 (buckling load constraint) and g_4 (maximum height constraint) are active.



MATLAB Code: 3.54

```
 [H,D]=meshgrid(1.0:1.0:600.0, 0.1:0.1:60.0);
f=6.59734e-3*(D.^2).*(sqrt(H.^2+4800));
g1=2.546479e4*sqrt(H.^2+4800)./((D.^2).*H)-1.5e4;
g2=2.0e4*sqrt(H.^2+4800)./H-1.816774e6*(D.^4)./(H.^2+4800);
g3=H-500;
g4=50-H;
g5=D-50;
g6=0.5-D;
cla reset
axis([-20,600,-2,60])
xlabel('H'),ylabel('D')
title('Exercise 3.54')
hold on
cv=[0 0];
const1=contour(H,D,g1,cv,'k');
text(550,3,'g1')
cv1=[1000.0:1000.0:20000.0];
const11=contour(H,D,g1,cv1,'m');
const2=contour(H,D,g2,cv,'k');
text(400,4.5,'g2')
cv2=[1000.0:1000.0:5000.0];
const21=contour(H,D,g2,cv2,'r');
const3=contour(H,D,g3,cv,'k');
cv3=[0.5:0.5:5.0];
text(515,30,'g3')
const31=contour(H,D,g3,cv3,'r');
const4=contour(H,D,g4,cv,'k');
text(20,30,'g4')
const41=contour(H,D,g4,cv3,'r');
const5=contour(H,D,g5,cv,'k');
cv5=[0.05:0.05:0.5];
text(250,53,'g5')
const51=contour(H,D,g5,cv5,'r');
const6=contour(H,D,g6,cv,'k');
text(250,-1,'g6')
const61=contour(H,D,g6,cv5,'m');
text(200,45,'Feasible Region')
fv=[6.6, 100, 500, 1000];
fs=contour(H,D,f,fv,'k--');
clabel(fs)
hold off
```

Alternate formulation

Cost Function: minimize mass; $f = 3(\rho A l)$, kg

Constraints:

$g_1 = \dfrac{P}{A} - \sigma_a \le 0$

$g_2 = P - \dfrac{P_{cr}}{FS} \le 0$

$g_3 = H - 100 \le 0;$

$g_4 = -H + 50 \le 0;$

$g_5 = D - 10 \le 0;$

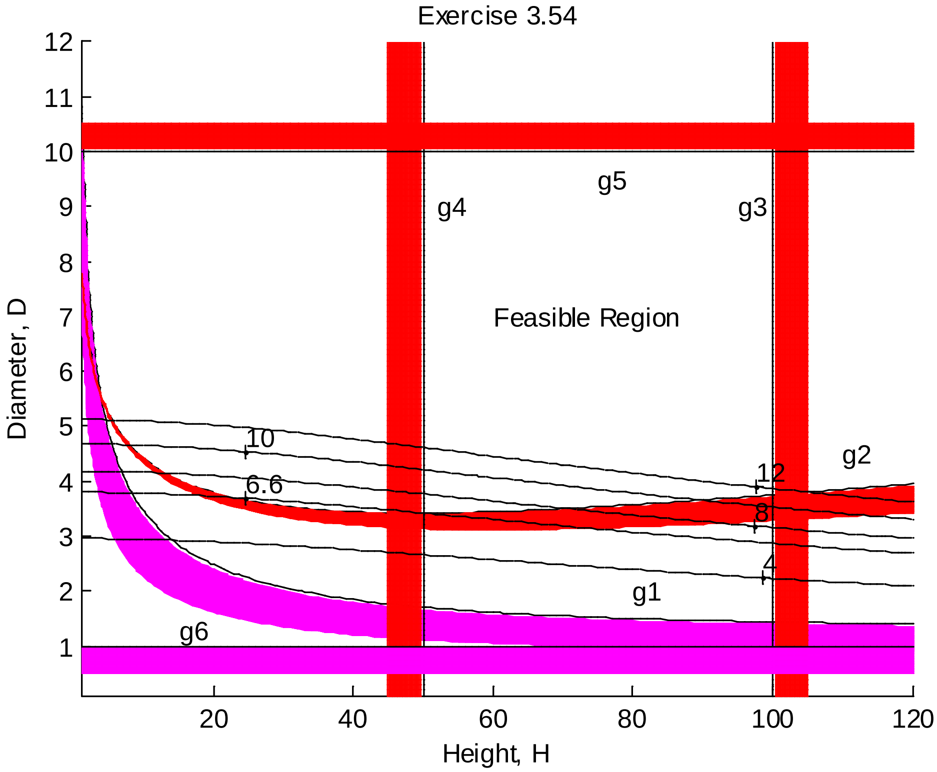
$g_6 = -D + 1 \le 0$

Solution is same as before.

MATLAB CODE FOR ALTERNATE FORMULATION

```
%Exercise 3.54 - Alternate Formulation
[H,D]=meshgrid(1.0:1.0:120.0, 0.1:0.1:12.0);

%Data for the problem
W=60000;  B=120;   ro=2.8/1000;  E=7.5e6;  sigma_a=15000;
D_min=1; D_max=10; H_min=50; H_max=100; FS=2;

%Analysis Expressions
L=(H.*H+B.*B./3).^0.5;
I=pi.*D.^4./64;
A=pi.*D.*D./4;
P_cr=pi.*pi.*E.*I./(L.*L);
P=W.*L./(3.*H);
Mass=3.*ro.*A.*L;
sigma=P./A;

%Formulation
f=Mass;
g1=sigma - sigma_a;
g2=P-P_cr./FS;

%f=6.59734e-3*(D.^2).*(sqrt(H.^2+4800));
%g1=2.546479e4*sqrt(H.^2+4800)./((D.^2).*H)-1.5e4;
%g2=2.0e4*sqrt(H.^2+4800)./H-1.816774e6*(D.^4)./(H.^2+4800);
g3=H - H_max;
g4=H_min - H;
g5=D - D_max;
```

```
g6=D_min - D;
cla reset
axis([1,120,.1,12])
xlabel('Height, H'),ylabel('Diameter, D')
title('Exercise 3.54')
hold on
cv=[0 0];
const1=contour(H,D,g1,cv,'k');
text(80,2,'g1')
cv1=[1000.0:100.0:20000.0];
const11=contour(H,D,g1,cv1,'m');
const2=contour(H,D,g2,cv,'k');
text(110,4.5,'g2')
cv2=[1000.0:100.0:10000.0];
const21=contour(H,D,g2,cv2,'r');
const3=contour(H,D,g3,cv,'k');
cv3=[0.5:0.1:5.0];
text(95,9,'g3')
const31=contour(H,D,g3,cv3,'r');
const4=contour(H,D,g4,cv,'k');
text(52,9,'g4')
const41=contour(H,D,g4,cv3,'r');
const5=contour(H,D,g5,cv,'k');
cv5=[0.05:0.01:0.5];
text(75, 9.5,'g5')
const51=contour(H,D,g5,cv5,'r');
const6=contour(H,D,g6,cv,'k');
text(15,1.3,'g6')
const61=contour(H,D,g6,cv5,'m');
text(60,7,'Feasible Region')
fv=[4, 6.6, 8, 10, 12];
fs=contour(H,D,f,fv,'k--');
clabel(fs)
hold off
```


Exercise 3.54
Diameter, D
Height, H
g1
g2
g3
g4
g5
g6
Feasible Region
10
6.6
12
8
4
20
40
60
80
100
120
1
2
3
4
5
6
7
8
9
10
11
12