C H A P T E R

8

Linear Programming Methods for Optimum Design

Section 8.2 Definition of Standard Linear Programming Problem

8.1

Answer True or False.

1. A linear programming problem having maximization of a function cannot be transcribed into the standard LP form. *False*
2. A surplus variable must be added to a "≤ type" constraint in the standard LP formulation. *False*
3. A slack variable for an LP constraint can have a negative value. *False*
4. A surplus variable for an LP constraint must be non-negative. *True*
5. If a "≤ type" constraint is active, its slack variable must be positive. *False*
6. If a "≥ type" constraint is active, its surplus variable must be zero. *True*
7. In the standard LP formulation, the resource limits are free in sign. *False*
8. Only "≤ type" constraints can be transcribed into the standard LP form. *False*
9. Variables that are free in sign can be treated in any LP problem. *True*
10. In the standard LP form, all the cost coefficients must be positive. *False*
11. All variables must be non-negative in the standard LP definition. *True*

8.2

Convert the following problem to the standard LP form:

Minimize $f = 5x_1 + 4x_2 - x_3$
Subject to $x_1 + 2x_2 - x_3 \geq 1$
$2x_1 + x_2 + x_3 \geq 4$
$x_1,\ x_2 \geq 0$; x_3 is unrestricted in sign.

Solution:
$x_3 = x_3^+ - x_3^-$; $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3^+$, $y_4 = x_3^-$, $y_5, y_6 =$ surplus variables for the 1st and 2nd constraints.
Minimize $f = 5y_1 + 4y_2 - y_3 + y_4$
Subject to $y_1 + 2y_2 - y_3 + y_4 - y_5 = 1$
$2y_1 + y_2 + y_3 - y_4 - y_6 = 4$
$y_i \geq 0; i = 1$ to 6

8.3

Convert the following problem to the standard LP form:

Maximize $z = x_1 + 2x_2$
Subject to $-x_1 + 3x_2 \leq 10$
$x_1 + x_2 \leq 6$
$x_1 - x_2 \leq 2$
$x_1 + 3x_2 \leq 6$
$x_1, x_2 \geq 0$

Solution:
x_3, x_4, x_5: slack variables for the 1st , 2nd and 3rd constraints respectively, x_6: a surplus variable for the 4th constraint.
Minimize $f = -x_1 - 2x_2$
Subject to $-x_1 - 3x_2 + x_3 = 10$
$x_1 + x_2 + x_4 = 6$
$x_1 - x_2 + x_5 = 2$
$x_1 + 3x_2 - x_6 = 6$
$x_i \geq 0; i = 1$ to 6

8.4

Convert the following problem to the standard LP form:

Minimize $f = 2x_1 - 3x_2$
Subject to $x_1 + x_2 \le 1$
$-2x_1 + x_2 \ge 2$
$x_1, x_2 \ge 0$

Solution:
x_3: a slack variable for the 1st constraint, x_4: a surplus variable for the 2nd constraint.
Minimize $f = 2x_1 - 3x_2$
Subject to $x_1 + x_2 + x_3 = 1$
$-2x_1 + x_2 - x_4 = 2$
$x_i \ge 0; i = 1$ to 4

8.5

Convert the following problem to the standard LP form:

Maximize $z = 4x_1 + 2x_2$
Subject to $-2x_1 + x_2 \le 4$
$x_1 + 2x_2 \ge 2$
$x_1, x_2 \ge 0$

Solution:
x_3: a slack variable for the 1st constraint, x_4: a surplus variable for the 2nd constraint.
Minimize $f = -4x_1 - 2x_2$
Subject to $-2x_1 + x_2 + x_3 = 4$
$x_1 + 2x_2 - x_4 = 2$
$x_i \ge 0; i = 1$ to 4

8.6

Convert the following problem to the standard LP form:

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \le 5$
$x_1 + x_2 = 4$
$x_1 - x_2 \ge 3$
$x_1, x_2 \ge 0$

Solution:
x_3: a slack variable for the 1st constraint, x_4: a surplus variable for the 3rd constraint.
Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + 2x_2 + x_3 = 5$
$x_1 + x_2 = 4$
$x_1 - x_2 - x_4 = 3$
$x_i \ge 0; i = 1$ to 4

8.7

Convert the following problem to the standard LP form:

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \le 5$
$2x_1 + x_2 = 4$
$x_1 - x_2 \ge 1$
$x_1, x_2 \ge 0$

Solution:

x_3: a slack variable for the 1st constraint, x_4: a surplus variable for the 3rd constraint.

Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + 2x_2 + x_3 = 5$
$2x_1 + x_2 = 4$
$x_1 - x_2 - x_4 = 1$
$x_i \ge 0; i = 1$ to 4

8.8

Convert the following problem to the standard LP form:

Minimize $f = 9x_1 + 2x_2 + 3x_3$
Subject to $-2x_1 - x_2 + 3x_3 \le -5$
$x_1 - 2x_2 + 2x_3 \ge -2$
$x_1, x_2, x_3 \ge 0$

Solution:

Reversing the sign on the RHS of both constraints by multiplying both sides with -1, then introducing a surplus variable x_4 for the 1st constraint and a slack variable x_5 for the 2nd constraint, we have the standard LP as:

Minimize $f = 9x_1 + 2x_2 + 3x_3$
Subject to $2x_1 + x_2 - 3x_3 - x_4 = 5$
$-x_1 + 2x_2 - 2x_3 + x_5 = 2$
$x_i \ge 0; i = 1$ to 5

8.9

Convert the following problem to the standard LP form:

Minimize $f = 5x_1 + 4x_2 - x_3$
Subject to $x_1 + 2x_2 - x_3 \ge 1$
$2x_1 + x_2 + x_3 \ge 4$
$x_1,\ x_2 \ge 0$; x_3 is unrestricted in sign.

Solution:

$x_3 = x_3^+ - x_3^-$; $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3^+$, $y_4 = x_3^-$, y_5, y_6 = surplus variables for the 1st and 2nd constraints.

Minimize $f = 5y_1 + 4y_2 - y_3 + y_4$
Subject to $y_1 + 2y_2 - y_3 + y_4 - y_5 = 1$
$2y_1 + y_2 + y_3 - y_4 - y_6 = 4$
$y_i \ge 0; i = 1$ to 6

8.10

Convert the following problem to the standard LP form:

Maximize $z = -10x_1 - 18x_2$
Subject to $x_1 - 3x_2 \leq -3$
$2x_1 + 2x_2 \geq 5$
$x_1, x_2 \geq 0$

Solution:

Multiply both sides of the 1st constraint with – 1; x_3, x_4 = surplus variables for the 1st and 2nd constraints.

Minimize $f = 10x_1 + 18x_2$
Subject to $-x_1 + 3x_2 - x_3 = 3$
$2x_1 + 2x_2 - x_4 = 5$
$x_i \geq 0; i = 1$ to 4

8.11

Convert the following problem to the standard LP form:

Minimize $f = 20x_1 - 6x_2$
Subject to $3x_1 - x_2 \geq 3$
$-4x_1 + 3x_2 = -8$
$x_1,\ x_2 \geq 0$

Solution:

x_3 = a surplus variable for the 1st constraint; change sign on the RHS of the 2nd constraint by multiplying by – 1.

Minimize $f = 20x_1 - 6x_2$
Subject to $3x_1 - x_2 - x_3 = 3$
$4x_1 - 3x_2 = 8$
$x_i \geq 0; i = 1$ to 3

8.12

Convert the following problem to the standard LP form:

Maximize $z = 2x_1 + 5x_2 - 4.5x_3 + 1.5x_4$
Subject to $5x_1 + 3x_2 + 1.5x_3 \leq 8$
$1.8x_1 - 6x_2 + 4x_3 + x_4 \geq 3$
$-3.6x_1 + 8.2x_2 + 7.5x_3 + 5x_4 = 15$
$x_i \geq 0; i = 1$ to 4

Solution:

x_5 = a slack variable for the 1st constraint; x_6 = a surplus variable for the 2nd constraint.

Minimize $f = -2x_1 - 5x_2 + 4.5x_3 - 1.5x_4$
Subject to $5x_1 + 3x_2 + 1.5x_3 + x_5 = 8$
$1.8x_1 - 6x_2 + 4x_3 + x_4 - x_6 = 3$
$-3.6x_1 + 8.2x_2 + 7.5x_3 + 5x_4 = 15$
$x_i \geq 0; i = 1$ to 6

8.13

Convert the following problem to the standard LP form:

Minimize $f = 8x_1 - 3x_2 + 15x_3$
Subject to $5x_1 - 1.8x_2 - 3.6x_3 \geq 2$
$3x_1 + 6x_2 + 8.2x_3 \geq 5$
$1.5x_1 - 4x_2 + 7.5x_3 \geq -4.5$
$-x_2 + 5x_3 \geq 1.5$
$x_1,\ x_2 \geq 0$; x_3 is unrestricted in sign.

Solution:
$x_3 = x_3^+ - x_3^-$; $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3^+$, $y_4 = x_3^-$; multiply by -1 on both sides of the 3rd constraint; y_5, y_6, y_8 = surplus variables for the 1st, 2nd and 4th constraints respectively; y_7 = a slack variable for the 3rd constraint.
Minimize $f = 8y_1 - 3y_2 + 15y_3 - 15y_4$
Subject to $5y_1 - 1.8y_2 - 3.6y_3 + 3.6y_4 - y_5 = 2$
$3y_1 + 6y_2 + 8.2y_3 - 8.2y_4 - y_6 = 5$
$-1.5y_1 + 4y_2 - 7.5y_3 + 7.5y_4 + y_7 = 4.5$
$-y_2 + 5y_3 - 5y_4 - y_8 = 1.5$
$y_i \geq 0; i = 1$ to 8

8.14

Convert the following problem to the standard LP form:

Maximize $z = 10x_1 + 6x_2$
Subject to $2x_1 + 3x_2 \leq 90$
$4x_1 + 2x_2 \leq 80$
$x_2 \geq 15$
$5x_1 + x_2 = 25$
$x_1,\ x_2 \geq 0$

Solution:
x_3, x_4 = slack variables for the 1st and 2nd constraints respectively; x_5 = a surplus variable for the 3rd constraint.
Minimize $f = -10x_1 - 6x_2$
Subject to $2x_1 + 3x_2 + x_3 = 90$
$4x_1 + 2x_2 + x_4 = 80$
$x_2 - x_5 = 15$
$5x_1 + x_2 = 25$
$x_1,\ x_2 \geq 0$
$x_i \geq 0; i = 1$ to 5

8.15

Convert the following problem to the standard LP form:

Maximize $z = -2x_1 + 4x_2$
Subject to $2x_1 + x_2 \geq 3$
$2x_1 + 10x_2 \leq 18$
$x_1,\ x_2 \geq 0$

Solution:

x_3 = a surplus variable for the 1st constraint, x_4 = a slack variable for the 2nd constraint.
Minimize $f = 2x_1 - 4x_2$
Subject to $2x_1 + x_2 - x_3 = 3$
$2x_1 + 10x_2 + x_4 = 18$
$x_i \geq 0; i = 1$ to 4

8.16

Convert the following problem to the standard LP form:

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \leq 5$
$2x_1 + x_2 = 4$
$x_1 - x_2 \geq 3$
$x_1 \geq 0$; x_2 is unrestricted in sign.

Solution:

$x_2 = x_2^+ - x_2^-$; $y_1 = x_1,\ y_2 = x_2^+, y_3 = x_2^-$; y_4 = a slack variable for the 1st constraint, and y_5 = a surplus variable for the 3rd constraint.
Mimimize $f = -y_1 - 4y_2 + 4y_3$
Subject to $y_1 + 2y_2 - 2y_3 + y_4 = 5$
$2y_1 + y_2 - y_3 = 4$
$y_1 - y_2 + y_3 - y_5 = 3$
$y_i \geq 0; i = 1$ to 5

8.17

Convert the following problem to the standard LP form:

Minimize $f = 3x_1 + 2x_2$
Subject to $x_1 - x_2 \geq 0$
$x_1 + x_2 \geq 2$
$x_1,\ x_2 \geq 0$

Solution:

x_3, x_4 = surplus variables for the 1st and 2nd constraints respectively.
Minimize $f = 3x_1 + 2x_2$
Subject to $x_1 - x_2 - x_3 = 0$
$x_1 + x_2 - x_4 = 2$
$x_i \geq 0; i = 1$ to 4

8.18

Convert the following problem to the standard LP form:

Maximize $z = 3x_1 + 2x_2$
Subject to $x_1 - x_2 \geq 0$
$x_1 + x_2 \geq 2$
$2x_1 + x_2 \leq 6$
$x_1,\ x_2 \geq 0$

Solution:

x_3, x_4 = surplus variables for the 1st and 2nd constraints, x_5 = a slack variable for the 3rd constraint.
Maximize $f = -3x_1 - 2x_2$
Subject to $x_1 - x_2 - x_3 = 0$
$x_1 + x_2 - x_4 = 2$
$2x_1 + x_2 + x_5 = 6$
$x_i \geq 0; i = 1$ to 5

8.19

Convert the following problem to the standard LP form:

Maximize $z = x_1 + 2x_2$
Subject to $3x_1 + 4x_2 \leq 12$
$x_1 + 3x_2 \geq 3$
$x_1 \geq 0$; x_2 is unrestricted in sign.

Solution:

$x_2 = x_2^+ - x_2^-$; $y_1 = x_1,\ \ y_2 = x_2^+, y_3 = x_2^-$; y_4 = a slack variable for the 1st constraint, and y_5 = a surplus variable for the 2nd constraint.
Minimize $f = -y_1 - 2y_2 + 2y_3$
Subject to $3y_1 + 4y_2 - 4y_3 + y_4 = 12$
$y_1 + 3y_2 - 3y_3 - y_5 = 3$
$y_i \geq 0; i = 1$ to 5

Section 8.3 Basic Concepts Related to LP Problems
Section 8.4 Calculation of Basic Solutions

8.20

Answer True or False.

1. In the standard LP definition, the number of constraint equations (i.e., rows in the matrix **A**) must be less than the number of variables. *True*
2. In an LP problem, the number of "≤ type" constraints cannot be more than the number of design variables. *False*
3. In an LP problem, the number of "≥ type" constraints cannot be more than the number of design variables. *False*
4. An LP problem has an infinite number of basic solutions. *False*
5. A basic solution must have zero value for some of the variables. *True*
6. A basic solution can have negative values for some of the variables. *True*
7. A degenerate basic solution has exactly m variables with nonzero values, where m is the number of equations. *False*
8. A basic feasible solution has all variables with non-negative values. *True*
9. A basic feasible solution must have m variables with positive values, where m is the number of equations. *False*
10. The optimum point for an LP problem can be inside the feasible region. *False*
11. The optimum point for an LP problem lies at a vertex of the feasible region. *True*
12. The solution to any LP problem is only a local optimum. *False*
13. The solution to any LP problem is a unique global optimum. *False*

8.21

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \leq 5$
$2x_1 + x_2 = 4$
$x_1 - x_2 \geq 1$
$x_1,\ x_2 \geq 0$

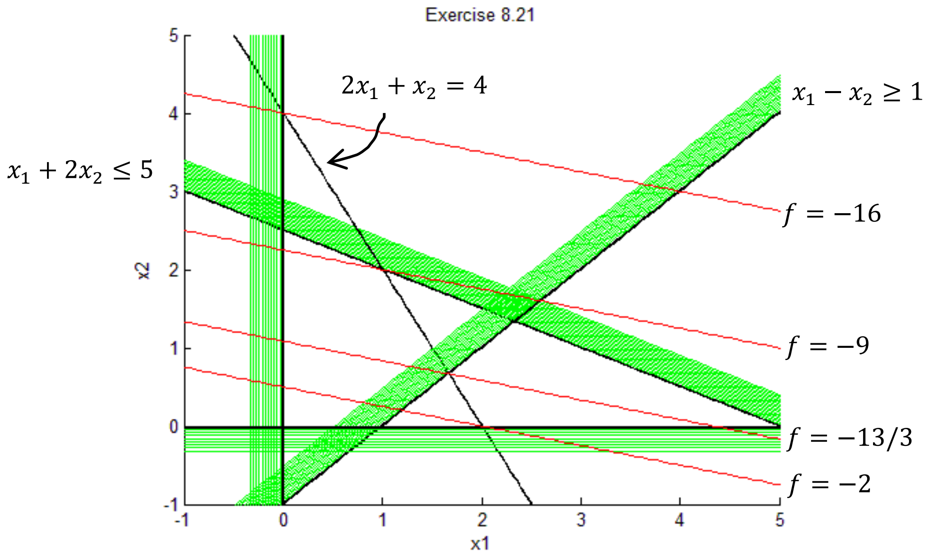
Solution:
Standard LP form:
Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + 2x_2 + x_3 = 5$
$2x_1 + x_2 = 4$
$x_1 - x_2 - x_4 = 1$
$x_i \geq 0; i = 1$ to 4

Since n = 4 and m = 3, the problem has $\frac{4!}{3!1!} = 4$ basic solutions. These solutions are given in Table E8.21 along with the corresponding cost function values. Basic feasible solutions are #2 and #4.

Table E8.21

	x_1	x_2	x_3	x_4	f	
1.	0	4	-3	-5	-16	infeasible
2.	2	0	3	1	-2	feasible
3.	1	2	0	-2	-9	infeasible
4.	5/3	2/3	2	0	-13/3	feasible

Figure E8.21



Exercise 8.21
$2x_1 + x_2 = 4$
$x_1 - x_2 \geq 1$
$x_1 + 2x_2 \leq 5$
$f = -16$
$f = -9$
$f = -13/3$
$f = -2$
x1
x2

```
clear all
[x1,x2]=meshgrid(-1:0.05:5, -1:0.05:5);
f=-x1-4*x2;
g1=x1+2*x2-5;
h1=2*x1+x2-4;
g2=-x1+x2+1;

g3=-x1;
g4=-x2;
cla reset
axis auto
xlabel('x1'),ylabel('x2')
title('Exercise 8.21')
hold on
cv1=[0:0.05:0.8];
const1=contour(x1,x2,g1,cv1,'g');
cv2=[0:0.01:0.02];
const1=contour(x1,x2,g1,cv2,'k');
cv3=[0:0.01:0.02];
const2=contour(x1,x2,h1,cv3,'k');
cv4=[0:0.03:0.5];
const3=contour(x1,x2,g2,cv4,'g');
cv5=[0:0.01:0.02];
const3=contour(x1,x2,g2,cv5,'k');

cv6=[0:0.03:0.35];
contour(x1,x2,g3,cv6,'g');
cv7=[0:0.04:0.35];
contour(x1,x2,g4,cv7,'g');
cv8=[0:0.01:0.01];
contour(x1,x2,g3,cv8,'k');
cv9=[0:0.01:0.02]
contour(x1,x2,g4,cv9,'k');

fv=[-16 -9 -13/3 -2];
fs=contour(x1,x2,f,fv,'r');

grid off
hold off
```

8.22

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = -10x_1 - 18x_2$
Subject to $x_1 - 3x_2 \leq -3$
$2x_1 + 2x_2 \geq 5$
$x_1,\ x_2 \geq 0$

Solution:

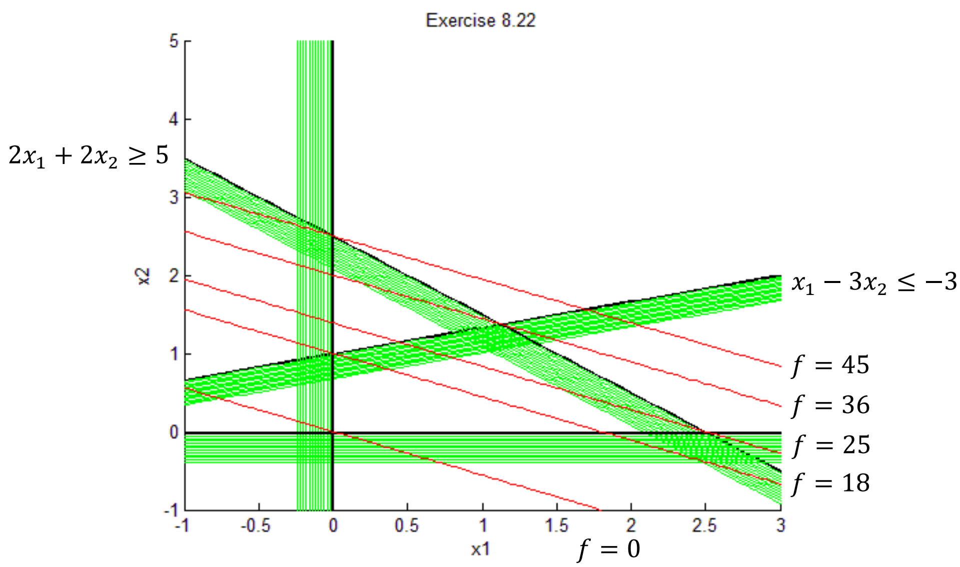
Standard LP form:
Minimize $f = 10x_1 + 18x_2$
Subject to $-x_1 + 3x_2 - x_3 = 3$
$2x_1 + 2x_2 - x_4 = 5$
$x_i \geq 0; i = 1 \text{ to } 4$

Since n = 4 and m = 2, the problem has $\frac{4!}{2!2!} = 6$ basic solutions. They are given in Table E8.22 along with the corresponding cost function values. Basic feasible solutions are #3 and #6.

Table E8.22

	x_1	x_2	x_3	x_4	f	
1.	0	0	-3	-5	0	infeasible
2.	0	1	0	-3	18	infeasible
3.	0	2.5	4.5	0	45	feasible
4.	-3	0	0	-11	-30	infeasible
5.	2.5	0	-5.5	0	25	infeasible
6.	9/8	11/8	0	0	36	feasible

Figure E8.22



```
clear all
[x1,x2]=meshgrid(-1:0.05:3, -1:0.05:5);
f=10*x1+18*x2;
g1=x1-3*x2+3;
g2=-2*x1-2*x2+5;
g3=-x1;
g4=-x2;
cla reset
axis auto
xlabel('x1'),ylabel('x2')
title('Exercise 8.22')
hold on
cv1=[0:0.07:1.0];
const1=contour(x1,x2,g1,cv1,'g');
cv2=[0:0.01:0.02];
const1=contour(x1,x2,g1,cv2,'k');
cv3=[0:0.07:0.85];
const3=contour(x1,x2,g2,cv3,'g');
cv4=[0:0.01:0.02];
const4=contour(x1,x2,g2,cv4,'k');

cv6=[0:0.02:0.25];
contour(x1,x2,g3,cv6,'g');
cv7=[0:0.03:0.4];
contour(x1,x2,g4,cv7,'g');
cv8=[0:0.01:0.01];
contour(x1,x2,g3,cv8,'k');
cv9=[0:0.01:0.02]
contour(x1,x2,g4,cv9,'k');

fv=[0 18 -30 25 45 36];
fs=contour(x1,x2,f,fv,'r');

grid off
hold off
```

8.23

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = x_1 + 2x_2$
Subject to $3x_1 + 4x_2 \leq 12$
$x_1 + 3x_2 \geq 3$
$x_1 \geq 0$, x_2 is unrestricted in sign.

Solution:
Standard LP form:
Minimize $f = -y_1 - 2y_2 + 2y_3$
Subject to $3y_1 + 4y_2 - 4y_3 + y_4 = 12$
$y_1 + 3y_2 - 3y_3 - y_5 = 3$
$y_i \geq 0; i = 1$ to 5

Since n = 5 and m = 2, the problem has at most $\frac{5!}{2!3!} = 10$ basic solutions. They are given in Table E8.23 along with the corresponding cost function values. Basic feasible solutions are #4, #5, #7, #8 and #9.

Table E8.23

	y_1	y_2	y_3	y_4	y_5	f	
1.	0	0	0	12	-3	0	infeasible
2.	0	0	-3	0	6	-6	infeasible
3.	0	0	-1	8	0	-2	infeasible
4.	0	3	0	0	6	-6	feasible
5.	0	1	0	8	0	-2	feasible
6.	0	-	-	0	0	-	no solution
7.	4	0	0	0	1	-4	feasible
8.	3	0	0	3	0	-3	feasible
9.	4.8	0	0.6	0	0	-3.6	infeasible
10.	4.8	-0.6	0	0	0	-3.6	infeasible

8.24

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Minimize $f = 20x_1 - 6x_2$
Subject to $3x_1 - x_2 \geq 3$
$-4x_1 + 3x_2 = -8$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = 20x_1 - 6x_2$
Subject to $3x_1 - x_2 - x_3 = 3$
$4x_1 - 3x_2 = 8$
$x_i \geq 0; i = 1$ to 3

Since n = 3 and m = 2, the problem has at most $\frac{3!}{2!1!} = 3$ basic solutions. They are given in Table E8.24 along with the corresponding cost function values. Basic feasible solution is #2.

Table E8.24

	x_1	x_2	x_3	f	
1.	0	-8/3	-1/3	16	infeasible
2.	2	0	3	40	feasible
3.	0.2	-2.4	0	18.4	infeasible

8.25

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = 5x_1 - 2x_2$
Subject to $2x_1 + x_2 \le 9$
$x_1 - 2x_2 \le 2$
$-3x_1 + 2x_2 \le 3$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -5x_1 + 2x_2$
Subject to $2x_1 + x_2 + x_3 = 9$
$x_1 - 2x_2 + x_4 = 2$
$-3x_1 + 2x_2 + x_5 = 3$
$x_i \ge 0; i = 1$ to 5

Since n = 5 and m = 3, the problem has $\frac{5!}{3!2!} = 10$ basic solutions as shown in Table E8.25. Basic feasible solutions are #1, 4, 6, 8 and 9.

Table E8.25

	x_1	x_2	x_3	x_4	x_5	f	
1.	0	0	9	2	3	0	feasible
2.	0	9	0	20	-15	18	infeasible
3.	0	-1	10	0	5	-2	infeasible
4.	0	1.5	7.5	5	0	3	feasible
5.	4.5	0	0	-2.5	16.5	-22.5	infeasible
6.	2	0	5	0	9	-10	feasible
7.	-1	0	11	3	0	5	infeasible
8.	4	1	0	0	13	-18	feasible
9.	15/7	33/7	0	65/7	0	-9/7	feasible
10.	-2.5	-2.25	16.25	0	0	8	infeasible

8.26

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \leq 5$
$x_1 + x_2 = 4$
$x_1 - x_2 \geq 3$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + 2x_2 + x_3 = 5$
$x_1 + x_2 = 4$
$x_1 - x_2 - x_4 = 3$
$x_i \geq 0; i = 1$ to 4

Since n = 4 and m = 3, the problem has $\frac{4!}{3!1!} = 4$ basic solutions as shown in the table. The basic feasible solutions are #2, and #4.

Table E8.26

	x_1	x_2	x_3	x_4	f	
1.	0	4	-3	-7	-16	infeasible
2.	4	0	1	1	-4	feasible
3.	3	1	0	-1	-7	infeasible
4.	3.5	0.5	0.5	0	-5.5	feasible

8.27

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Minimize $f = 5x_1 + 4x_2 - x_3$
Subject to $x_1 + 2x_2 - x_3 \geq 1$
$2x_1 + x_2 + x_3 \geq 4$
$x_1,\ x_3 \geq 0$, x_2 is unrestricted in sign.

Solution:
Standard LP form:
Minimize $f = 5y_1 + 4y_2 - 4y_3 - y_4$
Subject to $y_1 + 2y_2 - 2y_3 - y_4 - y_5 = 1$
$2y_1 + y_2 - y_3 + y_4 - y_6 = 4$
$y_i \geq 0; i = 1$ to 6
where $y_1 = x_1,\ y_2 = x_2^+, y_3 = x_2^-, y_4 = x_3$, and y_5 and y_6 are surplus variables. Since n = 6 and m = 2, the problem has $\frac{6!}{2!4!} = 15$basic solutions as shown in Table E8.27. The basic feasible solutions are #8, 9, 12, 13 and 14.

Table E8.27

	y_1	y_2	y_3	y_4	y_5	y_6	f	
1.	0	0	0	0	-1	-4	0	infeasible
2.	0	0	0	-1	0	-5	1	infeasible
3.	0	0	0	4	-5	0	-4	infeasible
4.	0	0	-0.5	0	0	-3.5	2	infeasible
5.	0	0	-4	0	7	0	16	infeasible
6.	0	0	-5/3	7/3	0	0	13/3	infeasible
7.	0	0.5	0	0	0	-3.5	2	infeasible
8.	0	4	0	0	7	0	16	feasible
9.	0	5/3	0	7/3	0	0	13/3	feasible
10.	0	-	-	0	0	0	-	no solution
11.	1	0	0	0	0	-2	5	infeasible
12.	2	0	0	0	1	0	10	feasible
13.	5/3	0	0	2/3	0	0	23/3	feasible
14.	7/3	0	2/3	0	0	0	9	feasible
15.	7/3	-2/3	0	0	0	0	9	infeasible

8.28

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Minimize $f = 9x_1 + 2x_2 + 3x_3$
Subject to $-2x_1 - x_2 + 3x_3 \leq -5$
$x_1 - 2x_2 + 2x_3 \geq -2$
$x_1, x_2, x_3 \geq 0$

Solution:
Standard LP form:
Minimize $f = 9x_1 + 2x_2 + 3x_3$
Subject to $2x_1 + x_2 - 3x_3 - x_4 = 5$
$-x_1 + 2x_2 - 2x_3 + x_5 = 2$
$x_i \geq 0; i = 1$ to 5

Since n = 5 and m = 2, the problem has $\frac{5!}{2!3!} = 10$ feasible solutions as shown in Table E8.28. The basic feasible solutions are #7 and #10.

Table E8.28

	x_1	x_2	x_3	x_4	x_5	f	
1.	0	0	0	-5	2	0	infeasible
2.	0	0	-5/3	0	-4/3	-15/3	infeasible
3.	0	0	-1	-2	0	-3	infeasible
4.	0	5	0	0	-8	10	infeasible
5.	0	1	0	-4	0	2	infeasible
6.	0	-1	-2	0	0	-8	infeasible
7.	2.5	0	0	0	4.5	22.5	feasible
8.	-2	0	0	-9	0	-18	infeasible
9.	4/7	0	-9/7	0	0	9/7	infeasible
10.	1.6	1.8	0	0	0	18	feasible

8.29

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = 4x_1 + 2x_2$
Subject to $-2x_1 + x_2 \le 4$
$x_1 + 2x_2 \ge 2$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -4x_1 - 2x_2$
Subject to $-2x_1 + x_2 + x_3 = 4$
$x_1 + 2x_2 - x_4 = 2$
$x_i \ge 0; i = 1$ to 4

Since n = 4 and m = 2, the problem has $\frac{4!}{2!2!} = 6$ basic solutions as shown in Table E8.29. The basic feasible solutions are #2, 3, and 5.

Table 8.29

	x_1	x_2	x_3	x_4	f	
1.	0	0	4	-2	0	infeasible
2.	0	4	0	6	-8	feasible
3.	0	1	3	0	-2	feasible
4.	-2	0	0	-4	8	infeasible
5.	2	0	8	0	-8	feasible
6.	-1.2	1.6	0	0	1.6	infeasible

8.30

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = 3x_1 + 2x_2$
Subject to $x_1 - x_2 \geq 0$
$x_1 + x_2 \geq 2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -3x_1 - 2x_2$
Subject to $x_1 - x_2 - x_3 = 0$
$x_1 + x_2 - x_4 = 2$
$x_i \geq 0; i = 1$ to 4

Since n = 4 and m = 2, the problem has $\frac{4!}{2!2!} = 6$ basic solutions as shown in Table E8.30. The basic feasible solutions are #5 and 6.

Table E8.30

	x_1	x_2	x_3	x_4	f	
1.	0	0	0	-2	0	infeasible
2.	0	0	0	-2	0	infeasible
3.	0	2	-2	0	-4	infeasible
4.	0	0	0	-2	0	infeasible
5.	2	0	2	0	-6	feasible
6.	1	1	0	0	-5	feasible

8.31

Find all the basic solutions for the following LP problem using the Gauss-Jordan elimination method. Identify basic feasible solutions and show them on graph paper.

Maximize $z = 4x_1 + 5x_2$
Subject to $-x_1 + 2x_2 \le 10$
$3x_1 + 2x_2 \le 18$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -4x_1 - 5x_2$
Subject to $-x_1 + 2x_2 + x_3 = 10$
$3x_1 + 2x_2 + x_4 = 18$
$x_i \ge 0; i = 1$ to 4
Since n = 4 and m = 2, this problem has $\frac{4!}{2!2!} = 6$ basic solutions as shown in Table E8.31. The basic feasible solutions are #1, 2, 5 and 6.

Table E8.31

	x_1	x_2	x_3	x_4	f	
1.	0	0	10	18	0	feasible
2.	0	5	0	8	-25	feasible
3.	0	9	-8	0	-45	infeasible
4.	-10	0	0	48	40	infeasible
5.	6	0	16	0	-24	feasible
6.	2	6	0	0	-38	feasible

Section 8.5 The Simplex Method

8.32

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = x_1 + 0.5x_2$
Subject to $6x_1 + 5x_2 \leq 30$
$3x_1 + x_2 \leq 12$
$x_1 + 3x_2 \leq 12$
$x_1, \; x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 0.5x_2$
Subject to $6x_1 + 5x_2 + x_3 = 30$
$3x_1 + x_2 + x_4 = 12$
$x_1 + 3x_2 + x_5 = 12$
$x_i \geq 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.32. The optimum solution is $x_1^* = \frac{10}{3}, x_2^* = 2$, and $f^* = -\frac{13}{3}$ where the 1st and 2nd constraints are active.

Table E8.32

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	6	5	1	0	0	30	$\frac{30}{6} = 5$
x_4	**3**	1	0	1	0	12	$\frac{12}{3} = \mathbf{4}$
x_5	1	3	0	0	1	12	$\frac{12}{1} = 12$
Cost	**-1**	-0.5	0	0	0	f-0	
x_3	0	**3**	1	-2	0	6	$\frac{6}{3} = \mathbf{2}$
x_1	1	1/3	0	1/3	0	4	$\frac{4}{1/3} = 12$
x_5	0	$8/3$	0	-1/3	1	8	$\frac{8}{8/3} = 3$
Cost	0	**-1/6**	0	$1/3$	0	f+4	0
x_2	0	1	$1/3$	$-2/3$	0	2	
x_1	1	0	$-1/9$	$5/9$	0	$10/3$	
x_5	0	0	$-8/9$	$13/9$	1	$8/3$	
Cost	0	0	$1/18$	$2/9$	0	f+3/13	

8.33

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 3x_1 + 2x_2$
Subject to $3x_1 + 2x_2 \le 6$
$-4x_1 + 9x_2 \le 36$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -3x_1 - 2x_2$
Subject to $3x_1 + 2x_2 + x_3 = 6$
$-4x_1 + 9x_2 + x_4 = 36$
$x_i \ge 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.33. The optimum solutions are points lying on the line segment between (0,3) and (2,0) with $f^* = 6$.

Table E8.33

Basic	x_1	x_2	x_3	x_4	**b**	ratio
x_3	3	2	1	0	6	$\frac{6}{3} = \underline{\mathbf{2}}$
x_4	-4	9	0	1	36	negative
Cost	-**<u>3</u>**	-2	0	0	$f - 0$	
x_1	1	**<u>2/3</u>**	1/3	0	2	$\frac{2}{2/3} = \underline{\mathbf{3}}$
x_4	0	35/3	4/3	1	44	$\frac{44}{35/3} = 3.77$
Cost	0	**<u>0</u>**	1	0	$f + 6$	
If x_2 is introduced into the basic set, we get the other solution as						
x_2	3/2	1	1/2	0	3	
x_4	-35/2	0	-9/2	1	9	
Cost	0	0	1	0	$f + 6$	

8.34

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = x_1 + 2x_2$
Subject to $-x_1 + 3x_2 \leq 10$
$x_1 + x_2 \leq 6$
$x_1 - x_2 \leq 2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 2x_2$
Subject to $-x_1 + 3x_2 + x_3 = 10$
$x_1 + x_2 + x_4 = 6$
$x_1 - x_2 + x_5 = 2$
$x_i \geq 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.34. The optimum solution is $x_1^* = 2.0,\ x_2^* = 4.0$, and $z^* = 10.0$, where the 1st and 2nd constraints are active.

Table E8.34

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	-1	**3**	1	0	0	10	10/3=**3.3**
x_4	1	1	0	1	0	6	6/1=6
x_5	1	-1	0	0	1	2	negative
Cost	-1	**-2**	0	0	0	f-0	
x_2	-1/3	1	1/3	0	0	10/3	negative
x_4	**4/3**	0	-1/3	1	0	8/3	(8/3)/(4/3)=**2**
x_5	2/3	0	1/3	0	1	16/3	(16/3)/(2/3)=8
Cost	**-5/3**	0	2/3	0	0	f+20/3	
x_2	0	1	1/4	1/4	0	4	
x_1	1	0	-1/4	3/4	0	2	
x_5	0	0	1/2	-1/2	1	4	
Cost	1	0	1/4	5/4	0	f+10	

8.35

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 2x_1 + x_2$
Subject to $-x_1 + 2x_2 \leq 10$
$3x_1 + 2x_2 \leq 18$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -2x_1 - x_2$
Subject to $-x_1 + 2x_2 + x_3 = 10$
$3x_1 + 2x_2 + x_4 = 18$
$x_i \geq 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.35. The optimum solution is $x_1^* = 6.0,\ x_2^* = 0$, and $z^* = 12.0$,where the 2nd constraint is active.

Table E8.35

Basic	x_1	x_2	x_3	x_4	b	ratio
x_3	-1	2	1	0	10	negative
x_4	**3**	2	0	1	18	$\frac{18}{3} = \mathbf{6}$
Cost	-**2**	-1	0	0	f-0	
x_3	0	8/3	1	1/3	16	
x_1	1	2/3	0	1/3	6	
Cost	0	1/3	0	2/3	f+12	

8.36

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 5x_1 - 2x_2$
Subject to $2x_1 + x_2 \leq 9$
$x_1 - x_2 \leq 2$
$-3x_1 + 2x_2 \leq 3$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -5x_1 + 2x_2$
Subject to $2x_1 + x_2 + x_3 = 9$
$x_1 - x_2 + x_4 = 2$
$-3x_1 + 2x_2 + x_5 = 3$
$x_i \geq 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.36. The optimum solution is $x_1^* = 3.667,\ x_2^* = 1.667$, and $z^* = 15.0$, where the 1st and 2nd constraints are active.

Table E8.36

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	2	1	1	0	0	9	9/2=4.5
x_4	**1**	-1	0	1	0	2	2/1=**2**
x_5	-3	2	0	0	1	3	negative
Cost	**-5**	2	0	0	0	f-0	
x_3	0	**3**	1	-2	0	5	**5/3**
x_1	1	-1	0	1	0	2	negative
x_5	0	-1	0	3	1	9	negative
Cost	0	**-3**	0	5	0	f+10	
x_2	0	1	1/3	-2/3	0	5/3	
x_1	1	0	1/3	1/3	0	11/3	
x_5	0	0	1/3	7/3	1	11	
Cost	0	0	1	3	0	f+15	

8.37

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Minimize $f = 2x_1 - x_2$
Subject to $-x_1 + 2x_2 \le 10$
$3x_1 + 2x_2 \le 18$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = 2x_1 - x_2$
Subject to $-x_1 + 2x_2 + x_3 = 10$
$3x_1 + 2x_2 + x_4 = 18$
$x_i \ge 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.37. The optimum solution is $x_1^* = 0.0$, $x_2^* = 5.0$, and $z^* = -5.0$, where the 1st constraint is active.

Table E8.37

Basic	x_1	x_2	x_3	x_4	b	ratio
x_3	-1	2	1	0	10	10/2=**5**
x_4	3	2	0	1	18	18/2=9
Cost	2	**-1**	0	0	f-0	
x_2	-1/2	1	1/2	0	5	
x_4	4	0	-1	1	8	
Cost	3/2	0	1/2	0	f+5	

8.38

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

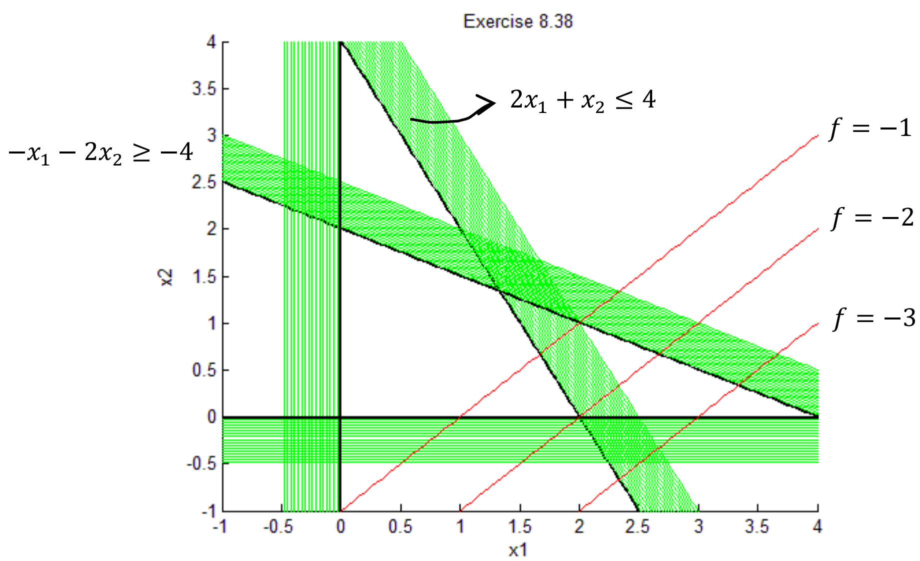
Minimize $f = -x_1 + x_2$
Subject to $2x_1 + x_2 \le 4$
$-x_1 - 2x_2 \ge -4$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -x_1 + x_2$
Subject to $2x_1 + x_2 + x_3 = 4$
$x_1 + 2x_2 + x_4 = 4$
$x_i \ge 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.38. The optimum solution is $x_1^* = 2.0$, $x_2^* = 0.0$, and $f^* = -2.0$, where the 1st constraint is active. The solution can be verified graphically.

Table E8.38

Basic	x_1	x_2	x_3	x_4	b	ratio
x_3	**2**	1	1	0	4	4/2=**2**
x_4	1	2	0	1	4	4/1=4
Cost	-**1**	1	0	0	f-0	
x_1	1	1/2	1/2	0	2	
x_4	0	3/2	-1/2	1	2	
Cost	0	3/2	1/2	0	f+2	

Figure E8.38



```
clear all
[x1,x2]=meshgrid(-1:0.05:4, -1:0.05:4);
f=-x1+x2;
g1=2*x1+x2-4;
g2=x1+2*x2-4;
g3=-x1;
g4=-x2;
cla reset
axis auto
xlabel('x1'),ylabel('x2')
title('Exercise 8.38')
hold on
cv1=[0:0.05:1.0];
contour(x1,x2,g1,cv1,'g');
cv2=[0:0.01:0.02];
contour(x1,x2,g1,cv2,'k');
cv3=[0:0.05:1.0];
contour(x1,x2,g2,cv3,'g');
cv4=[0:0.01:0.02];
contour(x1,x2,g2,cv4,'k');
cv5=[0:0.03:0.5];
contour(x1,x2,g3,cv5,'g');
contour(x1,x2,g4,cv5,'g');
contour(x1,x2,g3,cv4,'k');
contour(x1,x2,g4,cv4,'k');
fv=[-3 -2 -1];
fs=contour(x1,x2,f,fv,'r');

grid off
hold off
```

8.39 ——

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

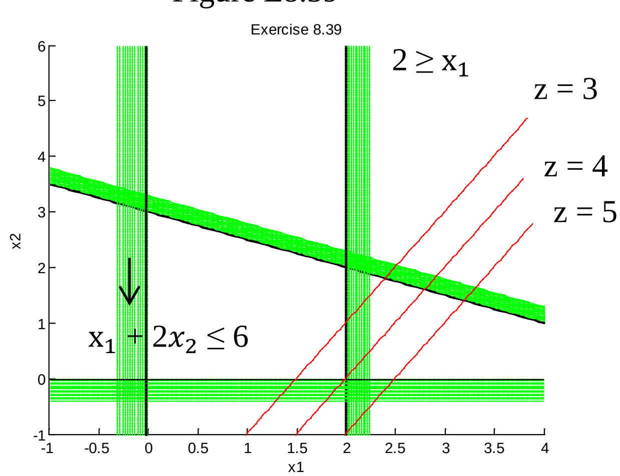
Maximize $z = 2x_1 - x_2$
Subject to $x_1 + 2x_2 \leq 6$
$2 \geq x_1$
$x_1, x_2 \geq 0$

Solution
Standard LP form:
Minimize $f = -2x_1 + x_2$
Subject to $x_1 + 2x_2 + x_3 = 6$
$x_1 + x_4 = 2$
$x_i \geq 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.39. The optimum solution is $x_1^* = 2.0$, and $x_2^* = 0.0$ and $z^* = 4.0$,where the 2^{nd} constraint is active. The solution can be verified graphically.

Table E8.39

Basic	x_1	x_2	x_3	x_4	b	ratio
x_3	1	2	1	0	6	6/1=6
x_4	**1**	0	0	1	2	2/1=**2**
Cost	**-2**	1	0	0	f-0	
x_3	0	2	1	-1	4	
x_1	1	0	0	1	2	
Cost	0	1	0	2	f+4	

Figure E8.39



```
clear all
[x1,x2]=meshgrid(-1:0.05:4, -1:0.05:6);
f=-2*x1+x2;
g1=x1+2*x2-6;
g2=x1-2;
g3=-x1;
g4=-x2;
cla reset
axis auto
xlabel('x1'),ylabel('x2')
title('Exercise 8.39')
hold on

cv1=[0:0.04:0.6];
contour(x1,x2,g1,cv1,'g');

cv2=[0:0.01:0.02];
contour(x1,x2,g1,cv2,'k');

cv3=[0:0.02:0.25];
contour(x1,x2,g2,cv3,'g');

cv4=[0:0.01:0.01];
contour(x1,x2,g2,cv4,'k');

cv5=[0:0.025:0.3];
contour(x1,x2,g3,cv5,'g');

cv6=[0:0.03:0.4];
contour(x1,x2,g4,cv6,'g');
contour(x1,x2,g3,cv4,'k');
contour(x1,x2,g4,cv4,'k');

fv=[-5 -4 -3];
fs=contour(x1,x2,f,fv,'r');

grid off;
hold off;
```

8.40

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = x_1 + x_2$
Subject to $4x_1 + 3x_2 \le 12$
$x_1 + 2x_2 \le 4$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - x_2$
Subject to $4x_1 + 3x_2 + x_3 = 12$
$x_1 + 2x_2 + x_4 = 4$
$x_i \ge 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.40. The optimum solution is $x_1^* = 2.4$, $x_2^* = 0.8$, and $z^* = 3.2$, where the 1st and 2nd constraints are active.

Table E8.40

Basic	x_1	x_2	x_3	x_4	b	ratio
x_3	**4**	3	1	0	12	12/4=**3**
x_4	1	2	0	1	4	4/1=4
Cost	**-1**	-1	0	0	f-0	
If x_1 is introduced into the basic set, we get the other solution as						
x_1	1	3/4	1/4	0	3	3/¾=4
x_4	0	**5/4**	-1/4	1	1	1/(5/4)=**0.8**
Cost	0	**-1/4**	1/4	0	f+3	
x_1	1	0	2/5	-3/5	12/5	
x_2	0	1	-1/5	4/5	4/5	
Cost	0	0	1/5	1/5	f+16/5	

8.41

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = -2x_1 + x_2$
Subject to $x_1 \le 2$
$x_1 + 2x_2 \le 6$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = 2x_1 - x_2$
Subject to $x_1 + x_3 = 2$
$x_1 + 2x_2 + x_4 = 6$
$x_i \ge 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.41. The optimum solution is $x_1^* = 0.0,\ x_2^* = 3.0$, and $z^* = 3.0$, where the 2nd constraint is active.

Table E8.41

Basic	x_1	x_2	x_3	x_4	b	ratio
x_3	1	0	1	0	2	∞
x_4	1	**2**	0	1	6	6/2=**3**
Cost	2	**-1**	0	0	f-0	
x_3	1	0	1	0	2	
x_2	1/2	1	0	1/2	3	
Cost	5/2	0	0	1/2	f+3	

8.42

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 2x_1 + x_2$
Subject to $4x_1 + 3x_2 \le 12$
$x_1 + 2x_2 \le 4$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -2x_1 - x_2$
Subject to $4x_1 + 3x_2 + x_3 = 12$
$x_1 + 2x_2 + x_4 = 4$
$x_i \ge 0; i = 1$ to 4
where x_3, and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.42. The optimum solution is $x_1^* = 0.0$, $x_2^* = 4.0$, and $z^* = 22/3$, where the 1st constraint is active.

Table E8.42

Basic	x_1	x_2	x_3	x_4	b	ratio
x_3	**4**	3	1	0	12	12/4=**3**
x_4	1	2	0	1	4	4/1=4
Cost	-**2**	-1	0	0	f-0	
x_1	1	3/4	1/4	0	3	3/¾=4
x_4	0	1/4	-1/4	1	1	1/¼=**4**
Cost	0	**-1/3**	1/2	0	f+6	
x_1	1	0	1	-2	0	
x_2	0	1	-1	4	4	
Cost	0	0	-1/6	4/3	f+22/3	

8.43

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Minimize $f = 9x_1 + 2x_2 + 3x_3$
Subject to $2x_1 + x_2 - 3x_3 \geq -5$
$-x_1 - 2x_2 + 2x_3 \geq -2$
$x_1,\ x_2,\ x_3 \geq 0$

Solution:
Standard LP form:
Minimize $f = 9x_1 + 2x_2 + 3x_3$
Subject to $-2x_1 - x_2 + 3x_3 + x_4 = 5$
$x_1 + 2x_2 - 2x_3 + x_5 = 2$
$x_i \geq 0; i = 1$ to 5
where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.43. The optimum solution is $x_1^* = 0.0,\ x_2^* = 0.0$, and $z^* = 0.0$.

Table E8.43

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_4	-2	-1	3	1	0	5	
x_5	1	2	-2	0	1	2	
Cost	9	2	3	0	0	f-0	

8.44

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = x_1 + x_2$
Subject to $4x_1 + 3x_2 \le 9$
$x_1 + 2x_2 \le 6$
$2x_1 + x_2 \le 6$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - x_2$
Subject to $4x_1 + 3x_2 + x_3 = 9$
$x_1 + 2x_2 + x_4 = 6$
$2x_1 + x_2 + x_5 = 6$
$x_i \ge 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.44. The optimum solution is $x_1^* = 0.0,\ x_2^* = 3.0$, and $z^* = 3.0$, where the 1st and 2nd constraints are active.

Table E8.44

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	**4**	3	1	0	0	9	9/4=**2.25**
x_4	1	2	0	1	0	6	6/1=6
x_5	2	1	0	0	1	6	6/2=3
Cost	-**1**	-1	0	0	0	f-0	
If x_1 is introduced into the basic set, we get the other solution as							
x_1	1	3/4	1/4	0	0	9/4	(9/4)/(3/4)=3
x_4	0	**5/4**	-1/4	1	0	15/4	(15/4)/(5/4)=**3**
x_5	0	-1/2	-1/2	0	1	3/2	negative
Cost	0	-**1/4**	1/4	0	0	f+9/4	
x_1	1	0	2/5	-3/5	0	0	
x_2	0	1	-1/5	4/5	0	3	
x_5	0	0	-3/5	2/5	1	3	
Cost	0	0	1/5	1/5	0	f+3	

8.45

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + x_2 \leq 16$
$x_1 + 2x_2 \leq 28$
$24 \geq 2x_1 + x_2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + x_2 + x_3 = 16$
$x_1 + 2x_2 + x_4 = 28$
$2x_1 + x_2 + x_5 = 24$
$x_i \geq 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.45. The optimum solution is $x_1^* = 0.0,\ x_2^* = 14.0$, and $f^* = -56.0$, where the 2nd constraint is active.

Table E8.45

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	1	1	1	0	0	16	16/1=16
x_4	1	**2**	0	1	0	28	28/2=**14**
x_5	2	1	0	0	1	24	24/1=24
Cost	-1	**-4**	0	0	0	f	
x_3	1/2	0	1	-1/2	0	2	
x_2	1/2	1	0	1/2	0	14	
x_5	3/2	0	0	-1/2	1	10	
Cost	1	0	0	2	0	f+56	

8.46

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Minimize $f = x_1 - x_2$
Subject to $4x_1 + 3x_2 \leq 12$
$x_1 + 2x_2 \leq 4$
$4 \geq 2x_1 + x_2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = x_1 - x_2$
Subject to $4x_1 + 3x_2 + x_3 = 12$
$x_1 + 2x_2 + x_4 = 4$
$2x_1 + x_2 + x_5 = 4$
$x_i \geq 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.46. The optimum solution is $x_1^* = 0.0,\ x_2^* = 2.0$, and $f^* = -2.0$, where the 2nd constraint is active.

Table E8.46

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	4	3	1	0	0	12	12/3=4
x_4	1	**2**	0	1	0	4	4/2=**2**
x_5	2	1	0	0	1	4	4/1=4
Cost	1	**-1**	0	0	0	f	
x_3	5/2	0	1	-3/2	0	6	
x_2	1/2	1	0	1/2	0	2	
x_5	3/2	0	0	-1/2	1	2	
Cost	3/2	0	0	1/2	0	f+2	

8.47

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 2x_1 + 3x_2$
Subject to $x_1 + x_2 \leq 16$
$-x_1 - 2x_2 \geq -28$
$24 \geq 2x_1 + x_2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -2x_1 - 3x_2$
Subject to $x_1 + x_2 + x_3 = 16$
$x_1 + 2x_2 + x_4 = 28$
$2x_1 + x_2 + x_5 = 24$
$x_i \geq 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.47. The optimum solution is $x_1^* = 0.0,\ x_2^* = 14.0$, and $z^* = 42.0$, where the 2nd constraint is active.

Table E8.47

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	1	1	1	0	0	16	16/1=16
x_4	1	**2**	0	1	0	28	28/2=**14**
x_5	2	1	0	0	1	24	24/1=24
Cost	-2	**-3**	0	0	0	f	
x_3	1/2	0	1	-1/2	0	2	
x_2	1/2	1	0	1/2	0	14	
x_5	3/2	0	0	-1/2	1	10	
Cost	-1/2	0	0	3/2	0	f+42	

8.48

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = x_1 + 2x_2$
Subject to $2x_1 - x_2 \geq 0$
$2x_1 + 3x_2 \geq -6$
$x_1, \ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 2x_2$
Subject to $2x_1 - x_2 - x_3 = 0$
$-2x_1 - 3x_2 + x_4 = 6$
$x_i \geq 0; i = 1$ to 4
where x_3 is surplus variable, and x_4 is slack variable. The initial tableau for the Simplex method is set up in Table E8.48. The problem is unbounded which can be verified graphically. Since the basic feasible solution is degenerate the Simplex method fails due cycling of iterations.

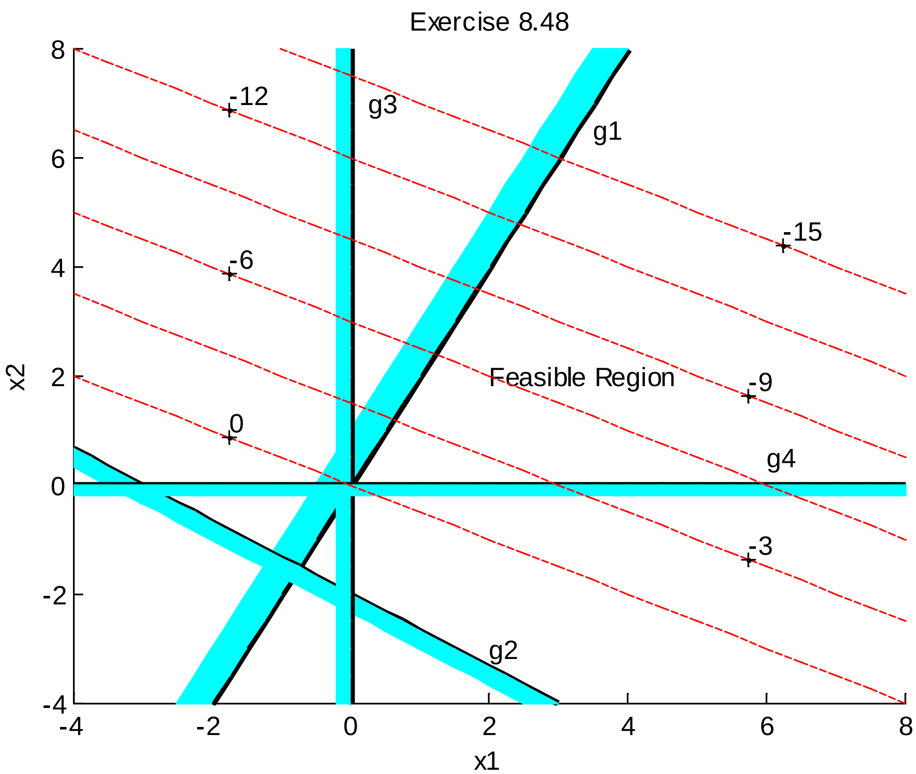
Table E8.48

Basic	x_1	x_2	x_3	x_4	**b**	ratio
x_3	2	-1	-1	0	0	negative
x_4	-2	-3	0	1	6	negative
Cost	-1	-**2**	0	0	f	

Matlab Code

```
%Exercise 8.48
%Create a grid from -4 to 8 with an increment of 0.5 for the variables x1 and x2
[x1,x2]=meshgrid(-4:0.5:8.0, -4:0.5:8.0);
%Enter functions for the minimization problem
f=-x1-2*x2;
g1=-2*x1+x2;
g2=-2*x1-3*x2-6;
g3=-x1;
g4=-x2;
cla reset
axis auto                   %Minimum and maximum values for axes are determined
automatically
                            %Limits for x- and y-axes may be specified with the command
                            %axis ([xmin xmax ymin ymax])
xlabel('x1'),ylabel('x2')        %Specifies labels for x- and y-axes
title ('Exercise 8.48')
hold on
cv1=[0 0];
cv12=[0.01:0.01:1];
```

```
const1=contour(x1,x2,g1,cv1,'k','LineWidth',4);
text(2,-3,'g2')
const1=contour(x1,x2,g1,cv12,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv12,'c');
text(3.5,6.5,'g1')
cv34=[0.01:0.01:0.2];
const3=contour(x1,x2,g3,cv1,'k','Linewidth',4);
const3=contour(x1,x2,g3,cv34,'c');
text(0.25,7,'g3')
const4=contour(x1,x2,g4,cv1,'k','LineWidth',3);
text(6,0.5,'g4')
const4=contour(x1,x2,g4,cv34,'c');
text(2,2,'Feasible Region')
fv=[0 -3 -6 -9 -12 -15];            %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'r--');       %'r' specifies red dashed lines for function
contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
                              %Subsequent plots will appear in separate windows
```



8.49

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 2x_1 + 2x_2 + x_3$
Subject to $10x_1 + 9x_3 \le 375$
$x_1 + 3x_2 + x_3 \le 33$
$2 \ge x_3$
$x_1,\ x_2,\ x_3 \ge 0$

Solution:
Standard LP form:
Minimize $f = -2x_1 - 2x_2 - x_3$
Subject to $10x_1 + 9x_3 + x_4 = 375$
$x_1 + 3x_2 + x_3 + x_5 = 33$
$x_3 + x_6 = 2$
$x_i \ge 0; i = 1$ to 6
where x_4, x_5 and x_6 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.49. The optimum solution is $x_1^* = 33.0$, $x_2^* = 0.0$, $x_3^* = 0.0$, and $z^* = 66.0$, where the 2nd constraint is active.

Table E8.49

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio
x_4	10	0	9	1	0	0	375	37.5
x_5	**1**	3	1	0	1	0	33	**33**
x_6	0	0	1	0	0	1	2	∞
Cost	**-2**	-2	-1	0	0	0	f	
x_4	0	-30	-1	1	-10	0	45	
x_1	1	3	1	0	1	0	33	
x_6	0	0	1	0	0	1	2	
Cost	0	4	1	0	2	0	f+66	

8.50

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = x_1 + 2x_2$
Subject to $-2x_1 - x_2 \geq -5$
$3x_1 + 4x_2 \leq 10$
$x_1 \leq 2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 2x_2$
Subject to $2x_1 + x_2 + x_3 = 5$
$3x_1 + 4x_2 + x_4 = 10$
$x_1 + x_5 = 2$
$x_i \geq 0; i = 1$ to 5
where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.50. The optimum solution is $x_1^* = 0.0$, $x_2^* = 2.5$, and $z^* = 5.0$, where the 2^{nd} constraint is active.

Table E8.50

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio
x_3	2	1	1	0	0	5	5/1=5
x_4	3	**4**	0	1	0	10	10/4=**2.5**
x_5	1	0	0	0	1	2	∞
Cost	-1	**-2**	0	0	0	f	
x_3	5/4	0	1	-1/4	0	5/2	
x_2	3/4	1	0	1/4	0	5/2	
x_5	1	0	0	0	1	2	
Cost	1/2	0	0	1/2	0	f+5	

8.51

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Minimize $f = -2x_1 - x_2$
Subject to $-2x_1 - x_2 \geq -5$
$3x_1 + 4x_2 \leq 10$
$x_1 \leq 3$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -2x_1 - x_2$
Subject to $2x_1 + x_2 + x_3 = 5$
$3x_1 + 4x_2 + x_4 = 10$
$x_1 + x_5 = 3$
$x_i \geq 0; i = 1$ to 5

where x_3, x_4 and x_5 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.51. The optimum solution is $x_1^* = 2.0$, $x_2^* = 1.0$, and $f^* = -5.0$, where the 1st and 2nd constraints are active.

Table E8.51

Basic	x_1	x_2	x_3	x_4	x_5	**b**	ratio
x_3	**2**	1	1	0	0	5	5/2=**2.5**
x_4	3	4	0	1	0	10	10/3=3.3
x_5	1	0	0	0	1	3	3/1=3
Cost	**-2**	-1	0	0	0	f	
x_1	1	1/2	0.5	0	0	5/2	2.5/0.5=5
x_4	0	**5/2**	-3/2	1	0	5/2	2.5/2.5=**1**
x_5	0	-1/2	-1/2	0	1	1/2	negative
Cost	0	**0**	1	0	0	f+5	f+5
x_1	1	0	4/5	-1/5	0	2	
x_2	0	1	-3/5	2/5	0	1	
x_5	0	0	-4/5	1/5	1	1	
Cost	0	0	1	0	0	f+5	

8.52

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 12x_1 + 7x_2$
Subject to $2x_1 + x_2 \leq 5$
$3x_1 + 4x_2 \leq 10$
$x_1 \leq 2$
$x_2 \leq 3$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -12x_1 - 7x_2$
Subject to $2x_1 + x_2 + x_3 = 5$
$3x_1 + 4x_2 + x_4 = 10$
$x_1 + x_5 = 2$
$x_2 + x_6 = 3$
$x_i \geq 0; i = 1 \text{ to } 6$
where x_3, x_4, x_5 and x_6 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.52. The optimum solution is $x_1^* = 2.0$, $x_2^* = 1.0$, and $z^* = 31.0$, where the 1st, 2nd and 3rd constraints are active.

Table E8.52

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio
x_3	2	1	1	0	0	0	5	5/2=2.5
x_4	3	4	0	1	0	0	10	10/3
x_5	**1**	0	0	0	1	0	2	2/1=**2**
x_6	0	1	0	0	0	1	3	∞
Cost	**-12**	-7	0	0	0	0	f-0	
x_3	0	**1**	1	0	-2	0	1	1/1=**1**
x_4	0	4	0	1	-3	0	4	4/4=1
x_1	1	0	0	0	1	0	2	∞
x_6	0	1	0	0	0	1	3	3/1=3
Cost	0	**-7**	0	0	12	0	f+24	
x_2	0	1	1	0	-2	0	1	
x_4	0	0	-4	1	5	0	0	
x_1	1	0	0	0	1	0	2	
x_6	0	0	-1	0	2	1	2	
Cost	0	0	7	0	-2	0	f+31	

8.53

Solve the following problem by the Simplex method and verify the solution graphically whenever possible.

Maximize $z = 10x_1 + 8x_2 + 5x_3$
Subject to $10x_1 + 9x_3 \le 375$
$5x_1 + 15x_2 + 3x_3 \le 35$
$3 \ge x_3$
$x_1,\ x_2,\ x_3 \ge 0$

Solution:
Standard LP form:
Minimize $f = -10x_1 - 8x_2 - 5x_3$
Subject to $10x_1 + 9x_3 + x_4 = 375$
$5x_1 + 15x_2 + 3x_3 + x_5 = 35$
$x_3 + x_6 = 3$
$x_i \ge 0; i = 1$ to 6

where x_4, x_5 and x_6 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.53. The optimum solution is $x_1^* = 7.0,\ x_2^* = 0.0,\ x_3^* = 0.0$ and $z^* = 70.0$, where the 2nd constraint is active.

Table E8.53

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio
x_4	10	0	9	1	0	0	375	375/10=37.5
x_5	**5**	15	3	0	1	0	35	35/5=**7**
x_6	0	0	1	0	0	1	3	∞
Cost	**-10**	-8	-5	0	0	0	f	
x_4	0	-30	3	1	-2	0	305	
x_1	1	3	3/5	0	1/5	0	7	
x_6	0	0	1	0	0	1	3	
Cost	0	22	1	0	2	0	f+70	

Section 8.6 The Two Phase Simplex Method − Artificial Variables

8.54

Answer True or False.

1. A pivot step of the Simplex method replaces a current basic variable with a nonbasic variable. *True*
2. The pivot step brings the design point to the interior of the constraint set. *False*
3. The pivot column in the Simplex method is determined by the largest reduced cost coefficient corresponding to a basic variable. *False*
4. The pivot row in the Simplex method is determined by the largest ratio of right-side parameters with the positive coefficients in the pivot column. *False*
5. The criterion for a current basic variable to leave the basic set is to keep the new solution basic and feasible. *False*
6. A move from one basic feasible solution to another corresponds to extreme points of the convex polyhedral set. *True*
7. A move from one basic feasible solution to another can increase the cost function value in the Simplex method. *False*
8. The right sides in the Simplex tableau can assume negative values. *False*
9. The right sides in the Simplex tableau can become zero. *True*
10. The reduced cost coefficients corresponding to the basic variables must be positive at the optimum. *False*
11. If a reduced cost coefficient corresponding to a nonbasic variable is zero at the optimum point, there may be multiple solutions to the problem. *True*
12. If all elements in the pivot column are negative, the problem is infeasible. *False*
13. The artificial variables must be positive in the final solution. *False*

14. If artificial variables are positive at the final solution, the artificial cost function is also positive. *True*
15. If artificial cost function is positive at the optimum solution, the problem is unbounded. *False*

8.55 ---

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = x_1 + 2x_2$
Subject to $-x_1 + 3x_2 \le 10$
$x_1 + x_2 \le 6$
$x_1 - x_2 \le 2$
$x_1 + 3x_2 \ge 6$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 2x_2$
Subject to $-x_1 + 3x_2 + x_3 = 10$
$x_1 + x_2 + x_4 = 6$
$x_1 - x_2 + x_5 = 2$
$x_1 + 3x_2 - x_6 + x_7 = 6$
$x_i \ge 0; i = 1$ to 7

where x_3, x_4 and x_5 are slack variables; x_6 and x_7 are the surplus and artificial variables for the 4th constraint. The problem is solved by the Simplex method, which is given in Table E8.55. The optimum solution is $x_1^* = 2.0,\ x_2^* = 4.0$, and $z^* = 10.0$, where the 1st and 2nd constraints are active. The solution can be verified graphically.

Table E8.55

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	ratio	
x_3	-1	3	1	0	0	0	0	10	10/3	
x_4	1	1	0	1	0	0	0	6	6	
x_5	1	-1	0	0	1	0	0	2	negative	
x_7	1	**3**	0	0	0	-1	1	6	**2**	
Cost	-1	-2	0	0	0	0	0	f-0		
Arti	-1	**-3**	0	0	0	1	0	w-6		
x_3	-2	0	1	0	0	**1**	-1	4	**4**	
x_4	2/3	0	0	1	0	1/3	- 1/3	4	12	
x_5	4/3	0	0	0	1	- 1/3	1/3	4	negative	
x_2	1/3	1	0	0	0	- 1/3	1/3	2	negative	
Cost	- 1/3	0	0	0	0	**- 2/3**	2/3	f+4		
Arti	0	0	0	0	0	0	1	w-0		End phs1
x_6	-2	0	1	0	0	1	-1	4	negative	
x_4	**4/3**	0	- 1/3	1	0	0	0	8/3	**2**	
x_5	2/3	0	1/3	0	1	0	0	16/3	8	
x_2	- 1/3	1	1/3	0	0	0	0	10/3	negative	
Cost	**-5/3**	0	2/3	0	0	0	0	f+20/3		
x_6	0	0	1/2	3/2	0	1	-1	8		
x_1	1	0	- 1/4	3/4	0	0	0	2		
x_5	0	0	1/2	- 1/2	1	0	0	4		
x_2	0	1	1/4	1/4	0	0	0	4		
Cost	0 (c_1')	0 (c_2')	1/4 (c_3')	5/4 (c_4')	0 (c_5')	0 (c_6')	0 (c_7')	f+10	Cost	End phs2

Exercise 8.88

From the final tableau for Exercise 8.55,

x_3, x_4 and x_5 are slack variables; x_6 and x_7 are the surplus and artificial variables.

For $-x_1 + 3x_2 \leq 10$: $y_1 = \frac{1}{4}$ (c_3' in the slack variable column x_3)

$x_1 + x_2 \leq 6$: $y_2 = \frac{5}{4}$ (c_4' in the slack variable column x_4)

$x_1 - x_2 \leq 2$: $y_3 = 0$ (c_5' in the slack variable column x_5)

$x_1 + 3x_2 \geq 6$: $y_4 = 0$ (c_7' in the artificial variable column x_7)

Therefore, y_1 = 0.25, y_2 = 1.25, y_3= 0.0, y_4 = 0.0

Exercise 8.110

Referring to Exercise 8.55 and the final tableau in Table E8.55, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 10$: $\max\{-8/(1/2), -4/(1/2), -4/(1/4)\} \le \Delta_1 \le 8$, or $-8 \le \Delta_1 \le 8$;

For $b_2 = 6$: $\max\{-8/(3/2), -2/(3/4), -4/(1/4)\} \le \Delta_2 \le 8$, or $-2.6667 \le \Delta_2 \le 8$;

For $b_3 = 2$: $-4 \le \Delta_3 \le \infty$;

For $b_4 = 6$: $-\infty \le \Delta_4 \le 8$

Exercise 8.132

Referring to Exercise 8.55 and final tableau in Table E8.55, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -1$: $-\frac{1/4}{1/4} \le \Delta c_1 \le \frac{5/4}{3/4}$, or $-1 \le \Delta c_1 \le 1.6667$;

For $c_2 = -2$: $-\infty \le \Delta c_2 \le \min\{\frac{1/4}{1/4}, \frac{5/4}{1/4}\}$, or $-\infty \le \Delta c_2 \le 1$

For the original form:

For $c_1 = 1$: $-1.6667 \le \Delta c_1 \le 1$;

For $c_2 = 2$: $1 \le \Delta c_2 \le \infty$

8.56

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = 4x_1 + 2x_2$
Subject to $-2x_1 + x_2 \leq 4$
$x_1 + 2x_2 \geq 2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -4x_1 - 2x_2$
Subject to $-2x_1 + x_2 + x_3 = 4$
$x_1 + 2x_2 - x_4 + x_5 = 2$
$x_i \geq 0; i = 1$ to 5

where x_3 is slack variable, x_4 and x_5 are surplus and artificial variables for the 2nd constraint. The problem is solved by the Simplex method, which is given in Table E8.56. From the final tableau we conclude that this problem is unbounded. The solution can be verified graphically.

Table E8.56

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio	
x_3	-2	1	1	0	0	4	4	
x_5	1	**2**	0	-1	1	2	**1**	
Cost	-4	-2	0	0	0	f-0		
Arti	-1	**-2**	0	1	0	w-2		
x_3	-2.5	0	1	0.5	-0.5	3	negative	
x_2	**0.5**	1	0	-0.5	0.5	1	**2**	
Cost	**-3**	0	0	-1	1	f+2		
Arti	0	0	0	0	1	w-0		End phs1
x_3	0	5	1	-2	2	8		
x_1	1	2	0	-1	1	2		
Cost	0	6	0	-4	4	f+8		End phs2

8.57

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \leq 5$
$x_1 + x_2 = 4$
$x_1 - x_2 \geq 3$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + 2x_2 + x_3 = 5$
$x_1 + x_2 + x_5 = 4$
$x_1 - x_2 - x_4 + x_6 = 3$
$x_i \geq 0; i = 1$ to 6

The optimum solution is $x_1^* = 3.5$, $x_2^* = 0.5$, and $z^* = 5.5$, where the 2nd and 3rd constraints are active.

Table E8.57

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio	
x_3	1	2	1	0	0	0	5	5	
x_5	1	1	0	0	1	0	4	4	
x_6	**1**	-1	0	-1	0	1	3	**3**	
Cost	-1	-4	0	0	0	0	f-0		
Arti	**-2**	0	0	1	0	0	w-7		
x_3	0	3	1	1	0	-1	2	2/3	
x_5	0	**2**	0	1	1	-1	1	**1/2**	
x_1	1	-1	0	-1	0	1	3	negative	
Cost	0	-5	0	-1	0	1	f+3		
Arti	0	**-2**	0	-1	0	2	w-1		
x_3	0	0	1	-0.5	-1.5	0.5	0.5		
x_2	0	1	0	0.5	0.5	-0.5	0.5		
x_1	1	0	0	-0.5	0.5	0.5	3.5		
Cost	0 (c_1')	0 (c_2')	0 (c_3')	1.5 (c_4')	2.5 (c_5')	-1.5 (c_6')	f+5.5		
Arti	0	0	0	0	1	1	w-0	End phs1	End phs2

Exercise 8.90

From the final tableau for Exercise 8.57,
x_3 and x_5 are slack variables; x_4 is surplus variable; x_6 is artificial variable.

For $x_1 + 2x_2 \le 5$: $y_1 = 0$ (c_3' in the slack variable column x_3)
$x_1 + x_2 = 4$: $y_2 = 2.5$ (c_5' in the slack variable column x_5)
$x_1 - x_2 \ge 3$: $y_3 = -1.5$ (c_6' in the artificial variable column x_6)

Therefore, y_1 = 0.0, y_2 = 2.5, y_3= -1.5

Exercise 8.112

Referring to Exercise 8.57 and the final tableau in Table E8.57, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 5$: $-\frac{0.5}{1} \le \Delta_1 \le \infty$ or $-0.5 \le \Delta_1 \le \infty$;

For $b_2 = 4$: $\max\{-\frac{0.5}{0.5}, -\frac{3.5}{0.5}\} \le \Delta_2 \le \frac{0.5}{1.5}$ or $-1.0 \le \Delta_2 \le 0.333$;

For $b_3 = 3$: $\max\{-\frac{0.5}{0.5}, -\frac{3.5}{0.5}\} \le \Delta_3 \le \frac{0.5}{0.5}$ or $-1.0 \le \Delta_3 \le 1.0$

Exercise 8.134

Referring to Exercise 8.57 and final tableau in Table E8.57, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -1$: $-\frac{1.5}{0.5} \le \Delta c_1 \le \infty$ or $-3.0 \le \Delta c_1 \le \infty$;

For $c_2 = -4$: $-\infty \le \Delta c_2 \le \frac{1.5}{0.5}$ or $-\infty \le \Delta c_2 \le 3.0$

For the original form:

For $c_1 = 1$: $-\infty \le \Delta c_1 \le 3.0$;

For $c_2 = 4$: $-3.0 \le \Delta c_2 \le \infty$

8.58

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \le 5$
$2x_1 + x_2 = 4$
$x_1 - x_2 \ge 1$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 4x_2$
Subject to $x_1 + 2x_2 + x_3 = 5$
$2x_1 + x_2 + x_5 = 4$
$x_1 - x_2 - x_4 + x_6 = 1$
$x_i \ge 0; i = 1$ to 6

The optimum solution is $x_1^* = 1.667,\ x_2^* = 0.667$, and $z^* = 4.333$,where the 2nd and 3rd constraints are active.

Table E8.58

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio	
x_3	1	2	1	0	0	0	5	5	
x_5	2	1	0	0	1	0	4	2	
x_6	**1**	-1	0	-1	0	1	1	**1**	
Cost	-1	-4	0	0	0	0	f-0		
Arti	**-3**	0	0	1	0	0	w-5		
x_3	0	3	1	1	0	-1	4	4/3	
x_5	0	**3**	0	2	1	-2	2	**2/3**	
x_1	1	-1	0	-1	0	1	1	-1	
Cost	0	-5	0	-1	0	1	f+1		
Arti	0	**-3**	0	-2	0	3	w-2		
x_3	0	0	1	-1	-1	1	2		
x_2	0	1	0	2/3	1/3	- 2/3	0.666667		
x_1	1	0	0	- 1/3	1/3	1/3	1.666667		
Cost	0 (c_1')	0 (c_2')	0 (c_3')	7/3 (c_4')	5/3 (c_5')	-7/3 (c_6')	f+4.333333		
Arti	0	0	0	0	1	1	w-0	End phs1	End phs2

Exercise 8.91

From the final tableau for Exercise 8.58,
x_3 is slack variable; x_4 is surplus variable; x_5 and x_6 are artificial variables.

For $x_1 + 2x_2 \leq 5$: $y_1 = 0$ (c_3' in the slack variable column x_3)
$2x_1 + x_2 = 4$: $y_2 = \frac{5}{3}$ (c_5' in the artificial variable column x_5)
$x_1 - x_2 \geq 1$: $y_3 = -\frac{7}{3}$ (c_6' in the artificial variable column x_6)

Therefore, y_1 = 0.0, y_2 = 1.667, y_3= – 2.333

Exercise 8.113

Referring to Exercise 8.58 and the final tableau in Table E8.58, we can find the ranges for RHS by *Theorem 8.6* as follows:
For b_1 = 5: $-\frac{2}{1} \leq \Delta_1 \leq \infty$ or $-2.0 \leq \Delta_1 \leq \infty$;
For b_2 = 4: $\max\{-\frac{0.6667}{1/3}, -\frac{1.6667}{1/3}\} \leq \Delta_2 \leq \frac{2}{1}$ or $-2.0 \leq \Delta_2 \leq 2.0$;
For b_3 = 1: $\max\{-\frac{2}{1}, -\frac{1.6667}{1/3}\} \leq \Delta_3 \leq \frac{0.6667}{2/3}$ or $-2.0 \leq \Delta_3 \leq 1.0$

Exercise 8.135

Referring to Exercise 8.58 and final tableau in Table E8.58, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:
For $c_1 = -1$: $-\frac{7/3}{1/3} \leq \Delta c_1 \leq \infty$ or $-7.0 \leq \Delta c_1 \leq \infty$;
For $c_2 = -4$: $-\infty \leq \Delta c_2 \leq \frac{7/3}{2/3}$ or $-\infty \leq \Delta c_2 \leq 3.5$

For the original form:
For c_1 = 1: $-\infty \leq \Delta c_1 \leq 7.0$;
For c_2 = 4: $-3.5 \leq \Delta c_2 \leq \infty$

8.59

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 3x_1 + x_2 + x_3$
Subject to $-2x_1 - x_2 + 3x_3 \le -5$
$x_1 - 2x_2 + 3x_3 \ge -2$
$x_1,\ x_2,\ x_3 \ge 0$

Solution:
Standard LP form:
Minimize $f = 3x_1 + x_2 + x_3$
Subject to $2x_1 + x_2 - 3x_3 - x_4 + x_6 = 5$
$-x_1 + 2x_2 - 3x_3 + x_5 = 2$
$x_i \ge 0; i = 1$ to 6

The optimum solution is $x_1^* = 1.6$, $x_2^* = 1.8$, $x_3^* = 0.0$ and $f^* = 6.6$ where the 2nd constraint is active.

Table E8.59A

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio	
x_6	**2**	1	-3	-1	0	1	5	**2.5**	
x_5	-1	2	-3	0	1	0	2	negative	
Cost	3	1	1	0	0	0	f-0		
Arti	**-2**	-1	3	1	0	0	w-5		
x_1	1	0.5	-1.5	-0.5	0	0.5	2.5	5	
x_5	0	**2.5**	-4.5	-0.5	1	0.5	4.5	**1.8**	
Cost	0	**-0.5**	5.5	1.5	0	-1.5	f-7.5		
Arti	0	0	0	0	0	1	w-0		End phs1
x_1	1	0	-0.6	-0.4	-0.2	0.4	1.6		
x_2	0	1	-1.8	-0.2	0.4	0.2	1.8		
Cost	0 (c_1')	0 (c_2')	4.6 (c_3')	1.4 (c_4')	0.2 (c_5')	-1.4 (c_6')	f-6.6		End phs2

Table E8.59B LP Solver

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$E\$11	Objective Fucntion:min Sum of LHS	0	6.6

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$10	variable value x1	0	1.6	Contin
\$C\$10	variable value x2	0	1.8	Contin
\$D\$10	variable value x3	0	0	Contin

Exercise 8.92

From the final tableau for Exercise 8.59,
x_4 and x_5 are surplus variables; x_6 and x_7 are artificial variables.

For $-2x_1 - x_2 + 3x_3 \le -5$ $\qquad y_1 = 1.4$ (c_6' in the artificial variable column x_6)
$x_1 - 2x_2 + 3x_3 \le -2$ $\qquad y_2 = -0.2$ (c_5' in the artificial variable column x_5)

Therefore, y_1 = 4.0, y_2 = -1.0

Table 8.92 LP Solver

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$12	Constraint1 Sum of LHS	5	1.4	5	1E+30	4
\$E\$13	Constraint2 Sum of LHS	2	-0.2	2	8	4.5

Exercise 8.114

Referring to Exercise 8.59 and the final tableau in Table E8.59, we can find the ranges for RHS by *Theorem 8.6* as follows:

For b_1 = 5 (in Table E8.59): $\quad \max\{-\frac{1.6}{0.4}, -\frac{1.8}{0.2}\} \le \Delta_1 \le \infty$ or $-4.0 \le \Delta_1 \le \infty$;

For b_2 = 2 (in Table E8.59): $\quad \max\{-\frac{1.8}{0.4}\} \le \Delta_2 \le \frac{1.6}{0.2}$ or $-4.5 \le \Delta_2 \le 8.0$

Therefore,

For $b_1 = -5$: $\quad -\infty \le \Delta_1 \le 4.0$;

For $b_2 = -2$: $\quad -8.0 \le \Delta_2 \le 4.5$

Exercise 8.136

Referring to Exercise 8.59 and final tableau in Table E8.59, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 3$: $\max\{-\frac{4.6}{0.6}, -\frac{1.4}{0.4}, -\frac{0.2}{0.2}\} \le \Delta c_1 \le \infty$ or $-1.0 \le \Delta c_1 \le \infty$;

For $c_2 = 1$: $\max\{-\frac{4.6}{1.8}, -\frac{1.4}{0.2}\} \le \Delta c_2 \le \frac{0.2}{0.4}$ or $-2.556 \le \Delta c_2 \le 0.5$;

For $c_3 = 1$: $-4.6 \le \Delta c_3 \le \infty$

Table 8.136 LP Solver

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B10	variable value x1	1.6	0	3	1E+30	1
C10	variable value x2	1.8	0	1	0.5	2.555555556
D10	variable value x3	0	4.6	1	1E+30	4.6

8.60

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 5x_1 + 4x_2 - x_3$
Subject to $x_1 + 2x_2 - x_3 \geq 1$
$2x_1 + x_2 + x_3 \geq 4$
$x_1,\ x_2 \geq 0$; x_3 is unrestricted in sign.

Solution:
Standard LP form:
Minimize $f = 5y_1 + 4y_2 - y_3 + y_4$
Subject to $y_1 + 2y_2 - y_3 + y_4 - y_5 + y_7 = 1$
$2y_1 + y_2 + y_3 - y_4 - y_6 + y_8 = 4$
$y_i \geq 0; i = 1$ to 8
The optimum solution is $x_1^* = 0.0,\ x_2^* = 1.667, x_3^* = 2.333$ and $f^* = 4.33,$ where both constraints are active.

Table E8.60

Basic	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	b	ratio
y_7	1	**2**	-1	1	-1	0	1	0	1	**0.5**
y_8	2	1	1	-1	0	-1	0	1	4	4
Cost	5	4	-1	1	0	0	0	0	f-0	
Arti	-3	**-3**	0	0	1	1	0	0	w-5	
y_2	0.5	1	-0.5	0.5	-0.5	0	0.5	0	0.5	negative
y_8	1.5	0	**1.5**	-1.5	0.5	-1	-0.5	1	3.5	**2.33333**
Cost	3	0	1	-1	2	0	-2	0	f-2	
Arti	-1.5	0	**-1.5**	1.5	-0.5	1	1.5	0	w-3.5	
y_2	1	1	0	0	- 1/3	- 1/3	1/3	1/3	1.6667	
y_3	1	0	1	-1	1/3	- 2/3	- 1/3	2/3	2.3333	
Cost	2 (c_1')	0 (c_2')	0 (c_3')	0 (c_4')	5/3 (c_5')	2/3 (c_6')	-5/3 (c_7')	- 2/3 (c_8')	f-4.3333	
Arti	0	0	0	0	0	0	1	1	w-0	End phs1

Exercise 8.93

From the final tableau for Exercise 8.60,
y_5 and y_6 are surplus variables; y_7 and y_8 are artificial variables.

For $x_1 + 2x_2 - x_3 \geq 1$: $Y_1 = -\frac{5}{3}$ (c_7' in the artificial variable column y_7)
$2x_1 + x_2 + x_3 \geq 4$: $Y_2 = -\frac{2}{3}$ (c_8' in the artificial variable column y_8)

Therefore, Y_1 = -1.667, Y_2 = -0.667

Exercise 8.115

Referring to Exercise 8.60 and the final tableau in Table E8.60, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 1$: $-\frac{5/3}{1/3} \le \Delta_1 \le \frac{7/3}{1/3}$ or $-5.0 \le \Delta_1 \le 7.0$;

For $b_2 = 4$: $\max\{-\frac{5/3}{1/3}, -\frac{8/3}{2/3}\} \le \Delta_2 \le \infty$ or $-4 \le \Delta_2 \le \infty$

Exercise 8.137

Referring to Exercise 8.60 and final tableau in Table E8.60, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 5$: $-2.0 \le \Delta c_1 \le \infty$;

For $c_2 = 4$: $\max\{-\frac{5/3}{1/3}, -\frac{2/3}{1/3}\} \le \Delta c_1 \le \frac{2}{1}$ or $-2.0 \le \Delta c_2 \le 2.0$;

For $c_3 = -1$: $\max\{-\frac{2/3}{2/3}, -\frac{0}{1}\} \le \Delta c_2 \le \min\{\frac{5/3}{1/3}, \frac{2}{1}\}$ or $0 \le \Delta c_3 \le 2.0$;

For $c_4 = 1$: $0 \le \Delta c_4 \le \infty$

8.61

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = -10x_1 - 18x_2$
Subject to $x_1 - 3x_2 \le -3$
$2x_1 + 2x_2 \ge 5$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = 10x_1 + 18x_2$
Subject to $-x_1 + 3x_2 - x_3 + x_5 = 3$
$2x_1 + 2x_2 - x_4 + x_6 = 5$
$x_i \ge 0; i = 1 \text{ to } 6$
The optimum solution is $x_1^* = 1.125,\ x_2^* = 1.375$ and $z^* = 36.0$ where both constraints are active.

Table E8.61

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio	
x_5	-1	**3**	-1	0	1	0	3	**1**	
x_6	2	2	0	-1	0	1	5	5/2	
Cost	10	18	0	0	0	0	f-0		
Arti	-1	**-5**	1	1	0	0	w-8		
x_2	- 1/3	1	- 1/3	0	1/3	0	1	negative	
x_6	**8/3**	0	2/3	-1	- 2/3	1	3	**9/8**	
Cost	16	0	6	0	-6	0	f-18		
Arti	**-8/3**	0	- 2/3	1	5/3	0	w-3		
x_2	0	1	- 1/4	- 1/8	1/4	1/8	1.375		
x_1	1	0	1/4	- 3/8	- 1/4	3/8	1.125		
Cost	0 (c_1')	0 (c_2')	2 (c_3')	6 (c_4')	-2 (c_5')	-6 (c_6')	f-36		
Arti	0	0	0	0	1	1	w-0	End phs1	End phs2

Exercise 8.94

Referring final tableau for Exercise 8.61,
x_3 and x_4 are surplus variables; x_5 and x_6 are artificial variables.

For $x_1 - 3x_2 \le -3$: $y_1 = -2$ (c_5' in the artificial variable column x_5)
$2x_1 + 2x_2 \ge 5$: $y_2 = -6$ (c_6' in the artificial variable column x_6)

Therefore, y_1 = 2.0, y_2 = -6.0

Exercise 8.116

Referring to Exercise 8.61 and the final tableau in Table E8.61, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 3$ (in Table E8.61): $-\frac{1.375}{1/4} \le \Delta_1 \le \frac{1.125}{1/4}$ or $-5.5 \le \Delta_1 \le 4.5$;

For $b_2 = 5$: $\max\{-\frac{1.667}{1/3}, -\frac{1.125}{3/8}\} \le \Delta_2 \le \infty$ or $-3.0 \le \Delta_2 \le \infty$;

Therefore,

For $b_1 = -3$: $-4.5 \le \Delta_1 \le 5.5$;

For $b_2 = 5$: $-3.0 \le \Delta_2 \le \infty$

<u>Exercise 8.138</u>

Referring to Exercise 8.61 and final tableau in Table E8.61, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 10$: $-\frac{6}{3/8} \le \Delta c_1 \le \frac{2}{1/4}$ or $-16.0 \le \Delta c_1 \le 8.0$;

For $c_2 = 18$: $\max\{-\frac{2}{1/4}, -\frac{6}{1/8}\} \le \Delta c_2 \le \infty$ or $-8.0 \le \Delta c_2 \le \infty$

For the original form:

For $c_1 = -10$: $-8.0 \le \Delta c_1 \le 16.0$;

For $c_2 = -18$: $-\infty \le \Delta c_2 \le \min\{\frac{2}{1/4}, \frac{6}{1/8}\}$ or $-\infty \le \Delta c_2 \le 8.0$

8.62

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 20x_1 - 6x_2$
Subject to $3x_1 - x_2 \geq 3$
$-4x_1 + 3x_2 = -8$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = 20x_1 - 6x_2$
Subject to $3x_1 - x_2 - x_3 + x_4 = 3$
$4x_1 - 3x_2 + x_5 = 8$
$x_i \geq 0; i = 1$ to 5
The optimum solution is $x_1^* = 2.0,\ x_2^* = 0.0$ and $f^* = 40.0$ where second constraint is active.

Table E8.62

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio	
x_4	**3**	-1	-1	1	0	3	**1**	
x_5	4	-3	0	0	1	8	2	
Cost	20	-6	0	0	0	f-0		
Arti	**-7**	4	1	0	0	w-11		
x_1	1	- 1/3	- 1/3	1/3	0	1	negative	
x_5	0	-5/3	**4/3**	-4/3	1	4	**3**	
Cost	0	2/3	20/3	-20/3	0	f-20		
Arti	0	5/3	**-4/3**	7/3	0	w-4		
x_1	1	- 3/4	0	0	1/4	2		
x_3	0	-5/4	1	-1	3/4	3		
Cost	0 (c_1')	9 (c_2')	0 (c_3')	0 (c_4')	-5 (c_5')	f-40		
Arti	0	0	0	1	1	w-0		End phs1

Exercise 8.95

Referring final tableau for Exercise 8.62
x_3 is surplus variable; x_4 and x_5 are artificial variables.

For $3x_1 - x_2 \geq 3$: $y_1 = 0$ (c_4' in the artificial variable column x_4)
$4x_1 - 3x_2 = 8$: $y_2 = -5$ (c_5' in the artificial variable column x_5)

Therefore, y_1 = 0.0, y_2 = 5.0 (for the original form $-4x_1 + 3x_2 = -8$)

Exercise 8.117

Referring to Exercise 8.62 and the final tableau in Table E8.62, we can find the ranges for RHS by *Theorem 8.6* as follows:

From Table E8.62,

For $b_1 = 3$: $\infty \le \Delta_1 \le -\frac{3}{1}$ or $-\infty \le \Delta_1 \le 3.0$;

For $b_2 = 8$ (in Table E8.62): $\max\{-\frac{2}{1/4}, -\frac{3}{3/4}\} \le \Delta_2 \le \infty$ or $-4.0 \le \Delta_2 \le \infty$;

Therefore,

For $b_1 = 3$: $-\infty \le \Delta_1 \le 3.0$;

For $b_2 = -8$: $-\infty \le \Delta_2 \le 4.0$

Exercise 8.139

Referring to Exercise 8.62 and final tableau in Table E8.62, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 20$: $-\frac{9}{3/4} \le \Delta c_1 \le \infty$ or $-12 \le \Delta c_1 \le \infty$;

For $c_2 = -6$: $-9.0 \le \Delta c_2 \le \infty$

8.63

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = 2x_1 + 5x_2 - 4.5x_3 + 1.5x_4$
Subject to $5x_1 + 3x_2 + 1.5x_3 \le 8$
$1.8x_1 - 6x_2 + 4x_3 + x_4 \ge 3$
$-3.6x_1 + 8.2x_2 + 7.5x_3 + 5x_4 = 15$
$x_i \ge 0; i = 1$ to 4

Solution:
Standard LP form:
Minimize $f = -2x_1 - 5x_2 + 4.5x_3 - 1.5x_4$
Subject to $5x_1 + 3x_2 + 1.5x_3 + x_5 = 8$
$1.8x_1 - 6x_2 + 4x_3 + x_4 - x_6 + x_7 = 3$
$-3.6x_1 + 8.2x_2 + 7.5x_3 + 5x_4 + x_8 = 15$
$x_i \ge 0; i = 1$ to 8
where x_5 is slack variable, x_6 is surplus variable, x_7 and x_8 are artificial variables for the 2nd and 3rd constraints. The problem is solved by the Simplex method, which is given in Table E8.63.
The optimum solution is $x_1^* = 1.3357$, $x_2^* = 0.4406$, $x_3^* = 0.0$, $x_4^* = 3.2392$ and $z^* = 9.7329$, where all the three constraints are active.

Table E8.63B

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	ratio
x_5	5	3	1.5	0	1	0	0	0	8	5.3333
x_3	1.8	-6	**4**	1	0	-1	1	0	3	**0.75**
x_8	-3.6	8.2	7.5	5	0	0	0	1	15	2
Cost	-2	-5	4.5	-1.5	0	0	0	0	f-0	
Arti	1.8	-2.2	**-11.5**	-6	0	1	0	0	w-18	
x_5	4.325	5.25	0	-0.375	1	0.375	-0.375	0	6.875	1.3095
x_3	0.45	-1.5	1	0.25	0	-0.25	0.25	0	0.75	negative
x_8	-6.975	**19.45**	0	3.125	0	1.875	-1.875	1	9.375	**0.4820**
Cost	-4.025	1.75	0	-2.625	0	1.125	-1.125	0	f-3.375	
Arti	6.975	**-19.45**	0	-3.125	0	-1.875	2.875	0	w-9.375	
x_5	**6.2077**	0	0	-1.2185	1	-0.1311	0.1311	-0.2699	4.3444	0.6999
x_3	-0.08792	0	1	0.4910	0	-0.1054	0.1054	0.07712	1.47301	negative
x_2	-0.35861	1	0	0.16067	0	0.0964	-0.0964	0.05141	0.48201	negative
Cost	**-3.3974**	0	0	-2.9062	0	0.9563	-0.9563	-0.09	f-4.2185	
Arti	0	0	0	0	0	0	1	1	w-0	End phs1
x_1	1	0	0	-0.1963	0.16109	-0.0211	0.0211	-0.0435	0.6999	negative
x_3	0	0	1	**0.47375**	0.01416	-0.1073	0.10726	0.0733	1.53454	**3.2392**
x_2	0	1	0	0.090277	0.05777	0.08883	-0.0888	0.03582	0.733	8.1193
Cost	0	0	0	**-3.5731**	0.54729	0.88455	-0.8846	-0.2377	f-1.841	
x_1	1	0	0.4143	0	0.16696	-0.0656	0.06556	-0.01311	1.3357	
x_4	0	0	2.1108	1	0.0299	-0.2264	0.2264	0.1547	3.2392	
x_2	0	1	-0.1906	0	0.0551	0.10927	-0.1093	0.02185	0.4406	
Cost	0 (c_1')	0 (c_2')	7.5421 (c_3')	0 (c_4')	0.65411 (c_5')	0.07561 (c_6')	-0.0756 (c_7')	0.3151 (c_8')	f+9.7329	End phs2

Table 8.63A LP Solver

Objective Cell (Max)

Cell	Name	Original Value	Final Value
F11	Objective Fucntion:max Sum of LHS	9.732867133	9.732867133

Variable Cells

Cell	Name	Original Value	Final Value	Integer
B10	variable value x1	1.335664336	1.335664336	Contin
C10	variable value x2	0.440559441	0.440559441	Contin
D10	variable value x3	0	0	Contin
E10	variable value x4	3.239160839	3.239160839	Contin

Exercise 8.96

Referring final tableau for Exercise 8.63,
x_5 is slack variable, x_6 is surplus variable, $x_7$and x_8 are artificial variables.

For $5x_1 + 3x_2 + 1.5x_3 \le 8$: $y_1 = 0.654$ (c_5' in the slack variable column x_5)
$1.8x_1 - 6x_2 + 4x_3 + x_4 \ge 3$: $y_2 = -0.0756$ (c_7' in the artificial variable column x_7)
$-3.6x_1 + 8.2x_2 + 7.5x_3 + 5x_4 = 15$: $y_3 = 0.3151$ (c_8' in the artificial variable column x_8)

Therefore, y_1 = 0.654, y_2 = – 0.076, y_3 = 0.315

Table 8.96 LP Solver

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F12	Constraint1 Sum of LHS	8	0.654108392	8	1E+30	8
F13	Constraint2 Sum of LHS	3	- 0.075611888	3	4.032	14.30733591
F14	Constraint3 Sum of LHS	15	0.315122378	15	101.8666667	20.16

Exercise 8.118

Referring to Exercise 8.63 and the final tableau in Table E8.63, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 8$: $\max\{-\frac{1.3357}{0.16696}, -\frac{3.2392}{0.0299}, -\frac{0.4406}{0.0551}\} \leq \Delta_1 \leq \infty$ or $-8.0 \leq \Delta_1 \leq \infty$;

For $b_2 = 3$: $\max\{-\frac{1.3357}{0.06556}, -\frac{3.2392}{0.2264}\} \leq \Delta_2 \leq \frac{0.4406}{0.1093}$ or $-14.307 \leq \Delta_2 \leq 4.032$;

For $b_3 = 15$: $\max\{-\frac{3.2392}{0.1547}, -\frac{0.4406}{0.02185}\} \leq \Delta_3 \leq \frac{1.3357}{0.01311}$ or $-20.160 \leq \Delta_3 \leq 101.867$

Exercise 8.140

Referring to Exercise 8.63 and final tableau in Table E8.63, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -2$: $-\frac{0.07561}{0.0656} \leq \Delta c_1 \leq \frac{0.65411}{0.16696}$ or $-1.153 \leq \Delta c_1 \leq 3.918$;

For $c_2 = -5$: $-\frac{7.5421}{0.1906} \leq \Delta c_2 \leq \min\{\frac{0.65411}{0.0551}, \frac{0.07561}{0.10927}\}$ or $-39.57 \leq \Delta c_2 \leq 0.692$;

For $c_3 = 4.5$: $-7.542 \leq \Delta c_3 \leq \infty$;

For $c_4 = -1.5$: $-\frac{0.07561}{0.2264} \leq \Delta c_4 \leq \min\{\frac{7.5421}{2.1108}, \frac{0.65411}{0.0299}\}$ or $-0.334 \leq \Delta c_4 \leq 3.573$

For the original form:

For $c_1 = 2$: $-3.918 \leq \Delta c_1 \leq 1.153$;

For $c_2 = 5$: $-0.692 \leq \Delta c_2 \leq 39.579$;

For $c_3 = -4.5$: $-\infty \leq \Delta c_3 \leq 7.542$;

For $c_4 = 1.5$: $-3.573 \leq \Delta c_4 \leq 0.334$

Table 8.140 LP Solver

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B10	variable value x1	1.335664336	0	2	1.153333333	3.917801047
C10	variable value x2	0.440559441	0	5	39.57889908	0.692
D10	variable value x3	0	-7.542132867	-4.5	7.542132867	1E+30
E10	variable value x4	3.239160839	0	1.5	0.333976834	3.573049528

8.64

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 8x_1 - 3x_2 + 15x_3$
Subject to $5x_1 - 1.8x_2 - 3.6x_3 \geq 2$
$3x_1 + 6x_2 + 8.2x_3 \geq 5$
$1.5x_1 - 4x_2 + 7.5x_3 \geq -4.5$
$-x_2 + 5x_3 \geq 1.5$
$x_1,\ x_2 \geq 0$; x_3 is unrestricted in sign.

Solution:
Standard LP form:
Minimize $f = 8y_1 - 3y_2 + 15y_3 - 15y_4$
Subject to $5y_1 - 1.8y_2 - 3.6y_3 + 3.6y_4 - y_5 + y_9 = 2$
$3y_1 + 6y_2 + 8.2y_3 - 8.2y_4 - y_6 + y_{10} = 5$
$-1.5y_1 + 4y_2 - 7.5y_3 + 7.5y_4 + y_7 = 4.5$
$-y_2 + 5y_3 - 5y_4 - y_8 + y_{11} = 1.5$
$y_i \geq 0; i = 1$ to 11
where $x_3 = x_3^+ - x_3^-$, $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3^+$, $y_4 = x_3^-$; y_5, y_6, y_8 are surplus variables, y_7 is slack variable, y_9, y_{10}, y_{11} are artificial variables for the 1st, 2nd and 4th constraints. The problem is solved by the Simplex method, which is given in Table E8.64.
The optimum solution is $x_1^* = 0.654$, $x_2^* = 0.076$, $x_3^* = 0.315$ and $f^* = 9.732$ where the 1st, 2nd and 4th constraints are active.

Table E8.64

Basic	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	b	ratio
y_9	5	-1.8	-3.6	3.6	-1	0	0	0	1	0	0	2	negative
y_{10}	3	6	8.2	-8.2	0	-1	0	0	0	1	0	5	0.6098
y_7	-1.5	4	-7.5	7.5	0	0	1	0	0	0	0	4.5	negative
y_{11}	0	-1	**5**	-5	0	0	0	-1	0	0	1	1.5	**0.3**
Cost	8	-3	15	-15	0	0	0	0	0	0	0	f-0	
Arti	-8	-3.2	**-9.6**	9.6	1	1	0	1	0	0	0	w-8.5	
y_9	**5**	-2.52	0	0	-1	0	0	-0.72	1	0	0.72	3.08	**0.616**
y_{10}	3	7.64	0	0	0	-1	0	1.64	0	1	-1.64	2.54	0.8467
y_7	-1.5	2.5	0	0	0	0	1	-1.5	0	0	1.5	6.75	negative
y_3	0	-0.2	1	-1	0	0	0	-0.2	0	0	0.2	0.3	∞
Cost	8	0	0	0	0	0	0	3	0	0	-3	f-4.5	
Arti	**-8**	-5.12	0	0	1	1	0	-0.92	0	0	1.92	w-5.62	
y_1	1	-0.504	0	0	-0.2	0	0	-0.144	0.2	0	0.144	0.616	negative
y_{10}	0	**9.152**	0	0	0.6	-1	0	2.072	-0.6	1	-2.072	0.692	**0.0756**
y_7	0	1.744	0	0	-0.3	0	1	-1.716	0.3	0	1.716	7.674	4.4002
y_3	0	-0.2	1	-1	0	0	0	-0.2	0	0	0.2	0.3	negative
Cost	0	4.032	0	0	1.6	0	0	4.152	-1.6	0	-4.152	f-9.428	
Arti	0	**-9.152**	0	0	-0.6	1	0	-2.072	1.6	0	3.072	w-0.692	
y_1	1	0	0	0	-0.167	-0.055	0	-0.03	0.167	0.055	0.023	0.654	21.88
y_2	0	1	0	0	0.0656	-0.109	0	0.2264	-0.066	0.1093	-0.226	0.076	negative
y_7	0	0	0	0	-0.414	0.1906	1	-2.111	0.414	-0.191	2.111	7.542	3.5731
y_3	0	0	1	-1	0.0131	-0.022	0	-0.155	-0.013	0.022	**0.154**	0.315	**2.0367**
Cost	0 (c_1')	0 (c_2')	0 (c_3')	0 (c_4')	1.336 (c_5')	0.441 (c_6')	0 (c_7')	3.239 (c_8')	-1.336 (c_9')	-0.441 (c_{10}')	-3.239 (c_{11}')	f-9.732	
Arti	0	0	0	0	0	0	0	0	1	1	1	w-0	End phs1

Exercise 8.97

Referring final tableau for Exercise 8.64,

y_5, y_6, y_8 are surplus variables, y_7 is slack variable, y_9, y_{10}, y_{11} are artificial variables

For $5x_1 - 1.8x_2 - 3.6x_3 \geq 2$: $Y_1 = -1.336$ (c_9' in the artificial variable column y_9)

$3x_1 + 6x_2 + 8.2x_3 \geq 5$: $Y_2 = -0.441$ (c_{10}' in the artificial variable column y_{10})

$1.5x_1 - 4x_2 + 7.5x_3 \geq -4.5$: $Y_3 = 0.0$ (c_7' in the slack variable column y_7)

$-x_2 + 5x_3 \geq 1.5$: $Y_4 = -3.239$ (c_{11}' in the artificial variable column y_{11})

Therefore, Y_1 = -1.336, Y_2 = -0.441, Y_3 = 0.0, Y_4= -3.329

Exercise 8.119

Referring to Exercise 8.64 and the final tableau in Table E8.64, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 2$: $\max\{-\frac{0.654}{0.167}, -\frac{7.542}{0.414}\} \le \Delta_1 \le \min\{\frac{0.076}{0.066}, \frac{0.315}{0.013}\}$ or $-3.918 \le \Delta_1 \le 1.153$;

For $b_2 = 5$: $\max\{-\frac{0.654}{0.055}, -\frac{0.076}{0.1093}, -\frac{0.315}{0.022}\} \le \Delta_2 \le \frac{7.542}{0.191}$ or $-0.692 \le \Delta_2 \le 39.579$;

For $b_3 = 4.5$ (in Table E8.64): $-\frac{7.542}{1} \le \Delta_3 \le \infty$ or $-7.542 \le \Delta_3 \le \infty$;

For $b_4 = 1.5$: $\max\{-\frac{0.654}{0.023}, -\frac{7.542}{2.111}, -\frac{0.315}{0.154}\} \le \Delta_4 \le \frac{0.076}{0.226}$ or $-2.037 \le \Delta_4 \le 0.334$

Therefore,

For $b_1 = 2$: $-3.918 \le \Delta_1 \le 1.153$;

For $b_2 = 5$: $-0.692 \le \Delta_2 \le 39.579$;

For $b_3 = -4.5$: $-\infty \le \Delta_3 \le 7.542$;

For $b_4 = 1.5$: $-2.037 \le \Delta_4 \le 0.334$

Exercise 8.141

Referring to Exercise 8.64 and final tableau in Table E8.64, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 8$: $\max\{-\frac{1.336}{0.167}, -\frac{0.441}{0.055}, -\frac{3.239}{0.03}\} \le \Delta c_1 \le \infty$ or $-8.0 \le \Delta c_1 \le \infty$;

For $c_2 = -3$: $-\frac{0.441}{0.109} \le \Delta c_2 \le \min\{\frac{1.336}{0.0656}, \frac{3.239}{0.2264}\}$ or $-4.032 \le \Delta c_2 \le 14.307$;

For $c_3 = 15$: $\max\{-\frac{0.441}{0.022}, -\frac{3.239}{0.155}, -\frac{0}{1}\} \le \Delta c_3 \le \frac{1.336}{0.0131}$ or $0 \le \Delta c_3 \le 101.867$;

For $c_4 = -15$: $0 \le \Delta c_4 \le \infty$

8.65

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = 10x_1 + 6x_2$
Subject to $2x_1 + 3x_2 \leq 90$
$4x_1 + 2x_2 \leq 80$
$x_2 \geq 15$
$5x_1 + x_2 = 25$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -10x_1 - 6x_2$
Subject to $2x_1 + 3x_2 + x_3 = 90$
$4x_1 + 2x_2 + x_4 = 80$
$x_2 - x_5 + x_6 = 15$
$5x_1 + x_2 + x_7 = 25$
$x_i \geq 0; i = 1$ to 7

where x_3, x_4 are slack variables, x_5 is surplus variable, x_6 and x_7 are artificial variables for the 3rd and 4th constraints. The problem is solved by the Simplex method, which is given in Table E8.65. The optimum solution is $x_1^* = 0.0,\ x_2^* = 25.0$ and $z^* = 150.0$ where the 4th constraint is active.

Table E8.65

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	ratio	
x_3	2	3	1	0	0	0	0	90	45	
x_4	4	2	0	1	0	0	0	80	20	
x_6	0	1	0	0	-1	1	0	15	∞	
x_7	**5**	1	0	0	0	0	1	25	**5**	
Cost	-10	-6	0	0	0	0	0	f-0		
Arti	**-5**	-2	0	0	1	0	0	w-40		
x_3	0	13/5	1	0	0	0	- 2/5	80	123/4	
x_4	0	6/5	0	1	0	0	- 4/5	60	50	
x_6	0	**1**	0	0	-1	1	0	15	**15**	
x_1	1	1/5	0	0	0	0	1/5	5	25	
Cost	0	-4	0	0	0	0	2	f+50		
Arti	0	**-1**	0	0	1	0	1	w-15		
x_3	0	0	1	0	13/5	-13/5	- 2/5	41	63/4	
x_4	0	0	0	1	6/5	-6/5	- 4/5	42	35	
x_2	0	1	0	0	-1	1	0	15	negative	
x_1	1	0	0	0	**1/5**	- 1/5	1/5	2	**10**	
Cost	0	0	0	0	**-4**	4	2	f+110		
Arti	0	0	0	0	0	1	1	w-0		End phs1
x_3	-13	0	1	0	0	0	-3	15		
x_4	-6	0	0	1	0	0	-2	30		
x_2	5	1	0	0	0	0	1	25		
x_5	5	0	0	0	1	-1	1	10		
Cost	20 (c_1')	0 (c_2')	0 (c_3')	0 (c_4')	0 (c_5')	0 (c_6')	6 (c_7')	f+150		End phs2

Exercise 8.98

Referring final tableau for Exercise 8.65,

x_3, x_4 are slack variables; x_5 is surplus variable, x_6 and x_7 are artificial variables.

For $2x_1 + 3x_2 \leq 90$: $y_1 = 0.0$ (c_3' in the slack variable column x_3)

$4x_1 + 2x_2 \leq 80$: $y_2 = 0.0$ (c_4' in the slack variable column x_4)

$x_2 \geq 15$: $y_3 = 0.0$ (c_6' in the artificial variable column x_6)

$5x_1 + x_2 = 25$: $y_4 = 6.0$ (c_7' in the artificial variable column x_7)

Therefore, y_1 = 0.0, y_2 = 0.0, y_3 = 0.0, y_4 = 6.0

<u>Exercise 8.120</u>

Referring to Exercise 8.65 and the final tableau in Table E8.65, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 90$: $-15.0 \le \Delta_1 \le \infty$;

For $b_2 = 80$: $-30.0 \le \Delta_2 \le \infty$;

For $b_3 = 15$: $-\infty \le \Delta_3 \le 10.0$;

For $b_4 = 25$: $\max\{-\frac{25}{1}, -\frac{10}{1}\} \le \Delta_4 \le \min\{\frac{15}{3}, \frac{30}{2}\}$ or $-10.0 \le \Delta_4 \le 5.0$

<u>Exercise 8.142</u>

Referring to Exercise 8.65 and final tableau in Table E8.65, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -10$: $-20.0 \le \Delta c_1 \le \infty$;

For $c_2 = -6$: $\infty \le \Delta c_2 \le \frac{20}{5}$ or $-\infty \le \Delta c_2 \le 4.0$

For the original form:

For $c_1 = 10$: $-\infty \le \Delta c_1 \le 20.0$;

For $c_2 = 6$: $-4.0 \le \Delta c_2 \le \infty$

8.66

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = -2x_1 + 4x_2$
Subject to $2x_1 + x_2 \geq 3$
$2x_1 + 10x_2 \leq 18$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = 2x_1 - 4x_2$
Subject to $2x_1 + x_2 - x_3 + x_5 = 3$
$2x_1 + 10x_2 + x_4 = 18$
$x_i \geq 0; i = 1$ to 5
where x_3 is surplus variable, x_4 is slack variable, x_5 is artificial variable for 1st constraint. The problem is solved by the Simplex method, which is given in Table E8.66.
The optimum solution is $x_1^* = 0.6667,\ x_2^* = 1.6667$ and $z^* = 5.3333$ where both the constraints are active.

Table E8.66

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio	
x_5	**2**	1	-1	0	1	3	**1.5**	
x_4	2	10	0	1	0	18	9	
Cost	2	-4	0	0	0	f-0		
Arti	**-2**	-1	1	0	0	w-3		
x_1	1	0.5	-0.5	0	0.5	1.5	3	
x_4	0	**9**	1	1	-1	15	**5/3**	
Cost	0	**-5**	1	0	-1	f-3		
Arti	0	0	0	0	1	w-0		End phs1
x_1	1	0	- 5/9	-5/90	5/9	0.666667		
x_2	0	1	1/9	1/9	- 1/9	1.666667		
Cost	0 (c_1')	0 (c_2')	14/9 (c_3')	5/9 (c_4')	-14/9 (c_5')	f+5.333333		End phs2

Exercise 8.99

Referring final tableau for Exercise 8.66,
x_3 is surplus variable, x_4 is slack variable, x_5 is artificial variable.

For $2x_1 + x_2 \geq 3$: $y_1 = -\frac{14}{9}$ (c_5' in the artificial variable column x_5)

$2x_1 + 10x_2 \leq 18$: $y_2 = \frac{5}{9}$ (c_4' in the slack variable column x_4)

Therefore, y_1 = -1.556, y_2 = 0.556

<u>Exercise 8.121</u>

Referring to Exercise 8.66 and the final tableau in Table E8.66, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 3$: $-\frac{2/3}{5/9} \le \Delta_1 \le \frac{5/3}{1/9}$ or $-1.2 \le \Delta_1 \le 15.0$;

For $b_2 = 18$: $-\frac{5/3}{1/9} \le \Delta_2 \le \frac{2/3}{5/90}$ or $-15.0 \le \Delta_2 \le 12.0$

<u>Exercise 8.143</u>

Referring to Exercise 8.66 and final tableau in Table E8.66, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 2$: $\max\{-\frac{14/9}{5/9}, -\frac{5/9}{5/90}\} \le \Delta c_1 \le \infty$ or $-2.8 \le \Delta c_1 \le \infty$;

For $c_2 = -4$: $\infty \le \Delta c_2 \le \min\{\frac{14/9}{1/9}, \frac{5/9}{1/9}\}$ or $-\infty \le \Delta c_2 \le 5.0$

For the original form:

For $c_1 = -2$: $-\infty \le \Delta c_1 \le 2.8$;

For $c_2 = 4$: $-5.0 \le \Delta c_2 \le \infty$

8.67

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = x_1 + 4x_2$
Subject to $x_1 + 2x_2 \le 5$
$2x_1 + x_2 = 4$
$x_1 - x_2 \ge 3$
$x_1 \ge 0$; x_2 is unrestricted in sign

Solution:
Standard LP form:
Minimize $f = -y_1 - 4y_2 + 4y_3$
Subject to $y_1 + 2y_2 - 2y_3 + y_4 = 5$
$2y_1 + y_2 - y_3 + y_6 = 4$
$y_1 - y_2 + y_3 - y_5 + y_7 = 3$
$y \ge 0; i = 1$ to 7

The optimum solution is $x_1^* = 2.3333$, $x_2^* = -0.6667$ and $z^* = -0.3333$ where 2nd and 3rd constraints are active.

Table E8.67

Basic	y_1	y_2	y_3	y_4	y_5	y_6	y_7	b	ratio
y_4	1	2	-2	1	0	0	0	5	5
y_6	**2**	1	-1	0	0	1	0	4	**2**
y_7	1	-1	1	0	-1	0	1	3	3
Cost	-1	-4	4	0	0	0	0	f-0	
Arti	**-3**	0	0	0	1	0	0	w-7	
y_4	0	3/2	-3/2	4	0	- 1/2	0	3	negative
y_1	1	1/2	- 1/2	0	0	1/2	0	2	negative
y_7	0	-3/2	**3/2**	0	-1	- 1/2	1	1	**2/3**
Cost	0	-7/2	7/2	0	0	1/2	0	f+2	
Arti	0	3/2	**-3/2**	0	1	3/2	0	w-1	
y_4	0	0	0	4	-1	-1	1	4	
y_1	1	0	0	0	- 1/3	1/3	1/3	2.3333	
y_3	0	-1	1	0	- 2/3	- 1/3	2/3	0.66667	
Cost	0 (c'_1)	0 (c'_2)	0 (c'_3)	0 (c'_4)	7/3 (c'_5)	5/3 (c'_6)	-7/3 (c'_7)	f-0.3333	
Arti	0	0	0	0	0	1	1	w-0	

End phs 1

Exercise 8.100

Referring final tableau for Exercise 8.67,
y_4 is slack variable; y_5 is surplus variable; y_6 and y_7 are artificial variables.

For $x_1 + 2x_2 \leq 5$: $Y_1 = 0.0$ (c_4' in the slack variable column y_4)
$2x_1 + x_2 = 4$: $Y_2 = \frac{5}{3}$ (c_6' in the artificial variable column y_6)
$x_1 - x_2 \geq 3$: $Y_3 = -\frac{7}{3}$ (c_7' in the artificial variable column y_7)

Therefore, $Y_1 = 0.0$, $Y_2 = 1.667$, $Y_3 = -2.233$

Exercise 8.122

Referring to Exercise 8.67 and the final tableau in Table E8.67, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 5$: $-\frac{4}{4} \leq \Delta_1 \leq \infty$ or $-4.0 \leq \Delta_1 \leq \infty$;
For $b_2 = 4$: $-\frac{7/3}{1/3} \leq \Delta_2 \leq \min\{\frac{4}{1}, \frac{2/3}{1/3}\}$ or $-7.0 \leq \Delta_2 \leq 2.0$;
For $b_3 = 3$: $\max\{-\frac{4}{1}, -\frac{7/3}{1/3}, -\frac{2/3}{2/3}\} \leq \Delta_3 \leq \infty$ or $-1.0 \leq \Delta_3 \leq \infty$

Exercise 8.144

Referring to Exercise 8.67 and final tableau in Table E8.67, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -1$: $-7.0 \leq \Delta c_1 \leq \infty$;
For $c_2 = -4$: $0 \leq \Delta c_2 \leq \infty$;
For $c_3 = 4$: $0 \leq \Delta c_3 \leq \infty$;

For the original form:
For $c_1 = 1$: $-\infty \leq \Delta c_1 \leq 7.0$;
For $c_2 = 4$: $-\infty \leq \Delta c_2 \leq 0$;
For $c_3 = -4$: $-\infty \leq \Delta c_3 \leq 0$

8.68

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 3x_1 + 2x_2$
Subject to $x_1 - x_2 \geq 0$
$x_1 + x_2 \geq 2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = 3x_1 + 2x_2$
Subject to $x_1 - x_2 - x_3 + x_5 = 0$
$x_1 + x_2 - x_4 + x_6 = 2$
$x_i \geq 0; i = 1$ to 6

The optimum solution is $x_1^* = 1.0,\ x_2^* = 1.0$ and $f^* = 5.0$ where both constraints are active.

Table E8.68

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	ratio
x_5	**1**	-1	-1	0	1	0	0	**0**
x_6	1	1	0	-1	0	1	2	2
Cost	3	2	0	0	0	0	f-0	
Arti	**-2**	0	1	1	0	0	w-2	
x_1	1	**-1**	-1	0	1	0	0	**0**
x_6	0	2	1	-1	-1	1	2	1
Cost	0	5	3	0	-3	0	f-0	
Arti	0	**-2**	-1	1	2	0	w-2	
x_2	-1	1	1	0	-1	0	0	negative
x_6	**2**	0	-1	-1	1	1	2	**1**
Cost	5	0	-2	0	2	0	f-0	
Arti	**-2**	0	1	1	0	0	w-2	
x_2	0	1	0.5	-0.5	-0.5	0.5	1	
x_1	1	0	-0.5	-0.5	0.5	0.5	1	
Cost	0 (c_1')	0 (c_2')	0.5 (c_3')	2.5 (c_4')	-0.5 (c_5')	-2.5 (c_6')	f-5	
Arti	0	0	0	0	1	1	w-0	

End phs1

Exercise 8.101

Referring final tableau for Exercise 8.68,
x_3 and x_4 are surplus variables; x_5 and x_6 are artificial variables.

For $x_1 - x_2 \geq 0$: $y_1 = -0.5$ (c_5' in the artificial variable column x_5)
$x_1 + x_2 \geq 2$: $y_2 = -2.5$ (c_6' in the slack variable column x_6)

Therefore, y_1 = -0.5, y_2 = -2.5

Exercise 8.123

Referring to Exercise 8.68 and the final tableau in Table E8.68, we can find the ranges for RHS by *Theorem 8.6* as follows:

For b_1 = 0: $-\frac{1}{0.5} \leq \Delta_1 \leq \frac{1}{0.5}$ or $-2.0 \leq \Delta_1 \leq 2.0$;

For b_2 = 2: $\max\{-\frac{1}{0.5}, -\frac{1}{0.5}\} \leq \Delta_2 \leq \infty$ or $-2.0 \leq \Delta_2 \leq \infty$

Exercise 8.145

Referring to Exercise 8.68 and final tableau in Table E8.68, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For c_1 = 3: $\max\{-\frac{0.5}{0.5}, -\frac{2.5}{0.5}\} \leq \Delta c_1 \leq \infty$ or $-1.0 \leq \Delta c_1 \leq \infty$;

For c_2 = 2: $-\frac{2.5}{0.5} \leq \Delta c_2 \leq \frac{0.5}{0.5}$ or $-5.0 \leq \Delta c_2 \leq 1.0$

8.69

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = 3x_1 + 2x_2$
Subject to $x_1 - x_2 \geq 0$
$x_1 + x_2 \geq 2$
$2x_1 + x_2 \leq 6$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -3x_1 - 2x_2$
Subject to $x_1 - x_2 - x_3 + x_6 = 0$
$x_1 + x_2 - x_4 + x_7 = 2$
$2x_1 + x_2 + x_5 = 6$
$x_i \geq 0; i = 1$ to 7

The optimum solution is $x_1^* = 2.0,\ x_2^* = 2.0$ and $z^* = 10.0$ where the 1st and 3rd constraints are active.

Table E8.69

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	ratio	
x_6	**1**	-1	-1	0	0	1	0	0	**0**	
x_7	1	1	0	-1	0	0	1	2	2	
x_5	2	1	0	0	1	0	0	6	3	
Cost	-3	-2	0	0	0	0	0	f-0		
Arti	**-2**	0	1	1	0	0	0	w-2		
x_1	1	-1	-1	0	0	1	0	0	negative	
x_7	0	**2**	1	-1	0	-1	1	2	**1**	
x_5	0	3	2	0	1	-2	0	6	2	
Cost	0	-5	-3	0	0	3	0	f-0		
Arti	0	**-2**	-1	1	0	2	0	w-2		
x_1	1	0	-0.5	-0.5	0	0.5	0.5	1	negative	
x_2	0	1	0.5	-0.5	0	-0.5	0.5	1	negative	
x_5	0	0	0.5	**1.5**	1	-0.5	-1.5	3	**2**	
Cost	0	0	-0.5	**-2.5**	0	0.5	2.5	f+5		
Arti	0	0	0	0	0	1	1	w-0		End phs1
x_1	1	0	- 1/3	0	1/3	1/3	0	2		
x_2	0	1	2/3	0	1/3	- 2/3	0	2		
x_4	0	0	1/3	1	2/3	- 1/3	-1	2		
Cost	0 (c_1')	0 (c_2')	1/3 (c_3')	0 (c_4')	5/3 (c_5')	- 1/3 (c_6')	0 (c_7')	f+10		End phs2

Exercise 8.102

Referring final tableau for Exercise 8.69,

x_3 and x_4 are surplus variables; x_5 is slack variable; x_6 and x_7 are artificial variables.

For $x_1 - x_2 \geq 0$: $y_1 = -\frac{1}{3}$ (c'_6 in the artificial variable column x_6)

$x_1 + x_2 \geq 2$: $y_2 = 0.0$ (c'_7 in the artificial variable column x_7)

$2x_1 + x_2 \leq 6$: $y_2 = \frac{5}{3}$ (c'_5 in the slack variable column x_5)

Therefore, $y_1 = -0.333$, $y_2 = 0.0$, $y_3 = 1.667$

Exercise 8.124

Referring to Exercise 8.69 and the final tableau in Table E8.69, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 0$: $\max\{-\frac{2}{1/3}\} \leq \Delta_1 \leq \min\{\frac{2}{2/3}, \frac{2}{1/3}\}$ or $-6.0 \leq \Delta_1 \leq 3.0$;

For $b_2 = 2$: $-\infty \leq \Delta_2 \leq \frac{2}{1}$ or $-\infty \leq \Delta_2 \leq 2.0$;

For $b_3 = 6$: $\max\{-\frac{2}{1/3}, -\frac{2}{1/3}, -\frac{2}{2/3}\} \leq \Delta_3 \leq \infty$ or $-3.0 \leq \Delta_3 \leq \infty$

Exercise 8.146

Referring to Exercise 8.69 and final tableau in Table E8.69, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -3$: $-\frac{1/3}{1/3} \leq \Delta c_1 \leq \frac{5/3}{1/3}$ or $-1.0 \leq \Delta c_1 \leq 5.0$;

For $c_2 = -2$: $-\infty \leq \Delta c_2 \leq \min\{\frac{5/3}{1/3}, \frac{1/3}{2/3}\}$ or $-\infty \leq \Delta c_2 \leq 0.5$

For the original form:

For $c_1 = 3$: $-5.0 \leq \Delta c_1 \leq 1.0$;

For $c_2 = 2$: $-0.5 \leq \Delta c_2 \leq \infty$

8.70

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = x_1 + 2x_2$
Subject to $3x_1 + 4x_2 \le 12$
$x_1 + 3x_2 \le 3$
$x_1 \ge 0$; x_2 is unrestricted in sign.

Solution:
Standard LP form:
Minimize $f = -y_1 - 2y_2 + 2y_3$
Subject to $3y_1 + 4y_2 - 4y_3 + y_4 = 12$
$y_1 + 3y_2 - 3y_3 + y_5 = 3$
$y_i \ge 0; i = 1$ to 5
The optimum solution is $x_1^* = 4.8,\ x_2^* = -0.6$ and $z^* = 3.6$, where both constraints are active.

Table E8.70

Basic	y_1	y_2	y_3	y_4	y_5	b	ratio
y_4	3	4	-4	1	0	12	3
y5	1	**3**	-3	0	1	3	**1**
Cost	-1	**-2**	2	0	0	f-0	
y_4	5/3	0	0	1	-4/3	8	4.8
y_2	**1/3**	1	-1	0	1/3	1	**3**
Cost	**- 1/3**	0	0	0	2/3	f+2	
y_4	0	-5	**5**	1	-3	3	
y_1	1	3	-3	0	1	3	negative
Cost	0	1	**-1**	0	1	f+3	
y_3	0	-1	1	0.2	-0.6	0.6	
y_1	1	0	0	0.6	-0.8	4.8	
Cost	0 (c_1')	0 (c_2')	0 (c_3')	0.2 (c_4')	0.4 (c_5')	f+3.6	

Exercise 8.103

Referring final tableau for Exercise 8.70,
y_4 and y_5 slack variables.

For $3x_1 + 4x_2 \le 12$: $Y_1 = 0.2$ (c_4' in the slack variable column y_4)
$x_1 + 3x_2 \le 3$: $Y_2 = 0.4$ (c_5' in the artificial variable column y_5)

Therefore, $Y_1 = 0.2$, $Y_2 = 0.4$

Exercise 8.125

Referring to Exercise 8.70 and the final tableau in Table E8.70, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 12$: $\max\{-\frac{0.6}{0.2}, -\frac{4.8}{0.6}\} \le \Delta_1 \le \infty$ or $-3.0 \le \Delta_1 \le \infty$;

For $b_2 = 3$: $-\infty \le \Delta_2 \le \min\{\frac{0.6}{0.6}, \frac{4.8}{0.8}\}$ or $-\infty \le \Delta_2 \le 1.0$

Exercise 8.147

Referring to Exercise 8.70 and final tableau in Table E8.70, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -1$: $-\frac{0.4}{0.8} \le \Delta c_1 \le \frac{0.2}{0.6}$ or $-0.5 \le \Delta c_1 \le 0.333$;

For $c_2 = -2$: $0 \le \Delta c_2 \le \infty$;

For $c_3 = 2$: $\max\{-\frac{0.4}{0.6}, -\frac{0}{1}\} \le \Delta c_3 \le \frac{0.2}{0.2}$ or $0 \le \Delta c_3 \le 1.0$

For the original form:

For $c_1 = 1$: $-0.333 \le \Delta c_1 \le 0.5$;

For $c_2 = 2$: $-\infty \le \Delta c_2 \le 0$;

For $c_3 = -2$: $-1 \le \Delta c_3 \le 0$

8.71

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = x_1 + 2x_2$
Subject to $-x_1 + 3x_2 \leq 20$
$x_1 + x_2 \leq 6$
$x_1 - x_2 \leq 12$
$x_1 + 3x_2 \geq 6$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = x_1 + 2x_2$
Subject to $-x_1 + 3x_2 + x_3 = 20$
$x_1 + x_2 + x_4 = 6$
$x_1 - x_2 + x_5 = 12$
$x_1 + 3x_2 - x_6 + x_7 = 6$
$x_i \geq 0; i = 1$ to 7
The optimum solution is $x_1^* = 0.0,\ x_2^* = 2.0$ and $f^* = 4.0$ where the 3rd constraint is active.

Table E8.71A

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	ratio	
x_3	-1	3	1	0	0	0	0	20	6.666667	
x_4	1	1	0	1	0	0	0	6	6	
x_5	1	-1	0	0	1	0	0	12	negative	
x_7	1	**3**	0	0	0	-1	1	6	**2**	
Cost	1	2	0	0	0	0	0	f-0		
Arti	-1	**-3**	0	0	0	1	0	w+6		
x_3	-2	0	1	0	0	1	-1	14		
x_4	2/3	0	0	1	0	1/3	- 1/3	4		
x_5	4/3	0	0	0	1	- 1/3	1/3	14		
x_2	1/3	1	0	0	0	- 1/3	1/3	2		
Cost	1/3 (c_1')	0 (c_2')	0 (c_3')	0 (c_4')	0 (c_5')	2/3 (c_6')	- 2/3 (c_7')	f-4		
Arti	0	0	0	0	0	0	1	w-0	End phs1	End phs2

Figure E8.71

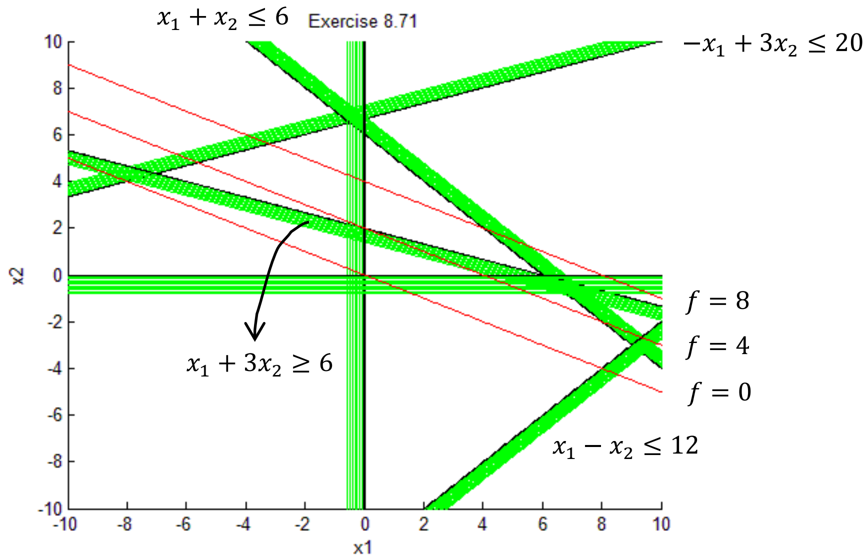


Table 8.71B LP Solver

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$D\$4	Objective Fucntion:min Sum of LHS	0	4

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$3	variable value x1	0	0	Contin
\$C\$3	variable value x2	0	2	Contin

Exercise 8.104

Referring final tableau for Exercise 8.71,
x_3, x_4 and x_5 are slack variables; x_6 is surplus variable; x_7 is artificial variable.

For $-x_1 + 3x_2 \le 20$: $y_1 = 0.0$ (c_3' in the slack variable column x_3)
$x_1 + x_2 \le 6$: $y_2 = 0.0$ (c_4' in the slack variable column x_4)
$x_1 - x_2 \le 12$: $y_3 = 0.0$ (c_5' in the slack variable column x_5)
$x_1 + 3x_2 \ge 6$: $y_4 = -\frac{2}{3}$ (c_7' in the artificial variable column x_7)

Therefore, $y_1 = 0.0$, $y_2 = 0.0$, $y_3 = 0.0$, $y_4 = -\frac{2}{3}$

Table 8.104 LP Solver

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D5	Constraint1 Sum of LHS	6	0	20	1E+30	14
D6	Constraint2 Sum of LHS	2	0	6	1E+30	4
D7	Constraint3 Sum of LHS	-2	0	12	1E+30	14
D8	Constraint4 Sum of LHS	6	0.666666667	6	12	6

Exercise 8.126

Referring to Exercise 8.71 and the final tableau in Table E8.71, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 20$: $-\frac{14}{1} \leq \Delta_1 \leq \infty$ or $-14.0 \leq \Delta_1 \leq \infty$;

For $b_2 = 6$: $-\frac{4}{1} \leq \Delta_2 \leq \infty$ or $-4.0 \leq \Delta_2 \leq \infty$;

For $b_3 = 12$: $-\frac{14}{1} \leq \Delta_3 \leq \infty$ or $-14.0 \leq \Delta_3 \leq \infty$;

For $b_4 = 6$: $\max\{-\frac{14}{1/3}, -\frac{2}{1/3}\} \leq \Delta_4 \leq \min\{\frac{14}{1}, \frac{4}{1/3}\}$ or $-6.0 \leq \Delta_4 \leq 12.0$

Exercise 8.148

Referring to Exercise 8.71 and final tableau in Table E8.71, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 1$: $-\frac{1}{3} \leq \Delta c_1 \leq \infty$ or $-0.333 \leq \Delta c_1 \leq \infty$;

For $c_2 = 2$: $-\frac{2/3}{1/3} \leq \Delta c_2 \leq \frac{1/3}{1/3}$ or $-2.0 \leq \Delta c_2 \leq 1$;

Table 8.148 LP Solver

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	variable value x1	0	0.333333333	1	1E+30	0.333333333
C3	variable value x2	2	0	2	1	2

```
clear all
[x1,x2]=meshgrid(-10:0.05:10, -10:0.05:10);
f=x1+2*x2;
g1=-x1+3*x2-20;
g2=x1+x2-6;
g3=x1-x2-12;
g4=-x1-3*x2+6;

g5=-x1;
g6=-x2;

cla reset
axis auto
xlabel('x1'),ylabel('x2')
title('Exercise 8.71')
hold on
cv1=[0:0.2:1.8];
contour(x1,x2,g1,cv1,'g');
cv2=[0:0.01:0.02];
contour(x1,x2,g1,cv2,'k');
cv3=[0:0.06:0.8];
contour(x1,x2,g2,cv3,'g');
cv4=[0:0.01:0.02];
contour(x1,x2,g2,cv4,'k');
cv5=[0:0.06:0.8];
contour(x1,x2,g3,cv5,'g');
cv6=[0:0.01:0.02];
contour(x1,x2,g3,cv6,'k');
cv7=[0:0.1:1.8];
contour(x1,x2,g4,cv7,'g');
cv8=[0:0.01:0.02];
contour(x1,x2,g4,cv8,'k');
cv9=[0:0.08:0.6];
contour(x1,x2,g5,cv9,'g');
cv10=[0:0.01:0.01];
contour(x1,x2,g5,cv10,'k');
cv11=[0:0.08:0.8];
contour(x1,x2,g6,cv11,'g');
cv12=[0:0.01:0.01];
contour(x1,x2,g6,cv12,'k');

fv=[0 4 8];
fs=contour(x1,x2,f,fv,'r');

grid off
hold off
```

8.72

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = 3x_1 + 8x_2$
Subject to $3x_1 + 4x_2 \le 20$
$x_1 + 3x_2 \ge 6$
$x_1 \ge 0$; x_2 is unrestricted in sign

Solution:
Standard LP form:
Minimize $f = -3y_1 - 8y_2 + 8y_3$
Subject to $3y_1 + 4y_2 - 4y_3 + y_4 = 20$
$y_1 + 3y_2 - 3y_3 - y_5 + y_6 = 6$
$y_i \ge 0;\ i = 1$ to 6
The optimum solution is $x_1^* = 0.0,\ x_2^* = 5.0$ and $z^* = 40.0,$ where the 1st constraint is active.

Table E8.72A

Basic	y_1	y_2	y_3	y_4	y_5	y_6	b	ratio	
y_4	3	4	-4	1	0	0	20	5	
y_6	1	**3**	-3	0	-1	1	6	**2**	
Cost	-3	-8	8	0	0	0	f-0		
Arti	-1	**-3**	3	0	1	0	w-6		
y_4	5/3	0	0	1	**4/3**	-4/3	12		
y_2	1/3	1	-1	0	- 1/3	1/3	2	negative	
Cost	- 1/3	0	0	0	**-8/3**	8/3	f+16		
Arti	0	0	0	0	0	1	w-0		End phs1
y_5	1.25	0	0	0.75	1	-1	9		
y_2	0.75	1	-1	0.25	0	0	5		
Cost	3 (c_1')	0 (c_2')	0 (c_3')	2 (c_4')	0 (c_5')	0 (c_6')	f+40		End phs2

Table 8.72B LP Solver

Objective Cell (Max)

Cell	Name	Original Value	Final Value
D11	Objective Fucntion:max Sum of LHS	0	40

Variable Cells

Cell	Name	Original Value	Final Value	Integer
B10	variable value x1	0	0	Contin
C10	variable value x2	0	5	Contin

Exercise 8.105

Referring final tableau for Exercise 8.72,
y_4 is slack variable; y_5 is surplus variable; y_6 is artificial variable.

For $3x_1 + 4x_2 \leq 20$: $Y_1 = 2.0$ (c'_4 in the slack variable column y_4)
$x_1 + 3x_2 \geq 6$: $Y_2 = 0.0$ (c'_6 in the artificial variable column y_6)
Therefore, $Y_1 = 2.0$, $Y_2 = 0.0$

Exercise 8.127

Referring to Exercise 8.72 and the final tableau in Table E8.72, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 20$: $\max\{-\frac{9}{0.75}, -\frac{5}{0.25}\} \leq \Delta_1 \leq \infty$ or $-12.0 \leq \Delta_1 \leq \infty$;

For $b_2 = 6$: $-\infty \leq \Delta_2 \leq \frac{9}{1}$ or $-\infty \leq \Delta_2 \leq 9.0$

Exercise 8.149

Referring to Exercise 8.72 and final tableau in Table E8.72, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -3$: $-3.0 \leq \Delta c_1 \leq \infty$;

For $c_2 = -8$: $\infty \leq \Delta c_2 \leq \min\{\frac{3}{0.75}, \frac{2}{0.25}\}$ or $\infty \leq \Delta c_2 \leq 4$;

For $c_3 = 8$: $0 \leq \Delta c_3 \leq \infty$

For the original form:

For $c_1 = 3$: $-\infty \leq \Delta c_1 \leq 3.0$;

For $c_2 = 8$: $-4.0 \leq \Delta c_2 \leq \infty$;

For $c_3 = -8$: $-\infty \leq \Delta c_3 \leq 0$

Table 8.149 LP Solver

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B10	variable value x1	0	-3	3	3	1E+30
C10	variable value x2	5	0	8	1E+30	4

8.73

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 2x_1 - 3x_2$
Subject to $x_1 + x_2 \leq 1$
$-2x_1 + x_2 \geq 2$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = 2x_1 - 3x_2$
Subject to $x_1 + x_2 + x_3 = 1$
$-2x_1 + x_2 - x_4 + x_5 = 2$
$x_i \geq 0; i = 1$ to 5
There is no feasible solution. This can be also verified graphically.

Table E8.73

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio
x_3	1	**1**	1	0	0	1	**1**
x_5	-2	1	0	-1	1	2	2
Cost	2	-3	0	0	0	f-0	
Arti	2	**-1**	0	1	0	w-2	
x_2	1	1	1	0	0	1	
x_5	-3	0	-1	-1	1	1	
Cost	5	0	3	0	0	f+3	
Arti	3	0	1	1	0	w-1	

8.74

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 3x_1 - 3x_2$
Subject to $-x_1 + x_2 \le 0$
$x_1 + x_2 \ge 2$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = 3x_1 - 3x_2$
Subject to $-x_1 + x_2 + x_3 = 0$
$x_1 + x_2 - x_4 + x_5 = 2$
$x_i \ge 0; i = 1$ to 5

There are infinite optimum points with $f^* = 0$.

Table E8.74

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio	
x_3	-1	1	1	0	0	0	negative	
x_5	**1**	1	0	-1	1	2	**2**	
Cost	3	-3	0	0	0	f-0		
Arti	**-1**	-1	0	1	0	w-2		
x_3	0	**2**	1	-1	1	2	**1**	
x_1	1	1	0	-1	1	2	2	
Cost	0	**-6**	0	3	-3	f-6		
Arti	0	0	0	0	1	w-0		End phs1
x_2	0	1	0.5	-0.5	0.5	1	negative	
x_1	1	0	-0.5	-0.5	0.5	1	negative	
Cost	0 (c'_1)	0 (c'_2)	3 (c'_3)	0 (c'_4)	0 (c'_5)	f-0		

Exercise 8.107

Referring final tableau for Exercise 8.74,
x_3 is slack variable; x_4 is surplus variable; x_5 is artificial variable.

For $-x_1 + x_2 \le 0$: $y_1 = 3.0$ (c'_3 in the slack variable column x_3)
$x_1 + x_2 \ge 2$: $y_2 = 0.0$ (c'_5 in the artificial variable column x_5)

Therefore, $y_1 = 3.0$, $y_2 = 0.0$

<u>Exercise 8.129</u>

Referring to Exercise 8.74 and the final tableau in Table E8.74, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 0$: $-\frac{1}{0.5} \leq \Delta_1 \leq \frac{1}{0.5}$ or $-2 \leq \Delta_1 \leq 2$;

For $b_2 = 2$: $\max\{-\frac{1}{0.5}, -\frac{1}{0.5}\} \leq \Delta_2 \leq \infty$ or $-2 \leq \Delta_2 \leq \infty$

<u>Exercise 8.151</u>

Referring to Exercise 8.74 and final tableau in Table E8.74, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = 3$: $\max\{-\frac{3}{0.5}, -\frac{0}{0.5}\} \leq \Delta c_1 \leq \infty$ or $0 \leq \Delta c_1 \leq \infty$;

For $c_2 = -3$: $-\frac{0}{0.5} \leq \Delta c_2 \leq \frac{3}{0.5}$ or $0 \leq \Delta c_2 \leq 6$

8.75

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Minimize $f = 5x_1 + 4x_2 - x_3$
Subject to $x_1 + 2x_2 - x_3 \geq 1$
$2x_1 + x_2 + x_3 \geq 4$
$x_1,\ x_2 \geq 0$; x_3 is unrestricted in sign.

Solution:
Standard LP form:
Minimize $f = 5y_1 + 4y_2 - y_3 + y_4$
Subject to $y_1 + 2y_2 - y_3 + y_4 - y_5 + y_7 = 1$
$2y_1 + y_2 + y_3 - y_4 - y_6 + y_8 = 4$
$y_i \geq 0;\ i = 1$ to 8
The optimum solution is $x_1^* = 0.0,\ x_2^* = 1.6667,\ x_3^* = 2.333$ and $f^* = 4.333$, where the 1st and 2nd constraints are active.

Table E8.75

Basic	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	b	ratio
y_7	**1**	2	-1	1	-1	0	1	0	1	**1**
y_8	2	1	1	-1	0	-1	0	1	4	2
Cost	5	4	-1	1	0	0	0	0	f-0	
Arti	**-3**	-3	0	0	1	1	0	0	w-5	
y_1	1	2	-1	1	-1	0	1	0	1	negative
y_8	0	-3	**3**	-3	2	-1	-2	1	2	
Cost	0	-6	4	-4	5	0	-5	0	f-5	
Arti	0	3	**-3**	3	-2	1	3	0	w-2	
y_1	1	**1**	0	0	-0.33333	-0.33333	0.333333	0.333333	1.666667	
y_3	0	-1	1	-1	0.666667	-0.33333	-0.66667	0.333333	0.666667	negative
Cost	0	**-2**	0	0	2.333333	1.333333	-2.33333	-1.33333	f-7.66667	
Arti	0	0	0	0	0	0	1	1	w-0	End phs1
y_2	1	1	0	0	-0.33333	-0.33333	0.333333	0.333333	1.666667	
y_3	1	0	1	-1	0.333333	-0.66667	-0.33333	0.666667	2.333333	
Cost	2 (c_1')	0 (c_2')	0 (c_3')	0 (c_4')	1.6667 (c_5')	0.6667 (c_6')	-1.6667 (c_7')	-0.6667 (c_8')	f-4.333	End phs2

Exercise 8.108

Referring final tableau for Exercise 8.75,

y_5, y_6 are surplus variables; y_7 and y_8 are artificial variables

For $x_1 + 2x_2 - x_3 \geq 1$: $Y_1 = -1.6667$(c_7' in the artificial variable column y_7)

$2x_1 + x_2 + x_3 \geq 4$: $Y_2 = -0.6667$ (c_8' in the artificial variable column y_8)

Therefore, Y_1 = -1.6667, Y_2 = -0.6667

Exercise 8.130

Referring to Exercise 8.75 and the final tableau in Table E8.75, we can find the ranges for RHS by *Theorem 8.6* as follows:

For b_1 = 1: $-\frac{5/3}{1/3} \leq \Delta_1 \leq \frac{2/3}{2/3}$ or $-5 \leq \Delta_1 \leq 1$;

For b_2 = 4: $\max\{-\frac{5/3}{1/3}, -\frac{2/3}{1/3}\} \leq \Delta_2 \leq \infty$ or $-2 \leq \Delta_2 \leq \infty$

Exercise 8.152

Referring to Exercise 8.75 and final tableau in Table E8.75, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For c_1 = 5: $-2.0 \leq \Delta c_1 \leq \infty$;

For c_2 = 4: $\max\{-\frac{5/3}{1/3}, -\frac{2/3}{1/3}\} \leq \Delta c_2 \leq \infty$ or $-2.0 \leq \Delta c_2 \leq \infty$;

For $c_3 = -1$: $-\frac{2/3}{2/3} \leq \Delta c_3 \leq \frac{5/3}{1/3}$ or $-1 \leq \Delta c_3 \leq 5$;

For c_4 = 1: $0 \leq \Delta c_4 \leq \infty$

8.76

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Maximize $z = 4x_1 + 5x_2$
Subject to $x_1 - 2x_2 \le -10$
$3x_1 + 2x_2 \le 18$
$x_1,\ x_2 \ge 0$

Solution:
Standard LP form:
Minimize $f = -4x_1 - 5x_2$
Subject to $-x_1 + 2x_2 - x_3 + x_5 = 10$
$3x_1 + 2x_2 + x_4 = 18$
$x_i \ge 0; i = 1$ to 5

The optimum solution is $x_1^* = 2.0,\ x_2^* = 6.0$ and $z^* = 38.0$ where the 1st and 2nd constraints are active.

Table E8.76

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio	
x_5	-1	**2**	-1	0	1	10	**5**	
x_4	3	2	0	1	0	18	9	
Cost	-4	-5	0	0	0	f-0		
Arti	1	**-2**	1	0	0	w-10		
x_2	-0.5	1	-0.5	0	0.5	5	negative	
x_4	**4**	0	1	1	-1	8	**2**	
Cost	**-6.5**	0	-2.5	0	2.5	f+25		
Arti	0	0	0	0	1	w-0		End phs1
x_2	0	1	-0.375	0.125	0.375	6	negative	
x_1	1	0	**0.25**	0.25	-0.25	2		
Cost	0	0	**-0.875**	1.625	0.875	f+38		
x_2	1.5	1	0	0.5	0	9		
x_3	4	0	1	1	-1	8		
Cost	3.5 (c_1')	0 (c_2')	0 (c_3')	2.5 (c_4')	0 (c_5')	f+45		End phs2

Exercise 8.109

Referring final tableau for Exercise 8.76,
x_3 is surplus variable; x_4 is slack variable; x_5 is artificial variable.

For $x_1 - 2x_2 \le -10$: $y_1 = 0.0$ (c_5' in the artificial variable column x_5)
$3x_1 + 2x_2 \le 18$: $y_2 = 2.5$ (c_4' in the slack variable column x_4)

Therefore, $y_1 = 0.0$, $y_2 = 2.5$

Exercise 8.131

Referring to Exercise 8.76 and the final tableau in Table E8.76, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 10$: $-\infty \le \Delta_1 \le \frac{8}{1}$ or $-\infty \le \Delta_1 \le 8$;

For $b_2 = 18$: $\max\{-\frac{8}{1}, -\frac{9}{0.5}\} \le \Delta_2 \le \infty$ or $-8 \le \Delta_2 \le \infty$

Exercise 8.153

Referring to Exercise 8.76 and final tableau in Table E8.76, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -1$: $-3.5 \le \Delta c_1 \le \infty$;

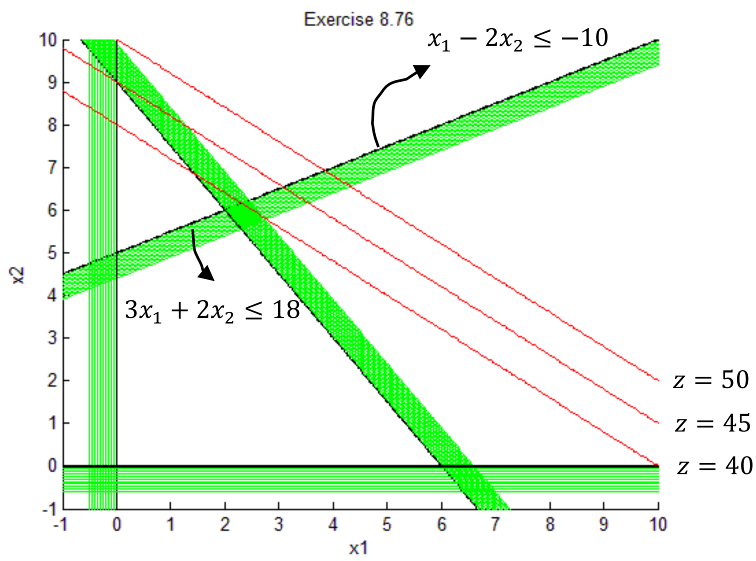
For $c_2 = -5$: $-\infty \le \Delta c_2 \le \frac{2.5}{0.5}$ or $-\infty \le \Delta c_2 \le 5$

For the original form:

For $c_1 = 1$: $-\infty \le \Delta c_1 \le 3.5$;

For $c_2 = 5$: $-5 \le \Delta c_2 \le \infty$

Figure E8.76



```
[x1,x2]=meshgrid(-1:0.05:10, -1:0.05:10);
f=-4*x1-5*x2;
g1=x1-2*x2+10;
g2=3*x1+2*x2-18;
g3=-x1;
g4=-x2;

axis auto
xlabel('x1'),ylabel('x2')
title('Exercise 8.76')
hold on
cv1=[0:0.1:1.2];
contour(x1,x2,g1,cv1,'g');
cv2=[0:0.01:0.02];
contour(x1,x2,g1,cv2,'k');
cv3=[0:0.1:1.8];
contour(x1,x2,g2,cv3,'g');
cv4=[0:0.01:0.02];
contour(x1,x2,g2,cv4,'k');
cv5=[0:0.05:0.5];
contour(x1,x2,g3,cv5,'g');
cv6=[0:0.01:0.01];
contour(x1,x2,g3,cv6,'k');
cv7=[0:0.06:0.6];
contour(x1,x2,g4,cv7,'g');
cv8=[0:0.01:0.02];
contour(x1,x2,g4,cv8,'k');

fv=[-50 -45 -40];
fs=contour(x1,x2,f,fv,'r');
```

8.77

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Exercise 2.2, we have:
Maximize $z = 48A + 28B$
Subject to $0.6A + 0.8B \leq 20{,}000$
$0.4A + 0.2B \leq 10{,}000$
$A \leq 20{,}000$
$B \leq 30{,}000$
$A, B \geq 0$

Solution:
Standard LP form:
Minimize $f = -48A - 28B$
Subject to $0.6A + 0.8B + x_1 = 20{,}000$
$0.4A + 0.2B + x_2 = 10{,}000$
$A + x_3 = 20{,}000$
$B + x_4 = 30{,}000$
$A, B, x_1, x_2, x_3, x_4 \geq 0$
where x_1, x_2, x_3 and x_4 are the slack variables. The problem is solved by the Simplex method, which is given in Table E8.77A.
The optimum solution is A^* = 20,000, B^* = 10,000 and f^*= – 1,240,000 (z^*= 1,240,000), where the 1st, 2nd and 3rd constraints are active. The solution can be also verified graphically. Note that a different choice of the pivot element at the second iteration gives a different value for reduced costs in the final tableau, as can be seen in Table E8.77B. This happens when there are redundant constraints at the optimum point (irregular point).

Table E8.77A

Basic	A	B	x_1	x_2	x_3	x_4	b	ratio
x_1	0.6	0.8	1	0	0	0	20000	33333.33
x_2	0.4	0.2	0	1	0	0	10000	25000
x_3	**1**	0	0	0	1	0	20000	**20000**
x_4	0	1	0	0	0	1	30000	∞
Cost	**-48**	-28	0	0	0	0	f-0	
x_1	0	0.8	1	0	-0.6	0	8000	10000
x_2	0	**0.2**	0	1	-0.4	0	2000	**10000**
A	1	0	0	0	1	0	20000	∞
x_4	0	1	0	0	0	1	30000	30000
Cost	0	**-28**	0	0	48	0	f-960000	
x_1	0	0	1	-4	**1**	0	0	**0**
B	0	1	0	5	-2	0	10000	negative
A	1	0	0	0	1	0	20000	20000
x_4	0	0	0	-5	2	1	20000	10000
Cost	0	0	0	140	**-8**	0	f-1240000	
x_3	0	0	1	-4	1	0	0	
B	0	1	2	-3	0	0	10000	
A	1	0	-1	4	0	0	20000	
x_4	0	0	-2	3	0	1	20000	
Cost	0	0	8	108	0	0	f-1240000	

Table E8.77B

Basic	A	B	x_1	x_2	x_3	x_4	b	ratio
x_1	0.6	0.8	1	0	0	0	20000	33333.33
x_2	0.4	0.2	0	1	0	0	10000	25000
x_3	**1**	0	0	0	1	0	20000	**20000**
x_4	0	1	0	0	0	1	30000	∞
Cost	**-48**	-28	0	0	0	0	f-0	
x_1	0	**0.8**	1	0	-0.6	0	8000	**10000**
x_2	0	0.2	0	1	-0.4	0	2000	10000
A	1	0	0	0	1	0	20000	∞
x_4	0	1	0	0	0	1	30000	30000
Cost	0	**-28**	0	0	48	0	f-960000	
B	0	1	1.25	0	-0.75	0	10000	
x_2	0	0	-0.25	1	-0.25	0	0	
A	1	0	0	0	1	0	20000	
x_4	0	0	-1.25	0	0.75	1	20000	
Cost	0	0	35	0	27	0	f-1240000	

Exercise 8.154

From final tableaus in Table E8.77A and Table E8.77B, we can find the ranges for the cost coefficients by *Theorem 8.8* as follows:

1. Use final tableau in Table E8.77A.
   For $c_1 = -48$: $\max\{-\frac{8}{1}\} \le \Delta c_1 \le \min\{\frac{108}{4}\}$ or $-8 \le \Delta c_1 \le 27$;
   For $c_2 = -28$: $\max\{-\frac{108}{3}\} \le \Delta c_2 \le \min\{\frac{8}{2}\}$ or $-36 \le \Delta c_2 \le 4$

2. Use final tableau in Table E8.77B.
   For $c_1 = -48$: $-\infty \le \Delta c_1 \le 27$;
   For $c_2 = -28$: $\max\{-\frac{27}{0.75}\} \le \Delta c_2 \le \{\frac{35}{1.25}\}$ or $-36 \le \Delta c_2 \le 28$

8.78

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Exercise 2.6, we have:
Maximize $z = 10A + 8B$
Subject to $0.4A + 0.5B \leq 100$
$0.6A + 0.5B \leq 80$
$A \leq 70$
$B \leq 110$
$A, B \geq 0$

Solution:
Standard LP form:
Minimize $f = -10A - 8B$
Subject to $0.4A + 0.5B + x_1 = 100$
$0.6A + 0.5B + x_2 = 80$
$A + x_3 = 70$
$B + x_4 = 110$
$A, B, x_1, x_2, x_3, x_4 \geq 0$

where x_1, x_2, x_3 and x_4 are slack variables. The problem is solved by the Simplex method, which is given in Table E8.78. The optimum solution is $A^* = 70$, $B^* = 76$ and $f^* = -1308$ ($z^* = 1308$), where the 2nd and 3rd constraints are active. The solution can be also verified graphically.

Table E8.78

Basic	A	B	x_1	x_2	x_3	x_4	b	ratio
x_1	0.4	0.5	1	0	0	0	100	250
x_2	0.6	0.5	0	1	0	0	80	133.3333
x_3	**1**	0	0	0	1	0	70	**70**
x_4	0	1	0	0	0	1	110	∞
Cost	**-10**	-8	0	0	0	0	f-0	
x_1	0	0.5	1	0	-0.4	0	72	144
x_2	0	**0.5**	0	1	-0.6	0	38	**76**
A	1	0	0	0	1	0	70	∞
x_4	0	1	0	0	0	1	110	110
Cost	0	**-8**	0	0	10	0	f-700	
x_1	0	0	1	-1	0.2	0	34	
B	0	1	0	2	-1.2	0	76	
A	1	0	0	0	1	0	70	
x_4	0	0	0	-2	1.2	1	34	
Cost	0	0	0	16	0.4	0	f-1308	

Exercise 8.155

From the final tableau in Table E8.78, we can find the ranges for cost coefficients by *Theorem 8.8* as follows:

For $c_1 = -10$: $-\infty \le \Delta c_1 \le 0.4$;

For $c_2 = -8$: $\max\left\{\frac{0.4}{-1.2}\right\} \le \Delta c_2 \le \min\left\{\frac{16}{2}\right\}$ or $-0.3333 \le \Delta c_2 \le 8$

8.79

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Exercise 2.7, we have

Minimize $f = 2B + M$

Subject to $B + 2M \geq 5$

$3B + 2M \geq 4$

$B, M \geq 0$

Solution:

Standard LP form:

Minimize $f = 2B + M$

Subject to $B + 2M - x_1 + x_3 = 5$

$3B + 2M - x_2 + x_4 = 4$

$B, M, x_1, x_2, x_3, x_4 \geq 0$

The optimum solution is $B^* = 0$, $M^* = 2.5$ and $f^* = 2.5$. Constraint #1 is active.

Table E8.79

Basic	B	M	x_1	x_2	x_3	x_4	b	ratio	
x_3	1	2	-1	0	1	0	5	2.5	
x_4	3	**2**	0	-1	0	1	4	**2**	
Cost	2	1	0	0	0	0	f-0		
Arti	-4	**-4**	1	1	0	0	w-9		
x_3	-2	0	-1	**1**	1	-1	1	1	
M	1.5	1	0	-0.5	0	0.5	2	negative	
Cost	0.5	0	0	0.5	0	-0.5	f-2		
Arti	2	0	1	**-1**	0	2	w-1		
x_2	-2	0	-1	1	1	-1	1		
M	0.5	1	-0.5	0	0.5	0	2.5		
Cost	1.5	0	0.5	0	-0.5	0	f-2.5		
Arti	0	0	0	0	1	1	w-0		End phs1

8.80

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Exercise 2.8, we have:
Maximize $z = x_1 + 2x_2$
Subject to $x_1 + x_2 \leq 800$
$0.1x_1 + 0.4x_2 \leq 225$
$\frac{x_1}{600} + \frac{x_2}{1200} \leq 1$
$x_1,\ x_2 \geq 0$

Solution:
Standard LP form:
Minimize $f = -x_1 - 2x_2$
Subject to $x_1 + x_2 + x_3 = 800$
$0.1x_1 + 0.4x_2 + x_4 = 225$
$2x_1 + x_2 + x_5 = 1200$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

The optimum solution is x_1^* = 316.67, x_2^* = 483.33 and f^*= -1283.3 (z^*= 1283.3). Constraints #1 and 2 are active.

Table E8.80

Basic	x_1	x_2	x_3	x_4	x_5	b	ratio
x_3	1	1	1	0	0	800	800
x_4	0.1	**0.4**	0	1	0	225	**562.5**
x_5	2	1	0	0	1	1200	1200
Cost	-1	**-2**	0	0	0	f-0	
x_3	**0.75**	0	1	-2.5	0	237.5	**316.6667**
x_2	0.25	1	0	2.5	0	562.5	2250
x_5	1.75	0	0	-2.5	1	637.5	364.2857
Cost	**-0.5**	0	0	5	0	f-1125	
x_1	1	0	1.333333	-3.33333	0	316.6667	
x_2	0	1	-0.33333	3.333333	0	483.3333	
x_5	0	0	-2.33333	3.333333	1	83.33333	
Cost	0	0	0.666667	3.333333	0	f-1283.333	

8.81

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Exercise 2.18, we have:

Maximize $z = 0.1x_1 + 0.08x_2 + 0.05x_3$

Subject to $\frac{2x_1}{3} \times \frac{1}{0.9} + \frac{3x_3}{5} \le 250{,}000$

$\frac{x_3}{5} \le 2000$

$\frac{x_1}{3} \times \frac{1}{0.9} + x_2 \times \frac{1}{0.95} + \frac{x_3}{5} \le 110{,}000$

$x_1 \ge 100{,}000$

$x_2 \ge 50{,}000$

$x_3 \ge 10{,}000$

Solution:

Standard LP form:

Minimize $f = -0.1x_1 - 0.08x_2 - 0.05x_3$

Subject to $\frac{2}{2.7}x_1 + \frac{3}{5}x_3 + x_4 = 250{,}000$

$\frac{1}{5}x_3 + x_5 = 2000$

$\frac{1}{2.7}x_1 + \frac{1}{0.95}x_2 + \frac{1}{5}x_3 + x_6 = 110{,}000$

$x_1 - x_7 + x_{10} = 100{,}000$

$x_2 - x_8 + x_{11} = 50{,}000$

$x_3 - x_9 + x_{12} = 10{,}000$

$x_i \ge 0; i = 1$ to 12

The optimum solution is $x_1^* = 149{,}499.2$, $x_2^* = 50{,}000$, $x_3^* = 10{,}000$ and $f^* = -19{,}499.2$;
where the 2nd and 3rd constraints are active as well as the 2nd and the 3rd simple bound constraints.

8.82

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Exercise 2.20, we have:

Maximize $z = 9900A + 18000B + 18900C$

Subject to $40{,}000A + 60{,}000B + 70{,}000C \le 2{,}000{,}000$

$3A + 6B + 6C \le 150$

$A + B + C \le 30$

$A, B, C \ge 0$

Solution:

Standard LP form:

Minimize $f = -9900A - 18000B - 18900C$

Subject to $40{,}000A + 60{,}000B + 70{,}000C + x_1 = 2{,}000{,}000$

$3A + 6B + 6C + x_2 = 150$

$A + B + C + x_3 = 30$

$A, B, C, x_1, x_2, x_3 \ge 0$

The optimum solution is $A^* = 10$, $B^* = 0$, $C^* = 20$ and $z^* = 477{,}000$; where the 2nd and 3rd constraints are active.

Table E8.82

Basic	A	B	C	x_1	x_2	x_3	b	ratio
x_1	40000	60000	70000	1	0	0	2000000	28.57143
x_2	3	6	**6**	0	1	0	150	**25**
x_3	1	1	1	0	0	1	30	30
Cost	-9900	-18000	**-18900**	0	0	0	f-0	
x_1	5000	-10000	0	1	-11666.7	0	250000	50
C	0.5	1	1	0	0.166667	0	25	50
x_3	**0.5**	0	0	0	-0.16667	1	5	**10**
Cost	**-450**	900	0	0	3150	0	f+472500	
x_1	0	-10000	0	1	-10000	-10000	200000	
C	0	1	1	0	0.333333	-1	20	
A	1	0	0	0	-0.33333	2	10	
Cost	0	900	0	0	3000	900	f+477000	

8.83

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Section 2.4, we have:
Minimize $f = 3.6x_1 + 3.075x_2 + 2.58x_3 + 2.7x_4$
Subject to $x_1 + x_2 \le 240$
$x_3 + x_4 \le 300$
$x_1 + x_3 \le 200$
$x_2 + x_4 \le 200$
$x_1 + x_2 + x_3 + x_4 \ge 300$
$x_i \ge 0; i = 1$ to 4

Solution:
Standard LP form:
Minimize $f = 3.6x_1 + 3.075x_2 + 2.58x_3 + 2.7x_4$
Subject to $x_1 + x_2 + x_5 = 240$
$x_3 + x_4 + x_6 = 300$
$x_1 + x_3 + x_7 = 200$
$x_2 + x_4 + x_8 = 200$
$x_1 + x_2 + x_3 + x_4 - x_9 + x_{10} = 300$
$x_i \ge 0; i = 1$ to 10
The optimum solution is $x_1^* = 0.0,\ x_2^* = 0.0,\ x_3^* = 200,\ x_4^* = 100$ and $f^* = 786$, where the 2nd, 3rd and 5th constraints are active.

Table E8.83

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	b	ratio
x_5	1	1	0	0	1	0	0	0	0	0	240	240
x_6	0	0	1	1	0	1	0	0	0	0	300	∞
x_7	**1**	0	1	0	0	0	1	0	0	0	200	**200**
x_8	0	1	0	1	0	0	0	1	0	0	200	∞
x_{10}	1	1	1	1	0	0	0	0	-1	1	300	300
Cost	3.6	3.075	2.58	2.7	0	0	0	0	0	0	f-0	
Arti	**-1**	-1	-1	-1	0	0	0	0	1	0	w-300	
x_5	0	**1**	-1	0	1	0	-1	0	0	0	40	**40**
x_6	0	0	1	1	0	1	0	0	0	0	300	∞
x_1	1	0	1	0	0	0	1	0	0	0	200	∞
x_8	0	1	0	1	0	0	0	1	0	0	200	200
x_{10}	0	1	0	1	0	0	-1	0	-1	1	100	100
Cost	0	3.075	-1.02	2.7	0	0	-3.6	0	0	0	f-720	
Arti	0	**-1**	0	-1	0	0	1	0	1	0	w-100	
x_2	0	1	-1	0	1	0	-1	0	0	0	40	negative
x_6	0	0	1	1	0	1	0	0	0	0	300	300
x_1	1	0	1	0	0	0	1	0	0	0	200	200
x_8	0	0	1	1	-1	0	1	1	0	0	160	160
x_{10}	0	0	**1**	1	-1	0	0	0	-1	1	60	**60**
Cost	0	0	2.055	2.7	-3.075	0	-0.525	0	0	0	f-843	
Arti	0	0	**-1**	-1	1	0	0	0	1	0	w-60	
x_2	0	1	0	1	0	0	-1	0	-1	1	100	∞
x_6	0	0	0	0	1	1	0	0	1	-1	240	240
x_1	1	0	0	-1	**1**	0	1	0	1	-1	140	**140**
x_8	0	0	0	0	0	0	1	1	1	-1	100	∞
x_3	0	0	1	1	-1	0	0	0	-1	**1**	60	negative
Cost	0	0	0	0.645	**-1.02**	0	-0.525	0	2.055	-2.055	f-966.3	
Arti	0	0	0	0	0	0	0	0	0	1	w-0	End phs1
x_2	0	1	0	**1**	0	0	-1	0	-1	1	100	**100**
x_6	-1	0	0	1	0	1	-1	0	0	0	100	100
x_5	1	0	0	-1	1	0	1	0	1	-1	140	negative
x_8	0	0	0	0	0	0	1	1	1	-1	100	∞
x_3	1	0	1	0	0	0	1	0	0	0	200	∞
Cost	1.02	0	0	**-0.375**	0	0	0.495	0	3.075	-3.075	f-823.5	
x_4	0	1	0	1	0	0	-1	0	-1	1	100	
x_6	-1	-1	0	0	0	1	0	0	1	-1	0	
x_5	1	1	0	0	1	0	0	0	0	0	240	
x_8	0	0	0	0	0	0	1	1	1	-1	100	

x_3	1	0	1	0	0	0	1	0	0	0	200	
Cost	1.02	0.375	0	0	0	0	0.12	0	2.7	-2.7	f-786	End phs2

8.84

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Referring to Formulation 1 of Exercise 2.21, we have

Minimize $f = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11} + y_{12}$;

subject to $0.3081x_1 + 0.3128x_3 + 0.2847x_5 + 0.3082x_7 + 0.2886x_9 + 0.285x_{11} + 0.3476y_1 + 0.3264y_3 + 0.3212y_5 + 0.3216y_7 + 0.3256y_9 + 0.2976y_{11} = 330{,}000$;

$0.3276x_2 + 0.3726x_4 + 0.3315x_6 + 0.3542x_8 + 0.2808x_{10} + 0.3588x_{12} + 0.3696y_2 + 0.3888y_4 + 0.3740y_6 + 0.3696y_8 + 0.3168y_{10} + 0.3744y_{12} = 125{,}000$;

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} \le 1{,}200{,}000$;

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11} + y_{12} \le 1{,}000{,}000$;

$0.39x_1 + 0.39x_2 + 0.46x_3 + 0.46x_4 + 0.44y_1 + 0.44y_2 + 0.48y_3 + 0.48y_4 \le 190{,}000$;

$0.39x_5 + 0.39x_6 + 0.46x_7 + 0.46x_8 + 0.44y_5 + 0.44y_6 + 0.48y_7 + 0.48y_8 \le 240{,}000$;

$0.39x_9 + 0.39x_{10} + 0.46x_{11} + 0.46x_{12} + 0.44y_9 + 0.44y_{10} + 0.48y_{11} + 0.48y_{12} \le 290{,}000$;

$x_i \ge 0$, $y_i \ge 0$; $i = 1$ to 12

The optimum solution is $x_7^* = 121622.4$, $x_{12}^* = 212056.5$, $y_1^* = 431818.2$, $y_6^* = 130786.4$, $y_9^* = 437395.4$ and all other variables are nonbasic. The optimum cost is 1,333,679.0.

8.85

Solve the following LP problem by the Simplex method and verify the solution graphically, whenever possible.

Obtain solutions for the three formulations of the "cabinet design" problem given in Section 2.6. Compare solutions for the three formulations.

Minimize $f = 3.6x_1 + 3.075x_2 + 2.58x_3 + 2.7x_4$

Subject to $x_1 + x_2 \le 240$

$x_3 + x_4 \le 300$

$x_1 + x_3 \le 200$

$x_2 + x_4 \le 200$

$x_1 + x_2 + x_3 + x_4 \ge 300$

$x_i \ge 0; i = 1$ to 4

The optimum solution for :

Formulation 1: $x_1^* = 0$, $x_2^* = 800$, $x_3^* = 0$, $x_4^* = 500$, $x_5^* = 1500$, $x_6^* = 0$, and $f^* = 7500$.

Formulation 2: $x_1^* = 0$, $x_2^* = 0$, $x_3^* = 4500$, $x_4^* = 4000$, $x_5^* = 3000$, $x_6^* = 0$, and $f^* = 7500$.

Formulation 3: $x_1^* = 0$, $x_2^* = 8$, $x_3^* = 0$, $x_4^* = 5$, $x_5^* = 15$, $x_6^* = 0$, and $f^* = 7500$.

The three formulations yield the same solution.

Section 8.7 Postoptimality Analysis

8.86/8.87

Formulate and solve the "crude oil" problem stated in Exercise 2.2. What is the effect on the cost function if the market for lubricating oil suddenly increases to 12,000 barrels? What is the effect on the solution if the price of Crude A drops to \$110/bbl? Verify the solutions graphically.

Minimize $f = 3.6x_1 + 3.075x_2 + 2.58x_3 + 2.7x_4$
Subject to $x_1 + x_2 \leq 240$
$x_3 + x_4 \leq 300$
$x_1 + x_3 \leq 200$
$x_2 + x_4 \leq 200$
$x_1 + x_2 + x_3 + x_4 \geq 300$
$x_i \geq 0; i = 1$ to 4

1. If market for lubricating oil increases to 12,000 barrels, the cost function will not change (since the Lagrange multiplier is zero).
2. If price of Crude A drops to \$24/bbl., the cost function will be decreased by (30-24)(20,000) = \$120,000.

8.88

Formulate and solve the problem stated in Exercise 2.6. What are the effects of the following changes? Verify your solutions graphically.

1. The supply of material C increases to 120kg.
2. The supply of material D increases to 100kg.
3. The market for product A decreases to 60.
4. The profit for A decreases to \$8/kg.

Minimize $f = 3.6x_1 + 3.075x_2 + 2.58x_3 + 2.7x_4$
Subject to $x_1 + x_2 \leq 240$
$x_3 + x_4 \leq 300$
$x_1 + x_3 \leq 200$
$x_2 + x_4 \leq 200$
$x_1 + x_2 + x_3 + x_4 \geq 300$
$x_i \geq 0; i = 1$ to 4

1. Supply of material C increases to 120 kg: There is no effect on cost function since the corresponding constraint is not active.
2. Supply of material D increases to 100 kg: Since the allowable increase is 17 kg, which is less than the intended increase 20 kg, the changed problem must be resolved. Alternatively, if we examine the graph for the problem and use the new data the new optimum solution is given as $A^* = 70$, $B^* = 110$ and $f^* = -1580$.
3. Market for product A decreases to 60 kg: Cost function will be increased by (70 – 60)(0.4) = \$4, thus the new cost function is $f^* = -1304$.

4. Profit for A reduces to \$8/kg: Since the allowable decrease is only \$0.4 which is less than the intended decrease of \$2, the basis will be changed. The changed problem must be resolved. Alternatively, we can examine the graph for the problem and use the new data to find the new optimum solution as $A^* = 41.6667$, $B^* = 110$ and $f^* = -1213.33$.

8.89

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.55

8.90

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
Unbounded problem

8.91

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.57

8.92

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.58

8.93

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.59

8.94

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.60

8.95

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.61

8.96

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.62

8.97

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.63

8.98

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.64

8.99

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.65

8.100

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.66

8.101

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.67

8.102

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.68

8.103

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.69

8.104

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.70

8.105

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.71

8.106

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.72

8.107

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
Infeasible problem

8.108

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.74

8.109

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.75

8.110

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.76

8.111

Solve the following problem and determine Lagrange multipliers for the constraints at the optimum point.
For solution refer to Exercise 8.55

8.112

Solve the following problem and determine ranges for the right-side parameters.
This problem is unbounded.

8.113

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.57

8.114

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.58

8.115

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.59

8.116

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.60

8.117

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.61

8.118

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.62

8.119

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.63

8.120

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.64

8.121

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.65

8.122

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.66

8.123

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.67

8.124

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.68

8.125

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.69

8.126

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.70

8.127

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.71

8.128

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.72

8.129

Solve the following problem and determine ranges for the right-side parameters.
This problem has no feasible solution.

8.130

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.74

8.131

Solve the following problem and determine ranges for the right-side parameters.
Solved after Exercise 8.75

8.132

Solve the following problem and determine ranges for the right-side parameters.
For solution refer to Exercise 8.76

8.133

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.55

8.134

Solve the following problem and determine ranges for the coefficients of the objective function. This problem is unbounded.

8.135

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.57

8.136

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.58

8.137

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.59

8.138

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.60

8.139

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.61

8.140

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.62

8.141

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.63

8.142

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.64

8.143

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.65

8.144

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.66

8.145

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.67

8.146

Solve the following problem and determine ranges for the coefficients of the objective function. For solution refer to Exercise 8.68

8.147

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.69

8.148

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.70

8.149

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.71

8.150

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.72

8.151

Solve the following problem and determine ranges for the coefficients of the objective function.
This problem has no feasible solution.

8.152

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.74

8.153

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.75

8.154

Solve the following problem and determine ranges for the coefficients of the objective function.
For solution refer to Exercise 8.76

8.155

Formulate and solve the optimum design problem of Exercise 2.2. Determine Lagrange multipliers for the constraints. Calculate the ranges for the right-side parameters, and the coefficients of the objective function. Verify your results graphically.
For solution refer to Exercise 8.77

8.156

Formulate and solve the optimum design problem of Exercise 2.6. Determine Lagrange multipliers for the constraints. Calculate the ranges for the right-side parameters, and the coefficients of the objective function. Verify your results graphically.
For solution refer to Exercise 8.78

8.157

Formulate and solve the "diet" problem stated in Exercise 2.7. Refer to Exercise 8.79. Investigate the effect on the optimum solution of the following changes:

1. Cost of milk increases to \$1.20/kg;
   $\Delta f = (1.2 - 1.0)(2.5) = 0.5$

2. Need for vitamin A increases to 6 units;
   $\Delta f = (6 - 5)(0.5) = 0.5$
   $(B^* = 0, M^* = 3 \text{ and } f^* = 3)$

3. Need for vitamin reduces to 3 units;
   $\Delta f = -(3 - 4)(0) = 0$

8.158

Formulate and solve the problem stated in Exercise 2.8. Refer to Exercise 8.80. Investigate the effect on the optimum solution of the following changes:

1. Supply of empty bottles reduces to 750
   $\Delta \mathrm{f} = -(750 - 800)(0.66667) = 33.3333$
   $(x_1^* = 250, x_2^* = 500, f^* = -1250)$

2. Profit on a bottle of wine reduces to \$0.80
   $\Delta \mathrm{f} = (1 - 0.8)(316.6667) = 63.3333$

3. Only 200 bottles of alcohol can be produced
   $\Delta \mathrm{f} = -(200 - 225)(3.3333) = 83.3333$
   $(\mathrm{x}_1^* = 400, \mathrm{x}_2^* = 400, f^* = -1200)$

8.159

Formulate and solve the problem stated in Exercise 2.18. Refer to Exercise 8.81. Investigate the effect on the optimum solution of the following changes:

1. Profit on margarine increases to \$0.06/kg;
   Increase in c_3: $\Delta c_3 = 0.01$; $-\infty \le \Delta c_3 \le 0.004$.
   Hence we need to resolve the problem.
   $f^* = -19549.47$, $x_1^* = 149494$, $x_2^* = 50000$, $x_3^* = 10000$.

2. Supply of milk base substances increases to 2500 kg;
   $\Delta f = 0$

3. Supply of soybeans reduces to 220,000 kg;
   No change on optimum solution, since the associated constraint is not active.

8.160

Solve the "saw mill" problem formulated in Section 2.4. Refer to Exercise 8.83. Investigate the effect on the optimum solution of the following changes:

1. Transportation cost for the logs increases to \$0.16 per kilometer per log;
   Cost function will increase \$52.4

2. Capacity of mill A reduces to 200 logs/day
   Optimum solution is not changed.

3. Capacity of mill B reduces to 270 logs/day
   Cost function is increased by \$11.25
   ($x_1^* = 0$, $x_2^* = 30$, $x_3^* = 200$ and $x_4^* = 70$)

8.161

Formulate and solve the problem stated in Exercise 2.20. Refer to Exercise 8.82. Investigate the effect on the optimum solution of the following changes:

1. Due to demand on the capital, the available cash reduces to \$1.8 million;
   $\Delta f = -(1{,}800{,}000 - 2{,}000{,}000)(0) = 0$

2. Initial investment for truck B increases to \$65,000; no change in the solution

3. Maintenance capacity reduces to 28 trucks;
   $\Delta f = -(28 - 30)(900) = 1800$
   ($A^* = 6$, $B^* = 0$, $C^* = 22$, $f^* = -475200$)

8.162

Formulate and solve the "steel mill" problem stated in Exercise 2.21. Refer to Exercise 8.84. Investigate the effect on the optimum solution of the following changes:

1. Capacity of the reduction plant 1 increases to 1,300,000;
   $\Delta f = -(1{,}300{,}000 - 1{,}200{,}000)(0) = 0$

2. Capacity of the reduction plant 2 reduces to 950,000;
   $\Delta \mathrm{f} = -(950{,}000 - 1{,}000{,}000)(0.049713) = 2485.65$

3. Capacity of fabricating plant 2 increases to 250,000;
   $\Delta f = 0$

4. Demand for product 2 increases to 130,000;
   $\Delta f = -(130{,}000 - 125{,}000)(-2.806718) = 14033.59$

5. Demand for product 1 reduces to 280,000;
   $\Delta f = -(280{,}000 - 330{,}000)(-3.244646) = -162232.3$

8.163

Obtain solutions for the three formulations of the “cabinet design” problem given in Section 2.6. Compare the three formulations. Refer to Exercise 8.85. Investigate the effect on the optimum solution of the following changes:

1. Bolting capacity is reduced to 5500/day;
   $\Delta f = -(5500 - 6000)(0) = 0$

2. Cost of riveting C increases to \$0.70;
   $\Delta f = (3.5 - 3.0)(800) = 400$

3. The company must manufacture only 95 devices per day;
   $\Delta f = -(-5)(8)(-3) - (-5)(5)(-4.8) - (-5)(15)(-1.8) = -375$

8.164

Minimize $f = 2x_1 - 4x_2$
subject to $g_1 = 10x_1 + 5x_2 \le 15$
$g_2 = 4x_1 + 10x_2 \le 36$
$x_1 \ge 0$, $x_2 \ge 0$

Slack variables for g_1 and g_2 are x_3 and x_4, respectively. The final tableau for the problem is given in Table E8.163. Using the given tableau:

1. Determine the optimum values of f and **x**.
2. Determine Lagrange multipliers for g_1 and g_2.
3. Determine the ranges for the right sides of g_1 and g_2.
4. What is the smallest value that f can have, with the current basis, if the right side of g_1 is changed? What is the right side of g_1 for that case?

1. $x_1^* = 0.0$, $x_2^* = 3.0$, $f^* = -12$;
2. 2. $y_1 = 4/5$, $y_2 = 0$
3. $-15.0 \le \Delta_1 \le 3.0$; $-6.0 \le \Delta_2 \le \infty$
4. $\Delta f = -y_1\Delta_1 = -0.8\Delta_1$;

When $\Delta_1 = 3.0$, we have the smallest objective function value with current basis unchanged.

The new objective function value is $f' = f^* + \Delta f = -12 + (-0.8)(3.0) = -14.4$. New right hand side of $g_1 = 15 + 3 = 18$.