CHAPTER

7

Optimum Design with MATLAB®

7.1

Formulate and solve Exercise 3.34

Optimum solution: $x_1^* \doteq 103.0$ mm, $x_2^* \doteq 0.955$, $f^* \doteq 2.9$ kg; shear stress, and buckling constraint are active.

7.2

Formulate and solve Exercise 3.35

Optimum solution: $d_o^* \doteq 103.0$ mm, $d_i^* \doteq 98.36$ mm, $f^* \doteq 2.9$ kg; shear stress, and buckling constraints are active.

7.3

Formulate and solve Exercise 3.36

Optimum solution: $R^* = 50.3$ mm, $t^* \doteq 2.35$ mm, $f^* \doteq 2.9$ kg; shearing stress, and buckling constraints are active.

7.4

Formulate and solve Exercise 3.50

Optimum solution: $A_1^* \doteq 300$ mm^2, $A_2^* \doteq 50.0$ mm^2, $f^* \doteq 7.0$ kg; member 1 stress constraint is active.

7.5

Formulate and solve Exercise 3.51

Optimum solution: $R^* \doteq 130$ cm, $t^* \doteq 2.86$ cm, $f^* \doteq 57000$ kg; combined stress constraint, and (diameter/thickness) ratio constraint are active.

7.6*

Formulate and solve Exercise 3.52

Optimum solution: $d_o^* \doteq 41.56$ cm, $d_i^* \doteq 40.19$ cm, $f^* = 680.0$ kg; top deflection constraint, and (diameter/thickness) ratio constraint are active.

7.7*

Formulate and solve Exercise 3.53

Optimum solution: $d_o^* \doteq 1310$ mm, $t^* \doteq 14.2$ mm, $f^* \doteq 92{,}500$ N; maximum deflection constraint, and (diameter/thickness) ratio constraint are active.

7.8*

Formulate and solve Exercise 3.54

Optimum solution: $H^* = 50.0$ cm, $D^* \doteq 3.42$ cm, $f^* \doteq 6.6$ kg; buckling load constraint, and maximum height constraint are active.

7.9

Problem formulation: Minimize $f = bh$;
subject to $g_1 = 1.0 - [3gkEI/(3WEI + kWL^3)]^{1/2}/8.0 \le 0$, where $I = bh^3/12$,
and $0.5 \le b \le 1.0$, $0.2 \le h \le 2.0$.
Solution: Initial design: $b = 0.5$, $h = 0.2$, optimum solution; $b^* = 0.5$ in, $h^* = 0.28107$ in, $f^* = 0.140536\ \text{in}^2$, active constraints (Lagrange multiplier); $g_1(0.54523)$, lower limit on $b(0.0936936)$.

7.10

<u>Formulation:</u> Units of N and cm are used

1. Design variables: $x_1 = b$, $x_2 = t_1$, $x_3 = t_2$, $x_4 = h$
2. Cost function: $f = L\,(2x_1x_2+x_3x_4) = 150(2x_1x_2+x_3x_4)$
3. Constraints:

<axial stress> $g_1 = (Mc/I + P\cos\theta/A)/\sigma_a - 1.0 \le 0$,

where $M = PL\sin\theta$, $c = x_2 + x_4/2$, $I = [x_1(2x_2+x_4)^3 - (x_1 - x_3)\,x_4^3]/12$, $A = 2x_1x_2 + x_3x_4$,

$P = 70000$, $L = 150$, $\theta = 45°$, $\sigma_a = 10000$;

<shear stress> $g_2 = (VQ/Ix_3)/\tau_a - 1.0 \le 0$,

where $V = P\sin\theta$, $Q = x_1x_2\,(x_2+x_4)/2 + x_3x_4^2/8$, $\tau_a = 6000$;

<deflection> $g_3 = [(P\sin\theta)L^3/(3EI)]/\Delta - 1.0 \le 0$, where $\Delta = 1.5$;

<buckling> $g_4 = 1.0 - \pi^2 EI\,/\,(4L^2\,P\cos\theta) \le 0$, $g_5 = 1.0 - \pi^2 EI\,'/(4L^2\,P\cos\theta) \le 0$,

where $I\,' = x_1^3x_2/6 + x_3^3x_4/12$;

<design limits> $x_1 \ge 10$, $x_2 \le 1$, $x_3 \le 1.5$, $x_4 \le 15$.

Initial design; $x_1 = 60$, $x_2 = 0.9$, $x_3 = 0.9$, $x_4 = 14$,

Optimum; $x_1^* = 50.4437$ cm, $x_2^* = 1.0$ cm, $x_3^* = 0.52181$ cm, $x_4^* = 15.0$ cm, $f^* = 16307.2\ \text{cm}^3$, active constraints (Lagrange multipliers); $g_1(15502.0)$, $g_2(805.224)$, upper limit of $x_2(154.797)$, upper limit of $x_4(14641.4)$.

7.11

Formulation:

1. Design variables; $b_i = A_i$, $b_{i+3} = x_i$, i = 1 to 3.

2. Cost function; f = volume of truss members = $\sum_{i=1}^{3} b_i L_i = \sum_{i=1}^{3} b_i [L^2 + b_{i+3}^2]^{1/2}$

3. Constraints (18 stress constraints)

$g_j = \sigma_{1j}/5000 - 1.0 \le 0$, $j = 1,2,3$; $g_{3+j} = -\sigma_{1j}/5000 - 1.0 \le 0$, $j = 1,2,3$;

$g_{6+j} = \sigma_{2j}/20000 - 1.0 \le 0$, $j = 1,2,3$; $g_{9+j} = -\sigma_{2j}/20000 - 1.0 \le 0$, $j = 1,2,3$;

$g_{12+j} = \sigma_{3j}/5000 - 1.0 \le 0$, $j = 1,2,3$; $g_{15+j} = -\sigma_{3j}/5000 - 1.0 \le 0$, $j = 1,2,3$;

4. Design variable limits (arbitrary) $1.0\text{E}{-10} \le b_i \le 20$, in^2; $-10.0 \le b_{i+3} \le 10.0$, in, $i = 1,2,3$

Solution: Initial design: $b_1 = b_2 = b_3 = 6.0$, $b_4 = b_6 = 0.5$, $b_5 = 0.0$,

Optimum solution: $b_1^* = A_1 = 1.4187$, $b_2^* = A_2 = 2.0458$, $b_3^* = A_3 = 2.9271$ in^2, $b_4^* = x_1 = -4.6716$, $b_5^* = x_2 = 8.9181$, $b_6^* = x_3 = 4.6716$ in, $f^* = 75.3782$ in^3, active constraints (Lagrange multipliers); $g_3(27.411)$, $g_7(4.86191)$, $g_{11}(0.0)$, $g_{13}(20.5489)$, $g_{17}(22.5562)$.

7.12

Formulation: Minimize $f = (x_2 - x_3)^2$;

subject to $h_1 = \phi^2 x_1(x_1 - x_2 + x_3)/x_2 x_3 - 1 = 0$, $h_2 = 1.0 - x_2(1 - x_1 + x_2)/\phi^3 x_1 = 0$,

and design bounds $1.0\text{E}{-10} \le x_1, x_2, x_3 \le 1000.0$

Solution for $\phi = \sqrt{2}$, $x^{(0)} = (1, 1, 1)$;

Optimum; $x_1^* = 2.4138$, $x_2^* = 3.4138$, $x_3^* = 3.4141$, $f^* = 1.2877 \times 10^{-7}$. Active constraints (Lagrange multipliers); $h_1(-0.007119)$, $h_2(0.003528)$. [Program used; IDESIGN; 8 iterations]

Solution for $\phi = 2^{1/3}$, $x^{(0)} = (1,1,1)$.

Optimum; $x_1^* = 2.2606$, $x_2^* = 2.8481$, $x_3^* = 2.8472$, $f^* = 8.03 \times 10^{-7}$.

Active constraints (Lagrange multipliers); $h_1(-5.47 \times 10^{-6})$, $h_2(1.6236 \times 10^{-6})$.