Cover page for answers.pdf
CSE512 Fall 2019 - Machine Learning - Homework 6

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① Deflated Matrix $\tilde{X}$ is given by.

$$\tilde{X} = (I - v_1 v_1^T) X$$

Here it is given that $(I - v_1 v_1^T)$ is symmetric

Hence when we take the $\tilde{X}$ transpose.

$$\tilde{X}^T = (I - v_1 v_1^T) X^T$$

Lets take the equation

$$\tilde{C} = \frac{1}{n} \tilde{X} \tilde{X}^T - \lambda v_1 v_1^T$$

substitute the value of $\tilde{X}^T$ and $\tilde{X}$ in $\tilde{C} = \frac{1}{n} \tilde{X} \tilde{X}^T$

$$\tilde{C} = \frac{1}{n} \left((I - v_1 v_1^T) X \right) \left(X^T (I - v_1 v_1^T) \right)$$

$$= \frac{1}{n} \left[X X^T + v_1 v_1^T X X^T v_1 v_1^T - X X^T v_1 v_1^T - v_1 v_1^T X X^T \right]$$

As $X X^T v_1 = n \lambda_1 v_1$ and $v_1^T v_1 = I$

$$\tilde{C} = \frac{1}{n} X X^T + \frac{1}{n} \left[n \lambda_1 v_1 v_1^T - n \lambda_1 v_1 v_1^T - v_1 v_1^T X X^T \right]$$

$$= \frac{-1}{n} \left[v_1 v_1^T X X^T \right] + \frac{1}{n} X X^T \, //$$

It is given that $(AB)^T = B^T A^T$ and.

$$v_1 v_1^T X X^T = (X X^T v_1 v_1^T)^T$$

$$= \frac{-1}{n} \left[X X^T v_1 v_1^T \right] + \frac{1}{n} X X^T$$

$$= \frac{1}{n} X X^T - [\cancel{n} \lambda_1 v_1 v_1^T] \frac{1}{\cancel{n}}$$

$$\Rightarrow \frac{1}{n} X X^T - \lambda_1 v_1 v_1^T \, //$$

(2). Given that $C v_j = \lambda_j v_j$ — (A)

From the solution to 1st question we get

$\Rightarrow \tilde{C} v_j = \left(\frac{1}{n} X X^T - \lambda_1 v_1 v_1^T \right) v_j$

when $j \neq 1$, we know than $v_1^T v_j = 0$

and also $\frac{1}{n} X X^T = C$

The above equation becomes.

$C v_j - \lambda_1 v_j \times 0$

$$\tilde{C} v_j = C v_j$$

From (A)

$$\tilde{C} v_j = \lambda_j v_j \quad \text{when } j \neq 1$$

(3). u is the first principal eigen vector of $\tilde{C}$

$u = v_2$ - to prove

$v_j \quad \forall j \neq 1$ is eigen vector of $\tilde{C}$

and $v_2 > v_3 > \dots v_k$.

Hence from this we can say v_2 is the largest eigen vector of $\tilde{C}$.

we know $\tilde{C} v_1 = C v_1 - \lambda_1 v_1 v_1^T v_1$

As $j=1$, $v_1 v_1^T = 1$.

$\therefore \tilde{C} v_1 = C v_1 - \lambda_1 v_1$

But $C v_1 = \lambda_1 v_1$

Hence $\tilde{C} v_1 = 0$ for $j=1$

As $\tilde{C} V_1 = 0$ and V_2 is largest eigen vector

we can say that V_2 is principal eigen vector of $\tilde{C}$

Hence $u = V_2$

(4). def find (C, k):

/* 2 list - one for storing eigen values and another to store lambda values */

e = [ ]

L = [ ]

for i in range (~~[redacted]~~ k):

value1, value2 = f(C)

e.append (value 2)

L.append (value1)

C = C - value1 * np.dot (value 2, transpose (value2))

return e, L

- This gives us the first k principal basis vector of X.