Arithmetic Progression(AP)

Definition: A sequence in which the difference between any two consecutive term is constant is called as *Arithimetic Progression(AP)*. The constant value that defines the difference between any two consecutive terms is called the *Common Difference*.
Let T be an arithmetic progression that has a as the first term and common difference of d, then we can write it's n^{th} term as:
T_n = a + (n-1)d

Sum of Arithimetic Series

Let the first term be a and and common difference d, then the sum of arithmetic series for n terms,S_n is:
S_n = a + (a+d) + (a+2d) +...+ (a+(n-2)d) + (a+(n-1)d)
Reversing the series:
S_n = (a+(n-1)d) + (a+(n-2)d) +...+ (a+2d) + (a+d) + a
Adding the corresponding terms, noting that they add up to 2a+(n-1)d everytime:
2*S_n=$\underbrace{(2a + (n-1)d) + ... + (2a + (n-1)d)}_{nterms}$
There are n of these terms, so:
2*S_n=n*(2a + (n-1)d)

S_n=$\frac{n(2a+(n-1)d)}{2}$

Also, the first term in the series is a, and the last one is a+(n-1)d, so we can say the sum of the series is the first term plus the last term multiplied by the number of terms divided by 2.
i.e, $S_n = \dfrac{n}{2}(a_1+a_n)$

Geometric Progression(GP)

Definition: A sequence in which the ratio of the a term to it's preceding term is the same for any term is called as *Geometric Progression*. The constant value of the ratio is called as the common ratio of the geometric progression .
Let T be an geometric progression that has a as the first term and common ratio is r, then we can write it's n^{th} term as:
T_n = a r^{n-1}

Sum of Geometric Series [1]

Let $S_n = \sum_{j=0}^{n-1}$ a r^j.
Then:

$$
\begin{aligned}
(r-1)S_n &= rS_n - S_n \\
&= a\left(r\sum_{j=0}^{n-1} r^j - \sum_{j=0}^{n-1} r^j\right) \\
&= a\left(\sum_{j=1}^{n} r^j - \sum_{j=0}^{n-1} r^j\right) \\
&= a\left(r^n + \sum_{j=1}^{n-1} r^j - \left(r^0 + \sum_{j=1}^{n-1} r^j\right)\right) \\
&= a(r^n - r^0)
\end{aligned}
$$

$$=\text{a}(r^n - 1)$$

Hence, S_n = a $(\dfrac{r^n - 1}{r - 1})$

References

[1] proofwiki.org. Sum of geometric progression.