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HW-3

Ans No, we cannot build a neural network of arbitrary length to learn XOR.

XOR is ~~linear~~ non-linearly separable. Neural network using just linear activation function is a generalized linear model.

∴ XOR can't be classified by it.

$$y = f(w_2 (w_1 x + b_1) + b_2)$$

It's same as SVM with linear function.

$$\begin{aligned} f(x) &= B(Ax + a) + b \\ &= \underbrace{BA}_{\downarrow \text{Constant}} x + \underbrace{Ba + b}_{\downarrow \text{Constant}} \end{aligned}$$

$$\therefore f(x) = C_1 x + C_2$$

$A_2 = \text{Sigmoid}$

Reasons :-

Sigmoid Saturation, Killing Gradients

When sigmoid neuron's activation saturates at either 0 or 1, the gradient at these regions is almost 0.

\therefore no signal flow through neuron.

Remedy : ① Initialize the weights by extra one.

② Take small value of weights initially.

ReLU : It will perform better than sigmoid.

ReLU also have problems like dead neuron. It can be fragile during training and die. Large gradients results in ~~update~~ update weight that gradient flowing could be zero. This kills neurons.

Remedy \Rightarrow Keep the learning rate low.

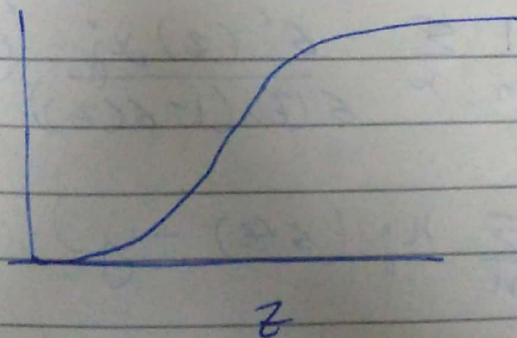
B
A3 = For quadratic, learning is slow when error is large.

$$C = \frac{(y - a)^2}{2}$$

$$a = \sigma(z)$$

$$\frac{\partial C}{\partial w} = (a - y) \sigma'(z) n = a \sigma'(z)$$

illy $\frac{\partial C}{\partial b} = a \sigma'(z)$



$\therefore \frac{\partial C}{\partial w}, \frac{\partial C}{\partial b}$ get very small

\therefore slow learning.

Cross entropy cost,

$$C = -\frac{1}{n} \sum_k [y_k \ln a_k + (1-y_k) \ln (1-a_k)]$$

$$\text{I} \quad \boxed{C > 0}$$

II If neuron's actual output is close to desired o/p, y_k , then cross entropy will be close to zero

III

$$\frac{\partial C}{\partial w_j} = -\frac{1}{n} \sum_k \left(\frac{y_k}{\sigma(z_k)} - \frac{(1-y_k)}{1-\sigma(z_k)} \right) \sigma'(z_k) x_{kj}$$

$$= -\frac{1}{n} \sum_k \frac{\sigma'(z_k) x_{kj} (\sigma(z_k) - y_k)}{\sigma(z_k) (1-\sigma(z_k))}$$

$$= \frac{1}{n} \sum_k x_{kj} (\sigma(z_k) - y_k)$$

This tells the rate at which weight learns is controlled by $\sigma(z_k) - y_k$

∴ Larger error \Rightarrow more it will learn