**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:**  B. 0.2676

We have a normal distribution with = 45 and = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).

Pr(X > 50)

= 1 - Pr(X ≤ 50).

Z = (X - )/ = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50)

= Pr(Z ≤ (50 - 45)/8.0)

= Pr(Z ≤ 0.625)

=73.4%

Probability that the service manager will not meet his demand will be

= 100-73.4

= 26.6% or 0.2676

So, option B is correct.

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**

**Ans:**

Probabilty of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X ≤ 44).

Z = (X - µ )/ σ = (X - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6)

= Pr(Z ≤ 1)

=84.1345%

Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age

= Pr(X<44)-0.5

=84.1345-0.5

= 34.1345%

Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

1. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

**Ans:**

Probabilty of employees less than age of 30 = Pr(X<30).

Z = (X - µ )/ σ = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X ≤ 30) =

Pr(Z ≤ (30 - 38)/6)

= Pr(Z ≤ -1.333)

=9.12%

So the number of employees with probability 0.912 of them being under age 30

= 0.0912\*400

=36.48( or 36 employees).

Therefore the statement B of the question is also TRUE.

1. **If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

**Ans:**

As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. **Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:** Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100

= -(-2.57)\*20+100

= 151.4

Z(+0.005)\*20+100

= (-2.57)\*20+100

= 48.6

So, option D is correct.

1. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

**Ans:** (17.504, 90.496)

1. **Specify the 5th percentile of profit (in Rupees) for the company**

**Ans:** Rs.24.07 million