

Figure 1: A sample series of moves at the end of a game of Betsy with n = 3. From the initial board in (a), red chooses to drop a pebble in the second column, resulting in (b). Blue then rotates the first column to give (c). Red drops another pebble in the second column to give (d). Blue drops a pebble in the first column to give (e). Finally, red rotates the first column, and wins the game by completing a row within the top n rows of the board.

## Part 1: Betsy

A popular game in a certain exotic, far-off land<sup>1</sup> is called *Betsy*. It's played on a vertical board that is n squares wide and n+3 squares tall (where n is often 5 for beginners, but can grow quite large in the professional tournaments). The board starts off empty, with each of the two players (red and blue) given  $\frac{1}{2}n \times (n+3)$  pebbles of their own color. Blue goes first, choosing one of two possible types of moves.

- Drop: Choose one of the n columns, and drop a blue pebble into that column. The pebble falls to occupy the bottom-most empty square in that column. The player is not allowed to choose a column that is already full (i.e., already has n + 3 pebbles in it).
- Rotate: Choose one of the n columns, remove the pebble from the bottom of that column (whether red or blue) so that all pebbles fall down one square, and then drop that same pebble into the top of that column. The player is not allowed to choose an empty column for this type of move.

After making a move, blue checks the top n rows of the board to see if they have completed a row of n blue pebbles, a column of n blue pebbles, or one of the two diagonals of blue pebbles. The bottom three rows of the board are ignored during this check. If a row, column, or diagonal has been completed in blue, blue wins! Otherwise, red makes the same check and wins if any row, column, or diagonal has been completed with red. Note this means that if blue completes a row, column, or diagonal of blue pebbles, they win *even* if they have also completed a row, column, or diagonal of red. If no one has won, player red takes their turn, either dropping a red pebble into an incomplete column or rotating a non-empty column. Figure 1 shows several sample moves from a game in progress, with n = 3.

For example, the encoding of the board in Figure 1(a) would be: