Part - 3 Linear Algebra.

1) $A = \begin{bmatrix} 2 & 4 & 6 \\ 0 & -2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -2 & 5 \\ 0 & 8 & 2 \end{bmatrix}$

Here A is 2×3 = inner dim = $3 \neq 2$
B is 2×3.

A.B not Possible

Here B is 2×3 $\Rightarrow$ inner dim = $3 \neq 2$
A is 2×3 B.A is also not Possible.

2) $A = \begin{bmatrix} 2 & 4 & 6 \\ 0 & -2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -0.5 \\ 1 & 0 \\ 1 & 0.5 \end{bmatrix}$

$A.B = \begin{bmatrix} 14 & 2 \\ 2 & 2 \end{bmatrix}$

Here A.B is possible because

A is 2×3 $\Rightarrow$ inner dim =
B is 3×2 $3 = 3$

So $A.B = 2 \times 2$ Matrix

③ $A = \begin{bmatrix} 4 & 0 & -2 \\ 1 & 3 & -2 \\ 1 & 2 & -1 \end{bmatrix}$ $\lambda_0 = 1$

polynomial of A & Deremine eigenvalues

$\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 0 & -2 \\ 1 & 3-\lambda & -2 \\ 1 & 2 & -1-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (4-\lambda)\begin{vmatrix} 3-\lambda & -2 \\ 2 & -1-\lambda \end{vmatrix} - 0 \begin{vmatrix} 1 & -2 \\ 1 & -1-\lambda \end{vmatrix}$$

$$+ (-2)\begin{vmatrix} 1 & 3-\lambda \\ 1 & 2 \end{vmatrix}$$

$$= (4-\lambda)\left[(3-\lambda)(-1-\lambda) + 4\right] - 2(2-(3-\lambda))$$

$$\Rightarrow (4-\lambda)\left[-(3+3\lambda-\lambda-\lambda^2)+4\right] - 2(2-3+\lambda)$$

$$= (4-\lambda)\left[1 - 2\lambda + \lambda^2\right] - (-2+2\lambda)$$

$$= (4 - 8\lambda + 4\lambda^2 - \lambda + 2\lambda^2 - \lambda^3) - (-2 + 2\lambda)$$

$$= (4 - 9\lambda + 6\lambda^2 - \lambda^3) - (-2 + 2\lambda)$$

$$= 4 - 9\lambda + 6\lambda^2 - \lambda^3 + 4 - 2\lambda$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\Rightarrow \cancel{\lambda^2 (\lambda - 6)}$$

Here we take $\lambda = 1$ so $\lambda - 1 = 0$

$$\Rightarrow (\lambda - 1)(\lambda^2 - 5\lambda + 6)$$

$$(\lambda - 1)(\lambda - 3)(\lambda - 2) = 0$$

$$\boxed{\lambda = 2 \quad \lambda = 3, \quad \lambda = 1}$$

$$\boxed{\text{so Eigenvalues } \lambda = 1, 2, 3}$$