In [1]:
```python
import numpy as np
import math
import matplotlib.pyplot as plt
from scipy.spatial.distance import cdist as cd
```

In [2]:
```python
pos = np.arange(-5.0,5,0.1)
```

Kernel Density Estimation Using Gaussian Kernel

In [3]:
```python
def kernel(x,xi,h):
    return 1/(2*math.pi*h**2)*math.exp((-(x-xi)**2)/(2*h**2))
```

In [4]:
```python
def kde(samples, h):
    size = np.size(samples)
    density = np.zeros(size)
    for i in range(0,size):
        x=0
        for j in range(0,size):
            x += kernel(samples[i],samples[j],h)
        density[i] = x/size

    return density
```

Visualization

In [5]:
```python
print(kde(pos,0.2))
```

```
[0.11962994 0.15474337 0.17887646 0.19179397 0.19717879 0.19892699
 0.199369   0.19945604 0.19946939 0.19947098 0.19947113 0.19947114
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 0.19947114 0.19947114 0.19947114 0.19947114 0.19947114 0.19947113
 0.19947098 0.19946939 0.19945604 0.199369   0.19892699 0.19717879
 0.19179397 0.17887646 0.15474337 0.11962994]
```
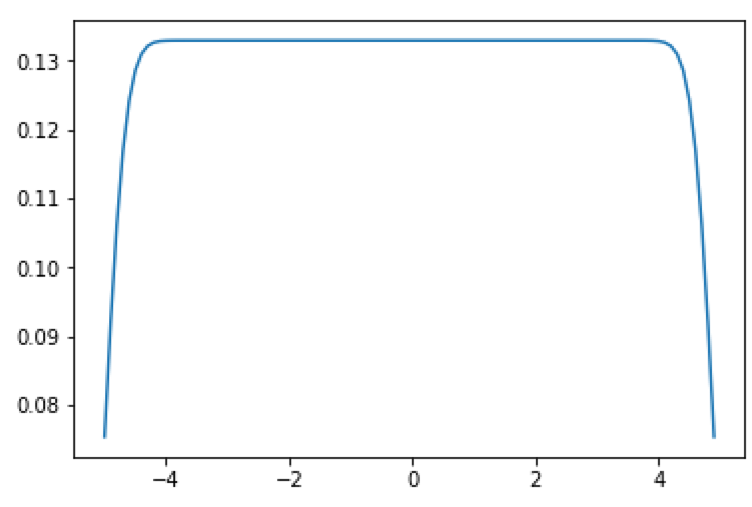
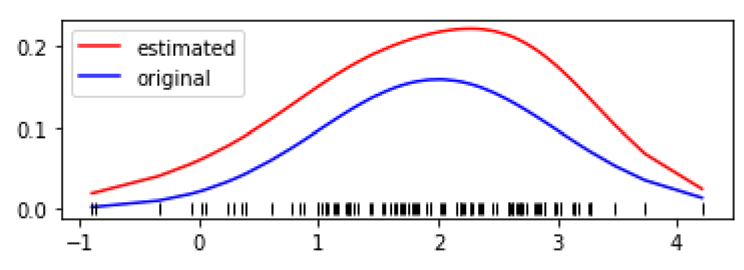

In [6]:
```python
plt.plot(pos,kde(pos,0.3))
plt.show()
```



Testing on data obtained using Normal Distribution

In [7]:
```python
rand_1 = np.random.normal(2,1,100)
rand_1 = np.sort(rand_1)
probability_density = np.zeros(np.size(rand_1))
for i in range(0,np.size(rand_1)):
    probability_density[i] = kernel(2,rand_1[i],1)
plt.subplot(2,1,1)
a = plt.plot(rand_1,kde(rand_1,0.6),'r',label='estimated')
b = plt.plot(rand_1,probability_density,'b',label='original')
plt.plot(rand_1, np.zeros(np.size(rand_1)), '|k',markeredgewidth=1)
plt.legend()
plt.show()
```



KNN

In [8]:
```python
def distance(a,b):
    return abs(a-b)
```

In [9]:
```python
def knn(samples, k):
    samples = np.sort(samples)
    size = np.size(samples)
    density = np.zeros(size)
    for i in range(0,size):
        distance_to_k = 0
        distance_vector= np.zeros(size)
        for  j in range(0,size):
            distance_vector[j] = distance(samples[i],samples[j])
            arg_distance = distance_vector.argsort()
            temp2 = arg_distance[k-1]
            distance_to_k = distance(samples[i],samples[temp2])
            density[i] = k
            density[i]/=size
            density[i]/=((4/3)*math.pi*distance_to_k**3)
    return density
```

Visualization

In [10]:
```python
print(knn(pos,4))
```

```
[0.35367765 1.19366207 1.19366207 1.19366207 1.19366207 1.19366207
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 1.19366207 1.19366207 1.19366207 0.35367765]
```
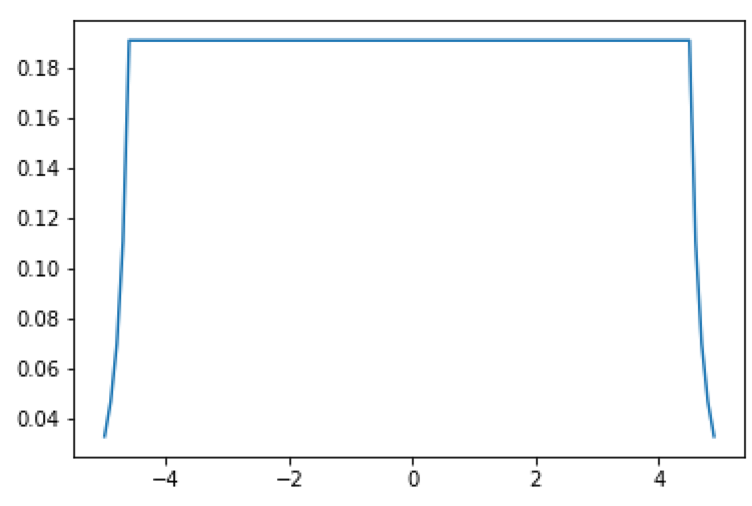

```
/Library/Frameworks/Python.framework/Versions/3.4/lib/python3.4/site-packages/ipykernel_launche
r.py:15: RuntimeWarning: divide by zero encountered in double_scalars
  from ipykernel import kernelapp as app
```

In [11]:
```python
plt.plot(pos,knn(pos,10))
plt.show()
```

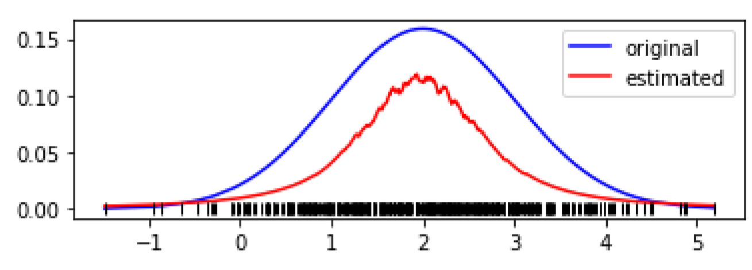
```
/Library/Frameworks/Python.framework/Versions/3.4/lib/python3.4/site-packages/ipykernel_launche
r.py:15: RuntimeWarning: divide by zero encountered in double_scalars
  from ipykernel import kernelapp as app
```



Testing on data obtained using Normal Distribution

In [12]:
```python
rand_1 = np.random.normal(2,1,500)
rand_1 = np.sort(rand_1)
probability_density = np.zeros(np.size(rand_1))
for i in range(0,np.size(rand_1)):
    probability_density[i] = kernel(2,rand_1[i],1)
plt.subplot(2,1,1)
plt.plot(rand_1, probability_density,'b',label='original')
plt.plot(rand_1,knn(rand_1,350),'r',label='estimated')
plt.plot(rand_1, np.zeros(np.size(rand_1)), '|k',markeredgewidth=1)
plt.legend()
plt.show()
```

```
/Library/Frameworks/Python.framework/Versions/3.4/lib/python3.4/site-packages/ipykernel_launche
r.py:15: RuntimeWarning: divide by zero encountered in double_scalars
  from ipykernel import kernelapp as app
```



Apply method

In [13]:
```python
def parameters():
    k = 50  # knn neighbors
    h = 0.6  # kde windowsize / radius
    return h, k


def gauss1D(m, v, N, w):
    pos = np.arange(-w, w - w / N, 2 * w / N)
    insE = -0.5 * ((pos - m) / v) ** 2
    norm = 1 / (v * np.sqrt(2 * np.pi))
    res = norm * np.exp(insE)
    realDensity = np.stack((pos, res), axis=1)
    return realDensity


h, k = parameters()

print('Question: Kernel/K-Nearest Neighborhood Density Estimators')

# Produce the random samples
samples = np.random.normal(0, 1, 100)
samples = np.sort(samples)

# Compute the original normal distribution
realDensity = gauss1D(0, 1, 100, 5)

# Estimate the probability density using the KDE
estDensity = kde(samples, h)

# plot results
plt.subplot(2, 1, 1)
plt.plot(samples, estDensity, 'r', linewidth=1.5, label='KDE Estimated Distribution')
plt.plot(realDensity[:, 0], realDensity[:, 1], 'b', linewidth=1.5, label='Real Distribution')
plt.legend()

# Estimate the probability density using KNN
estDensity = knn(samples, k)

# Plot the distributions
plt.subplot(2, 1, 2)
plt.plot(samples, estDensity, 'r', linewidth=1.5, label='KNN Estimated Distribution')
plt.plot(realDensity[:, 0], realDensity[:, 1], 'b', linewidth=1.5, label='Real Distribution')
plt.legend()
plt.show()
```

```
Question: Kernel/K-Nearest Neighborhood Density Estimators
```

```
/Library/Frameworks/Python.framework/Versions/3.4/lib/python3.4/site-packages/ipykernel_launche
r.py:15: RuntimeWarning: divide by zero encountered in double_scalars
  from ipykernel import kernelapp as app
```