

**COM S 311 SPRING 2019
HOMEWORK 6**

Due: April 26, 11:59PM

Late Submission Due: April 27, 11:59PM (25% penalty)

- (1) (10 pts) We are given a directed graph $G = (V, E)$, where each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \leq r(u, v) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v . We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.
- (2) (10 pts) Suppose that we represent the graph $G = (V, E)$ as an adjacency matrix. Give a simple implementation of Prim's algorithm for this case that runs in $O(|V|^2)$ time.
- (3) (10 pts) Given a graph G and minimum spanning tree T , suppose that we decrease the weight of one of the edges in T . Show that T is still a minimum spanning tree for G . More formally, let T be a minimum spanning tree for G with edge weights given by weight function w . Choose one edge $(x, y) \in T$ and a positive number k , and define the weight function w' by

$$w'(u, v) = \begin{cases} w(u, v), & \text{if } (u, v) \neq (x, y), \\ w(u, v) - k, & \text{if } (u, v) = (x, y). \end{cases}$$

Show that T is a minimum spanning tree for G with edge weights given by w' .

GUIDELINES:

- It is important to know whether you really know! For each problem, if you write the statement "I do not know how to solve this problem" (and nothing else), you will receive 20% credit for that problem. If you do write a solution, then your grade could be anywhere between 0% to 100%. To receive this 20% credit, you must explicitly state that you do not know how to solve the problem.
- You are allowed to discuss with classmates, but you must work on the homework problems on your own. You should write the final solutions alone, without consulting anyone. Your writing should demonstrate that you understand the proofs completely.
- When proofs are required, you should make them both clear and rigorous. Do not hand waive. If your handwriting is not legible, then your homework will not be graded.
- **Please submit your HW via Canvas. If you type your solutions, then please submit the PDF version. If you hand-write your solutions, then please scan or take a picture of your solutions and submit a PDF version. Please make sure that the quality of the scan/picture is good, and your hand writing is legible. If you take a picture and convert it to a PDF file, please make sure the file size is reasonable (e.g., roughly at most 10-11MB). If we cannot open your file, then your homework will not be graded.**
- Any concerns about grading should be expressed within one week of returning the homework.

Note: We reserve the right to grade only a subset of the problems assigned. Which problems will be graded will be decided after the submission deadline.