AN class 02/12/2020

Wednesday, 2 December 2020 11:08 AM

$x_0, x_1 \dots x_t$ - states

$u_1, u_2 \dots u_t$ control

$z_1, z_2 \dots z_t$ measurements

Joint prob $P(x,y) = P(X=x, Y=y)$

$= p(x)\,P(y)$

x, y are independent

Conditional prob.

$$p(x|y) = \frac{p(x,y)}{p(y)} \quad \text{— (1)}$$

Theorem of total prob.

$$p(x) = \sum_y p(x|y)\,p(y) \quad \text{discrete}$$

$$= \int p(x|y)\,p(y)\,dy \quad \text{— (2)}$$

Bayes rule:

$$p(x|y) = \frac{p(y|x)\,p(x)}{p(y)} \quad \text{— (3)}$$

y: dat

infer x from y

→ $p(y)$ does n't depend on x

$1/p(y) = \eta$

$$p(x|y) = \eta\, p(y|x)\,p(x) \quad \text{— (4)}$$

η ↓ normalization constant

→ $$p(x|y,z) = \frac{p(y|x,z)\,p(x|z)}{p(y|z)} \quad \text{— (5)}$$

$$p(x,y|z) = p(x|z)\,p(y|z) \quad \text{— (6)}$$

Conditional independence

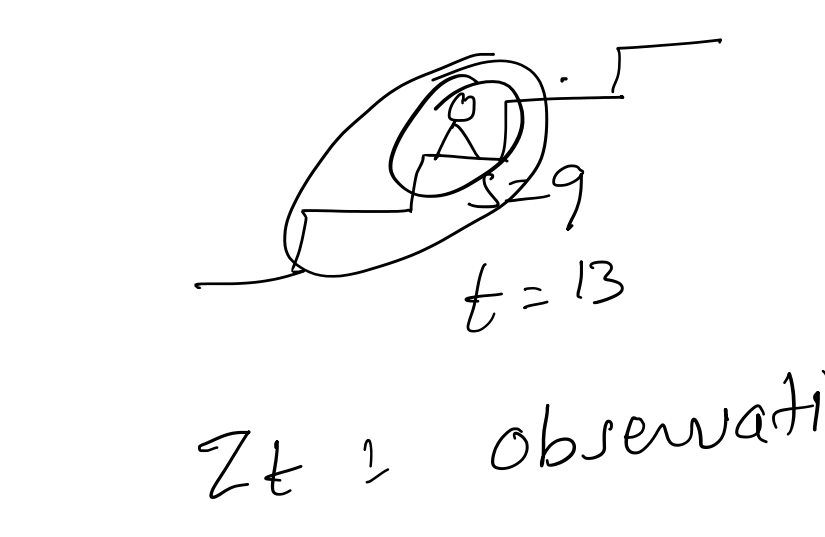
$p(x,y|z) = p(x|z)\,p(y|z)$

$\Rightarrow p(x,y) = p(x)\,p(y)$

States

* Pose: 3D: x, y, z, roll, pitch, yaw

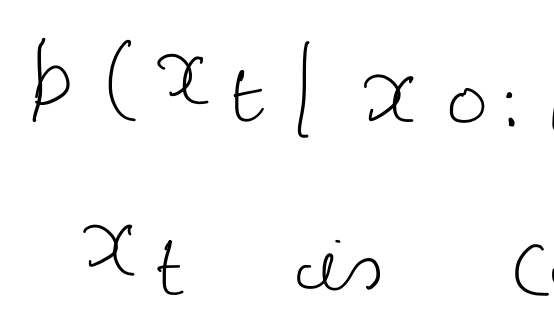
2D: x, y, yaw



- battery
- working

x_t : complete

$p(x_t | x_{0:t-1}) = p(x_t | x_{t-1})$ (Markov chain)



t = 13

z_t : observation / measurement.

Probabilistic generative law

- evolution of state

$$p(x_t | x_{0:t-1}, u_{1:t}) \quad \text{— (7)}$$

→ x_t is complete (Markov).

$$p(x_t | x_{0:t-1}, u_{1:t}) = p(x_t | x_{t-1}, u_t) \quad \text{— (8)}$$

(conditional independence).

→ model measurements

$$p(z_t | x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t | x_t) \quad \text{— (9.)}$$

$p(x_t | x_{t-1}, u_t) \Rightarrow$ state transition prob.

$p(z_t | x_t) \Rightarrow$ measurement prob.

Belief distribution

↓

→ reflects robot's internal knowledge about the state of environment

⇒ State cant be measured directly

$$bel(x_t) = p(x_t | z_{1:t}, u_{1:t}) \quad \text{— (10)}$$

measurement z_t is already taken

$$\overline{bel}(x_t) = p(x_t | z_{1:t-1}, u_{1:t}) \quad \text{— (11)}$$

(Prediction)

Bayes Filter Algorithms

- calculates belief distribution from measurement & control data.
- Recursive $bel(x_t) \leftarrow bel(x_{t-1})$.

Two steps:-

(1.) Control update/ prediction.

$$\overline{bel}(x_t) = \int p(x_t | u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1} \quad \text{— (12)}$$

(2.) measurement update:-

$$bel(x_t) = \eta\, p(z_t | x_t)\, \overline{bel}(x_t) \quad \text{— (13)}$$

Bayes filter: derivation

assumption: x_t : complete.

u_t : random

$$bel(x_t) = p(x_t | z_{1:t}, u_{1:t})$$

$$= \frac{p(z_t | x_t, z_{1:t-1}, u_{1:t}) \times p(x_t | z_{1:t-1}, u_{1:t})}{p(z_t | z_{1:t-1}, u_{1:t})} \quad \text{— (14)}$$

$$p(x|y,z) = \frac{p(y|x,z)\,p(x|z)}{p(y|z)}$$

↳ $x = x_t$, $y = z_t$, $z = z_{1:t-1}, u_{1:t}$

$$\frac{p(z_t | x_t, z_{1:t-1}, u_{1:t})\, p(x_t | z_{1:t-1}, u_{1:t})}{p(z_t | z_{1:t-1}, u_{1:t})}$$

→ x_t is complete,

$$p(z_t | x_t, z_{1:t-1}, u_{1:t}) = P(z_t | x_t) \quad \text{— (15.)}$$

$$p(x_t | z_{1:t}, u_{1:t}) = \eta\, p(z_t | x_t)\, P(x_t | z_{1:t-1}, u_{1:t}) \quad \text{— (16)}$$

$$bel(x_t) = \eta\, p(z_t | x_t)\, \overline{bel}(x_t) \quad \text{— (17)}$$

eq (2) $p(x) = \int p(x|y)\,p(y)\,dy$

$$\overline{bel}(x_t) = p(x_t | z_{1:t-1}, u_{1:t})$$

$$= \int p(x_t | x_{t-1}, z_{1:t-1}, u_{1:t}) \times p(x_{t-1} | z_{1:t-1}, u_{1:t})\, dx_{t-1} \quad \text{— (18)}$$

Conditional independence,

$$p(x_t | x_{t-1}, z_{1:t-1}, u_{1:t}) = p(x_t | x_{t-1}, u_t) \quad \text{— (19)}$$

$$\overline{bel}(x_t) = \int p(x_t | x_{t-1}, u_t) \cdot p(x_{t-1} | z_{1:t-1}, u_{1:t-1})\, dx_{t-1} \quad \text{— (20)}$$