binary PSK signaling scheme.

The M-ary PSK signal can be demodulated using a coherent demodulation scheme if a phase reference is available at the receiver. For purposes of illustration we will discuss the demodulation of four-phase PSK (also known as QPSK or quadrature PSK) in detail and then present the results for the general M-ary PSK.

In four-phase PSK, one of four possible waveforms is transmitted during each signaling interval T_s. These waveforms are:

$$\left.\begin{aligned} s_1(t) &= A\cos\omega_c t \\ s_2(t) &= -A\sin\omega_c t \\ s_3(t) &= -A\cos\omega_c t \\ s_4(t) &= A\sin\omega_c t \end{aligned}\right\} \text{ for } 0 \leq t \leq T \tag{8.54}$$

These waveforms correspond to phase shifts of 0°, 90°, 180°, and 270° as shown in the phasor diagram in Figure 8.15. The receiver for the system is shown in Figure 8.16. The receiver requires two local reference waveforms

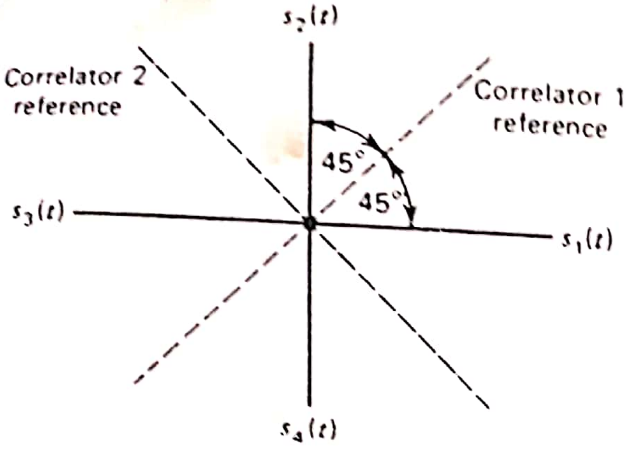


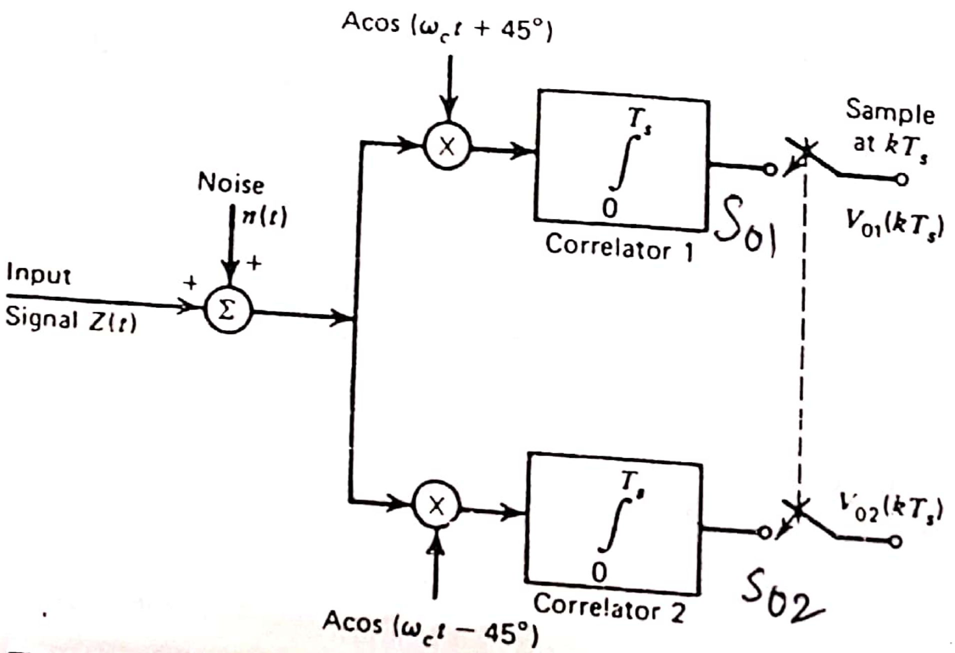
Figure 8.15 Phasor diagram for QPSK.

$A\cos(\omega_c t + 45°)$ and $A\cos(\omega_c t - 45°)$ that are derived from a coherent local carrier reference $A\cos\omega_c t$.

For purposes of analysis, let us consider the operation of the receiver during the signaling interval $(0, T_s)$. Let us denote the signal component at the output of the correlators by s_{01} and s_{02}, respectively, and the noise component by $n_0(t)$. If we assume that $s_1(t)$ was the transmitted signal during the signaling interval $(0, T_s)$, then we have

$$s_{01}(T_s) = \int_0^{T_s} (A\cos\omega_c t) A\cos\left(\omega_c t + \frac{\pi}{4}\right) dt$$

$$= \frac{A^2}{2} T_s \cos\frac{\pi}{4} = L_0$$



$W_c t$

Figure 8.16 Receiver for QPSK scheme. Polarities of $V_{01}(kT_s)$ and $V_{02}(kT_s)$ determine the signal present at the receiver input during the kth signaling interval as shown in Table 8.4.

$$s_{02}(T_s) = \int_0^{T_s} (A \cos \omega_c t)\, A \cos\left(\omega_c t - \frac{\pi}{4}\right) dt$$

$$= \frac{A^2}{2} T_s \cos \frac{\pi}{4} = L_0$$

Table 8.4 shows s_{01} and s_{02} corresponding to each of the four possible signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$.

Output signal levels shown in Table 8.4 indicate that the transmitted signal can be recognized from the polarities of the outputs of both correlators (i.e., the threshold levels are zero). In the presence of noise, there will be some probability that an error will be made by one or both correlators. An expression for the probability of incorrectly decoding the transmitted signal can be derived as follows.

The outputs of the correlators at time $t = T_s$ are

$$V_{01}(T_s) = s_{01}(T_s) + n_{01}(T_s)$$

$$V_{02}(T_s) = s_{02}(T_s) + n_{02}(T_s)$$

where $n_{01}(T_s)$ and $n_{02}(T_s)$ are zero mean Gaussian random variables defined by

$$n_{01}(T_s) = \int_0^{T_s} n(t) A \cos(\omega_c t + 45°)\, dt$$

$$n_{02}(T_s) = \int_0^{T_s} n(t) A \cos(\omega_c t - 45°)\, dt,$$

and $n(t)$ is a zero mean Gaussian random process with a power spectral density of $\eta/2$. With our assumption that $\omega_c = k2\pi r_s$ (k an integer > 0), we can show that $n_{01}(T_s)$ and $n_{02}(T_s)$ are independent Gaussian random variables with equal variance N_0 given by (see Problems 8.1, and 8.24)

$$\boxed{N_0 = \frac{\eta}{4} A^2 T_s} \tag{8.55}$$

Let us now calculate the probability of error assuming that $s_1(t)$ was the

Table 8.4. Output signal levels at sampling times.

	Input			
Output	$s_1(t)$	$s_2(t)$	$s_3(t)$	$s_4(t)$
$s_{01}(kT_s)$	L_0	$-L_0$	$-L_0$	L_0
$s_{02}(kT_s)$	L_0	L_0	$-L_0$	$-L_0$

transmitted signal. If we denote the probability that correlator 1 (Figure 8.16) makes an error by P_{ec1}, then

$$\begin{aligned} P_{ec1} &= P(n_{01}(T_s) < -L_0) \\ &= P(n_{01}(T_s) > L_0) \\ &= Q\left(\frac{L_0}{\sqrt{N_0}}\right) = Q\left(\sqrt{\frac{A^2T_s}{2\eta}}\right) \end{aligned} \tag{8.56}$$

By symmetry, the probability that the correlator 2 makes an error is

$$P_{ec2} = P_{ec1} = Q(\sqrt{A^2T_s/2\eta}) \tag{8.57}$$

The probability P_c that the transmitted signal is received correctly is

$$\begin{aligned} P_c &= (1 - P_{ec1})(1 - P_{ec2}) \\ &= 1 - 2P_{ec1} + P_{ec1}^2 \end{aligned}$$

We have made use of the fact that the noise outputs of the correlators are statistically independent at sampling times, and that $P_{ec1} = P_{ec2}$. Now, the probability of error P_e for the system is

$$\begin{aligned} P_e &= 1 - P_c \\ &= 2P_{ec1} - P_{ec1}^2 \\ &\simeq 2P_{ec1} \end{aligned}$$

since P_{ec1} will normally be $\ll 1$. Thus for the QPSK system we have

$$\boxed{P_e = 2Q(\sqrt{A^2T_s/2\eta})} \tag{8.58}$$

We can extend this result to the M-ary PSK signaling scheme when $M > 4$.

QPSK Receiver: (Calculating Pe)

→ Consider a coherent Receiver, QPSK - 'Mary Receiver. where M=4 (or Quadrature PSK. So, 4 different carriers, (m carriers). We take a combination

At the O/p of two bits - transmit a carrier.

(ie) (00, 01, 10, 11) → signalling adopted in QPSK.

4 carriers → 0° to 360° are divided into 4 phases.

0, 90°, 180°, 360°

↓

reference carrier - $A \cos w_c t$. (00)

01 - ($-A \sin w_c t$)

10 - ($-A \cos w_c t$)

11 - $A \sin w_c t$

Obj: To study performance of Receiver:

Pe of coherent QPSK:

Tx: signal + noise 'x' by locally generated carrier

↳ produces a signal that is sampled at Tb units of time. & compared with threshold.

$> T \rightarrow 1$ } Binary.

$< T \rightarrow 0$

In QPSK → we have four combinations so we have two locally generated carriers, two deciding blocks...

First bit will be decided by upper match filter

Second " " " lower "

Suppose, if the receiver makes an error under two circumstances:

→ upper matched filter makes an error. & second " may detect correctly

→ lower makes an error & upper matched filter

Pe depends on the individual performance of the two matched filters.

$Pe = 1 - P(\text{making correct decision})$

prob of 1st filter making correct decision $\cdot Pc_1$.

" " 2nd " " " $: Pc_2$

$Pe = 1 - Pc_1 \cdot Pc_2$.

$= 1 - Pc_1 \cdot Pc_2$

$Pc_1 = 1 - Pe_1$

prob of 1st filter making wrong decision

$Pc_2 = 1 - Pe_2$

$Pe = 1 - (1 - Pe_1)(1 - Pe_2)$

when will the match filter makes an error with the integrated o/p?

noise o/p > signal o/p : match filter makes an error.

$(y > x)$

Pe = P(noise o/p > signal o/p) $\quad Pe \propto \frac{1}{SNR}$.

$\Downarrow$ $= P(N_0 > L_0)$

prob of matched filter making an error

Pe of matched filter will make an error. $= Q\left(\sqrt{SNR}\right)$

$$= Q\left(\frac{L_0}{\sqrt{N_0}}\right)$$

$$= Q\left(\frac{\frac{A^2 T_b \cos\frac{\pi}{4}}{2}}{\frac{A}{2}\sqrt{\eta T_b}}\right)$$

$$= Q\left(\frac{A\, T_b \cos\frac{\pi}{4}}{\sqrt{\eta T_b}}\right)$$

$$= Q\left(\sqrt{\frac{A^2 T_b}{2\eta}}\right) \qquad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Assume Pe of MF1 $\simeq$ Pe of MF_2

$\therefore$ Total Pe = $2 \times$ Pe of MF

$$\boxed{\text{Total Pe} = 2Q\left(\sqrt{\frac{A^2 T_b}{2\eta}}\right)}$$