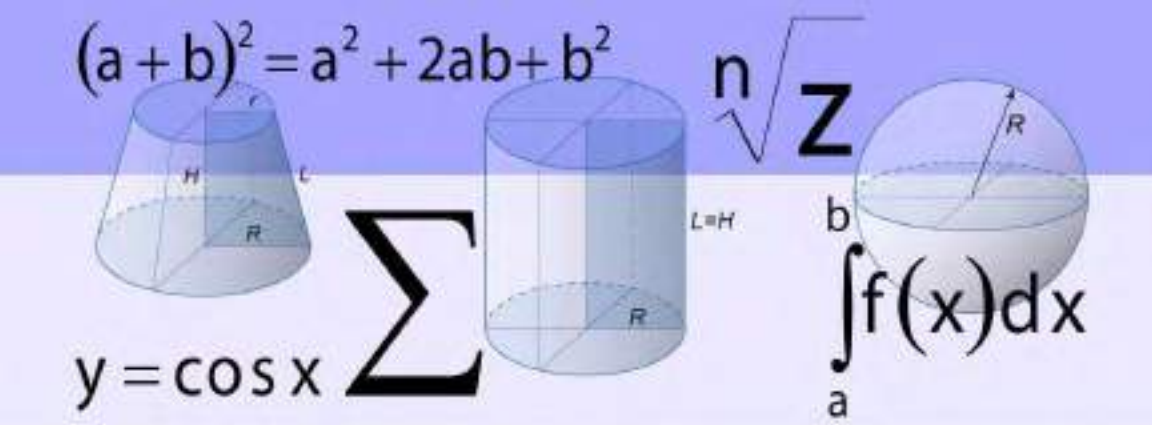

1300 Math Formulas

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Preface

This handbook is a complete desktop reference for students and engineers. It has everything from high school math to math for advanced undergraduates in engineering, economics, physical sciences, and mathematics. The ebook contains hundreds of formulas, tables, and figures from Number Sets, Algebra, Geometry, Trigonometry, Matrices and Determinants, Vectors, Analytic Geometry, Calculus, Differential Equations, Series, and Probability Theory.
The structured table of contents, links, and layout make finding the relevant information quick and painless, so it can be used as an everyday online reference guide.

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Chapter 1

Number Sets

1.1 Set Identities

Sets: A, B, C
Universal set: I
Complement : A'
Proper subset: $A \subset B$
Empty set: $\varnothing$
Union of sets: $A \cup B$
Intersection of sets: $A \cap B$
Difference of sets: $A \setminus B$

1. $A \subset I$

2. $A \subset A$

3. $A = B$ if $A \subset B$ and $B \subset A$.

4. Empty Set
   $\varnothing \subset A$

5. Union of Sets
   $C = A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

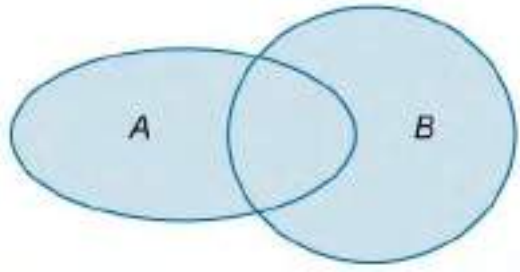

Figure 1.

6. Commutativity
$$A \cup B = B \cup A$$

7. Associativity
$$A \cup (B \cup C) = (A \cup B) \cup C$$

8. Intersection of Sets
$$C = A \cup B = \{x \mid x \in A \text{ and } x \in B\}$$

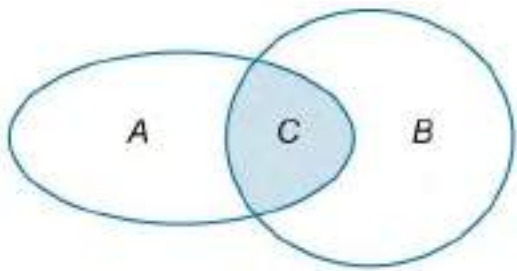

Figure 2.

9. Commutativity
$$A \cap B = B \cap A$$

10. Associativity
$$A \cap (B \cap C) = (A \cap B) \cap C$$

11. Distributivity
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

12. Idempotency
$$A \cap A = A,$$
$$A \cup A = A$$

13. Domination
$$A \cap \varnothing = \varnothing,$$
$$A \cup I = I$$

14. Identity
$$A \cup \varnothing = A,$$
$$A \cap I = A$$

15. Complement
$$A' = \{x \in I \mid x \notin A\}$$

16. Complement of Intersection and Union
$$A \cup A' = I,$$
$$A \cap A' = \varnothing$$

17. De Morgan's Laws
$$(A \cup B)' = A' \cap B',$$
$$(A \cap B)' = A' \cup B'$$

18. Difference of Sets
$$C = B \setminus A = \{x \mid x \in B \text{ and } x \notin A\}$$

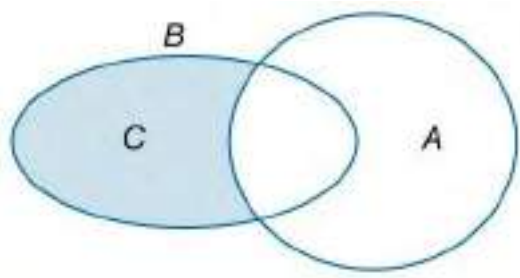

Figure 3.

19. $B \setminus A = B \setminus (A \cap B)$

20. $B \setminus A = B \cap A'$

21. $A \setminus A = \varnothing$

22. $A \setminus B = A$ if $A \cap B = \varnothing$.

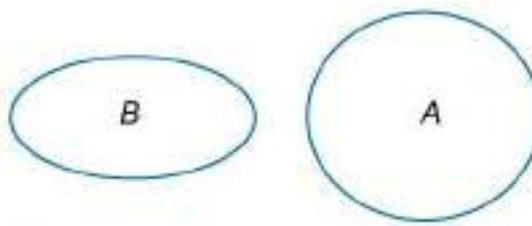

Figure 4.

23. $(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C)$

24. $A' = I \setminus A$

25. Cartesian Product
$C = A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

1.2 Sets of Numbers

Natural numbers: N
Whole numbers: N_0
Integers: Z
Positive integers: Z^+
Negative integers: Z^-
Rational numbers: Q
Real numbers: R
Complex numbers: C

26. Natural Numbers
Counting numbers: $N = \{1, 2, 3, \ldots\}$.

27. Whole Numbers
Counting numbers and zero: $N_0 = \{0, 1, 2, 3, \ldots\}$.

28. Integers
Whole numbers and their opposites and zero:
$Z^+ = N = \{1, 2, 3, \ldots\}$,
$Z^- = \{\ldots, -3, -2, -1\}$,
$Z = Z^- \cup \{0\} \cup Z^+ = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

29. Rational Numbers
Repeating or terminating decimals:
$$Q = \left\{ x \mid x = \frac{a}{b} \text{ and } a \in Z \text{ and } b \in Z \text{ and } b \neq 0 \right\}.$$

30. Irrational Numbers
Nonrepeating and nonterminating decimals.

31. Real Numbers
Union of rational and irrational numbers: R.

32. Complex Numbers
$C = \{x + iy \mid x \in R \text{ and } y \in R\}$,
where i is the imaginary unit.

33. $N \subset Z \subset Q \subset R \subset C$

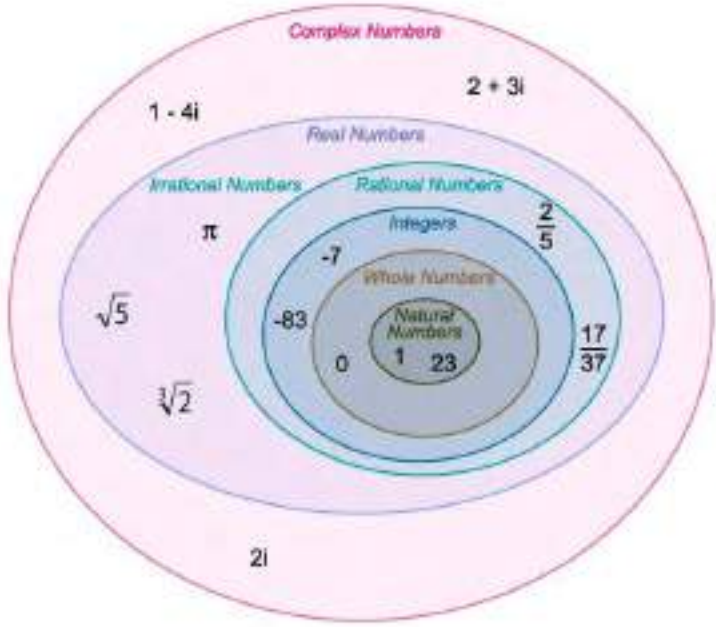

Figure 5.

1.3 Basic Identities

Real numbers: a, b, c

34. Additive Identity
$a + 0 = a$

35. Additive Inverse
$a + (-a) = 0$

36. Commutative of Addition
$a + b = b + a$

37. Associative of Addition
$(a + b) + c = a + (b + c)$

38. Definition of Subtraction
$a - b = a + (-b)$

39. Multiplicative Identity
$a \cdot 1 = a$

40. Multiplicative Inverse
$a \cdot \frac{1}{a} = 1, \ a \neq 0$

41. Multiplication Times 0
$a \cdot 0 = 0$

42. Commutative of Multiplication
$a \cdot b = b \cdot a$

43. Associative of Multiplication
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

44. Distributive Law
$a(b + c) = ab + ac$

45. Definition of Division
$\frac{a}{b} = a \cdot \frac{1}{b}$

1.4 Complex Numbers

Natural number: n
Imaginary unit: i
Complex number: z
Real part: a, c
Imaginary part: bi, di
Modulus of a complex number: r, r_1, r_2
Argument of a complex number: φ, φ_1, φ_2

46.

$i^1 = i$	$i^5 = i$	$i^{4n+1} = i$
$i^2 = -1$	$i^6 = -1$	$i^{4n+2} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{4n+3} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{4n} = 1$

47. $z = a + bi$

48. Complex Plane

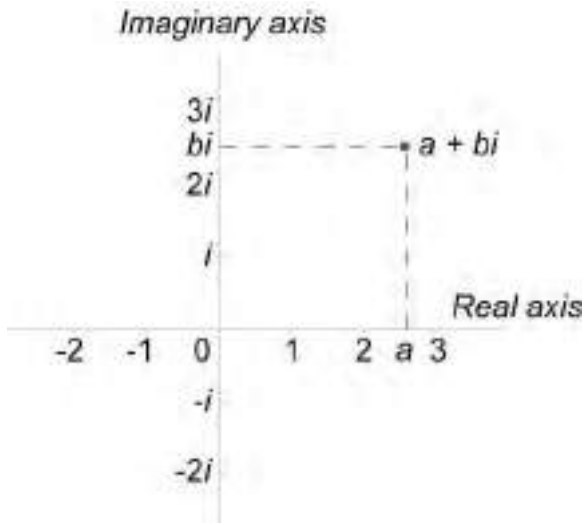

Figure 6.

49. $(a+bi)+(c+di)=(a+c)+(b+d)i$

50. $(a+bi)-(c+di)=(a-c)+(b-d)i$

51. $(a+bi)(c+di)=(ac-bd)+(ad+bc)i$

52. $\dfrac{a+bi}{c+di}=\dfrac{ac+bd}{c^2+d^2}+\dfrac{bc-ad}{c^2+d^2}\cdot i$

53. Conjugate Complex Numbers

$\overline{a+bi}=a-bi$

54. $a=r\cos\varphi,\ b=r\sin\varphi$

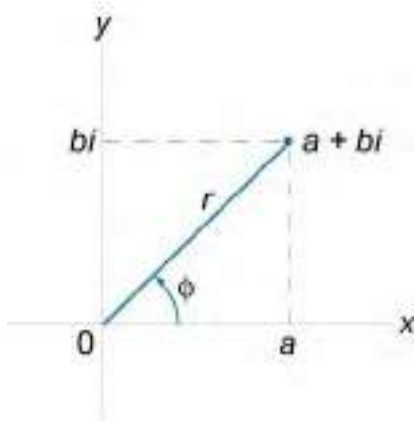

Figure 7.

55. Polar Presentation of Complex Numbers

$$a + bi = r(\cos\varphi + i\sin\varphi)$$

56. Modulus and Argument of a Complex Number

If $a + bi$ is a complex number, then

$$r = \sqrt{a^2 + b^2} \quad \text{(modulus)},$$

$$\varphi = \arctan\frac{b}{a} \quad \text{(argument)}.$$

57. Product in Polar Representation

$$\begin{aligned} z_1 \cdot z_2 &= r_1(\cos\varphi_1 + i\sin\varphi_1)\cdot r_2(\cos\varphi_2 + i\sin\varphi_2) \\ &= r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)] \end{aligned}$$

58. Conjugate Numbers in Polar Representation

$$\overline{r(\cos\varphi + i\sin\varphi)} = r[\cos(-\varphi) + i\sin(-\varphi)]$$

59. Inverse of a Complex Number in Polar Representation

$$\frac{1}{r(\cos\varphi + i\sin\varphi)} = \frac{1}{r}[\cos(-\varphi) + i\sin(-\varphi)]$$

60. Quotient in Polar Representation

$$\frac{z_1}{z_2}=\frac{r_1(\cos\varphi_1+i\sin\varphi_1)}{r_2(\cos\varphi_2+i\sin\varphi_2)}=\frac{r_1}{r_2}[\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)]$$

61. Power of a Complex Number

$$z^n=[r(\cos\varphi+i\sin\varphi)]^n=r^n[\cos(n\varphi)+i\sin(n\varphi)]$$

62. Formula "De Moivre"

$$(\cos\varphi+i\sin\varphi)^n=\cos(n\varphi)+i\sin(n\varphi)$$

63. Nth Root of a Complex Number

$$\sqrt[n]{z}=\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}\left(\cos\frac{\varphi+2\pi k}{n}+i\sin\frac{\varphi+2\pi k}{n}\right),$$

where

$k=0,1,2,\ldots,n-1.$

64. Euler's Formula

$$e^{ix}=\cos x+i\sin x$$

Chapter 2

Algebra

2.1 Factoring Formulas

Real numbers: a, b, c
Natural number: n

65. $a^2 - b^2 = (a+b)(a-b)$

66. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

67. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

68. $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a-b)(a+b)(a^2 + b^2)$

69. $a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

70. $a^5 + b^5 = (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

71. If n is odd, then
$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots - ab^{n-2} + b^{n-1})$.

72. If n is even, then
$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1})$,