Paper reading(18) - CL-686

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Title: Model Predictive Idle Speed Control: Design, Analysis, and Experimental Evaluation

Idle Speed: It is the rotational speed of the engine when vehicle is not in motion.

2 controllers: Single input(throttle) and multi-input(throttle and spark timing).

Design Flow

1. MPC prediction model is developed, model type and order of model are design parameters selected.
2. States are augmented, The cost function weights and the prediction horizon are the design parameters selected. And obtained controller is simulated to evaluate the performance and the relation between closed-loop performance and tuning parameters.
3. The MPC controller is synthesized in a suitable way for implementation in automotive ECUs.Computational feasibility of the controller is verified.
4. Experimental tests are performed to validate the nominal and non-nominal performance of the closed-loop system

Engine Model for ISC

$$\dot{N}(t) = \frac{1}{J}\frac{30}{\pi}(M_e(t) - M_L(t)) \tag{12a}$$

$$M_e(t) = \kappa_{\text{spk}}(t - t_{ds})M_{e,\delta}(t - t_d) - M_{\text{fr}}(t) - M_{\text{pmp}}(t) \tag{12b}$$

$$\kappa_{\text{spk}}(t) = (\cos(\alpha(t) - \alpha_{MBT}(t)))^{\xi} \tag{12c}$$

$$t_d(t) = \frac{60}{N(t)} \tag{12d}$$

$$\dot{M}_{e,\delta}(t) = -\gamma_2\frac{RT_{\text{im}}}{V_1}\frac{N(t)}{\gamma_1}M_{e,\delta}(t) + \gamma_2\frac{RT_{\text{im}}}{V_{\text{im}}}\gamma_3\vartheta(t) - \frac{\gamma_0 \cdot \gamma_1}{N^2}\dot{N}(t) \tag{12e}$$

Figure: Engine model from 1st principle neal idle speed

Engine Model for ISC(Contd...)

Where,

$N[RPM]$: engine speed,

$M_e[Nm], M_L[Nm]$: engine brake torque, load torque on crankshaft,

$k_{spk}, \alpha[rad]$: torque ratio, spark ignition angle,

$M_{e,\delta}(t)[Nm]$: indicated engine torque,

$v[deg]$: throttle position

Control oriented model

From the above model,
To remove nonlinearity - control variable is torque ratio k_{spk}
Control inputs are throttle position and torque ratio achieved via spark retard.
Linearized model: $Y(s) = G_{thr}(s)e^{st_d}U_1(s) + G_{spk}(s)e^{st_{ds}}U_2(s)$(Direct result from another reference paper)
where,
$y(t) = N(t) - \bar{N}, u_1(t) = v(t) - \bar{v}, u_2(t) = k_{spk}(t) - \bar{k_{spk}}$
u_1, u_2 are throttle and spark input.

Control oriented model(Contd...)

Discretized state space model

$$\begin{aligned} x_{thr}(k+1) &= A_{thr}x_{thr}(k) + B_{thr}u^{\delta}_{thr}(k) \\ y_{thr}(k) &= C_{thr}x_{thr}(k) \end{aligned} \tag{1}$$

$$\begin{aligned} x_{spk}(k+1) &= A_{spk}x_{spk}(k) + B_{spk}u^{\delta}_{spk}(k) \\ y_{spk}(k) &= C_{spk}x_{spk}(k) \end{aligned} \tag{2}$$

where, $k \in \mathbb{Z}$, $x_{thr}, x_{spk} \in \mathbb{R}^2$
$u^{\delta}_{spk}(k), u^{\delta}_{thr}(k)$ are delay free torque ratio and throttle command.

Control oriented model(Contd...)

By cascading a fourth-order airflow delay model with (1), and a first-order spark delay model with (2), the complete linear model of the engine is obtained.

$$\begin{aligned} x^p(k+1) &= A^p x^p(k) + B^p u^p(k) \\ y_p(k) &= C^p x^p(k) \end{aligned} \tag{3}$$

where, for multi input controller

$x^p = \begin{bmatrix} x^f_{thr} \\ x^f_{spk} \end{bmatrix}, u^p = \begin{bmatrix} u_{thr} \\ u_{spk} \end{bmatrix}, A^p = \begin{bmatrix} A^f_{thr} & 0 \\ 0 & A^f_{spk} \end{bmatrix},$

$\mathrm{B}^p = \begin{bmatrix} B^f_{thr} \\ B^f_{spk} \end{bmatrix}, C^p = \begin{bmatrix} C^f_{thr} & C^f_{spk} \end{bmatrix},$

$\mathrm{x}^f_{thr} \in \mathbb{R}^6, x^f_{spk} \in \mathbb{R}^3$ and $u^p \in \mathbb{R}^2$

for single input controller

$x^p = x^f_{thr}, u^p = u_{thr}, A^p = A^f_{thr}, B^p = B^f_{thr}, C^p = C^f_{thr}$

MPC design

Started with single input (throttle) controller
adding the dynamics $q_N(k+1) = q_N(k) + T_s(y_p(k) - r_{y_p}(k))$
where $q_N \in \mathbb{R}$ is the discrete-time integral of the output. $r_{y_p} \in \mathbb{R}$ is the desired speed set-point offset.
After adding this to eqn (3) of single input controller, state becomes

$$
\begin{gathered}
x = \begin{bmatrix} x_p \\ q_N \end{bmatrix}, x \in \mathbb{R}^7 \\
A = \begin{bmatrix} A_p & 0 \\ T_s C_p & 1 \end{bmatrix}, B = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, C = \begin{bmatrix} C_p & 0 \\ 0 & 1 \end{bmatrix}
\end{gathered} \tag{4}
$$

MPC design(Contd...)

Constraints and Cost function

Output constraints: $-200 \leq y_p(k) - r_{y_p}(k) \leq 200$
Input Constraints: $0 \leq u + u_{FF} \leq 10$, where $u_{FF} = \bar{v}$ and
$v(t) = u_{FF} + u(t)$

Cost function:

$$J(y(k), u(k), u(k-1)) = \sum_{i=0}^{h-1} (y(i|k) - r_y)^T Q (y(i|k) - r_y) + \Delta u(i|k) S \Delta u(i|k) \quad (5)$$

where, $(i|k)$ indicates i steps ahead from k,
$h \in \mathbb{Z}$ is prediction horizon,
$y(k)$ and $u(k)$ are output and input sequences from 0 to $h-1$ predicted at step k, $r_y = [r_{y_p} 0]^T$ is the output setpoint,
$\Delta u(i|k) = u(i|k) - u(i-1|k)$, Q and S are positive definite matrices.

MPC design(Contd...)

Optimal control problem

$$
\begin{aligned}
\min_{\sigma, u(k)} \quad & \rho\sigma^2 + \sum_{i=0}^{h-1} (y(i|k) - r_y)^T Q (y(i|k) - r_y) + \Delta u(i|k)^T S \Delta u(i|k) \\
\text{s.t.} \quad & x(i+1|k) = Ax(i|k) + Bu(i|k) \\
& y(i|k) = Cx(i|k), i = 0, ..., h-1 \\
& u_{min} \leq u(i|k) \leq u_{max}, i = 0, ..., h_u - 1 \\
& y_{min} - \sigma 1 \leq y(i|k) \leq y_{max} + \sigma 1, i = 0, ..., h_c \\
& u(i|k) = u((h_u) - 1|k), i = h_u, ..., h-1 \\
& \sigma \geq 0 \\
& u(-1|k) = u(k-1), x(0|k) = x(k)
\end{aligned}
\tag{6}
$$

The output constraints are “softened” by the additional optimization variable $\sigma \in \mathbb{R}$ and $\rho > 0$

Redefining the constraints
The constraints values are computed by the software running in the vehicle ECU.
For single input,

$$
\begin{aligned}
y_{min} &= \begin{bmatrix} -200 + r_{y_p}(k) \\ -\infty \end{bmatrix} \\
u_{min} &= -u_{FF} \\
y_{max} &= \begin{bmatrix} 200 + r_{y_p}(k) \\ +\infty \end{bmatrix} \\
u_{max} &= 10 - u_{FF}
\end{aligned}
\tag{7}
$$

MPC design(Contd...)

For multi-input,

$$
\begin{aligned}
y_{min} &= \begin{bmatrix} -200 + r_{y_p}(k) \\ -\infty \\ -\infty \end{bmatrix}, u_{min} = \begin{bmatrix} -u_{FF} \\ -0.1 \end{bmatrix} \\
y_{max} &= \begin{bmatrix} 200 + r_{y_p}(k) \\ +\infty \\ +\infty \end{bmatrix}, u_{max} = \begin{bmatrix} 10 - u_{FF} \\ \Delta k_{spk}^{max}(k) \end{bmatrix}
\end{aligned} \tag{8}
$$

Kalman filter

For the idle speed controllers, a stationary Kalman filter has been used for estimating x^p

$$\begin{aligned} x^p(k|k-1) &= A^p x^p(k-1) + B^p u^p(k-1) \\ x^p(k) &= x^p(k|k-1) + K_{KF}(y_p(k) - C^p x^p(k|k-1)) \end{aligned} \tag{9}$$

where K_{KF} is the stationary Kalman filter gain $K_{KF} = PC^{p^T}(C^p PC^{p^T} + R_{KF})^{-1}$ and P is the solution of the algebraic Riccati equation $P = A^p PA^{p^T} + QKF - A_p PC^{p^T}(C^p PC^{p^T} + RKF)^{-1} C^p PA^{p^T}$. And Kalman filter is tuned by Q_{KF} and R_{KF}, that represents here a process and measurement noise covariances.

Closed loop simulations

Defined parameters are,
$\bar{N} = 600r/min, h = 30, h_c = 3, h_u = 3$
$\epsilon_N(t) = \hat{N}(t) - N(t)$, Torque disturbance is of $20Nm$

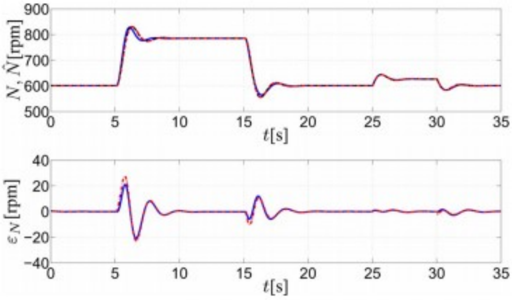

Figure: Upper plot: engine speed vs time, Lower plot: errot in control speeds vs time, nonlinear model (dashed line),continuous-time linearized model (solid line), discrete-time linearized model (dashed-dotted line)

Closed loop simulations(Contd...)

For single-input MPC, spark retard k_{spk} was kept constant and throttle v was the only available degree of freedom

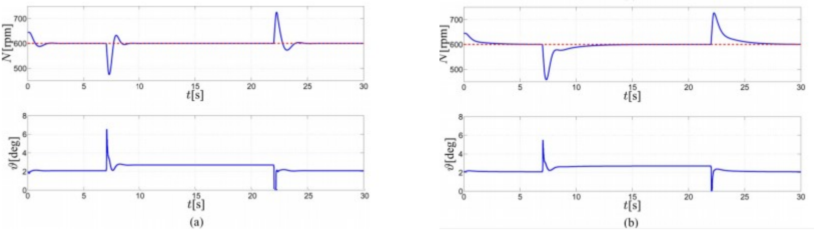

Figure: Single-input MPC with nonlinear model

Part (a) - aggressive tuning - Underdamped behaviour
Part (b) - slower overdamped behavior

Closed loop simulations(Contd...)

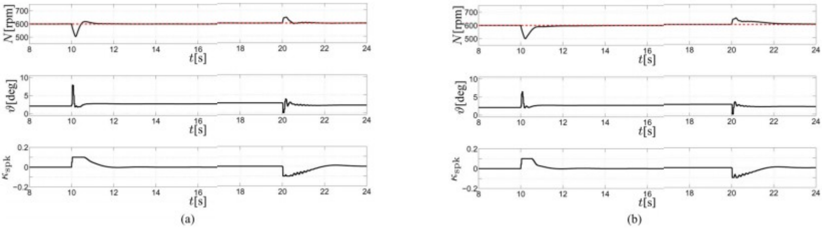

Figure: Multi-input MPC with nonlinear model

In both cases the multi-input controller achieve better disturbance rejection compared to the corresponding single-input MPC in terms of maximum deviation from the setpoint and settling time.