

KAN based Autoencoders for Factor models

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Introduction

- Factor models describe relationships between asset returns and factors.
- Standard form:
$$r_i = \beta_i factor_i + \epsilon_i$$
- Some popular models: Fama French, CAPM, etc.

What's the problem?

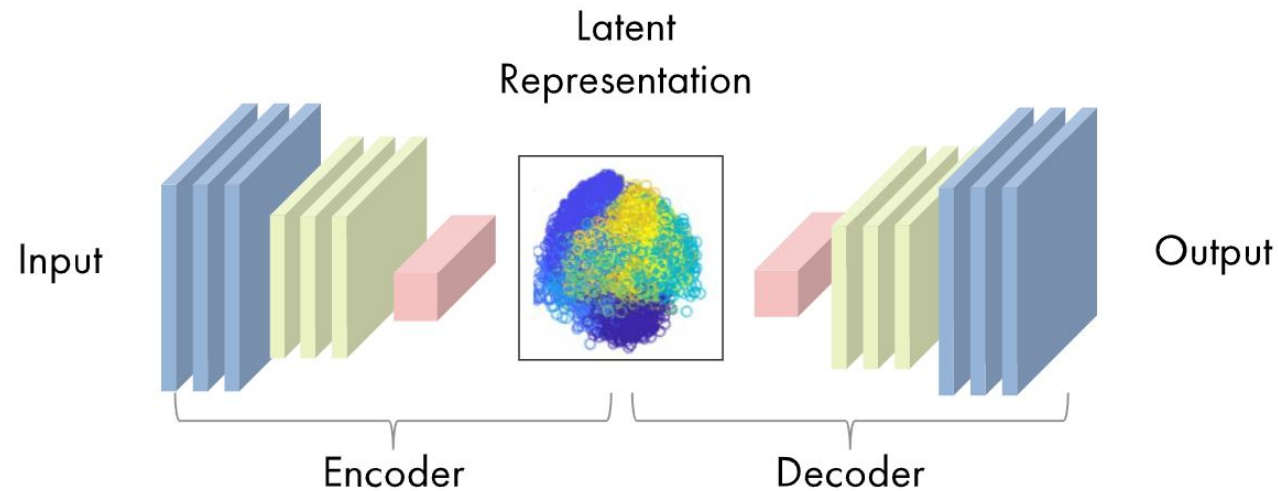
- Real world data isn't linear or independent.
- It is:
 1. Collinearity - Gram Schmidt Process, PCA
 2. Multidimensional representation - Feature engineering
 3. Noise - PCA
- Factors aren't enough, PCA can't go back

Experiment Setup

- Risk Weighted Portfolio Construction of Russell 3000 dataset
- Winsorized Outliers
- OLS for getting factor weights, $r_i = \beta_i factor_i + \epsilon_i$
- One reason to do this, faster and efficient, however entire dataset **could be** better (However, [1.5 million * 26 factors] v/s [15120 * 26 factors]).

Autoencoders

- Neural networks that learn to compress and reconstruct data
- Goal: Learn efficient representations while preserving important information



Kolmogorov Arnold Networks

Based on Kolmogorov-Arnold Representation theorem, any continuous function of several variables can be constructed using only:

- Addition, Composition, Functions of a single variable

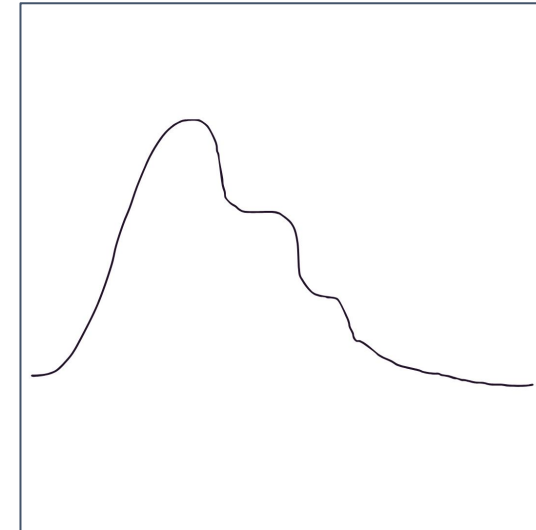
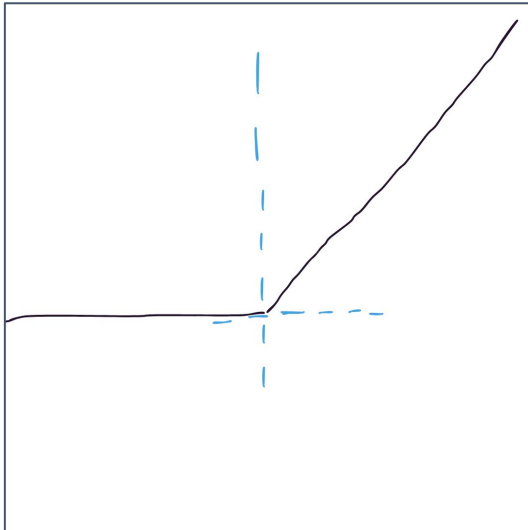
It showed complex multivariate functions could be built from simple building blocks.

Neural nets are black-box equation solvers, Researchers at MIT thought why not use the kolmogorov's work in neural nets.

Kolmogorov Arnold Networks

Activation functions in DL: ReLU, sigmoid, GeLU, etc.

KAN has flexible and learnable activation functions.



Kolmogorov Arnold Networks

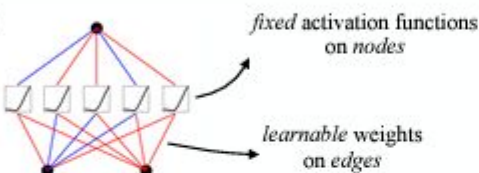
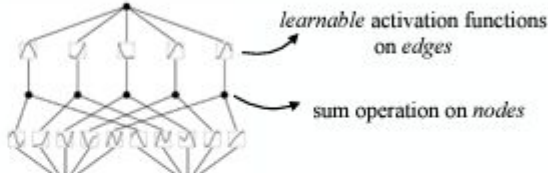
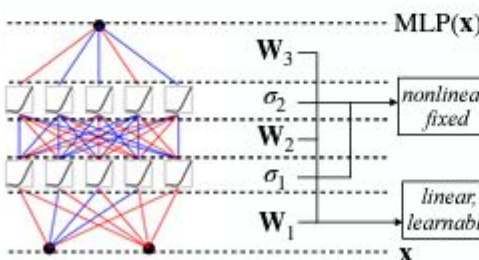
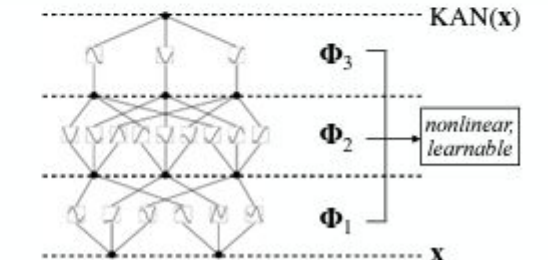
Model	Multi-Layer Perceptron (MLP)	Kolmogorov-Arnold Network (KAN)
Theorem	Universal Approximation Theorem	Kolmogorov-Arnold Representation Theorem
Formula (Shallow)	$f(\mathbf{x}) \approx \sum_{i=1}^{N(e)} a_i \sigma(\mathbf{w}_i \cdot \mathbf{x} + b_i)$	$f(\mathbf{x}) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^n \phi_{q,p}(x_p) \right)$
Model (Shallow)	<div>(a) </div>	<div>(b) </div>
Formula (Deep)	$\text{MLP}(\mathbf{x}) = (\mathbf{W}_3 \circ \sigma_2 \circ \mathbf{W}_2 \circ \sigma_1 \circ \mathbf{W}_1)(\mathbf{x})$	$\text{KAN}(\mathbf{x}) = (\Phi_3 \circ \Phi_2 \circ \Phi_1)(\mathbf{x})$
Model (Deep)	<div>(c) </div>	<div>(d) </div>

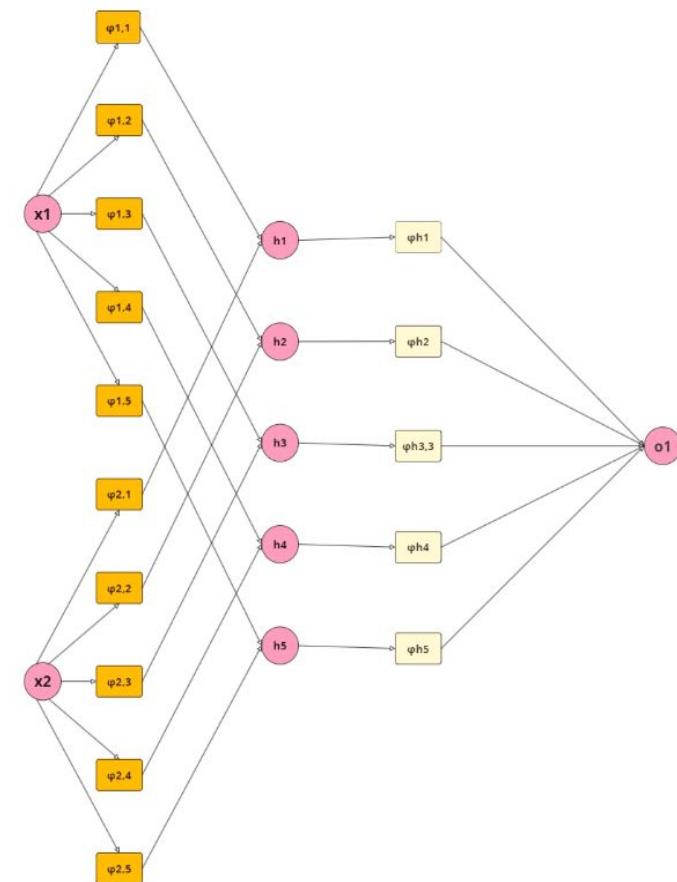
Figure 0.1: Multi-Layer Perceptrons (MLPs) vs. Kolmogorov-Arnold Networks (KANs)

KANs are flexible and interpretable, paper has examples.

Unfortunately, our work hasn't proved interpretability yet.

Model Design

- KAN based encoder for getting latent representations as model factors
- Use a simple decoder to reconstruct a factor model explaining returns
- Solve to minimize the loss function (Actual returns vs approximated returns)



Loss

Creating a loss function for factor model

- predicted return $\rightarrow r_{\square}^{\text{predicted}} = \boldsymbol{\beta}_{\square} \text{factor}_{\square} + \epsilon_{\square}$

$$\text{Loss} = r_{\square}^{\text{actual}} - r_{\square}^{\text{predicted}}$$

Results

Dataset: 60 years of stock data (1957-2016)

Training: 1957-1987 (30 yr) Validation: 1987-1999 (12 yr) Testing: 2000-2016 (16yr)

Table 1: R² Scores

Model	FF	CA	KAN-CA
1 factor	<0	11.06	11.02
3 factors	<0	11.39	11.26
6 factors	<0	11.29	11.32

Table 2: Predictive R² Scores

Model	FF	CA	KAN-CA
1 factor	<0	0.202	0.203
3 factors	<0	0.168	0.203
6 factors	<0	0.188	0.214

Table 3: Sharpe Ratio

Model	1 factor	3 factors	6 factors
CA	0.86	0.87	0.91
KAN-CA	0.84	0.86	0.96

Future Research Directions

- Demonstrate KAN's interpretability, so we have better interpretable factor models.
- Establish statistical significance of performance improvements through extended validation periods and comprehensive sensitivity analysis across market conditions.
- Optimize model architecture through systematic hyperparameter studies, adding more neurons and longer training periods, with standardized benchmarking against more state-of-the-art factor models.
- Add Mean Variance Portfolio, Random Portfolio Tests, Q-test

Feedback and questions

Paper link: <https://www.arxiv.org/abs/2408.02694>

Thank you!