CS6124D: Topics in Programming Languages

Tuples
18.03.2020

- generalization of binary products (pairs) to *n-ary* products ($n \geq 0$)
- $\{true,\ 1,\ \lambda x : Bool\ .\ x\}$ is a 3-tuple, $\{true\}$ is a one-element tuple, $\{\ \}$ is the empty tuple.
- operation - projecting the i^{th} element, where $1 \leq i \leq n$
- typing rules and evaluation rules are to be generalized to the *n-ary* case

New syntactic forms

$t ::=$ ... *terms*
$\{t_i\ ^{i \in 1..n}\}$
$t.i$

$v ::=$ ... *values*
$\{v_i\ ^{i \in 1..n}\}$

$T ::=$ ... *types*
$\{T_i\ ^{i \in 1..n}\}$

Evaluation Rules

$$\frac{t_j \rightarrow t'_j}{\{v_i\ ^{i \in 1..j-1},\ t_j,\ t_k\ ^{k \in j+1..n}\} \rightarrow \{v_i\ ^{i \in 1..j-1},\ t'_j,\ t_k\ ^{k \in j+1..n}\}} \quad \text{E-TUPLE}$$

$$\frac{t_1 \rightarrow t'_1}{t_1\ .\ i \rightarrow t'_1\ .\ i} \quad \text{E-PROJ}$$

$$\{v_i\ ^{i \in 1..n}\}\ .\ j \rightarrow v_j \quad \text{E-PROJTUPLE}$$

Typing Rules

$$\frac{for\ each\ i \quad \Gamma \vdash t_i : T_i}{\Gamma \vdash \{t_i\ ^{i \in 1..n}\} : \{T_i\ ^{i \in 1..n}\}} \quad \text{T-TUPLE}$$

$$\frac{\Gamma \vdash t_1 : \{T_i\ ^{i \in 1..n}\}}{\Gamma \vdash t_1\ .\ j : T_j} \quad \text{T-PROJ}$$

- E-TUPLE - evaluation follows left to right order, the j^{th} component is evaluated only after evaluating the first $j-1$ components to values.
- T-TUPLE - for n-tuple, n sub derivations

Exercise: Derive the type of the tuple $\{true,\ f\ true,\ \lambda x : Bool\ .\ x,\ f\}$ under the typing context $f : Bool \rightarrow T_1$