Dortmund Dilemma

In this problem for given N and K, you need to calculate the number of ways to build a string of length N with **exactly** K different characters, having at least one proper prefix and proper suffix in common. Following some steps will lead you to the solution.

Step 1: Find the number of strings of length N consisting of characters from the first K letters of english alphabet (not necessarily all of them), having **no** proper prefix and proper suffix in common. Let, $F(N,K)$ is the desired number of this step, then

$$F(N,K) = \begin{cases} K & \text{if } N = 1 \\ F(N-1,K) \times K & \text{if } N \text{ is odd} \\ F(N-1,K) \times K - F(N \div 2, K) & \text{if } N \text{ is even} \end{cases}$$

Step 2: Find the number of strings of length N consisting of characters from K of the first characters from english alphabet (not necessarily all of them), having **at least one** proper prefix and proper suffix in common. Let, $G(N,K)$ is the desired number of this step, then

$$G(N,K) = N^K - F(N,K)$$

Step 3: Find the number of strings of length N consisting of K of the first letters from english alphabet (all of them should be present), having **at least one** proper prefix and proper suffix in common. Let, $P(N,K)$ is the desired number of this step, then

$$P(N,K) = \begin{cases} G(N,K) & \text{if } K = 1 \\ G(N,K) - \sum_{j=1}^{K-1} G(N,j) \times \binom{K}{j} & \text{if } K > 1 \end{cases}$$

Step 4: Remember that, we need to find the number of strings having exactly K different characters from the 26 of English alphabet (Not only the first K) . So, for given N and K the final answer will be, $\binom{26}{K} \times P(N, K)$.