2) $f(n) = n$, $g(n) = n$, prove $f(n) = \Theta(g(n))$

Big O $\longrightarrow$ $f(n) <= C_1 \cdot g(n)$

$C_1 = 1$ $\longrightarrow$ Constant

$n <= 1 \cdot n$

$f(n) = O(g(n))$ $\longrightarrow$ True

Omega $\longrightarrow$ $f(n) >= C_2 \cdot g(n)$

$C_2 = 1$ $\longrightarrow$ constant

$n \geq 1 \cdot n$

$f(n) = \Omega(g(n)) \longrightarrow$ True

$f(n) = O(g(n))$ AND $f(n) = \Omega(g(n)) \longrightarrow$ True

$\therefore f(n) = \Theta(g(n)) \longrightarrow$ True

3) $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^3)$

Big O $\longrightarrow f(n) <= c \cdot g(n)$

$2^{6\log_2 n} \cdot 2^{5\log_2 n} = O(n^3)$

$\cancel{2^{\log_2}} \; 2^{\log_2 n^6} \cdot 2^{\log_2 n^5} = O(n^3)$

$n^6 \cdot n^5 = O(n^3)$

$\boxed{n^{11} = O(n^3)}$

$\boxed{n^{11} <= c \cdot n^3} \rightarrow$ Not a valid Big O

$c = n^8 \longrightarrow$ Not constant

$f(n) = O(g(n)) \longrightarrow$ false

4) $\frac{4^n}{2^n} = O(2^n)$

Big O $\longrightarrow f(n) <= c \cdot g(n)$

$\frac{4^n}{2^n} <= c \cdot 2^n$

$2^n <= c \cdot 2^n$

$c = 1 \longrightarrow$ constant

$2^n = O(2^n) \longrightarrow$ True

$f(n) <= c \cdot g(n) \longrightarrow$ True

$\frac{4^n}{2^n} = O(2^n) \longrightarrow$ True

5) $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

$2^{7\log_2 n} \cdot n^2 = \Theta(n^9)$

$2^{\log_2 n^7} \cdot n^2 = \Theta(n^9)$

$n^7 \cdot n^2 = \Theta(n^9)$

$n^9 = \Theta(n^9)$

Big O $\longrightarrow$ $f(n) <= g(n) \cdot c$

$n^9 <= c \cdot n^9$

$c = 1 \longrightarrow$ constant

$n^9 = O(n^9) \rightarrow$ True

omega $\longrightarrow$ $f(n) >= c \cdot g(n)$

$n^9 >= c \cdot n^9$

$c = 1 \longrightarrow$ constant

$n^9 = \Omega(n^9) \longrightarrow$ True

$f(n) = O(g(n))$ AND $f(n) = \Omega(g(n)) \rightarrow$ True

$\therefore f(n) = \Theta(g(n)) \longrightarrow$ True.

1) $f(n) = n - 10$ $\quad g(n) = n + 10$

Prove $f(n) = \Theta(g(n))$

Large values of n i.e. neglect '-10' & '+10'

Big O	omega
$f(n) <= c \cdot g(n)$	$f(n) >= c \cdot g(n)$
$n - 10 <= c \cdot (n+10)$	$n - 10 >= c \cdot (n+10)$
$c = 1$	$c = 1/2$
$f(n) = O(g(n))$	$n = 1000$
	$990 >= \frac{\cancel{1010}\ 505}{\cancel{2}}$
$f(n) = \Theta(g(n))$	$f(n) = \Omega(g(n))$