* Substitution Method Assignment Problem:

1) $T(n) = \begin{cases} 1 & n=1 \\ 2T(n/2)+n & n>1 \end{cases}$

$T(n) = 2T(n/2)+n$

$= 2\left(2T\left(\frac{n}{2^2}\right)+\frac{n}{2}\right)+n$

$\Rightarrow 2^2T\left(\frac{n}{2^2}\right)+2n \longrightarrow 2^{nd}$ term

$\Rightarrow 2^2\left(2T\left(\frac{n}{2^3}\right)+\frac{n}{2^2}\right)+2n \rightarrow 3^{rd}$ term

$= 2^3T\left(\frac{n}{2^3}\right)+3n$

$\frac{n}{2^k}=1$ ⟶ k times

$n = 2^k$

$\log_2 n = k$ $\qquad 2^kT\left(\frac{n}{2^k}\right)+k\cdot n$

$2^{\log_2 n}\cdot T\left(\frac{n}{2^{\log_2 n}}\right)+\log_2 n\cdot n$

$= n^{\cancel{\log_2 2}\,1}\cdot T\left(\frac{\cancel{n}\,1}{\cancel{n^{\log_2 2}}}\right)+\log_2 n\cdot n$

$T(1)=1$ $\qquad n\cdot T(1)+\log_2 n * n$

$= n+n*\log_2 n$

$= \underline{O(n\log_2 n)}$

2) $T(n) = \begin{cases} 1 & n=1 \\ 8T(n/2)+n^2 & n>1 \end{cases}$

$$T(n) = 8T(n/2) + n^2$$

2nd term $= 8\left(8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right) + n^2$

$= 8^2 T\left(\frac{n}{2^2}\right) + 3n^2$

3rd term $= 8^2\left(8T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right) + 3n^2$

$= 8^3 T\left(\frac{n}{2^3}\right) + 7n^2$

$\frac{n}{2^k} = 1$ k times

$\boxed{k = \log_2 n}$

$8^k T(n/2^k) + (2^k - 1)n^2$

$\left[\log_2 8 = \log_2 2^3\right] = 8^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + (2^{\log_2 n} - 1)n^2$

$= n^3 T\left(\frac{n}{n^{\cancel{\log_2 2}}}\right) + (n^{\cancel{\log_2 2}} - 1)n^2$

$[T(1) = 1]$ $= n^3 T(\cancel{n}/\cancel{n}) + n^3 - n^2$

$= n^3 + n^3 - n^2$

$= O(n^3)$