Table of Contents

Introduction

```
% SHUBHAM ASHOK KARKAR
% ASU ID: 1223319344
% Homework - 05
% MAE-598: Design Optimization

%%%%%%%%%%%%%% Main Entrance %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Instruction: Please read through the code and fill in blanks
% (marked by ***). Note that you need to do so for every involv
% function, i.e., m files.
```

Optional overhead

```
clear; % Clear the workspace
close all; % Close all windows
```

Optimization settings

```
% Here, we indicate the objective function by assigning a % variable
% to the function handle, for example:
f = @(x) x(1)^2+ (x(2)-3)^2; % Updated objective function
% In the same way, we also provide the gradient of the
% objective:
df =@(x) [2*x(1), 2*x(2)-6]; % Updated gradient

g =@(x) [x(2)^2-2*x(1); (x(2)-1)^2+5*x(1)-15]; % Adding constraints
dg =@(x) [-2 2*x(2); 5 2*x(2)-2 ];

% Note that explicit gradient and Hessian information is only optional.
% However, providing these information to the search algorithm will save
% computational cost from finite difference calculations for them.

% % Specify algorithm
opt.alg = 'matlabqp'; % 'myqp' or 'matlabqp'

% Turn on or off line search. You could turn on line search once other
% parts of the program are debugged.
opt.linesearch = true; % false or true
```

```
% Set the tolerance to be used as a termination criterion:
opt.eps = 1e-3; % Not altered

% Set the initial guess:
x0 = [1;1];

% Checking the feasibility of the first point.
if max(g(x0)>0)
    errordlg('Infeasible intial point! You need to start from a feasible
 one!');
    return
end
```

Run optimization

```
% Run your implementation of SQP algorithm. See mysqp.m
solution = mysqp(f, df, g, dg, x0, opt);
```

Report

```
%report(solution(),f,g);
for i = 1:length(solution.x)
    sol(i) = f(solution.x(:, i));  % Store all f(x1, x2) values here
    z = g(solution.x(:, i));  % Store all g(x1, x2) values here
    con1(i) = z(1);  % Store g1 constraint here
    con2(i) = z(2);  % Store g2 constraint here
end

count = 1:length(solution.x); % Iteration through each x1 and x2

tiledlayout(3, 1)
nexttile
plot(count, sol)
grid on
title('f(x1, x2) vs Iterations')
xlabel('Iterations')
ylabel('f(x1, x2)')

nexttile
hold on
plot(count, sol)
plot(count, con1)
plot(count, con2)
grid on
legend('f(x) value', 'g1(x)', 'g2(x)')
title('f(x), g1(x) and g2(x) vs. Iterations')
xlabel('No of Iterations')
ylabel('Functions')
hold off

nexttile
plot(solution.x(1, :), solution.x(2, :))
```

```
grid on
title('x2 vs x1')
xlabel('X1')
ylabel('X2')

disp("Values of x1 & x2 = ");
disp(solution.x(:, end));  % The optimum x1 and x2

disp("Function value (F) = ");
disp(sol(end));  % The objective function at x1 and x2

disp("g1(x1, x2) = ");
disp(con1(end));  % The first constraint function at x1 and x2

disp("g2(x1, x2) = ");
disp(con2(end));  % The second constraint function at x1 and x2

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



%%%%%%%%%%%%%% Sequential Quadratic Programming Implementation with BFGS %%%%%
%%%%%%%%%%%
%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak %%%%%%%%%%%%%%%%%%%%%%%%%%%

function solution = mysqp(f, df, g, dg, x0, opt)
    % Set initial conditions

    x = x0; % Set current solution to the initial guess

    % Initialize a structure to record search process
    solution = struct('x',[]);
    solution.x = [solution.x, x]; % save current solution to solution.x

    % Initialization of the Hessian matrix
    W = eye(numel(x));             % Start with an identity Hessian matrix
    % Initialization of the Lagrange multipliers
    mu_old = zeros(size(g(x)));    % Start with zero Lagrange multiplier
 estimates
    % Initialization of the weights in merit function
    w = zeros(size(g(x)));         % Start with zero weights

    % Set the termination criterion
    gnorm = norm(df(x) + mu_old'*dg(x)); % norm of Largangian gradient

    while gnorm>opt.eps % if not terminated

        % Implement QP problem and solve
        if strcmp(opt.alg, 'myqp')
            % Solve the QP subproblem to find s and mu (using your own method)
            [s, mu_new] = solveqp(x, W, df, g, dg);
```

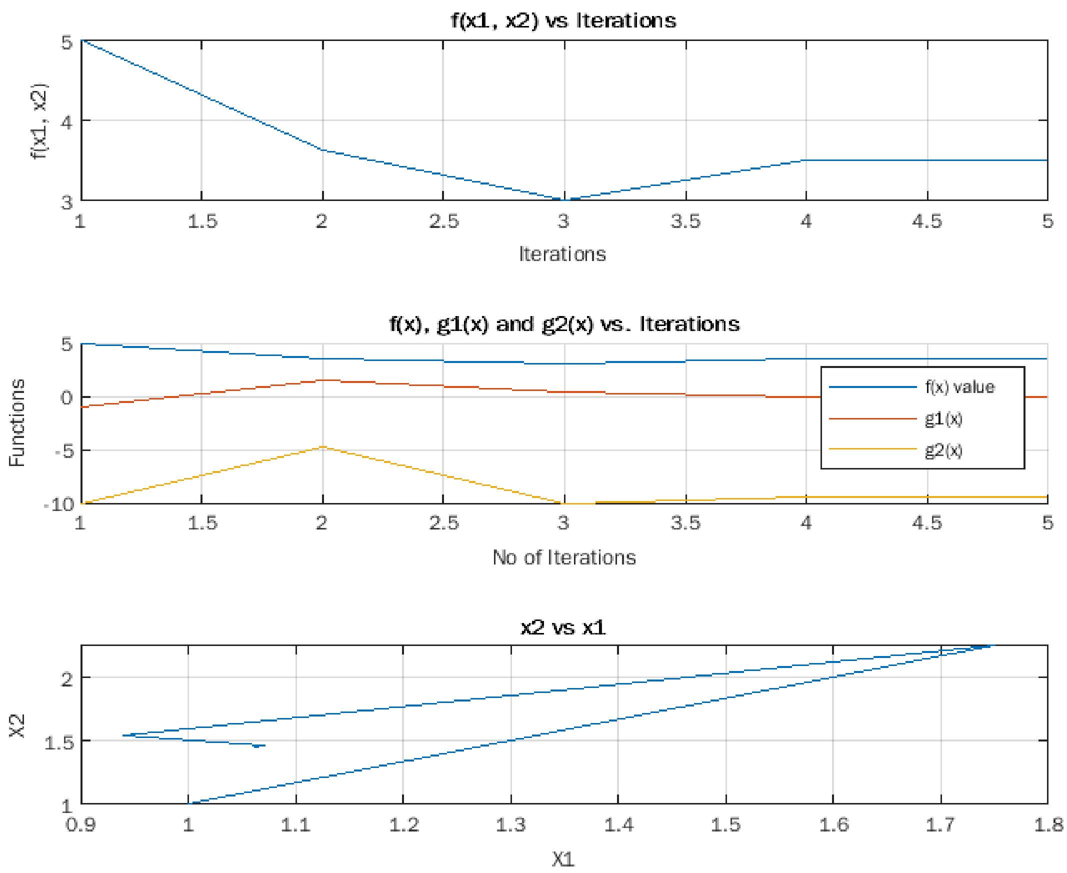
```
        else
            % Solve the QP subproblem to find s and mu (using MATLAB's solver)
            qpalg = optimset('Algorithm', 'active-set', 'Display', 'off');
            [s,~,~,~,lambda] = quadprog(W,[df(x)]', dg(x), -g(x), [], [], [],
 [], x, qpalg);
            mu_new = lambda.ineqlin;
        end

        % opt.linesearch switches line search on or off.
        % You can first set the variable "a" to different constant values and
 see how it
        % affects the convergence.
        if opt.linesearch
            [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w);
        else
            a = 0.1;
        end

        % Update the current solution using the step
        dx = a*s;               % Step for x
        x = x + dx;             % Update x using the step

        % Update Hessian using BFGS. Use equations (7.36), (7.73) and (7.74)
        % Compute y_k
        y_k = [df(x) + mu_new'*dg(x) - df(x-dx) - mu_new'*dg(x-dx)]';
        % Compute theta
        if dx'*y_k >= 0.2*dx'*W*dx
            theta = 1;
        else
            theta = (0.8*dx'*W*dx)/(dx'*W*dx-dx'*y_k);
        end
        % Compute  dg_k
        dg_k = theta*y_k + (1-theta)*W*dx;
        % Compute new Hessian
        W = W + (dg_k*dg_k')/(dg_k'*dx) - ((W*dx)*(W*dx)')/(dx'*W*dx);

        % Update termination criterion:
      gnorm = norm(df(x) + mu_new'*dg(x)); % norm of Largangian gradient
      mu_old = mu_new;

        % save current solution to solution.x
      solution.x = [solution.x, x];
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%The following code performs line search on the merit function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Armijo line search
function [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w_old)
    t = 0.1; % scale factor on current gradient: [0.01, 0.3]
    b = 0.8; % scale factor on backtracking: [0.1, 0.8]
    a = 1; % maximum step length

    D = s;                  % direction for x

    % Calculate weights in the merit function using eaution (7.77)
    w = max(abs(mu_old), 0.5*(w_old+abs(mu_old)));
    % terminate if line search takes too long
    count = 0;
    while count<100
        % Calculate phi(alpha) using merit function in (7.76)
        phi_a = f(x + a*D) + w'*abs(min(0, -g(x+a*D)));

        % Caluclate psi(alpha) in the line search using phi(alpha)
        phi0 = f(x) + w'*abs(min(0, -g(x)));        % phi(0)
        dphi0 = df(x)*D + w'*((dg(x)*D).*(g(x)>0)); % phi'(0)
        psi_a = phi0 +  t*a*dphi0;                  % psi(alpha)
        % stop if condition satisfied
        if phi_a<psi_a
            break;
        else
            % backtracking
            a = a*b;
            count = count + 1;
        end
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%The active set strategy is used to solve the QP subproblem in the following
 code.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [s, mu0] = solveqp(x, W, df, g, dg)
    % Implement an Active-Set strategy to solve the QP problem given by
    % min     (1/2)*s'*W*s + c'*s
    % s.t.    A*s-b <= 0
    %
    % where As-b is the linearized active contraint set

    % Strategy should be as follows:
    % 1-) Start with empty working-set
    % 2-) Solve the problem using the working-set
    % 3-) Check the constraints and Lagrange multipliers
```

```
   % 4-) If all constraints are staisfied and Lagrange multipliers are
positive, terminate!
   % 5-) If some Lagrange multipliers are negative or zero, find the most
negative one
   %     and remove it from the active set
   % 6-) If some constraints are violated, add the most violated one to the
working set
   % 7-) Go to step 2


   % Compute c in the QP problem formulation
   c = [df(x)]';

   % Compute A in the QP problem formulation
   A0 = dg(x);

   % Compute b in the QP problem formulation
   b0 = -g(x);

   % Initialize variables for active-set strategy
   stop = 0;           % Start with stop = 0
   % Start with empty working-set
   A = [];         % A for empty working-set
   b = [];         % b for empty working-set
   % Indices of the constraints in the working-set
   active = [];    % Indices for empty-working set

   while ~stop  % Continue until stop = 1
       % Initialize all mu as zero and update the mu in the working set
       mu0 = zeros(size(g(x)));

       % Extact A corresponding to the working-set
       A = A0(active,:);
       % Extract b corresponding to the working-set
       b = b0(active);

       % Solve the QP problem given A and b
       [s, mu] = solve_activeset(x, W, c, A, b);
       % Round mu to prevent numerical errors (Keep this)
       mu = round(mu*1e12)/1e12;

       % Update mu values for the working-set using the solved mu values
       mu0(active) = mu;

       % Calculate the constraint values using the solved s values
       gcheck = A0*s-b0;

       % Round constraint values to prevent numerical errors (Keep this)
       gcheck = round(gcheck*1e12)/1e12;

       % Variable to check if all mu values make sense.
       mucheck = 0;        % Initially set to 0

       % Indices of the constraints to be added to the working set
```

```
        Iadd = [];              % Initialize as empty vector
        % Indices of the constraints to be added to the working set
        Iremove = [];           % Initialize as empty vector

        % Check mu values and set mucheck to 1 when they make sense
        if (numel(mu) == 0)
            % When there no mu values in the set
            mucheck = 1;         % OK
        elseif min(mu) > 0
            % When all mu values in the set positive
            mucheck = 1;         % OK
        else
            % When some of the mu are negative
            % Find the most negaitve mu and remove it from acitve set
            [~,Iremove] = min(mu);  % Use Iremove to remove the constraint
        end

        % Check if constraints are satisfied
        if max(gcheck) <= 0
            % If all constraints are satisfied
            if mucheck == 1
                % If all mu values are OK, terminate by setting stop = 1
                stop = 1;
            end
        else
            % If some constraints are violated
            % Find the most violated one and add it to the working set
            [~,Iadd] = max(gcheck); % Use Iadd to add the constraint
        end
        % Remove the index Iremove from the working-set
        active = setdiff(active, active(Iremove));
        % Add the index Iadd to the working-set
        active = [active, Iadd];

        % Make sure there are no duplications in the working-set (Keep this)
        active = unique(active);
    end
end

function [s, mu] = solve_activeset(x, W, c, A, b)
    % Given an active set, solve QP

    % Create the linear set of equations given in equation (7.79)
    M = [W, A'; A, zeros(size(A,1))];
    U = [-c; b];
    sol = M\U;          % Solve for s and mu

    s = sol(1:numel(x));                % Extract s from the solution
    mu = sol(numel(x)+1:numel(sol));    % Extract mu from the solution

end

Values of x1 & x2 =
    1.0604
```

```
    1.4563

Function value (F) =
    3.5074

g1(x1, x2) =
   7.9687e-05

g2(x1, x2) =
   -9.4897
```



Published with MATLAB® R2021b