MAE 598 DESIGN OPTIMIZATION

PROJECT – 3

TOPOLOGY OPTIMIZATION

NAME: Shubham Ashok Karkar

ASU ID: 1223319344

What is Topology Optimization? :

Topology Optimization is a technique for decreasing the amount of material utilized in a system as well as the strain energy it generates. Topology optimization is a mathematical method for spatially optimizing the distribution of material within a defined region by minimizing a predefined cost function while satisfying previously established constraints.

Problem Definition:

In the following way, the function is minimized:

$$\min_{\mathbf{x}} \quad \mathbf{f} := \mathbf{d}^T\mathbf{K}(\mathbf{x})\mathbf{d}$$

$$\text{subject to:} \quad \mathbf{h} := \mathbf{K}(\mathbf{x})\mathbf{d} = \mathbf{u},$$

$$\mathbf{g} := V(\mathbf{x}) \leq v,$$

$$\mathbf{x} \in [0, 1].$$

Where V(x) = Total volume

V = upper bond of the volume

K(x) = stiffness matrix

x = Set of the densities

d = Displacement of structure which is under load ‘u’

The Design Sensitivity:

The reduced gradient (often called design sensitivity) can be calculated as

$$\frac{df}{d\mathbf{x}} = \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial f}{\partial \mathbf{u}}\left(\frac{\partial \mathbf{h}}{\partial \mathbf{u}}\right)^{-1}\frac{\partial \mathbf{h}}{\partial \mathbf{x}},$$

which leads to

$$\frac{df}{d\mathbf{x}} = -\mathbf{u}^T\frac{\partial \mathbf{K}}{\partial \mathbf{x}}\mathbf{u}.$$

Recall the relation between $\mathbf{K}$ and $\mathbf{x}$, and notice that

$$\mathbf{u}^T\mathbf{K}\mathbf{u} = \sum_{i=1}^{n}\mathbf{u}_i^T\mathbf{K}_i\mathbf{u}_i,$$

i.e., the total compliance (strain energy) is the summation of element-wise compliance. We can further simplify the gradient as follows:

$$\begin{aligned}
-\mathbf{u}^T\frac{\partial \mathbf{K}}{\partial \mathbf{x}}\mathbf{u} &= -\frac{\partial \mathbf{u}^T\mathbf{K}\mathbf{u}}{\partial \mathbf{x}} \\
&= -\frac{\partial \sum_{i=1}^{n}\mathbf{u}_i^T\mathbf{K}_i\mathbf{u}_i}{\partial \mathbf{x}} \\
&= [\ldots, -\frac{\partial \mathbf{u}_i^T\mathbf{K}_i\mathbf{u}_i}{\partial x_i}, \ldots] \\
&= [\ldots, -\mathbf{u}_i^T\frac{\partial \mathbf{K}_i}{\partial x_i}\mathbf{u}_i, \ldots] \\
&= [\ldots, -\mathbf{u}_i^T\frac{\partial \bar{\mathbf{K}}_e \Delta E x_i^3}{\partial x_i}\mathbf{u}_i, \ldots] \\
&= [\ldots, -3\Delta E x_i^2\mathbf{u}_i^T\bar{\mathbf{K}}_e\mathbf{u}_i, \ldots]
\end{aligned}$$

The penalty parameter aids in the reduction of topologies to binary values; when xi approaches 0, the cubic function reduces it to 0.