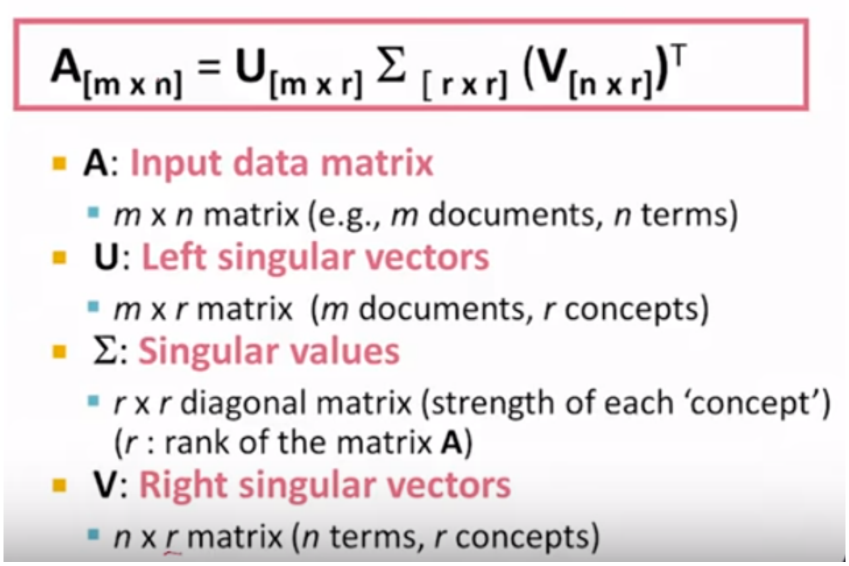
SVD - Singular Value Decomposition

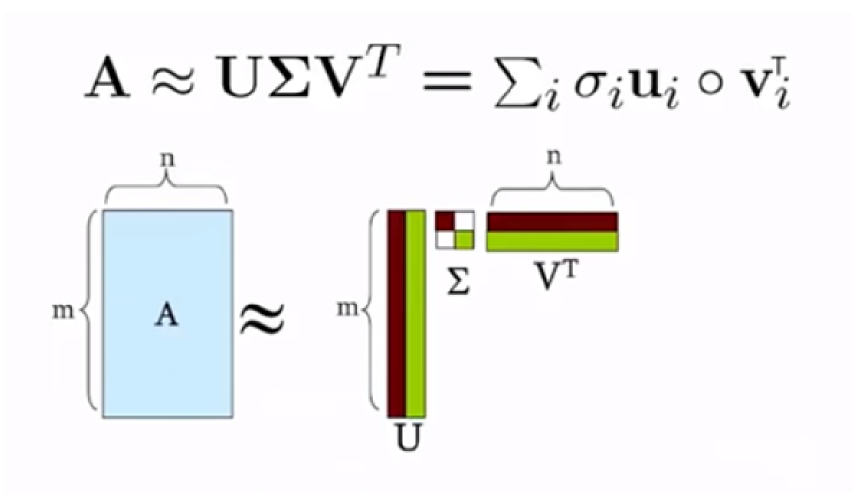
SVD is also called a *data reduction technique.*

The workflow of SVD:
As per mathematical formulae :



Where, m- rows
n - columns
r - concepts (rows), will look after this parameter later on in the document.

In the SVD we take our matrix and show it as a product of the 3 matrix



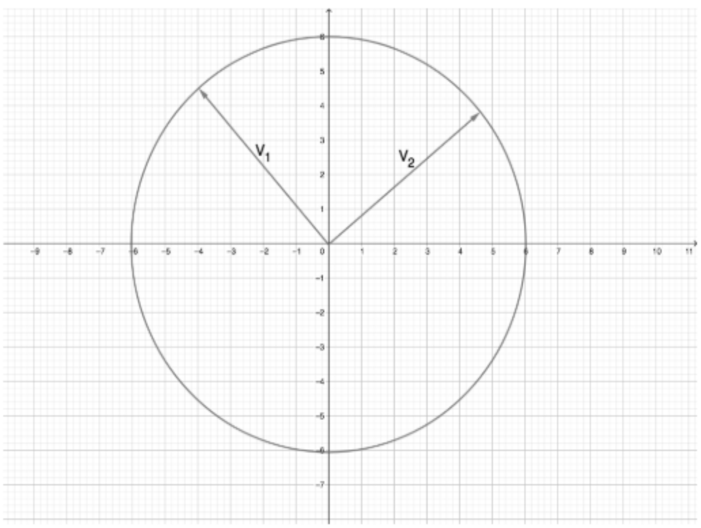
In the singular values, we have all the zeros only in the diagonal we have non zero values in descending order. I.e Largest first followed by second largest and so on..

Equation:

$$A = U\Sigma V^T$$

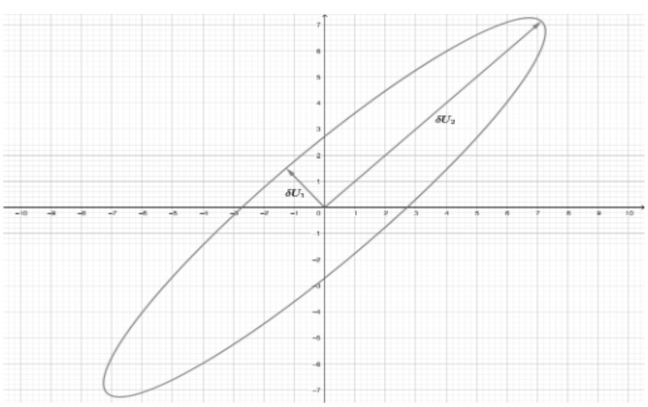
Where ***A*** *is an m x n matrix,* ***U*** *is an (m x m) orthogonal matrix,* ***Σ*** *is an (m x n) nonnegative rectangular diagonal matrix, and* ***V*** *is an (n x n) orthogonal matrix.*

Imagine a circle in two dimensions represented by vectors `v1` and `v2` undergoing a matrix transformation as illustrated on the cartesian coordinates below:



Two Dimensional Circle

After Matrix Multiplication



An Ellipse

From the images above, you can tell that when a matrix multiples a vector, it simply stretches it and then rotates it.

$$V_1, V_2, ...V_n \quad becomes \quad U_1, U_2, ...U_n$$

after the multiplication, and we have:

$$\sigma_1, \sigma_2, ...\sigma_n$$

representing the space of the individual stretching factors.

Therefore from this we can write the equation:

$$AV_n = \sigma_n U_n$$

which we can write more generally as:

$$AV = U\Sigma$$

Where **Σ** represents the space of all stretching factors(σ's).

But for an orthogonal matrix,

$$V^{-1} = V^T$$

Also, note that the product of a matrix and its inverse is the **identity matrix** (An identity matrix is a diagonal matrix with only 1's). This concept can be represented by the equation below:

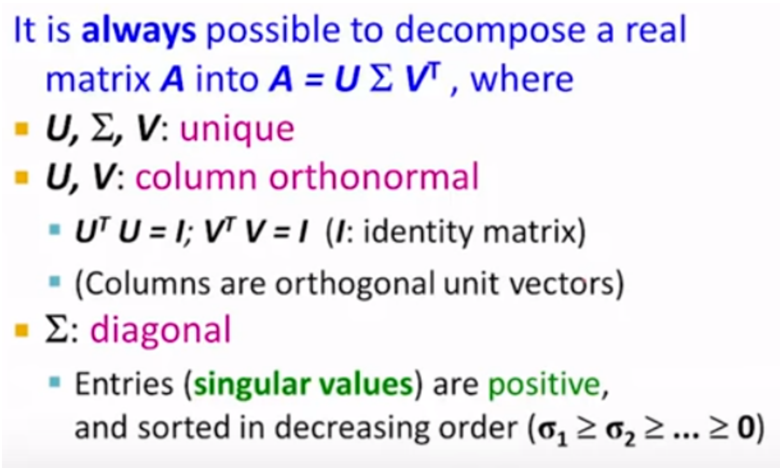
$$VV^{-1} = 1$$

Combining the above three equations leads us to the **Reduced Singular Value Decomposition**.

$$A = U\Sigma V^T$$

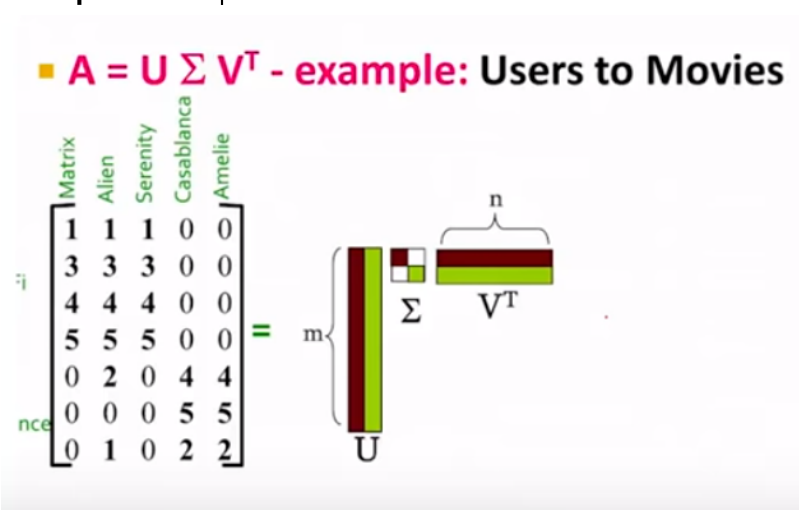
This how the equation got created for SVD.

Properties of SVD:

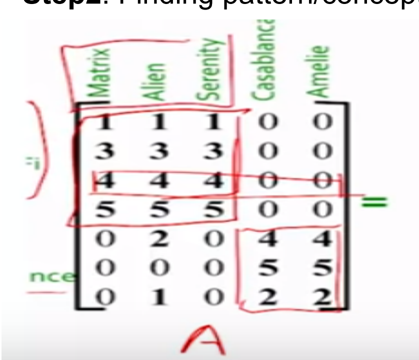


Where U and V are vectors and Sigma is scalar.
Note: Singular values are positive.

Example: users x movies
  Step1: Decompose to 3 matrix



  Step2: Finding pattern/concepts



Step3: Decomposed matrix

- $A = U \Sigma V^T$ - **example: Users to Movies**

$$
\begin{array}{c}
\begin{array}{ccccc} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \end{array} \\
\begin{bmatrix}
\mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 \\
\mathbf{3} & \mathbf{3} & \mathbf{3} & 0 & 0 \\
\mathbf{4} & \mathbf{4} & \mathbf{4} & 0 & 0 \\
\mathbf{5} & \mathbf{5} & \mathbf{5} & 0 & 0 \\
0 & \mathbf{2} & 0 & \mathbf{4} & \mathbf{4} \\
0 & 0 & 0 & \mathbf{5} & \mathbf{5} \\
0 & \mathbf{1} & 0 & \mathbf{2} & \mathbf{2}
\end{bmatrix}
\end{array}
=
\overset{U}{\begin{bmatrix}
\mathbf{0.13} & 0.02 & -0.01 \\
\mathbf{0.41} & 0.07 & -0.03 \\
\mathbf{0.55} & 0.09 & -0.04 \\
\mathbf{0.68} & 0.11 & -0.05 \\
0.15 & \mathbf{-0.59} & \mathbf{0.65} \\
0.07 & \mathbf{-0.73} & \mathbf{-0.67} \\
0.07 & \mathbf{-0.29} & \mathbf{0.32}
\end{bmatrix}}
\times
\overset{\Sigma}{\begin{bmatrix}
\mathbf{12.4} & 0 & 0 \\
0 & \mathbf{9.5} & 0 \\
0 & 0 & \mathbf{1.3}
\end{bmatrix}}
\times
\overset{V^T}{\begin{bmatrix}
\mathbf{0.56} & \mathbf{0.59} & \mathbf{0.56} & 0.09 & 0.09 \\
0.12 & -0.02 & 0.12 & \mathbf{-0.69} & \mathbf{-0.69} \\
0.40 & \mathbf{-0.80} & 0.40 & 0.09 & 0.09
\end{bmatrix}}
$$

SVD: Interpretation

'**movies**', '**users**' and '**concepts**':

- ***U***: user-to-concept similarity matrix
- ***V***: movie-to-concept similarity matrix
- Σ: its diagonal elements:
  'strength' of each concept