2014

Mathematics II

Time Alloted: 3 Hours

Full Marks: 70

The figure in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable

GROUP - A (Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

10x1=1

i) If the order of the matrices A, B and AB are 2×3 , m \times n and 2×2 respectively, then the value of m and n are

a) 2, 2

b) 3, 2

c) 2. 3

d) 3. 3

ii) The matrix $\begin{pmatrix} 1 & t \\ 5 & 10 \end{pmatrix}$ is non-singular, if t is not equal to

a) 1

b) 2

c) 3

d) 4

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[Turn over]

- The value of $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)$ iii)
 - a) 1
 - b) 1/e
 - c) 0 d) e
- iv) A matrix A is said to be an idempotent matrix if
 - a) $A^2 = 0$
 - b) A A Common signature and the sugar of

d) none of these,
$$f(x,y) = x \cos y + y e^{x}$$
, then $\left(\frac{\partial f}{\partial x}\right)_{\left(0,\frac{\pi}{2}\right)}$ is

- b) The state of the contract of the contract of
- d) π/2
- The eccentricity of the rectangular hyperbola is:

 - **b)** $\sqrt{2}$
 - c) 2
 - d) none of these

$$vii) \int_{0}^{4} \tan^{2} x dx =$$

a)
$$1 - \frac{\pi}{4}$$

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- viii) Rank of the matrix 1
 - a) 0 b) 1

 - c) 2 d) 3
- The greatest value f(x) =sinx+cosx is
 - a) $\sqrt{2}$
 - **b)** $2\sqrt{2}$

 - d) none of these
- The value of a skew-symmetric determinant of odd order is X)

 - b) 1
 - c) 2
 - d) 0.
- The length of latus rectum of the hyperbola $9x^2-25y^2=225$ is

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[Turn over]

d) $\frac{5}{9}$ xii) Which of the following functions continuous for all real

- **b**) |x|
- d) $\frac{x}{x-1}$

xiii) If $A = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$, then (2A + A') is

- $\mathbf{a})\begin{pmatrix} 3 & 2 \\ -2 & 9 \end{pmatrix}$
- b) $\begin{pmatrix} 2 & 3 \\ -2 & 9 \end{pmatrix}$
- c) $\begin{pmatrix} 2 & 9 \\ -2 & 3 \end{pmatrix}$

d) none of these

xiv) The focus of $y^2 = 36x$ is

- a) (3,0)
- b) (9,0)
- c) (3,9)
- d) None of these.
- xv) The function $u(x, y) = \frac{(x+y)^2}{(x-y)^2}$ is a homogeneous function of degree.
 - a) 1
 - b) 0
 - c) 2
 - d)-I

GROUP - B (Short Answer Type Questions) Answer any three of the following. 3x5=1

- 2. Solve the following system of equation by Cramer's Rule x + 2y + 3z = 6. 2x + 4y + z = 7, 3x + 2y + 9z = 14
- 3. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(ab+bc+ca)$
- 4. Find the maximum value of $x^{1/x}$
- 5. Find the Adjugate matrix of the square matrix $\begin{pmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{pmatrix}$

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[Turn over]

6. Evalute
$$f \frac{dx}{(1-x)\sqrt{1-x^2}}$$

7. Find the equation of an ellipse for which the principal axes are along the coordinate axis, the length of latus rectum is 4 unit and

eccentricity is
$$\frac{1}{3}$$

GROUP - C

(Long Answer Type Questions)

Answer any three of the following. 3x15=4

8. a) Solve the following system of equations by Matrix Inversion Method:

3x+y+z=4

x-y+2z =6

x+2v-z=3

b) find The latus rectum and the co-ordinates of the foci of the equilateral hyperbola $x^2 - y^2 = 25$

c) Express the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 3 & -2 \end{pmatrix}$ as a sum of a symmetric

and skew-symmetric matrices.

9. i) Find the equation of the parabola whos focus is at the point (3,-2) and directrix is the line 2x-y+3=0.

(ii) Find the points of local maxima and minima and the corresponding maximum and minimum values of the function

$$f(x) = x + \frac{1}{x}$$

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- (iii) A function is defined as f(x) = |x|, for all $x \in \mathbb{R}$. Show from definition that f'(0) does not exist. 5+5+5
- 10. a) For the matrix $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, Show that $A^2 5A + 71_2 = 0$

b) If
$$v = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$$
, then show that $x\frac{\delta v}{\delta x} + y\frac{\delta v}{\delta y} = \tan v$

- c) If $y=x^x$, then find $\frac{dy}{dx}$
- 11. a) If $y = \sin(a + bx)$, then find yn.

b) Evaluate
$$\lim_{n\to\infty} \left\{ \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right\}$$

- c) Evaluate $\int_{-1}^{1} x |x| dx$
- 12. a) If $y = \sin(m\sin^{-1}x)$, then prove that $(1-x^2)y^2 xy_1 + m^2y = 0$
 - b) If $u = x\phi \frac{y}{x} + \psi \left(\frac{y}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\phi \left(\frac{y}{x}\right)$
 - c) Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1 x^2}}$.

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