

# MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code: BBA-202
MATHEMATICS-II

Time Allotted: 3 Hours

Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

# GROUP - A ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any ten of the following:  $10 \times 1 = 10$ 

- i) The value of  $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$  is
  - a) e
  - b) 1/e
  - c) 0
  - d) 1.

2/20057

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- . ii) The matrix A is said to be orthogonal matrix, if
  - a) det(A) = 0
- b)  $det(A) \neq 0$
- c)  $A^T A = I$
- d) none of these.
- iii) The focus of  $y^2 = 36x$  is
  - a) (3,0)
- b) (9,0)
- c) (3,9)
- d) none of these.
- iv) The value of  $\int xe^x dx$  is
  - a) xe<sup>x</sup>
- b)  $e^{x}(x-1)$
- c)  $xe^x + x$
- d) none of these.
- v) The eccentricity of the ellipse  $3x^2 + 4y^2 = 24$  is
  - a)  $\frac{1}{4}$

b)  $\frac{3}{4}$ 

c)  $\frac{1}{2}$ 

- d)  $\frac{\sqrt{3}}{2}$ .
- vi) A matrix A is said to be an idempotent matrix, if
  - a)  $A^2 = A$
- b)  $A^2 = I$
- c)  $A^2 0$
- d) none of these.

2/20057

9

- vii) A function f(x) is said to be an odd function of x if f(-x) is equal to
  - a) -f(x)
- b) f(x)
- c) f(-x)

- viii) The value of  $\int \frac{dx}{x^2 a^2}$  is
  - a)  $\frac{1}{2a} \log \frac{a+x}{a-x}$  b)  $\sin^{-1} \frac{x}{a}$ c)  $\log \sqrt{x+x^2+a^2}$  d)  $\frac{1}{2a} \log \frac{x-a}{x+a}$ .
- The cofactor or 'c' in the determinant  $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$  is
  - a)  $(-1)^{3+3} \begin{vmatrix} a & h \\ h & b \end{vmatrix}$  b)  $(-1)^{3+2} \begin{vmatrix} a & h \\ h & b \end{vmatrix}$
  - c)  $(-1)^{2+3}\begin{vmatrix} a & h \\ h & b \end{vmatrix}$  d) none of these.
- x) The function  $U(x,y) = \frac{(x+y)^2}{(x-y)^2}$  is a homogeneous

function of degree

a)

b) 1

c) 2

d) 3.

2/20057

3

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- xi) Rank of the matrix  $\begin{bmatrix} -1 & 3 & 1 \\ 3 & -9 & -3 \end{bmatrix}$  is
  - a) 0

**b**) 1

c) 2

- d) 3.
- xii) The curve y = |x| is
  - a) differentiable everywhere
  - b) continuous everywhere
  - c) discontinuous at x = 0
  - d) not derivable at x = 1.

**GROUP - B** 

## (Short Answer Type Questions)

Answer any three of the following  $3 \times 5 = 15$ 

- 2. If  $f(x) = \sin(\log x)$ , then find f'(x).
- 3. Prove that  $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc.$
- 4. Evaluate the integral  $\int \frac{x-22}{3x^2-2x-8} dx$ .

2/20057

- 5. For the matrix  $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ , show that  $A^2 5A + 7I_2 = 0$  and hence find  $A^{-1}$ .
- 6. Find the equation of the parabola whose focus is (2, 1) and directrix is 3x y + 1 = 0.
- 7. Evaluate  $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

## **GROUP - C**

## (Long Answer Type Questions)

Answer any three of the following.  $3 \times 15 = 45$ 

8. a) Obtain the inverse of the matrix 
$$\begin{bmatrix} 2 & 4 & -1 \\ 3 & 1 & 2 \\ 1 & 3 & -3 \end{bmatrix}$$

and hence solve the following system of equations:

$$2x + 4y - z = 9$$
$$3x + y + 2z = 7$$
$$x + 3y - 3z = 4$$

- b) Show that the vectors  $\alpha_1 = (5, 7, 11)$ ,  $\alpha_2 = (2, 1, 3)$  and  $\alpha_3 = (3, 6, 8)$  are linearly dependent.
- c) If  $x = a \cos 2t$ ,  $y = a \sin 2t$ , then find  $\frac{d^2y}{dx^2}$ . 7 + 3 + 5

9. a) If 
$$y = 3^{4x} + \frac{3}{\sqrt[3]{x}}$$
, find  $\frac{dy}{dx}$ .

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b) If 
$$y = e^x \log x$$
, prove that  $xy_2 - (2x - 1)y_1 + (x - 1)y = 0$ 

c) Verify Euler's theorem for the function

$$f(x, y) = x^3 + y^3 + 3xy^2 + 3x^2y$$
.  $5 + 5 + 5$ 

- 10. a) Evaluate  $\int \frac{8^{2+x}-4^{2-x}}{2^{x+3}} dx$ .
  - b) Evaluate  $\int e^{x^3} \cdot x^5 dx$ .
  - c) If the centre, vertex and eccentricity of a hyperbola be (2, 4), (6, 4) and  $\sqrt{5}$ , then find its equation.

11. a) Evaluate the determinant 
$$\begin{vmatrix} 1 & \omega^3 & \omega^2 \\ \omega^3 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$$
 where  $\omega$ 

is the cube root of unity.

b) Reduce the matrix 
$$\begin{bmatrix} 0 & 0 & 5 & -3 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$
 to the matrix  $\begin{bmatrix} 2 & 4 & 3 & 5 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  by elementary row

operations.

2/20057

- c) Taking y-axis as major axis and x-axis as minor axis, find the equation of the ellipse whose sum of the squares of the lengths of the major and minor axes is 20 and eccentricity is  $\frac{1}{\sqrt{3}}$ . 5+5+5
- 12. a) Evaluate  $\int_{0}^{1} x^{3} \sqrt{1 + 3x^{4}} dx$ .
  - b) Find the area of the region bounded by  $y = 4x^2$ , y = 0, x = 1 and x = 3.
  - c) Find the point on the parabola  $y = x^2$  at which the tangent is parallel to the line y = 4x 5. 5 + 5 + 5

2/20057