**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans :- Since work beings 10 mins after the car is dropped, the time left to complete work is**

**50 mins. Probability that Service Manager cannot meet his commitment = P(X&gt;50) = 1-Pr(x&lt;=50)**

**(X is the time taken to complete work). Convert 50 to z-score Standard normal variable Z-(X-u)/=**

**(x-45)/8 P(X&lt;=50)=P(Z&lt;=(50-45)/8)-PR(Z&lt;=0.625)=0.73237-73.237% (the number in z-table)**

**Probability that service manager will not meet his commitment is 100-73.237-26.763% -0.2676**

**So, the answer is B.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans :- False**

**Reason : If more employees are older than 44, this will shift the  towards 44 with considering**

**sd. Which is not possible as  is given 38 with sd 6.**

**B. A training program for employees under the age of 30 at the center would be**

**expected to attract about 36 employees.**

**Ans : True**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans :- As both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,σ1 2 +σ2 2 ).**

**And 2X1 will just scale the normal distribution by 2 times.)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans :- Since we need to find out the values of a and b, which are symmetric about**

**the mean, such that the probability of the random variable taking a value between them**

**is 0.99.**

**The Probability of getting value between a and b should be 0.99.**

**So the Probability of going wrong, or the Probability outside the a and b area is 0.01**

**(ie. 1-0.99).**

**The Probability towards left from a = -0.005 (ie. 0.01/2).**

**The Probability towards right from b = +0.005 (ie. 0.01/2).**

**So since we have the probabilities of a and b, we need to calculate X, the random**

**variable at a and b which has got these probabilities.**

**By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.**

**Z=(X- μ) / σ**

**For Probability 0.005 the Z Value is -2.57 (from Z Table)**

**Z \* σ + μ = X**

**Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4**

**Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6**

**So, option D is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans :- Solution : &gt; qnorm(0.025,45\*5,3)**

**[1] 219.1201**

**&gt; qnorm(0.975,45\*5,3)**

1. **230.8799**

**&gt; qnorm(0.025,45\*7,3)**

**(1) 309.1201**

**&gt; qnorm(0.975,45\*7,3)**

**(1)320.8799**

**So the Rupee range with 95% probability for the annual profit of the company is given by,**

**=Profit1 + Profit2**

**= [219.12, 230.87] + [309.12, 320.87]**

**= [528.24, 551.74]**

**So Profit of the company in Rupees(in Million) is between range [528.24, 551.74]**

**B. Specify the 5 th percentile of profit (in Rupees) for the company**

**Ans :- &gt; qnorm(0.05,45\*7,3)**

**[1] 310.0654**

**&gt; qnorm(0.05,45\*5,3)**

**[1] 220.0654**

**5 th percentile of profit = 310.0654+ 220.0654 = 530.1308 Million Rs**

**C. Which of the two divisions has a larger probability of making a loss in a given year?**

**Ans :- Division 2 with distribution N(7, 4 2 )**.