SGD Derivation Process for Reformulation-Aware Metrics (RAMs)

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1 Reformulation Selecting

Add, Delete, Keep, Transform, Others, First (query):

$$\Omega \in \{A, D, K, T, O, F\} \ (Reformulation)$$

$$P(I = I_k \mid \Omega) = i_{\omega,k}, \text{ where } \sum_{k=1}^{K} i_{\omega,k} = 1$$

$$\text{Let } i_{\omega,k} = \text{softmax}(\pi_{\omega,k}) = \frac{e^{\pi_{\omega,k}}}{\sum_{j=1}^{K} e^{\pi_{\omega,j}}}$$

2 Modified Click Model

2.1 DBN

According to the assumptions of DBN, we have:

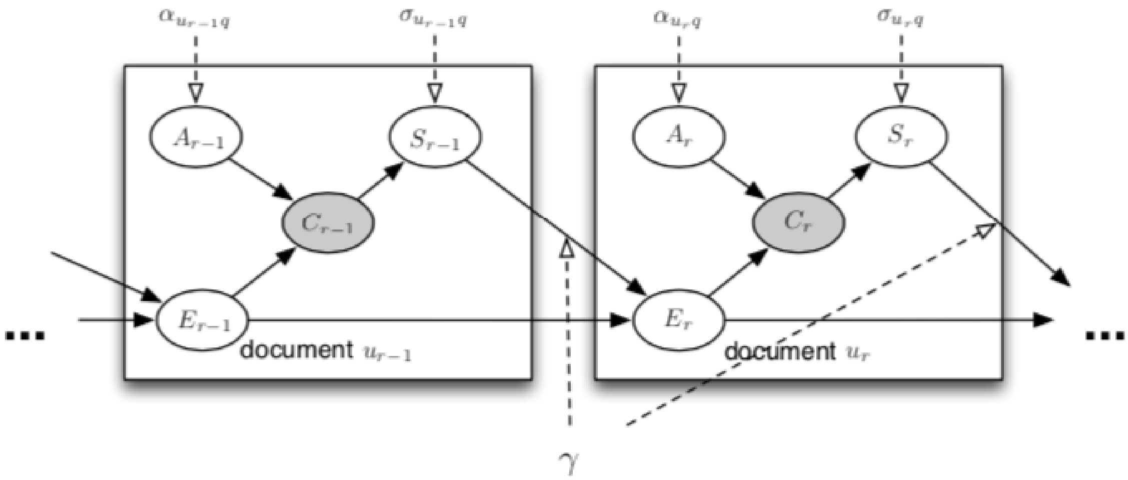

Figure 1: DBN PGM.

$$
\begin{aligned}
C_r = 1 \iff E_r = 1 \text{ and } A_r = 1 \\
P(A_r = 1) = \frac{2^{R_{u_r q}} - 1}{2^{R_{max}}}(or\ \alpha_{R_{u_r q}}) \\
P(E_1 = 1) = 1 \\
P(E_r = 1 \mid E_{r-1} = 0) = 0 \\
P(S_r = 1 \mid C_r = 1, I = I_k) = \sigma_{R_{u_r q} k} \\
P(E_r = 1 \mid S_{r-1} = 1) = 0 \\
P(E_r = 1 \mid E_{r-1} = 1, S_{r-1} = 0, I = I_k) = \gamma_k
\end{aligned}
$$

2.2 UBM

According to the assumptions of UBM, we have:

$$
\begin{aligned}
C_r = 1 \iff E_r = 1 \text{ and } A_r = 1 \\
P(A_r) = \frac{2^{R_{u_r q}} - 1}{2^{R_{max}}}(or\ \alpha_{R_{u_r q}}) \\
P(E_1 = 1) = 1 \\
P(E_r = 1 \mid C_1 = c_1, ..., C_{r-1} = c_{r-1}, I = I_k) = \gamma_{rr'k} \\
\text{where } r' = max\{k \in \{0, ..., r-1\} : c_k = 1\} \\
\rightarrow P(E_r = 1 \mid \mathbf{C}_{<r_u}) = P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, ..., C_{r-1} = 0, I = I_k) = \gamma_{rr'k}
\end{aligned}
$$

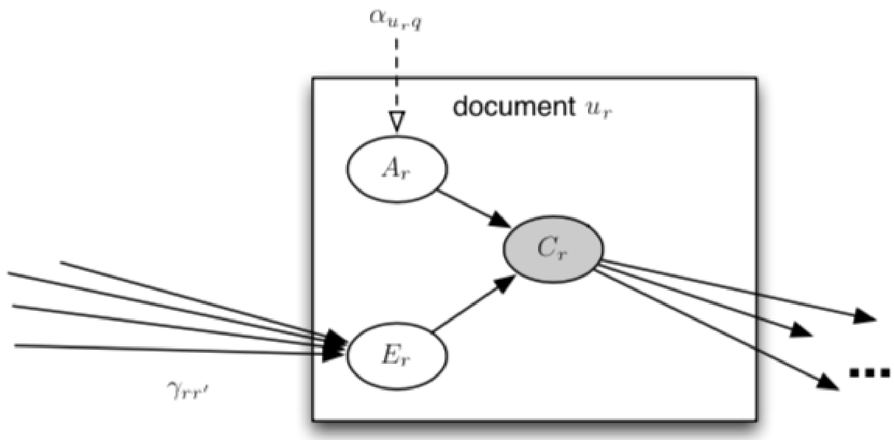

Figure 2: UBM PGM.

2.3 PBM

According to the assumptions of PBM, we have:

$$
\begin{aligned}
C_r = 1 &\iff E_r = 1 \text{ and } A_r = 1 \\
P(A_r) &= \frac{2^{R_{u_r q}} - 1}{2^{R_{max}}} (or\ \alpha_{R_{u_r q}}) \\
P(E_r = 1 \mid I = I_k) &= \gamma_{rk}
\end{aligned}
$$

3 Satisfaction Modeling

According to [1]:

$$
\begin{aligned}
uMetric_k &= \sum_{r=1}^{N} P(C_r = 1 \mid I = I_k) \cdot R_r \\
rrMetric_k &= \sum_{r=1}^{N} P(S_r = 1 \mid I = I_k) \cdot \frac{1}{r} = \sum_{r=1}^{N} \sigma_{R_{u_r q} k} \cdot P(C_r = 1 \mid I = I_k) \cdot \frac{1}{r}
\end{aligned}
$$

$$
\begin{cases}
uSat = \sum_{k=1}^{K} P(I = I_k) \cdot \beta_k \cdot (\sum_{r=1}^{N} P(C_r = 1 \mid I = I_k) \cdot R_r + \psi_k) \\
rrSat = \sum_{k=1}^{K} P(I = I_k) \cdot \beta_k \cdot (\sum_{r=1}^{N} P(S_r = 1 \mid I = I_k) \cdot \frac{1}{r} + \psi_k) \\
\qquad = \sum_{k=1}^{K} P(I = I_k) \cdot \beta_k \cdot (\sum_{r=1}^{N} \sigma_{R_{u_r q} k} \cdot P(C_r = 1 \mid I = I_k) \cdot \frac{1}{r} + \psi_k)
\end{cases}
$$

4 Preparation

- Observations: Ω, C, R, S;
- Parameters: $\pi_{\omega,k}$,$i_{\omega,k}$, $\sigma_{R_{u_r q} k}$, γ_k, ψ_k, β_k;

5 Probabilities

5.1 DBN

$$
\begin{aligned}
&P(C_u = 1) = \alpha_{R_{u_r q}} \epsilon_{r_u}, \\
&\text{where } \epsilon_{r+1} = \sum_{k=1}^{K} i_{\omega,k} \gamma_k \epsilon_{<r,k>} \cdot [\alpha_{R_{u_r q}} (1 - \sigma_{R_{u_r q} k}) + (1 - \alpha_{R_{u_r q}})] \\
&= \sum_{k=1}^{K} i_{\omega,k} \gamma_k \epsilon_{<r,k>} \cdot (1 - \alpha_{R_{u_r q}} \sigma_{R_{u_r q} k}) \\
&\text{and } \epsilon_{<r+1,k>} = \gamma_k \cdot (1 - \alpha_{R_{u_r q}} \sigma_{R_{u_r q} k}) \cdot \epsilon_{<r,k>}, \ \epsilon_{<1,k>} = 1, \ \forall k \\
&P(C_u = 1 \mid \mathbf{C}_{<r_u}) = \alpha_{R_{u_r q}} \epsilon_{r_u}, \\
&\text{where } \epsilon_{r+1} = \sum_{k=1}^{K} i_{\omega,k} \gamma_k \cdot [c_r^{(s)} \cdot (1 - \sigma_{R_{u_r q} k}) + (1 - c_r^{(s)}) \cdot \frac{(1 - \alpha_{R_{u_r q}}) \cdot \epsilon_{<r,k>}}{1 - \alpha_{R_{u_r q}} \epsilon_{<r,k>}}] \\
&\text{and } \epsilon_{<r+1,k>} = \gamma_k \cdot [c_r^{(s)} \cdot (1 - \sigma_{R_{u_r q} k}) + (1 - c_r^{(s)}) \cdot \frac{(1 - \alpha_{R_{u_r q}}) \cdot \epsilon_{<r,k>}}{1 - \alpha_{R_{u_r q}} \epsilon_{<r,k>}}], \ \epsilon_{<1,k>} = 1, \ \forall k
\end{aligned}
$$

5.2 UBM

$$
\begin{aligned}
&P(C_u = 1 \mid I = I_k) = \sum_{i=0}^{r_u - 1} \cdot P(C_i = 1 \mid I = I_k) \cdot (\prod_{j=i+1}^{r_u - 1} (1 - \alpha_{R_{u_j q}} \gamma_{jik})) \cdot \alpha_{R_{u_r q}} \gamma_{r_u i k}, \\
&\text{where } P(C_0 = 1) = 1 \\
&P(C_u = 1 \mid \mathbf{C}_{<r_u}, I = I_k) = \alpha_{R_{u_r q}} \gamma_{r r' k}
\end{aligned}
$$

5.3 PBM

$$
\begin{aligned}
P(C_u = 1 \mid I = I_k) &= \alpha_{R_{u_r q}} \gamma_{rk} \\
P(C_u = 1 \mid \mathbf{C}_{<r_u}, I = I_k) &= \alpha_{R_{u_r q}} \gamma_{rk}
\end{aligned}
$$

6 Parameter Estimation

$$
\begin{aligned}
&\min_{\Theta} f(\Theta) \\
&\mathit{where} \ f(\Theta) = (1 - \lambda) \cdot \mathcal{L}_1 + \lambda \mathcal{L}_2, \\
&\quad \mathcal{L}_1 = -LL_1, \ \mathcal{L}_2 = -LL_2 \ \mathit{or} \ \sum_{s \in S} ||Sat^{(s)} - sat^{(s)}||^2
\end{aligned}
$$

6.1 DBN

For LL_1:

$$LL_1 = \sum_{s \in S} \log(\prod_{r=1}^{N} P(C_r = c_r^{(s)} \mid \mathbf{C}_{<r}^{(s)}))$$

$$= \sum_{s \in S} \sum_{r=1}^{N} \log(\alpha_{R_{urq}} \epsilon_r \cdot c_r^{(s)} + (1 - \alpha_{R_{urq}} \epsilon_r) \cdot (1 - c_r^{(s)}))$$

(1) $$\frac{\partial LL_1}{\partial \sigma_{Rk}} = \sum_{s \in S} \sum_{r=1}^{N} [\frac{1}{\epsilon_r^{(s)}} \cdot c_r^{(s)} - (\frac{\alpha_{R_{urq}}}{1 - \alpha_{R_{urq}} \epsilon_r^{(s)}}) \cdot (1 - c_r^{(s)})] \cdot i_{\omega,k} \cdot \frac{\partial \epsilon_{<r,k>}^{(s)}}{\partial \sigma_{Rk}},$$

where $\frac{\partial \epsilon_{<r,k>}^{(s)}}{\partial \sigma_{Rk}} = 0, r \leq min\_r(R)$

and $\frac{\partial \epsilon_{<r+1,k>}^{(s)}}{\partial \sigma_{Rk}} = \gamma_k \cdot [-c_r^{(s)} \cdot \mathcal{I}(R_{urq} = R) + (1 - c_r^{(s)}) \cdot \frac{1 - \alpha_{R_{urq}}}{(1 - \alpha_{R_{urq}} \epsilon_{<r,k>}^{(s)})^2} \cdot \frac{\partial \epsilon_{<r,k>}^{(s)}}{\partial \sigma_{Rk}}]$

(2) $$\frac{\partial LL_1}{\partial \gamma_k} = \sum_{s \in S} \sum_{r=1}^{N} [\frac{1}{\epsilon_r^{(s)}} \cdot c_r^{(s)} - (\frac{\alpha_{R_{urq}}}{1 - \alpha_{R_{urq}} \epsilon_r^{(s)}}) \cdot (1 - c_r^{(s)})] \cdot i_{\omega,k} \cdot \frac{\partial \epsilon_{<r,k>}^{(s)}}{\partial \gamma_k},$$

where $\frac{\partial \epsilon_{<1,k>}^{(s)}}{\partial \gamma_k} = 0,$

and $\frac{\partial \epsilon_{<r+1,k>}^{(s)}}{\partial \gamma_k} = [c_r^{(s)}(1 - \sigma_{R_{urq}k}) + (1 - c_r^{(s)})(1 - \alpha_{R_{urq}}) \frac{\partial g(\gamma_k)}{\partial \gamma_k}],$

$$g(\gamma_k) = \frac{\gamma_k \epsilon_{<r,k>}^{(s)}}{1 - \alpha_{R_{urq}} \epsilon_{<r,k>}^{(s)}}, \therefore \frac{\partial g(\gamma_k)}{\partial \gamma_k} = \frac{\epsilon_{<r,k>}^{(s)} - (\epsilon_{<r,k>}^{(s)})^2 \alpha_{R_{urq}} + \gamma_k \cdot \frac{\partial \epsilon_{<r,k>}^{(s)}}{\partial \gamma_k}}{(1 - \alpha_{R_{urq}} \epsilon_{<r,k>}^{(s)})^2}$$

(3) $$\frac{\partial LL_1}{\partial \pi_{\omega,k}} = \frac{\partial LL_1}{\partial i_{\omega,k}} \cdot \frac{\partial i_{\omega,k}}{\partial \pi_{\omega,k}}$$

$$\frac{\partial LL_1}{\partial i_{\omega,k}} = \sum_{s \in S} \sum_{r=1}^{N} [\frac{1}{\epsilon_r^{(s)}} \cdot c_r^{(s)} - (\frac{\alpha_{R_{urq}}}{1 - \alpha_{R_{urq}} \epsilon_r^{(s)}}) \cdot (1 - c_r^{(s)})] \cdot \epsilon_{<r,k>},$$

where $\frac{\partial i_{\omega,k}}{\partial \pi_{\omega,k}} = \frac{e^{\pi_{\omega,k}} \cdot \sum_{j=1}^{K} \pi_{\omega j} - e^{2\pi_{\omega,k}}}{(\sum_{j=1}^{K} \pi_{\omega j})^2}$

For uSat:

$$L_2 = \sum_{s \in S} ||Sat^{(s)} - sat^{(s)}||^2$$

$$uSat = \sum_{k=1}^{K} i_{\omega,k} \beta_k \cdot (\sum_{r=1}^{N} \alpha_{R_{urq}} P(E_r = 1 \mid I = I_k) \cdot R_r + \psi_k)$$

$$= \sum_{k=1}^{K} i_{\omega,k} \beta_k \cdot (\sum_{r=1}^{N} \alpha_{R_{urq}} \epsilon_{<r,k>} \cdot R_r + \psi_k)$$

$$(1)\frac{\partial uL_2}{\partial \beta_k} = 2\cdot\sum_{s\in S}\Delta uSat \cdot i_{\omega,k}(\sum_{r=1}^{N}\alpha_{R_{urq}}\epsilon_{<r,k>}R_r+\psi_k), \Delta uSat = (Sat^{(s)}-sat^{(s)})$$

$$(2)\frac{\partial uL_2}{\partial \psi_k} = 2\cdot\sum_{s\in S}\Delta uSat \cdot i_{\omega_k}\beta_k$$

$$(3)\frac{\partial uL_2}{\partial \sigma_{Rk}} = 2\cdot\sum_{s\in S}\Delta uSat \cdot i_{\omega,k}\beta_k\cdot\sum_{r=1}^{N}[\alpha_{R_{urq}}R_r\frac{\partial\epsilon_{<r,k>}}{\partial\sigma_{Rk}}]$$

$$,\text{where } \frac{\partial\epsilon_{<r,k>}}{\partial\sigma_{Rk}} = 0, r\leq min\_r(R)$$

$$,\text{and } \frac{\partial\epsilon_{<r+1,k>}}{\partial\sigma_{Rk}} = [-\mathcal{I}(R=R_{urq})\cdot\gamma_k\alpha_{R_{urq}}\epsilon_{<r,k>}+\gamma_k(1-\alpha_{R_{urq}}\sigma_{R_{urq}k})\cdot\frac{\partial\epsilon_{<r,k>}}{\partial\sigma_{Rk}}], r>min\_r(R)$$

$$(4)\frac{\partial uL_2}{\partial \gamma_k} = 2\cdot\sum_{s\in S}\Delta uSat \cdot i_{\omega,k}\beta_k\cdot\sum_{r=1}^{N}[\alpha_{R_{urq}}R_r\cdot\frac{\partial\epsilon_{<r,k>}}{\partial\gamma_k})]$$

$$,\ \text{where } \frac{\partial\epsilon_{<1,k>}}{\partial\gamma_k}=0$$

$$,\ \text{and } \frac{\partial\epsilon_{<r+1,k>}}{\partial\gamma_k} = (1-\alpha_{R_{urq}}\sigma_{R_{urq}k})(\epsilon_r^{(s)}+\gamma_k\frac{\partial\epsilon_{<r,k>}}{\partial\gamma_k})$$

$$(5)\frac{\partial uL_2}{\partial \pi_{\omega,k}} = 2\cdot\sum_{s\in S}\Delta uSat \cdot \beta_k\cdot(\sum_{r=1}^{N}\alpha_{R_{urq}}R_r\cdot\epsilon_{<r,k>}+\psi_k)\cdot\frac{\partial i_{\omega_k}}{\partial\pi_{\omega_k}}$$

For rrSat:

$$rrSat = \sum_{k=1}^{K} i_{\omega,k}\beta_k\cdot(\sum_{r=1}^{N}\sigma_{R_{urq}k}\alpha_{R_{urq}}\epsilon_{<r,k>}\cdot\frac{1}{r}+\psi_k)$$

$$(1)\frac{\partial rrL_2}{\partial \beta_k} = 2\cdot\sum_{s\in S}\Delta rrSat\cdot i_{\omega,k}\cdot(\sum_{r=1}^{N}\sigma_{R_{urq}k}\alpha_{R_{urq}}\cdot\epsilon_{<r,k>}\cdot\frac{1}{r}+\psi_k)$$

$$(2)\frac{\partial rrL_2}{\partial \psi_k} = 2\cdot\sum_{s\in S}\Delta rrSat\cdot i_{\omega_k}\beta_k$$

$$(3)\frac{\partial rrL_2}{\partial \sigma_{Rk}} = 2\cdot\sum_{s\in S}\Delta rrSat\cdot i_{\omega,k}\beta_k\cdot\sum_{r=1}^{N}\alpha_{R_{urq}}\cdot\frac{1}{r}\cdot(\mathcal{I}(R=R_{urq})\cdot\epsilon_{<r,k>}+\alpha_{R_{urq}}\cdot\frac{\partial\epsilon_{<r,k>}}{\partial\sigma_{Rk}})$$

$$,\ \text{where}\frac{\partial\epsilon_{<r,k>}}{\partial\alpha_R} = 0, r\leq min\_r(R)$$

$$,\ \text{and } \frac{\partial\epsilon_{<r+1,k>}}{\partial\alpha_R} = [-\mathcal{I}(R=R_{urq})\cdot\gamma_k\sigma_{R_{urq}k}\epsilon_{<r,k>}-\gamma_k\alpha_{R_{urq}}\sigma_{R_{urq}k}\cdot\frac{\partial\epsilon_{<r,k>}}{\partial\alpha_R}], r>min\_r(R)$$

$$(4)\frac{\partial rrL_2}{\partial \gamma_k} = 2\cdot\sum_{s\in S}\Delta rrSat\cdot i_{\omega,k}\beta_k\cdot\sum_{r=1}^{N}\sigma_{R_{urq}k}\alpha_{R_{urq}}\cdot\frac{1}{r}\cdot\frac{\partial\epsilon_{<r,k>}}{\partial\gamma_k}$$

$$,\ \text{where } \frac{\partial\epsilon_{<1,k>}}{\partial\gamma_k}=0$$

$$,\ \text{and } \frac{\partial\epsilon_{<r+1,k>}}{\partial\gamma_k} = (1-\alpha_{R_{urq}}\sigma_{R_{urq}k})(\epsilon_r^{(s)}+\gamma_k\frac{\partial\epsilon_{<r,k>}}{\partial\gamma_k})$$

$$(5)\frac{\partial rrL_2}{\partial \pi_{\omega,k}} = 2\cdot\sum_{s\in S}\Delta rrSat\cdot\beta_k\cdot(\sum_{r=1}^{N}\sigma_{R_{urq}k}\alpha_{R_{urq}}\epsilon_{<r,k>}\cdot\frac{1}{r}+\psi_k)\cdot\frac{\partial i_{\omega_k}}{\partial\pi_{\omega_k}}$$

6.2 UBM

For LL_1:

$$P(C_u = 1 \mid I = I_k) = \sum_{i=0}^{r_u-1} \cdot P(C_i = 1 \mid I = I_k) \cdot (\prod_{j=i+1}^{r_u-1} (1 - \alpha_{R_{u_j q}} \gamma_{jik})) \cdot \alpha_{R_{u_r q}} \gamma_{r_u ik},$$

where $P(C_0 = 1 \mid I = I_k) = 1, \forall k$

$$P(C_u = 1 \mid \mathbf{C}_{<r_u}, I = I_k) = \alpha_{R_{u_r q}} \gamma_{rr'k}$$

$$LL_1 = \sum_{s \in S} \log(\prod_{r=1}^{N} P(C_r = c_r^{(s)} \mid \mathbf{C}_{<r}^{(s)}))$$

$$= \sum_{s \in S} \sum_{r=1}^{N} \log(\alpha_{R_{u_r q}} \cdot \sum_{k=1}^{K} (i_{\omega,k} \gamma_{r_u r'_u k}) \cdot c_r^{(s)} + (1 - \alpha_{R_{u_r q}} \cdot \sum_{k=1}^{K} (i_{\omega,k} \gamma_{r_u r'_u k})) \cdot (1 - c_r^{(s)}))$$

$$(1) \frac{\partial LL_1}{\partial \gamma_{rr'k}} = \sum_{s \in S} \sum_{r=1}^{N} [\frac{i_{\omega,k}}{\sum_{k=1}^{K} (i_{\omega,k} \gamma_{r_u r'_u k})} \cdot c_r^{(s)} + \frac{-\alpha_{R_{u_r q}} \cdot i_{\omega,k}}{1 - \alpha_{R_{u_r q}} \sum_{k=1}^{K} (i_{\omega,k} \gamma_{r_u r'_u k})} \cdot (1 - c_r^{(s)})] \cdot \mathcal{I}(r_u = r, r'_u = r')$$

$$(2) \frac{\partial LL_1}{\partial \pi_{\omega,k}} = \sum_{s \in S} \sum_{r=1}^{N} [\frac{\gamma_{r_u r'_u k}}{\sum_{k=1}^{K} (i_{\omega,k} \gamma_{r_u r'_u k})} \cdot c_r^{(s)} + \frac{-\alpha_{R_{u_r q}} \cdot \gamma_{r_u r'_u k}}{1 - \alpha_{R_{u_r q}} \sum_{k=1}^{K} (i_{\omega,k} \gamma_{r_u r'_u k})} \cdot (1 - c_r^{(s)})] \cdot \frac{\partial i_{\omega,k}}{\partial \pi_{\omega,k}}$$

For uSat:

$$(1) \frac{\partial L_2}{\partial \gamma_{rr'k}} = 2 \cdot \sum_{s \in S} \Delta uSat \cdot i_{\omega,k} \beta_k \cdot (\sum_{r=1}^{N} \frac{\partial P(C_r = 1 \mid I = I_k)}{\partial \gamma_{rr'k}} \cdot R_r)$$

where $\dfrac{\partial P(C_0 = 1 \mid I = I_k)}{\partial \gamma_{rr'k}} = 0, \ \forall r, r', k$

$$\frac{\partial P(C_r = 1 \mid I = I_k)}{\partial \gamma_{rr'k}}$$

$$= 0, \text{ if } r > r_u$$

$$= \sum_{i=0}^{r_u-1} P(C_i = 1 \mid I = I_k) \cdot [\mathcal{I}(i = r') \cdot \prod_{j=i+1}^{r_u-1} (1 - \alpha_{R_{u_j q}} \cdot \gamma_{jik}) \cdot \alpha_{R_{u_r q}}], \text{ if } r = r_u$$

$$= \sum_{i=0}^{r_u-1} \{ \frac{\partial P(C_i = 1 \mid I = I_k)}{\partial \gamma_{rr'k}} \cdot \prod_{j=i+1}^{r_u-1} (1 - \alpha_{R_{u_j q}} \gamma_{jik}) \cdot \alpha_{R_{u_r q}} \gamma_{r_u ik}$$

$$+ P(C_i = 1 \mid I = I_k) \cdot [\mathcal{I}(condition) \cdot \prod_{j=i+1, j \neq r, i \neq r'}^{r_u-1} (1 - \alpha_{R_{u_j q}} \gamma_{jik}) \cdot (-\alpha_{R_{u_r q}})] \}, \text{ if } r < r_u$$

$condition: \exists \ i, j, \ s.t. \ j = r, i = r'.$

$$(2) \frac{\partial L_2}{\partial \pi_{\omega,k}} = 2 \cdot \sum_{s \in S} \Delta uSat \cdot \beta_k \cdot (\sum_{r=1}^{N} P(C_r = 1 \mid I = I_k) \cdot R_r + \psi_k) \cdot \frac{\partial i_{\omega,k}}{\partial \pi_{\omega,k}}$$

$$(3) \frac{\partial L_2}{\partial \beta_k} = 2 \cdot \sum_{s \in S} \Delta uSat \cdot i_{\omega,k} \cdot (\sum_{r=1}^{N} P(C_r = 1 \mid I = I_k) \cdot R_r + \psi_k)$$

$$(4) \frac{\partial L_2}{\partial \psi_k} = 2 \cdot \sum_{s \in S} \Delta uSat \cdot i_{\omega,k} \beta_k$$

6.3 PBM

For LL_1:

$$
\begin{aligned}
LL_1 &= \sum_{s\in S}\log(\prod_{r=1}^{N} P(C_r = c_r^{(s)} \mid \mathbf{C}_{<r}^{(s)})) \\
&= \sum_{s\in S}\sum_{r=1}^{N}\log(\alpha_{R_{u_r q}} \cdot \sum_{k=1}^{K}(i_{\omega,k}\gamma_{r_u k}) \cdot c_r^{(s)} + (1-\alpha_{R_{u_r q}} \cdot \sum_{k=1}^{K}(i_{\omega,k}\gamma_{r_u k})) \cdot (1-c_r^{(s)})) \\
(1)\frac{\partial LL_1}{\partial \gamma_{rk}} &= \sum_{s\in S}\sum_{r=1}^{N}[\frac{i_{\omega,k}}{\sum_{k=1}^{K}(i_{\omega,k}\gamma_{r_u k})} \cdot c_r^{(s)} + \frac{-\alpha_{R_{u_r q}} \cdot i_{\omega,k}}{1-\alpha_{R_{u_r q}}\sum_{k=1}^{K}(i_{\omega,k}\gamma_{r_u k})} \cdot (1-c_r^{(s)})] \cdot \mathcal{I}(r_u = r) \\
(2)\frac{\partial LL_1}{\partial \pi_{\omega,k}} &= \sum_{s\in S}\sum_{r=1}^{N}[\frac{\gamma_{r_u k}}{\sum_{k=1}^{K}(i_{\omega,k}\gamma_{r_u k})} \cdot c_r^{(s)} + \frac{-\alpha_{R_{u_r q}} \cdot \gamma_{r_u k}}{1-\alpha_{R_{u_r q}}\sum_{k=1}^{K}(i_{\omega,k}\gamma_{r_u k})} \cdot (1-c_r^{(s)})] \cdot \frac{\partial i_{\omega,k}}{\partial \pi_{\omega,k}}
\end{aligned}
$$

For uSat:

$$
\begin{aligned}
&(1)\frac{\partial L_2}{\partial \gamma_{rk}} = 2\cdot\sum_{s\in S}\Delta uSat \cdot \beta_k \cdot \alpha_{R_{u_r q}} \cdot R_r \\
&(2)\frac{\partial L_2}{\partial \pi_{\omega,k}} = 2\cdot\sum_{s\in S}\Delta uSat \cdot \beta_k \cdot (\sum_{r=1}^{N} P(C_r = 1 \mid I = I_k) \cdot R_r + \psi_k) \cdot \frac{\partial i_{\omega,k}}{\partial \pi_{\omega,k}} \\
&(3)\frac{\partial L_2}{\partial \beta_k} = 2\cdot\sum_{s\in S}\Delta uSat \cdot i_{\omega,k} \cdot (\sum_{r=1}^{N} P(C_r = 1 \mid I = I_k) \cdot R_r + \psi_k) \\
&(4)\frac{\partial L_2}{\partial \psi_k} = 2\cdot\sum_{s\in S}\Delta uSat \cdot i_{\omega,k}\beta_k
\end{aligned}
$$

Reference

Chuklin, Aleksandr, Pavel Serdyukov, and Maarten De Rijke. "Click model-based information retrieval metrics." Proceedings of the 36th international ACM SIGIR conference on Research and development in information retrieval. 2013.

Chen, Jia, et al. "Incorporating Query Reformulating Behavior into Web Search Evaluation." Proceedings of the 30th ACM International Conference on Information and Knowledge Management. 2021.