Sardar Patel Institute of Technology,Mumbai
Department of Electronics and Telecommunication Engineering
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ETL54-Statistical Computational Laboratory

Lab-2: Probability Distributions

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Objective:To compute probability density function (pdf) and cumulative distribution function (cdf)

Outcomes:

1. To list and describe the well-known probability distributions with their characteristics.
2. To compute the probability distributions which are frequently occurs in Statistical Study

System Requirements: Ubuntu OS with R and RStudio installed

Introduction to Probability distribution:

A probability distribution describes how the values of a random variable is distributed. There are two types of probability distributions: Discrete and Continuous Well-known probability distributions which are frequently occurred in statistical study:

- Binomial Distribution
- Poisson Distribution
- Continuous Uniform Distribution
- Exponential Distribution
- Normal Distribution
- Chi-squared Distribution
- Student t Distribution
- F Distribution

R Functions for Probability Distributions:

Every distribution that R handles has four functions. There is a root name, for example, the root name for the normal distribution is norm. This root is prefixed by one of the letters

p for "probability", the cumulative distribution function (c. d. f.)
q for "quantile", the inverse c. d. f.
d for "density", the density function (p. f. or p. d. f.)
r for "random", a random variable having the specified distribution

For the normal distribution, these functions are pnorm, qnorm, dnorm, and rnorm. For the binomial distribution, these functions are pbinom, qbinom, dbinom, and rbinom. And so forth.

For a continuous distribution (like the normal), the most useful functions for doing problems involving probability calculations are the "p" and "q" functions (c. d. f. and inverse c. d. f.), because the the density (p. d. f.) calculated by the "d" function can only be used to calculate probabilities via integrals and R doesn't do integrals.

For a discrete distribution (like the binomial), the "d" function calculates the density (p. f.), which in this case is a probability
$f(x) = P(X = x)$
and hence is useful in calculating probabilities.
R has functions to handle many probability distributions. The table below gives the names of the functions for each distribution.
Table-1:Probability Distributions

Distribution	Functions			
Binomial	pbinom	qbinom	dbinom	rbinom
Cauchy	pcauchy	qcauchy	dcauchy	rcauchy
Chi-Square	pchisq	qchisq	dchisq	rchisq
Exponential	pexp	qexp	dexp	rexp
F	pf	qf	df	rf
Gamma	pgamma	qgamma	dgamma	rgamma
Geometric	pgeom	qgeom	dgeom	rgeom
Hypergeometric	phyper	qhyper	dhyper	rhyper
Logistic	plogis	qlogis	dlogis	rlogis
Log Normal	plnorm	qlnorm	dlnorm	rlnorm
Normal	pnorm	qnorm	dnorm	rnorm
Poisson	ppois	qpois	dpois	rpois
Student t	pt	qt	dt	rt
Uniform	punif	qunif	dunif	runif
Weibull	pweibull	qweibull	dweibull	rweibull

Procedure:
1. Open RStudio
2. Go to RConsole (>)
3. Probability distribution in R

```
>help(rnorm) #The normal Distribution
>help(dbinom) # The Binomial Distribution
```

Probability Distributions in R:
In R, probability functions take the form
[dpqr]distribution_abbreviation ()
where the first letter refers to the aspect of the distribution returned:
d = density
p = distribution function
q = quantile function
r = random generation (random deviates)

1. Binomial Distribution

The binomial distribution is a discrete probability distribution. It describes the outcome of n independent trials in an experiment. Each trial is assumed to have only two outcomes, either success or failure. If the probability of a successful trial is p, then the probability of having x successful outcomes in an experiment of n independent trials is as follows.

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)} \quad where\ x = 0, 1, 2, ..., n$$

Problem

Suppose there are twelve multiple choice questions in an English class quiz. Each question has five possible answers, and only one of them is correct. Find the probability of having four or less correct answers if a student attempts to answer every question at random.

Example Solution:

Since only one out of five possible answers is correct, the probability of answering a question correctly by random is 1/5=0.2. We can find the probability of having exactly 4 correct answers by random attempts as follows.

```
> dbinom(4, size=12, prob=0.2)
[1] 0.1329
```

To find the probability of having four or less correct answers by random attempts, we apply the function dbinom with x = 0,…,4.

```
> dbinom(0, size=12, prob=0.2) +
+ dbinom(1, size=12, prob=0.2) +
+ dbinom(2, size=12, prob=0.2) +
+ dbinom(3, size=12, prob=0.2) +
+ dbinom(4, size=12, prob=0.2)
[1] 0.9274
```

Alternatively, we can use the cumulative probability function for binomial distribution pbinom.

```
> pbinom(4, size=12, prob=0.2)
[1] 0.92744
```

Answer: The probability of four or less questions answered correctly by random in a twelve question multiple choice quiz is 92.7%.

2. Poisson Distribution

The Poisson distribution is the probability distribution of independent event occurrences in an interval. If λ is the mean occurrence per interval, then the probability of having x occurrences within a given interval is:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad where\ x = 0, 1, 2, 3, ...$$

Problem

If there are twelve cars crossing a bridge per minute on average, find the probability of having seventeen or more cars crossing the bridge in a particular minute.

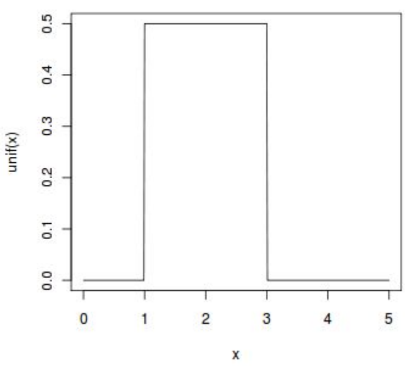
Answer:
>ppois(16, 12)
[1] 0.898709
> 1-ppois(16, 12)
[1] 0.101291

3. **Continuous Uniform Distribution**

The continuous uniform distribution is the probability distribution of random number selection from the continuous interval between a and b. Its density function is defined by the following.

$$f(x) = \begin{cases} \frac{1}{b-a} & when\ a \le x \le b \\ 0 & when\ x < a\ or\ x > b \end{cases}$$

Here is a graph of the continuous uniform distribution with a = 1, b = 3.



Problem

Select ten random numbers between one and three.

Answer:

>runif(10,1,3)

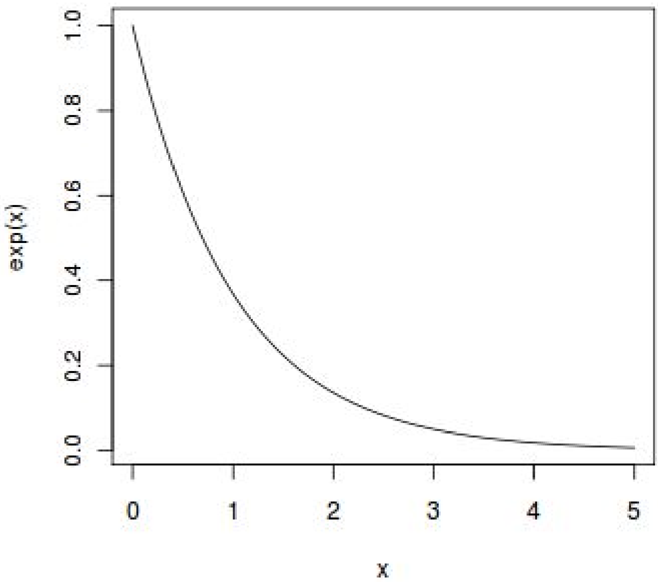
[1] 2.014956 1.613537 1.853815 2.386204 1.170272 1.450873 1.549061 1.544610 2.231659 1.859343

4. **Exponential Distribution**

The exponential distribution describes the arrival time of a randomly recurring independent event sequence. If μ is the mean waiting time for the next event recurrence, its probability density function is:

$$f(x) = \begin{cases} \frac{1}{\mu}e^{-x/\mu} & when\ x \ge 0 \\ 0 & when\ x < 0 \end{cases}$$

Here is a graph of the exponential distribution with $\mu = 1$.



Problem

Suppose the mean checkout time of a supermarket cashier is three minutes. Find the probability of a customer checkout being completed by the cashier in less than two minutes.

Answer:

```
>pexp(2,1/3)

[1] 0.4865829
```

5.Normal Distribution

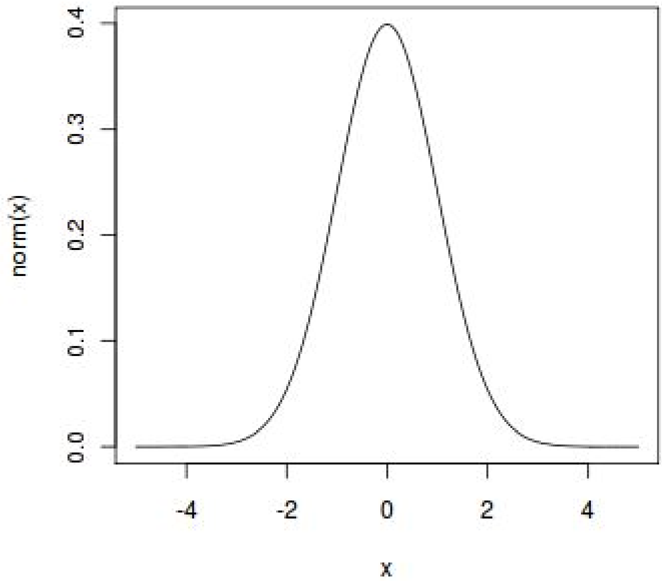
The normal distribution is defined by the following probability density function, where μ is the population mean and σ^2 is the variance.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

If a random variable X follows the normal distribution, then we write:

$$X \sim N(\mu, \sigma^2)$$

In particular, the normal distribution with $\mu = 0$ and $\sigma = 1$ is called the standard normal distribution, and is denoted as N(0,1). It can be graphed as follows.



The normal distribution is important because of the Central Limit Theorem, which states that the population of all possible samples of size n from a population with mean μ and variance σ^2 approaches a normal distribution with mean μ and σ^2/n when n approaches infinity.

Problem

Assume that the test scores of a college entrance exam fits a normal distribution. Furthermore, the mean test score is 72, and the standard deviation is 15.2. What is the percentage of students scoring 84 or more in the exam?

Answer:

> pnorm(84,72,15.2)

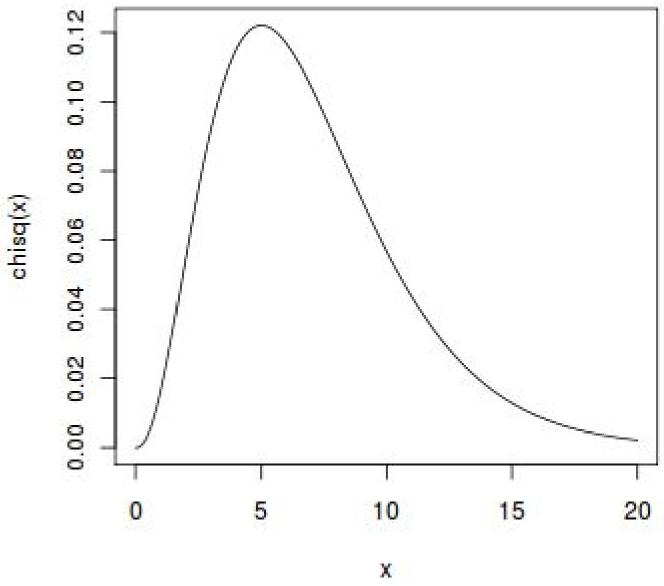
[1] 0.7850824

6.Chi-squared Distribution

If $X_1, X_2, \ldots, X_m$ are m independent random variables having the standard normal distribution, then the following quantity follows a Chi-Squared distribution with m degrees of freedom. Its mean is m, and its variance is 2m.

$$V = X_1^2 + X_2^2 + \cdots + X_m^2 \sim \chi^2_{(m)}$$

Here is a graph of the Chi-Squared distribution 7 degrees of freedom.



Problem

Find the 95th percentile of the Chi-Squared distribution with 7 degrees of freedom.

Answer:

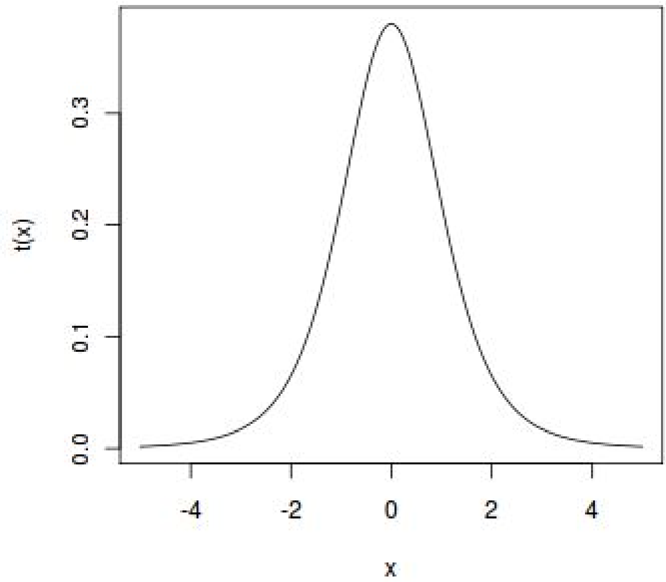
```
> qchisq(.95,7)

[1] 14.06714
```

8.Student t Distribution

Assume that a random variable Z has the standard normal distribution, and another random variable V has the Chi-Squared distribution with m degrees of freedom. Assume further that Z and V are independent, then the following quantity follows a Student t distribution with m degrees of freedom.

$$t = \frac{Z}{\sqrt{V/m}} \sim t_{(m)}$$

Here is a graph of the Student t distribution with 5 degrees of freedom.



Problem

Find the 2.5th and 97.5th percentiles of the Student t distribution with 5 degrees of freedom.

Answer:

> qt(0.025,5)

[1] -2.570582

> qt(0.975,5)

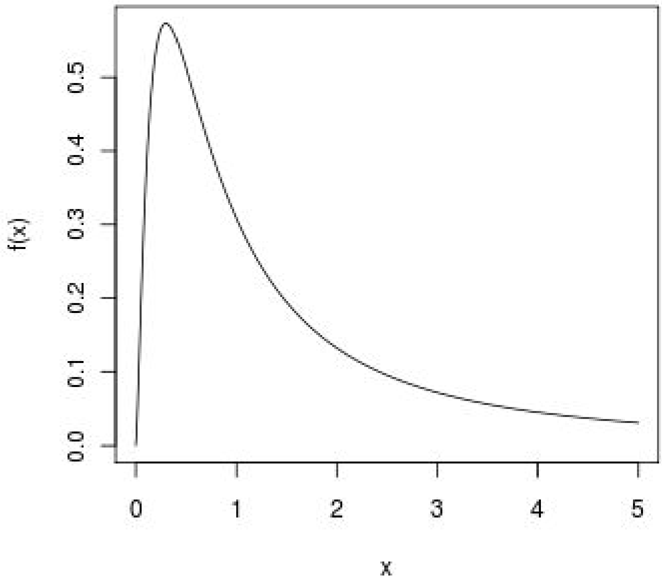
[1] 2.570582

8. F Distribution

If V_1 and V_2 are two independent random variables having the Chi-Squared distribution with m_1 and m_2 degrees of freedom respectively, then the following quantity follows an F distribution with m_1 numerator degrees of freedom and m_2 denominator degrees of freedom, i.e., (m_1,m_2) degrees of freedom.

$$F = \frac{V_1/m_1}{V_2/m_2} \sim F_{(m_1,m_2)}$$

Here is a graph of the F distribution with (5, 2) degrees of freedom.



Problem

Find the 95th percentile of the F distribution with (5, 2) degrees of freedom.

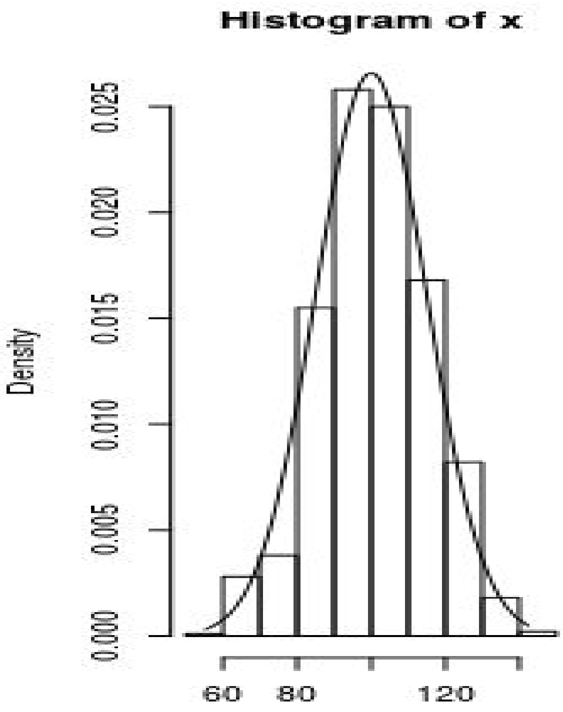
Answer:

```
>qf(.95,5,2)
[1] 19.29641
```

Describe the following with respect to probability distributions:

1.

```
>x <- rnorm(1000, mean=100, sd=15)
>hist(x, probability=TRUE)
>xx <- seq(min(x), max(x), length=100)
>lines(xx, dnorm(xx, mean=100, sd=15))
```



2. What is ***P*(*X* > 19)** when *X* has the **N(17.46, 375.67)** distribution?

ANSWER:

```
> pnorm(19, mean=17.46, sd=sqrt(375.67))
[1] 0.5316644
```

3. Interpret the following

```
> pnorm(1.96, lower.tail=TRUE)
[1] 0.9750021
> pnorm(1.96, lower.tail=FALSE)
[1] 0.0249979
```

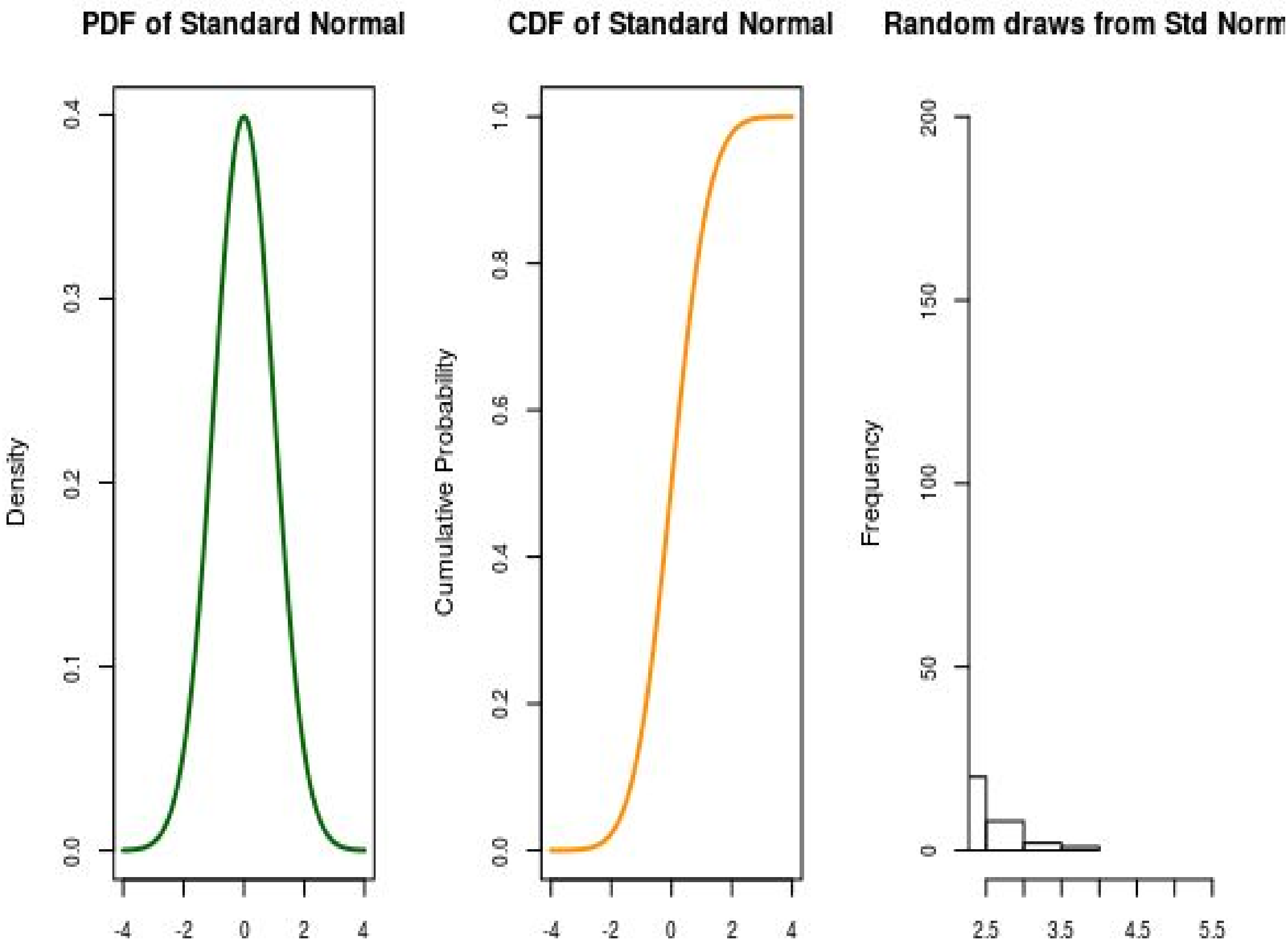
ANSWER:

1. The function pnorm creates a normal distribution of the vector of quantile=1.96 having mean=0 and standard deviation=1
2. when lower.tail=TRUE, probabilities are P[X ≤ x]
3. when lower.tail=FALSE, probabilities are P[X > x].

4. Run this in RStudio Script editor and explain it from plot

```
set.seed(3000)
xseq<- seq(-4,4,.01)
densities<- dnorm(xseq, 0,1)
cumulative<- pnorm(xseq, 0, 1)
randomdeviates<- rnorm(1000,0,1)
 par(mfrow=c(1,3), mar=c(3,4,4,2))
plot(xseq, densities, col="darkgreen",xlab="", ylab="Density", type="l",lwd=2,
cex=2, main="PDF of Standard Normal", cex.axis=.8)
plot(xseq, cumulative, col="darkorange", xlab="", ylab="Cumulative
Probability",type="l",lwd=2, cex=2, main="CDF of Standard Normal", cex.axis=.8)
hist(randomdeviates, main="Random draws from Std Normal", cex.axis=.8,
xlim=c(4,4))
```

ANSWER:



Conclusion:

1. In this experiment we learned about various probability density functions like binomial, poissons, continuous uniform, normal, exponential, chi-squared, student t and f distribution
2. we even applied the various forms of probability distributions in various real life applications and numericals
3. we learned about the different arguments accepted by the distribution functions and used it as per our need
4. we computed the various probability distributions which frequently occurs in Statistical Study
5. we learned normal distribution, and its variety of functions like pnorm, qnorm, dnorm, and rnorm. For the binomial distribution, these functions are pbinom, qbinom, dbinom, and rbinom. And so forth.