Sardar Patel Institute of Technology,Mumbai

Department of Electronics and Telecommunication Engineering

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ETL54-Statistical Computational Laboratory

Lab-3: Regression Analysis and Modeling

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Objective: To carry out linear regression (including multiple regression) and build a regression model

Outcomes:

1. To carry out linear regression (including multiple regression)
2. To build a regression model using both forward and backward step wise processes
3. To plot regression models
4. To add lines of best-fit to regression plots

System Requirements: Ubuntu OS with R and RStudio installed

Introduction to Linear Regression

Regression analysis is a statistical tool to determine relationships between different types of variables. Variables that remain unaffected by changes made in other variables are known as *independent variables*, also known as a *predictor* or *explanatory variables* while those that are affected are known as *dependent variables* also known as the *response variable*.

Linear regression is a statistical procedure which is used to predict the value of a response variable, on the basis of one or more predictor variables.

There are two types of linear regressions in R:

- **Simple Linear Regression –** Value of response variable depends on a single explanatory variable.
- **Multiple Linear Regression –** Value of response variable depends on more than 1 explanatory variables.

Some common examples of linear regression are calculating GDP, CAPM, oil and gas prices, medical diagnosis, capital asset pricing etc.

Simple Linear Regression in R

R Simple linear regression enables us to find a relationship between a continuous dependent variable Y and a continuous independent variable X. It is assumed that values of X are controlled and not subject to measurement error and corresponding values of Y are observed.

The **general simple linear regression model** to evaluate the value of Y for a value of X:

$y_i = \beta_0 + \beta_1 x + \varepsilon$

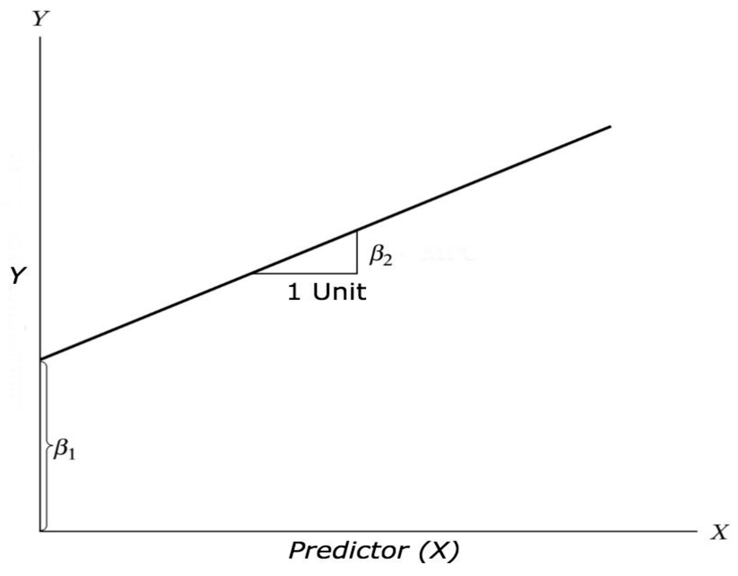
Here, the i[th] data point, y_i, is determined by the variable x_i;

β_0 and β_1 are regression coefficients;

ε_i is the error in the measurement of the i[th] value of x.

Regression analysis is implemented to do the following:

- Establish a relationship between independent (x) and dependent (y) variables.
- Predict the value of y based on a set of values of x1, x2…xn.
- Identify independent variables to understand which of them are important to explain the dependent variable, and thereby establishing a more precise and accurate causal relationship between the variables.



Multiple Linear Regression in R

In the real world, you may find situations where you have to deal with more than 1 predictor variable to evaluate the value of response variable. In this case, simple linear models cannot be used and you need to use R multiple linear regressions to perform such analysis with multiple predictor variables.

R multiple linear regression models with two explanatory variables can be given as:

$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{1i} + \varepsilon_i$

Here, the i^{th} data point, y_i, is determined by the levels of the two continuous explanatory variables x_{1i} and x_{1i}, by the three parameters β_0, β_1, and β_2 of the model, and by the residual ε_1 of point i from the fitted surface.

General Multiple regression models can be represented as:

$y_i = \Sigma\beta_1 x_{1i} + \varepsilon_i$

Procedure:

Step-1: Open R Studio and go to R console (>)

```
>sessionInfo()

>install.packages("DAAG")

>library(lattice)

>library(DAAG)

>?cars # built-in data set in car
```

Example Problem

For this analysis, we will use the *cars* dataset that comes with R by default. cars is a standard built-in dataset, that makes it convenient to demonstrate linear regression in a simple and easy to understand fashion. You can access this dataset simply by typing in cars in your R console. You will find that it consists of 50 observations(rows) and 2 variables (columns) – dist and speed. Lets print out the first six observations here..

head(cars) # display the first 6 observations#>

Ans :

```
speed dist
1     4    2
2     4   10
3     7    4
4     7   22
5     8   16
6     9   10
```

Before we begin building the regression model, it is a good practice to analyze and understand the variables. The graphical analysis and correlation study below will help with this.

Graphical Analysis

The aim of this exercise is to build a simple regression model that we can use to predict Distance (dist) by establishing a statistically significant linear relationship with Speed (speed). But before jumping in to the syntax, lets try to understand these variables graphically. Typically, for each of the independent variables (predictors), the following plots are drawn to visualize the following behavior:

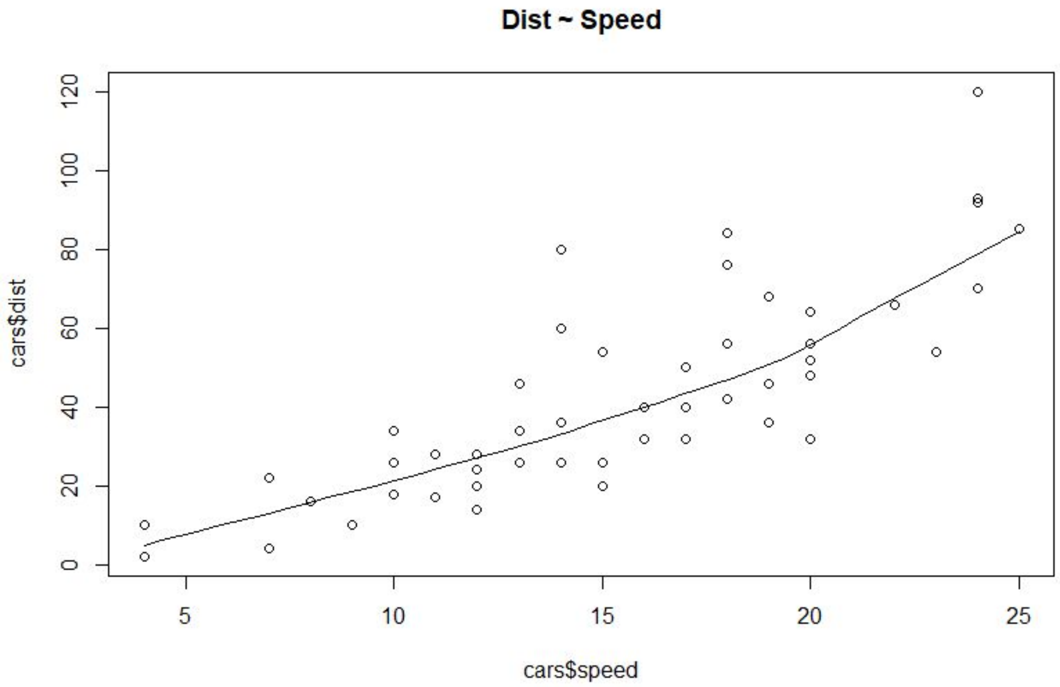
1. **Scatter plot**: Visualize the linear relationship between the predictor and response
2. **Box plot**: To spot any outlier observations in the variable. Having outliers in your predictor can drastically affect the predictions as they can easily affect the direction/slope of the line of best fit.
3. **Density plot**: To see the distribution of the predictor variable. Ideally, a close to normal distribution (a bell shaped curve), without being skewed to the left or right is preferred. Let us see how to make each one of them.

Scatter Plot

Scatter plots can help visualize any linear relationships between the dependent (response) variable and independent (predictor) variables. Ideally, if you are having multiple predictor variables, a scatter plot is drawn for each one of them against the response, along with the line of best as seen below.

scatter.smooth(x=cars$speed, y=cars$dist, main="Dist ~ Speed") # scatterplot

Ans :



Correlation

Correlation is a statistical measure that suggests the level of linear dependence between two variables, that occur in pair – just like what we have here in speed and dist. Correlation can take values between -1 to +1. If we observe for every instance where speed increases, the distance also increases along with it, then there is a high positive correlation between them and therefore the correlation between them will be closer to 1. The opposite is true for an inverse relationship, in which case, the correlation between the variables will be close to -1.

A value closer to 0 suggests a weak relationship between the variables. A low correlation ($-0.2 < x < 0.2$) probably suggests that much of variation of the response variable (*Y*) is unexplained by the predictor (*X*), in which case, we should probably look for better explanatory variables.

cor(cars$speed, cars$dist) # calculate correlation between speed and distance #>
Ans : 0.8068949

To Build Linear Model

Refer the following online regression tutorial and perform all the steps and interpret.

1. http://r-statistics.co/Linear-Regression.html

&

2. Read the PPT shared on Google Classroom

Important Points to remember:

1. Understanding lm() function
2. Linear Regression Diagnostics using summary() function
3. Statistical Significance: The p-Value: Null and Alternate Hypothesis
4. To calculate the t Statistic and p-Values
5. To calculate AIC and BIC
6. To know if the model is best fit for your data:

The most common metrics to look at while selecting the model are:

STATISTIC	CRITERION
R-Squared	Higher the better *(> 0.70)*
Adj R-Squared	Higher the better
F-Statistic	Higher the better
Std. Error	Closer to zero the better
t-statistic	Should be greater 1.96 for p-value to be less than 0.05
AIC	Lower the better
BIC	Lower the better
Mallows cp	Should be close to the number of predictors in model
MAPE (Mean absolute percentage error)	Lower the better
MSE (Mean squared error)	Lower the better
Min_Max Accuracy => mean(min(actual, predicted)/max(actual, predicted))	Higher the better

7. **Predicting Linear Models:**

Step 1: Create the training (development) and test (validation) data samples from original data.

Step 2: Develop the model on the training data and use it to predict the distance on test data

Step 3: Review diagnostic measures.

Step 4: Calculate prediction accuracy and error rates

8. Cross validation: k- Fold Cross validation

Build a regression model using the forward stepwise procedure.

1. Look at the mtcars data item. This is built into R.

> str(mtcars)

Ans :

```
'data.frame':	32 obs. of  11 variables:
 $ mpg : num  21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
 $ cyl : num  6 6 4 6 8 6 8 4 4 6 ...
 $ disp: num  160 160 108 258 360 ...
 $ hp  : num  110 110 93 110 175 105 245 62 95 123 ...
 $ drat: num  3.9 3.9 3.85 3.08 3.15 2.76 3.21 3.69 3.92 3.92 ...
 $ wt  : num  2.62 2.88 2.32 3.21 3.44 ...
 $ qsec: num  16.5 17 18.6 19.4 17 ...
 $ vs  : num  0 0 1 1 0 1 0 1 1 1 ...
 $ am  : num  1 1 1 0 0 0 0 0 0 0 ...
 $ gear: num  4 4 4 3 3 3 3 4 4 4 ...
 $ carb: num  4 4 1 1 2 1 4 2 2 4 ...
```

2. Start by creating a blank model using mpg as the response variable:

> mtcars.lm = lm(mpg ~ 1, data = mtcars)

Ans :

```
Call:
lm(formula = mpg ~ 1, data = mtcars
Coefficients:
(Intercept)
     20.09
```

3. Determine which predictor variable is the best starting candidate:

> add1(mtcars.lm, mtcars, test = 'F')

Ans :

Single term additions

```
Model:
mpg ~ 1
       Df Sum of Sq     RSS     AIC F value    Pr(>F)
<none>              1126.05 115.943
cyl     1    817.71  308.33  76.494 79.5610 6.113e-10 ***
disp    1    808.89  317.16  77.397 76.5127 9.380e-10 ***
hp      1    678.37  447.67  88.427 45.4598 1.788e-07 ***
drat    1    522.48  603.57  97.988 25.9696 1.776e-05 ***
wt      1    847.73  278.32  73.217 91.3753 1.294e-10 ***
qsec    1    197.39  928.66 111.776  6.3767  0.017082 *
vs      1    496.53  629.52  99.335 23.6622 3.416e-05 ***
am      1    405.15  720.90 103.672 16.8603  0.000285 ***
gear    1    259.75  866.30 109.552  8.9951  0.005401 **
carb    1    341.78  784.27 106.369 13.0736  0.001084 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

4. Add the best predictor variable to the blank model:

> mtcars.lm = lm(mpg ~ wt, data = mtcars)

Ans :

```
Call:
lm(formula = mpg ~ wt, data = mtcars)

Coefficients:
(Intercept)           wt
     37.285       -5.344
```

5.Do a quick check of the model summary:

> summary(mtcars.lm)

Ans:

```
Call:
lm(formula = mpg ~ wt, data = mtcars)

Residuals:
    Min      1Q  Median      3Q     Max
-4.5432 -2.3647 -0.1252  1.4096  6.8727

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  37.2851     1.8776  19.858  < 2e-16 ***
wt           -5.3445     0.5591  -9.559 1.29e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Residual standard error: 3.046 on 30 degrees of freedom
Multiple R-squared:  0.7528,	Adjusted R-squared:  0.7446
F-statistic: 91.38 on 1 and 30 DF,  p-value: 1.294e-10
```

6.Now look again at the remaining candidate predictor variables:

> add1(mtcars.lm, mtcars, test = 'F')

Ans:

```
Single term additions

Model:
mpg ~ wt
       Df Sum of Sq    RSS    AIC F value   Pr(>F)
<none>              278.32 73.217
cyl     1    87.150 191.17 63.198 13.2203 0.001064 **
disp    1    31.639 246.68 71.356  3.7195 0.063620 .
hp      1    83.274 195.05 63.840 12.3813 0.001451 **
drat    1     9.081 269.24 74.156  0.9781 0.330854
qsec    1    82.858 195.46 63.908 12.2933 0.001500 **
vs      1    54.228 224.09 68.283  7.0177 0.012926 *
am      1     0.002 278.32 75.217  0.0002 0.987915
gear    1     1.137 277.19 75.086  0.1189 0.732668
carb    1    44.602 233.72 69.628  5.5343 0.025646 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

7.Add the next best predictor variable to your regression model:

> ***mtcars.lm = lm(mpg ~ wt + cyl, data = mtcars)***

Ans:

```
Call:
lm(formula = mpg ~ wt + cyl, data = mtcars)

Coefficients:
(Intercept)           wt          cyl
     39.686       -3.191       -1.508
```

8.Now check the model summary once more:

> summary(mtcars.lm)

Ans:

```
Call:
lm(formula = mpg ~ wt + cyl, data = mtcars)
```

```
Residuals:
    Min      1Q  Median      3Q     Max 
-4.2893 -1.5512 -0.4684  1.5743  6.1004 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  39.6863     1.7150  23.141  < 2e-16 ***
wt           -3.1910     0.7569  -4.216 0.000222 ***
cyl          -1.5078     0.4147  -3.636 0.001064 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.568 on 29 degrees of freedom
Multiple R-squared:  0.8302,	Adjusted R-squared:  0.8185 
F-statistic: 70.91 on 2 and 29 DF,  p-value: 6.809e-12
```

9. Check the remaining variables to see if there are any other candidate predictors to add:

> add1(mtcars.lm, mtcars, test = 'F')

Ans:

```
Single term additions

Model:
mpg ~ wt + cyl
       Df Sum of Sq    RSS    AIC F value Pr(>F)
<none>              191.17 63.198               
disp    1    2.6796 188.49 64.746  0.3980 0.5332
hp      1   14.5514 176.62 62.665  2.3069 0.1400
drat    1    0.0010 191.17 65.198  0.0001 0.9903
qsec    1   10.5674 180.60 63.378  1.6383 0.2111
vs      1    0.7059 190.47 65.080  0.1038 0.7497
am      1    0.1249 191.05 65.177  0.0183 0.8933
gear    1    3.0281 188.14 64.687  0.4507 0.5075
carb    1   13.7724 177.40 62.805  2.1738 0.1515
```

10. The current model remains the most adequate.

Comments on Result:

Ans : In this example we wanted to build a model for linear regression. To do that we need to know the best candidates from the dataset that can contribute to make the best model for linear regression. To find the best candidates we check the significance of each element of the dataset and the element having more significance i.e. more number of asterisk (*) is added to the linear model. We repeat this till we find that no more element is significant enough to be added in the linear model. Hence the final linear model created is the best model that can be created from any given dataset.

Part-II: Logistic Regression

We use the logistic regression equation to predict the probability of a dependent variable taking the dichotomy values 0 or 1. Suppose x1, x2, ..., xp are the independent variables, α and βk (k = 1, 2, ..., p) are the parameters, and E(y) is the expected value of the dependent variable y, then the logistic regression equation is:

$$\boxed{E(y) = 1/(1 + e^{-(\alpha + \sum_k \beta_k x_k)})}$$

For example, in the built-in data set ***mtcars,*** the data column am represents the transmission type of the automobile model (0 = automatic, 1 = manual).

With the logistic regression equation, we can model the probability of a manual transmission in a vehicle based on its engine horsepower and weight data.

$$P(Manual\ Transmission) = 1/(1 + e^{-(\alpha + \beta_1 * Horsepower + \beta_2 * Weight)})$$

Estimated Logistic Regression Equation

Using the generalized linear model, an estimated logistic regression equation can be formulated as below. The coefficients a and bk (k = 1, 2, ..., p) are determined according to a maximum likelihood approach, and it allows us to estimate the probability of the dependent variable y taking on the value 1 for given values of xk (k = 1, 2, ..., p).

$$Estimate\ of\ P(y = 1 \mid x_1, ...x_p) = 1/(1 + e^{-(a + \sum_k b_k x_k)})$$

Example Problem:

By use of the logistic regression equation of vehicle transmission in the data set ***mtcars***, estimate the probability of a vehicle being fitted with a manual transmission if it has a 120hp engine and weights 2800 lbs.

Solution:

We apply the function ***glm*** to a formula that describes the transmission type (am) by the horsepower (hp) and weight (wt). This creates a generalized linear model (GLM) in the binomial family.

In R:

#Build a model:

am.glm = glm(formula=am ~ hp + wt, data=mtcars, family=binomial)

Ans :

Call: glm(formula = am ~ hp + wt, family = binomial, data = mtcars)

Coefficients:

(Intercept)	hp	wt
18.86630	0.03626	-8.08348

Degrees of Freedom: 31 Total (i.e. Null); 29 Residual

Null Deviance: 43.23

Residual Deviance: 10.06   AIC: 16.06

#Test data

newdata = data.frame(hp=120, wt=2.8)

Ans :

hp wt

1 120 2.8

#Predict

predict(am.glm, newdata, type="response")

Ans :

1

0.6418125

Answer

For an automobile with 120hp engine and 2800 lbs weight, the probability of it being fitted with a manual transmission is about 0.6418125.

Further detail of the function predict for generalized linear model can be found in the R documentation.

> ***help(predict.glm)***

Significance Test for Logistic Regression

We can decide whether there is any significant relationship between the dependent variable y and the independent variables xk (k = 1, 2, ..., p) in the logistic regression equation. In particular, if any of the null hypothesis that βk = 0 (k = 1, 2, ..., p) is valid, then xk is statistically insignificant in the logistic regression model.

Problem

At .05 significance level, decide if any of the independent variables in the logistic regression model of vehicle transmission in data set mtcars is statistically insignificant.

Solution

We apply the function glm to a formula that describes the transmission type (am) by the horsepower (hp) and weight (wt). This creates a generalized linear model (GLM) in the binomial family.

> am.glm = glm(formula=am ~ hp + wt, data=mtcars, family=binomial)

We then print out the summary of the generalized linear model and check for the p-values of the hp and wt variables.

> *summary(am.glm)*
Answer

As the p-values of the hp and wt variables are both less than 0.05 neither hp or wt is insignificant in the logistic regression model.

Further detail of the function summary for the generalized linear model can be found in the R documentation.

> *help(summary.glm)*

Describe the following with respect to Linear Regression and Building linear model and Prediction

1. List types of regression

Ans :

a) Linear Regression
b) Logistic Regression
c) Polynomial Regression
d) Stepwise Regression
e) Ridge Regression
f) Lasso Regression
g) ElasticNet Regression

2. What is statistical significance test?

Ans :

Statistical significance is the likelihood that a relationship between two or more variables is caused by something other than chance. Statistical hypothesis testing is used to determine whether the result of a data set is statistically significant. This test provides a p-value, representing the probability that random chance could explain the result. In general, a p-value of 5% or lower is considered to be statistically significant.

3. How to know if the model is best fit for your data?

Ans :

The most common metrics to look at while selecting the model are:

STATISTIC	**CRITERION**
R-Squared	Higher the better (> 0.70)
Adj R-Squared	Higher the better
F-Statistic	Higher the better
Std. Error	Closer to zero the better
t-statistic	Should be greater 1.96 for p-value to be less than 0.05
AIC	Lower the better
BIC	Lower the better
Mallows cp	Should be close to the number of predictors in model
MAPE (Mean absolute percentage error)	Lower the better
MSE (Mean squared error)	Lower the better
Min_Max Accuracy => mean(min(actual, predicted)/max(actual, predicted))	Higher the better

4. How to test model's performance?

Ans : There are three main errors (metrics) used to evaluate models, Mean absolute error, Mean Squared error and R2 score.

The Mean Absolute Error measures the average of the absolute difference between each ground truth and the predictions.

Mean Squared Error solves differentiability problem of the MAE.

R2 score answers the question that if this simple model has a larger error than the linear regression model. However, it terms of metrics the answer we need is how much larger. The R2 score answers this question. R2 score is 1 — (Error from Linear Regression Model/Simple average model).

Conclusion: In this experiment we learnt to create a model for linear as well as logistic regression. We were able to create a best model using the statistic metrics and the significance measure and p value. Then we were able to predict the result of the dependent variable for any particular independent variable. We also calculated the AIC and BIC values. Also, we plotted the linear regression graph and fitted the best fit line for the values evaluated by the model. We installed the ggplot2 and ggally packages and used the gpairs function to analyse regression relation between two regression model candidates.