1. $\underline{t=0}$:

Borrow 1000 Swiss Franc and convert to dollars at spot rate (1 swiss franc = \$ 1.05) to get \$1050 and put in bank. Enter into a long forward contract with expiry 2 mth and strike price (\$1.05) to buy swiss francs worth $1000e^{rt}$ =

(5) $1000e^{0.02\times\frac{2}{12}} = 1053.5$ dollars. $\left(\frac{1053.5 \leftarrow \text{swiss franc}}{1.05 \leftarrow \text{forward rate}}\right)$

$\underline{t = 2\text{mths}}$:

\$1050 dollars in bank are worth $1000e^{0.02\cdot\frac{2}{12}}$
$= 1053.5$

(5) Close the contract position to obtain

$\frac{1053.5}{1.05} = 1003.39$ Swiss franc

Return the borrowed money totalling

$1000e^{0.01\times\frac{2}{12}} = 1001.66$ francs.

Arbitrage = 1003.33 − 1001.66 = 1.66 swiss franc.

2. $\underline{t=0:}$

Borrow \$3.2 from bank

(i) Buy put option for \$1.19

(6) (ii) Purchase $\frac{1}{2}$ unit of stock for $\frac{1}{2}4 = \$2$

(iii) Put 0.01 in a separate bank a/c

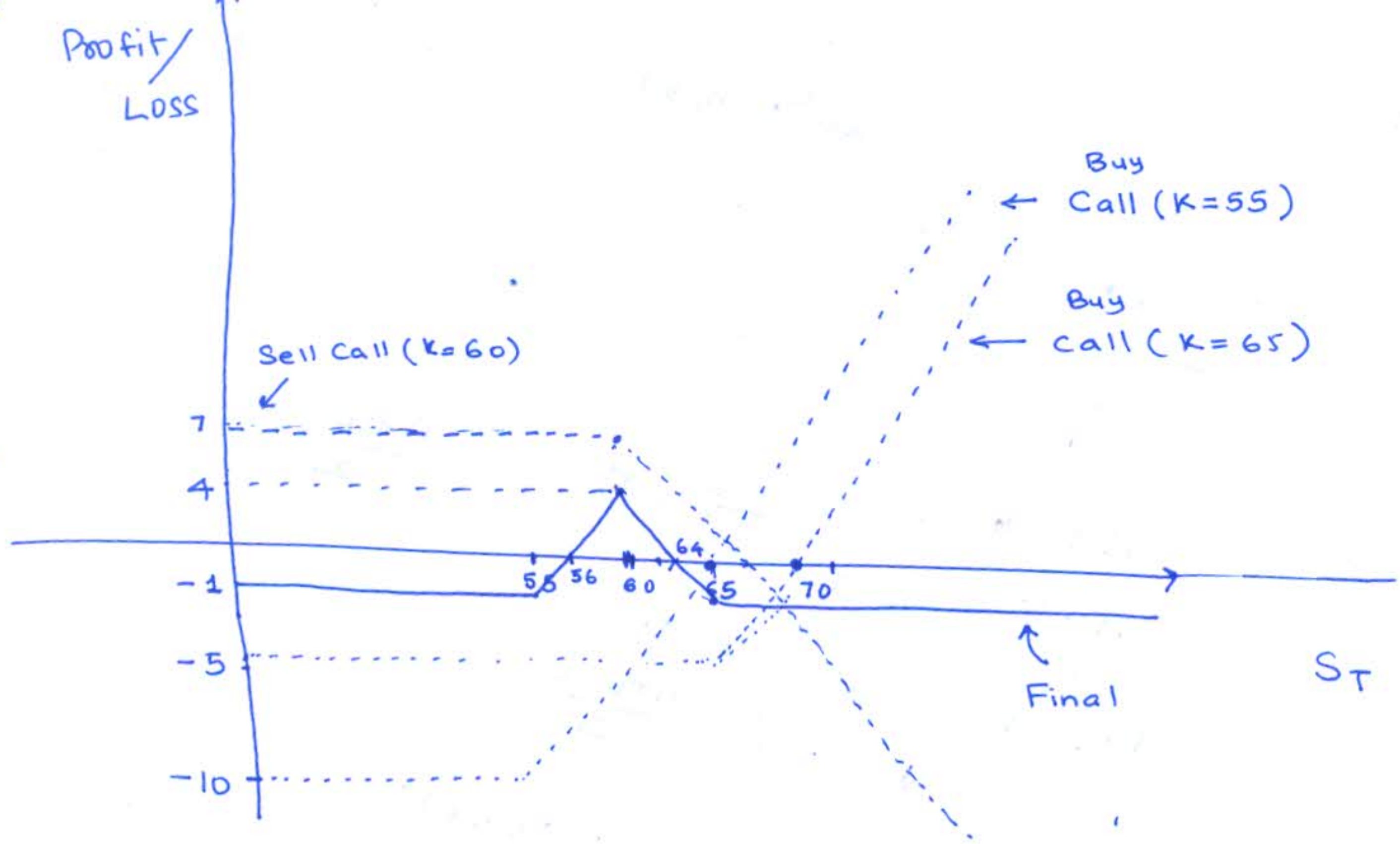
$\underline{t=1:}$

	H	T
Bank	$-3.2.(1+\frac{1}{4}) = -4$	$-3.2(1+\frac{1}{4}) = -4$
Stock	$\frac{1}{2}\times 8 = 4$	$\frac{1}{2}\times 2 = 1$
Other Bank a/c	$0.01\times 5/4 = 0.0125$	$0.01\times 5/4 = 0.0125$
Worth of put option	0	+3
Total	0	0

(4) Either case (H or T) we have hedged the put option with buying a stock. The stock offsets the option.

We make a risk free profit of 0.0125 either way.

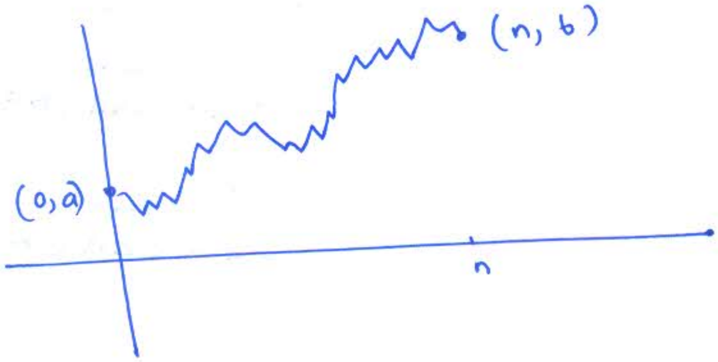
(6)



(ii) Trader strategy:

(4) The trader bets on his judgement that the stock price in the next six months will be around the current stock price (\$61). For this strategy the profitable range is when S_T is in the range (56, 64) which is within a small range of the current stock price.

4) (i)



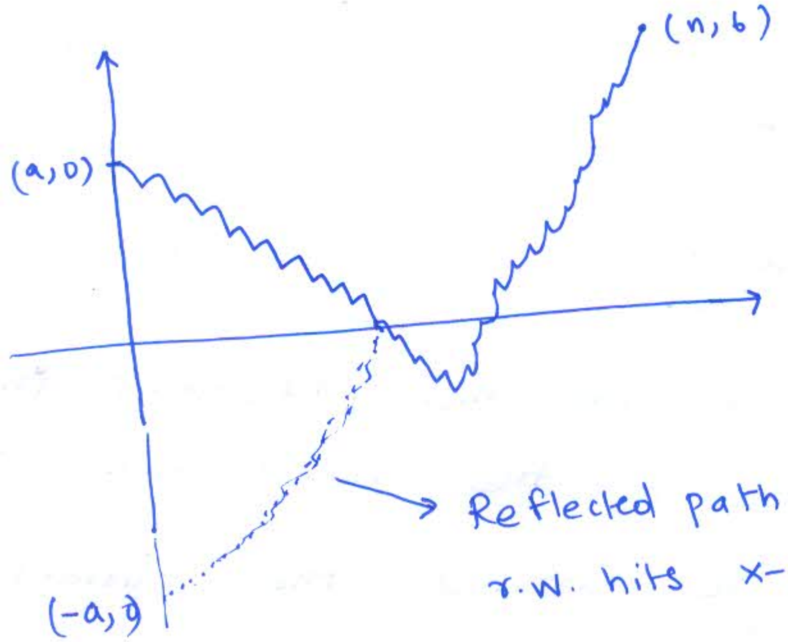
(1) $H + T = n \Longrightarrow H = \dfrac{n+b-a}{2}$

$H - T = b - a$

$\therefore \; N_n(a,b) = \dbinom{n}{\frac{n+b-a}{2}}$

ii)

(3)



→ Reflected path from first time r.w. hits x-axis

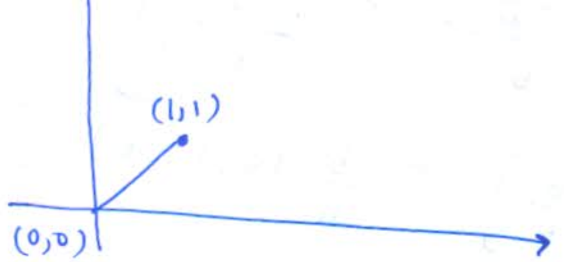
There is 1-1 correspondence between reflect paths and $N^0(a,b)$.

Reflected paths ~ start $(-a, 0)$ that touch x-axis ~~and~~ and arrive at (n, b)

Therefore Reflected paths $= N_n(-a, b)$

$\therefore \; N^0(a,b) = N_n(-a,b)$

a)



First step has to be to (1,1), so we need to find the number of paths from (1,1) to (n, b) that do not does the origin, that is $\overline{N^{0}_{n-1}(1,b)}$

$$\overline{N^{0}_{n-1}(1,b)} = N_{n-1}(1,b) - \text{Paths from } (1,1) \text{ to } (n,b) \text{ that } \cancel{\text{do}}\ \overset{\text{touch}}{\cancel{\text{not}}} \text{ or cross the origin}$$

(3)

$$= N_{n-1}(1,b) - N^{0}_{n-1}(1,b)$$

$$= N_{n-1}(1,b) - N_{n-1}(-1,b) \quad (\text{By part ii})$$

b) From (i) we get

$$\overline{N^{0}_{n-1}(1,b)} = \binom{n-1}{\frac{n-1+b-1}{2}} - \binom{n-1}{\frac{n-1+b+1}{2}}$$

$$= \binom{n-1}{\frac{n+b}{2}-1} - \binom{n-1}{\frac{n+b}{2}}$$

$$= \frac{(n-1)!}{\left(\frac{n+b}{2}-1\right)!\left(\frac{n-b}{2}\right)!} - \frac{(n-1)!}{\left(\frac{n+b}{2}\right)!\left(\frac{n-b}{2}-1\right)!}$$

$$= \frac{b}{n}\left[\frac{n!}{b\left(\frac{n+b}{2}-1\right)!\left(\frac{n-b}{2}\right)!} - \frac{n!}{b\left(\frac{n+b}{2}\right)!\left(\frac{n-b}{2}-1\right)!}\right]$$

$$= \frac{b}{n}\,\frac{n!}{\left(\frac{n+b}{2}\right)!\left(\frac{n-b}{2}\right)!}\left[\frac{\left(\frac{n+b}{2}\right)}{b} - \frac{\left(\frac{n-b}{2}\right)}{b}\right]$$

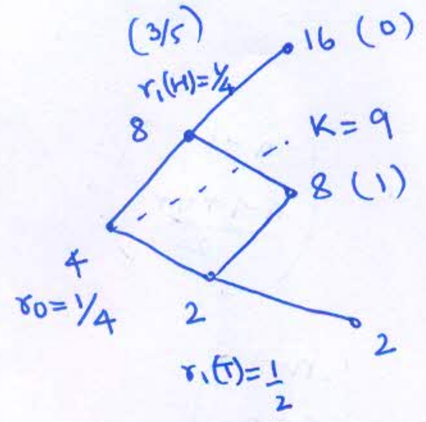
$$= \frac{b}{n}\,N_n(0,b)\,.\left[\frac{b}{b}\right] = \frac{b}{n}\,N_n(0,b)$$

iv) This answer is the probability of the paths that go from $(0,0)$ to $(\alpha+\beta, \alpha-\beta)$ that do not ~~do~~ touch the x-axis.

(3) Prob. (Narendra Modi always ahead of Rahul Gandhi) $= \dfrac{\frac{\alpha-\beta}{\alpha+\beta}\,N_n(0,b)}{N_n(0,b)}$

$$= \frac{\alpha-\beta}{\alpha+\beta}$$

5)



$V_n \equiv$ option price at time n

$V_2(HH)=0$, $V_2(TH)=V_2(HT)=1$, $V_2(TT)=7$

$$V_1(H) = \cancel{\tilde{q}_u V_2(HT) + \tilde{q}_d}\;\frac{q_u^{(H)} V_2(HH) + q_d^{(H)} V_2(HT)}{1-r}$$

where $q_u(H)$, $q_d(H)$ are the risk neutral probabilities given by $q_u(H) = \dfrac{1+r_1(H)-d(H)}{u(H)-d(H)}$, $q_d(H) = \cancel{1-q}\; 1-q_u(H)$

$$\frac{1+1/4-1}{2-1} = \frac{1}{4}, \quad q_d(H) = \frac{3}{4}$$

$$V_1(H) = \frac{q_u(H)\, V_2(HH) + q_d(H)\, V_2(HT)}{1+r_1(H)}$$

$$= \frac{\frac{1}{4}\cdot 0 + \frac{3}{4}\cdot 1}{1+\frac{1}{4}} = \frac{3}{5}$$

Now $q_u(T) = \dfrac{1+r_1(T)-d(T)}{u(T)-d(T)} = \dfrac{1+\frac{1}{2}-1}{4-1} = \dfrac{1}{6}$

$\therefore\ q_d(T) = 5/6$

$$V_1(T) = \frac{q_u(T)\ V_2(TH) + q_d(T)\ V_2(TT)}{1+r_1(T)}$$

$$= \frac{\frac{1}{6}\cdot 1 + \frac{5}{6}\cdot 7}{1+\frac{1}{2}} = 4$$

$$q_u^0 = \frac{1+r_0-d_0}{u_0-d_0} = \frac{1+1/4-1/2}{2-\frac{1}{2}} = 1/2$$

$$q_d^0 = 1/2$$

$$V_0 = \frac{q_u^0\ V_1(H) + q_d^0\ V_1(T)}{1+r_0}$$

$$= \frac{\frac{1}{2}\cdot\frac{3}{5} + \frac{1}{2}\cdot 4}{1 + \frac{1}{4}} = \frac{46}{25} = 1.84$$

Hence option price at time 0 is 1.84