In Sem 1

Computational finance (CS401) Winter 2019

Time: 1:30 hr

1. **Put call parity and arbitrage**

   a) State the Put-Call parity. Explain briefly why it should hold

   b)A call option of 3 month maturity and strike price \$30 is available for \$3 . The current stock price is \$31 and the risk free interest rate is 10%. If a 3 month put option with the same strike price and maturity is selling for \$2.25 what is the arbitrage opportunity available.

2. **Gambler's ruin**

   Consider a gambler who starts with K dollars of capital. If a head occurs he/she gains a dollar and if a tail occurs he/she loses a dollar. He wins this gambling gain if he/she makes N dollars where $N >> K$ and loses the game (is ruined) if ends up with zero dollars. Let p_K be the probability of ruin starting with K dollars of capital. Show that

   a)$p_K = \frac{1}{2}p_{K+1} + \frac{1}{2}p_{K-1}$

   b)Observing that $p_0 = 1$ and $p_N = 0$, solve the recurrence to compute p_K

   c)Use a symmetry argument considering a random walker to compute the same probability of ruin

   d)What happens to a gambler playing this game for a long duration?

   e)Give an idea to solve the same problem if the random walk was not symmetric (Just give a brief idea)

3. **Option trading strategy**

   a)You are an options trader with an outlook on 2019 elections. Call options for XYZ company expiring right after election results are available in the market for strike prices Rs 90 and Rs 100. You envisage that only two scenarios are likely: Post the elections if BJP wins majority then the stock price of XYZ is going to get a big boost (Price likely to be well over Rs 90), otherwise if BJP does not win a majority then the stock price of XYZ company is likely to go down significantly (Price likely to be well below 90). Devise an option trading strategy (a combination of buying and selling call/put options) for XYZ stock to profit from either of these scenarios at a small initial cost. Draw the payoff diagrams at maturity.

   b)If you envisage that there are three possibilities one 1. BJP wins majority (XYZ share goes up well above 100) 2. Congress wins majority (XYZ share remains stable between 90 and 100) 3. Coalition govt (XYZ share goes down well below 90). Revise your strategy such that you profit from scenarios 1) and 3) but not lose if scenario 2) happens. Again draw Payoff diagrams at maturity.

4. **Binomial model Path dependent options: Asian option**

   Consider a three period model $S_0 = 4, u = 2, d = \frac{1}{2}$ and $r = \frac{1}{4}$. Define $Y_n = \sum_{k=0}^{3} S_k$. Consider an Asian option with strike price $K = 4$ whose payoff is $(\frac{1}{4}Y_3 - 4)^+$. Compute the value of this Asian call option at time zero.