In Sem 2

Computational finance (CS401) Winter 2018

Time: 2hr

- Ito's lemma $F(t, S(t))$ where $dS = \mu S dt + \sigma S dW$ then $dF = \left(\frac{\partial F}{\partial t} + \mu S \frac{\partial F}{\partial S} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 F}{\partial S^2}\right) dt + \sigma S \frac{\partial F}{\partial S} dW$
- Black Scholes Merton formula for European call $c(x,t) = xN(d_1) - Ke^{-r\tau}N(d_2)$ $\tau = T - t,\ d_1 = \frac{\log \frac{x}{K} + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}},\ d_2 = d_1 - \sigma\sqrt{\tau}$

1. **Geometric Brownian motion**
   A stock follows a GBM. The stock is currently priced 50. Its expected return and volatality are 12% and 30% per annum respectively. What is the probability that the stock will be greater than 80 in two years time?

2. **Ito's lemma**
   Let the stock price be given by $dS = \mu S dt + \sigma S dW$. The price of a forward contract that matures at time T at time t is given by $F(t, S(t)) = S(t)e^{r(T-t)}$. Show using Ito's lemma that the forward contract price also evolves like a geometric motion with parameters $\mu' = \mu - r$ and $\sigma' = \sigma$.

3. **Conditional expectation**
   A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. If we assume that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until he reaches safety?

4. **Martingales**
   Let $S_n = \sum_{i=1}^{n} X_i$ be the position of the random walker at time n. Show that $Y_n = S_n^2 - n$ is a martingale with respect to $X_1, X_2, \ldots X_n$.

5. **Black-Scholes-Merton option formula**
   a.Show that the probability that the European call option will be exercised is $N(d_2)$.
   b.A European asset-or-nothing call pays its holder a fixed amount F if the price at expiration is larger than K and pays 0 otherwise. Find the value of the call option that expires in 6 months time and has $F = 100$ $K = 40$ if the present price of the security is 38, its volatality is 32% per annum and the risk free interest rate is 6% per annum.