DHIRUBHAI AMBANI INSTITUTE OF INFORMATION AND COMMUNICATION TECHNOLOGY
Gandhinagar 382007, Gujarat, India

Introduction to Nonlinear Dynamics (SC401)

Full End-Semester Examination Syllabus [1] – Autumn Semester, 2022

Topics covered chronologically (the first twenty items pertain to the mid-semester examination):

1. Introduction to the course and its overview.
2. Basic principles of differential equations (ordinary and partial). Orders of differential equations.
3. First order linear systems of one variable. Rate $\propto$ state. Transformation of variables, separation of variables, rescaling into dimensionless forms.
4. Scales, approximations and basic plotting techniques ($\dot{x} = a \pm bx$). Stokes's law of terminal velocity.
5. Atomic waste disposal. Viscoelastic deformation of rocks.
6. The Duckworth-Lewis method in cricket. Radioactivity. Radioactive series.
7. Detecting art forgery. Radio-carbon dating.
8. Q-R-C circuit. Flows on the line. Phase plots, fixed points (equilibrium points), attractors and repellers. Practice plotting of second-degree equations.
9. Plotting of a polynomial series and transcendental equation. Linear stability analysis and small perturbations.
10. Half-stable fixed points and power-law convergence.
11. The p-n diode. Plotting cubic polynomials. The logistic equation. Rescaling of variables. Integral solution.
12. Plotting of the logistic equation. Higher orders on nonlinearity. The Fermi-Dirac form of equation.
13. Modifications of the logistic equation. Nonlinear non-autonomous systems. Power laws and Zipf's law.
14. Population dynamics. Malthusian law of population growth. Logistic modelling of global demographics.
15. Country-wise examples of population growth and policy implications. Criticisms of the logistic equation.
16. The laws of social dynamics. Example of sharks and salmon. Critical population growth of New York city.
17. Turbulence. Free fall of a parachutist.
18. Item response theory. Sigmoid functions in neuron activity and positive cooperativity in haemoglobin. Spread of agricultural innovations.
19. Spread of industrial innovations. Growth of free living dividing cells. Gompertz law of tumour growth.
20. Bacteria versus toxin. Autocatalysis. The Allee effect.

    Post mid-semester examination.
21. Saddle-node bifurcation and its bifurcation diagram.
22. Examples of saddle-node bifurcation. Taylor expansion and the normal form of the saddle-node bifurcation.

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23. Transcritical bifurcation and its bifurcation diagram. Examples of transcritical bifurcation. Taylor expansion and the normal form of the transcritical bifurcation.

24. Laser action as an example of transcritical bifurcation. Supercritical pitchfork bifurcation and its bifurcation diagram. An example of supercritical pitchfork bifurcation.

25. Subcritical pitchfork bifurcation and its bifurcation diagram. Examples of pitchfork bifurcation. Taylor expansion and the normal form of the pitchfork bifurcation.

26. Hysteresis in subcritical pitchfork bifurcation. Imperfect bifurcation.

27. A model of a fishery and its improvement. A biochemical switch. Insect outbreak.

28. Flows on the circle. Uniform oscillator. Non-uniform oscillator. Bottlenecks.

29. Overdamped pendulum. Fireflies and synchronisation. Superconductivity (basic principles).

30. Josephson junctions. Two-dimensional systems. The oscillator equation. A coupled system.

31. Classification of linear systems. Saddle points, spirals, nodes and centre-type points. Parameter space.

32. Phase plots, eigenvalues and eigenvectors with examples. Nonlinear systems, Jacobian matrix, existence and uniqueness, Poincare-Bendixson theorem (statement only).

33. Conservative and irreversible systems.

34. Mathematical theories of war: Richardson's theory of conflict.

35. Mathematical theories of war: Lanchester's models of combat.

36. The principle of competitive exclusion. The predator-prey model.

37. Modelling love affairs. Epidemiology (basic principles).

38. Threshold theorem of epidemiology.

39. The epidemic curve. The Königsberg Bridge problem. Random networks and small-world networks.

40. Clustering coefficients in small-world clusters. Power laws. Scale-free networks.

Books:

1. *Nonlinear Dynamics and Chaos: Steven Strogatz*
   Chapter 1: Sections 1.0, 1.1, 1.2, 1.3 (general reading).
   Chapter 2: Sections 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6. Relevant exercises of Sections 2.2, 2.3, 2.4.

   Post mid-semester examination.

   Chapter 3: Sections 3.0, 3.1, 3.2, 3.3, 3.4, 3.6 (imperfect bifurcations), 3.7 (insect outbreak).
   Relevant exercises of Sections 3.1, 3.2, 3.4. Also check Questions 3.7.3, 3.7.4, 3.7.5, 3.7.6.
   Chapter 4: Sections 4.0, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6. Check Questions 4.3.1, 4.3.2.
   Chapter 5: Sections 5.0, 5.1, 5.2, 5.3. Relevant exercises of Sections 5.1, 5.2.
   Chapter 6: Sections 6.0, 6.1, 6.2, 6.3, 6.5, 6.6. Relevant exercises of Sections 6.1, 6.3, 6.5, 6.6.

2. *Differential Equations and Their Applications: Martin Braun*
   Chapter 1: Sections 1.3 (at the end of the section read the note on C-14 dating that follows Question 6, and the problems in Questions 7 & 8), 1.5 (also Questions 7 & 8 at the end of the section), 1.6, 1.7, 1.8.

   Post mid-semester examination.

   Chapter 4: Sections 4.5 (Both subsections 4.5.1 & 4.5.2), 4.10, 4.11, 4.12.