Financial Decision Making

Introduction

- A financial manager's job is to make decisions on behalf of the firm's investors. For good decisions, the benefits exceed the costs.
- The first step in decision making is to identify the costs and benefits of a decision. The next step is to quantify these costs and benefits. In order to compare the costs and benefits, we need to evaluate them in the same terms—cash today.
- Whenever a good trades in a **competitive market**—by which we mean a market in which it can be bought *and* sold at the same price—that price determines the cash value of the good. As long as a competitive market exists, the value of the good will not depend on the views or preferences of the decision maker.
- The value of an asset to the firm or its investors is determined by its competitive market price. The benefits and costs of a decision should be evaluated using these market prices, and when the value of the benefits exceeds the value of the costs, the decision will increase the market value of the firm.

Interest Rates and Time Value of Money

- For most financial decisions, unlike in the examples presented so far, costs and benefits occur at different points in time. For example, typical investment projects incur costs upfront and provide benefits in the future.
- A rupee today is worth more than a rupee in one year. If you have Re 1 today, you can invest it. For example, if you deposit it in a bank account paying 7% interest, you will have Rs 1.07 at the end of one year. We call the difference in value between money today and money in the future the **time value of money**.
- An exchange rate allows us to convert money from one currency to another, the interest rate allows us to convert money from one point in time to another. In essence, an interest rate is like an exchange rate across time.

- Suppose the current annual interest rate is 7%. By investing or borrowing at this rate, we can exchange Rs 1.07 in one year for each rupee today. More generally, the **risk-free interest rate**, $\boldsymbol{r_f}$, for a given period is the interest rate at which money can be borrowed or lent without risk over that period.
- After we know the risk-free interest rate, we can use it to evaluate other decisions in which costs and benefits are separated in time. We can exchange (1 + $\boldsymbol{r_f}$) rupee in the future per rupee today, and vice versa, without risk. We refer to **(1 + $\boldsymbol{r_f}$) as the interest rate factor** for risk free cash flows.
- Consider the benefit of Rs 1,05,000 in one year (a 5 per cent return) from an investment of 1,00,000. What is the equivalent amount in terms of rupees today? That is, how much would we need to have in the bank today so that we would end up with Rs 1,05,000 in the bank in one year? We find this amount by dividing by the interest rate factor.
- Our decision is the same whether we express the value of the investment in terms of rupees in one year (future value) or rupees today (present value)

- When computing a present value we can interpret the term 1/(1 + *r)* as the price of one rupee in one year. Note that the value is less than Re 1—money in the future is worth less today, and so its price reflects a discount.
- Because it provides the discount at which we can purchase money in the future, the amount 1/(1 + *r)* is called the one year **discount factor**. The risk-free interest rate is also referred to as the **discount rate** for a risk-free investment.

Q. Suppose you plan to invest Rs 20,000 in an account paying 8% interest. How much will you have in the account in 15 years?

- To compute the solution in Excel, we enter the four variables we know (*NPER* = 15, *RATE* = 8%, *PV* = -20,000, *PMT* = 0) and solve for the one we want to determine (*FV*) using the Excel function *FV*(*RATE*, *NPER*, *PMT*, *PV*).

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	15	8.00%	−20,000	0		
Solve for FV					63,443	=FV(0.08,15,0,−20000)

Q. Suppose that you invest Rs 20,000 in an account paying 8% interest. You plan to withdraw Rs 2000 at the end of each year for 15 years. How much money will be left in the account after 15 years?

- We can think of the deposit and the withdrawals as being separate accounts. In the account with the Rs 20,000 deposit, our savings will grow to Rs 63,443 in 15 years. Using the formula for the future value of an annuity, if we borrow Rs 2000 per year for 15 years at 8%, at the end our debt will have grown to Rs 54,304.

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	15	8.00%	−20,000	2000		
Solve for FV					9139	=FV(0.08,15,2000,−20000)

Law of One Price

- The practice of buying and selling equivalent goods in different markets to take advantage of a price difference is known as **arbitrage**. More generally, any situation in which it is possible to make a profit without taking any risk or making any investment as an **arbitrage opportunity**.
- Because an arbitrage opportunity has a positive NPV, whenever an arbitrage opportunity appears in financial markets, investors will race to take advantage.
- The same logic applies more generally whenever equivalent investment opportunities trade in two different competitive markets. If the prices in the two markets differ, investors will profit immediately by buying in the market where it is cheap and selling in the market where it is expensive. In doing so, they will equalize the prices. As a result, prices will not differ (at least not for long). This important property is the **Law of One Price.**
- No Arbitrage **Price(Security) =** ***PV*****(All cash flows paid by the security)**

Perpetuities

- A **perpetuity** is a stream of equal cash flows that occur at regular intervals and last forever. One example is the British government bond called a **consol** (or perpetual bond). Consol bonds promise the owner a fixed cash flow every year, forever.

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots = \sum_{n=1}^{\infty} \frac{C}{(1+r)^n}$$

- To find the value of a perpetuity one cash flow at a time would take forever—literally! You might wonder how, even with a shortcut, the sum of an infinite number of positive terms could be finite. The answer is that the cash flows in the future are discounted for an ever increasing number of periods, so their contribution to the sum eventually becomes negligible.

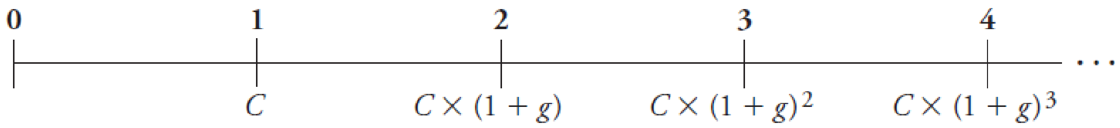
- By investing Rs 100 in the bank today, you can create a perpetuity paying Rs 5 per year. The Law of One Price tells us that the same good must have the same price in every market. Because the bank will allow us to create the perpetuity for Rs 100, the present value of the Rs 5 per year in perpetuity is this "do-it-yourself " cost of Rs 100.
- Now let's generalize this argument. Suppose we invest an amount *P* in the bank. Every year we can withdraw the interest we have earned, *C* = *r* * *P*, leaving the principal, *P*, in the bank. The present value of receiving *C* in perpetuity is therefore the upfront cost *P* = *C*/*r*. Therefore

$$PV(C \text{ in perpetuity}) = \frac{C}{r}$$

- In other words, by depositing the amount *C*/*r* today, we can withdraw interest of (*C*/*r*) * *r* = *C* each period in perpetuity. Thus, the present value of the perpetuity is *C*/*r*.

Growing Perpetuity

- A **growing perpetuity** is a stream of cash flows that occur at regular intervals and grow at a constant rate forever. In general, a growing perpetuity with a first payment *C* and a growth rate *g* will have the following series of cash flows.



- As with perpetuities with equal cash flows, we adopt the convention that the first payment occurs at date 1. Note a second important convention: The first payment does not include growth. That is, the first payment is *C*, even though it is one period away. Similarly, the cash flow in period *n* undergoes only *n* - 1 periods of growth.

- Substituting the cash flows from the preceding timeline into the general formula for the present value of a cash flow stream gives

$$PV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots = \sum_{n=1}^{\infty} \frac{C(1+g)^{n-1}}{(1+r)^n}$$

- Suppose $g \geq r$. Then the cash flows grow even faster than they are discounted; each term in the sum gets larger, rather than smaller. An infinite present value means that no matter how much money you start with, it is impossible to sustain a growth rate of g forever and reproduce those cash flows on your own. **Growing perpetuities of this sort cannot exist in practice because no one would be willing to offer one at any finite price.**
- The only viable growing perpetuities are those where the perpetual growth rate is less than the interest rate, so that each successive term in the sum is less than the previous term and the overall sum is finite. Consequently, we assume that $g<r$ for a growing perpetuity.

- To derive the formula for the present value of a growing perpetuity, we follow the same logic used for a regular perpetuity: **Compute the amount you would need to deposit today to create the perpetuity yourself**. In the case of a regular perpetuity, we created a constant payment forever by withdrawing the interest earned each year and reinvesting the principal.
- To increase the amount we can withdraw each year, the principal that we reinvest each year must grow. Therefore, we withdraw less than the full amount of interest earned each period, using the remaining interest to increase our principal.

- Suppose you want to create a perpetuity with cash flows that grow by 2% per year, and you invest Rs 100 in a bank account that pays 5% interest. At the end of one year, you will have Rs 105 in the bank—your original Rs 100 plus Rs 5 in interest. If you withdraw only Rs 3, you will have Rs 102 to reinvest—2% more than the amount you had initially. This amount will then grow to Rs 102 * 1.05 = Rs 107.10 in the following year, and you can withdraw Rs 3 * 1.02 = Rs 3.06, which will leave you with principal of Rs 107.10 - Rs 3.06 = Rs 104.04. Note that Rs 102 * 1.02 = Rs 104.04. **That is, both the amount you withdraw and the principal you reinvest grow by 2% each year.**

- If we want to increase the amount we withdraw from the bank each year by *g*, then the principal in the bank will have to grow by the same factor *g*. So, instead of withdrawing all of the interest *rP*, we leave *gP* in the bank in addition to our original principal *P*, and only withdraw

$$\mathbf{C = (r - g)P}.$$

- Solving this last equation for *P*, the initial amount deposited in the bank account, gives the present value of a growing perpetuity with initial cash flow C

$$PV(\text{growing perpetuity}) = \frac{C}{r-g}$$

- We can generalize this argument. In the case of an equal-payment perpetuity, we deposited an amount *P* in the bank and withdrew the interest each year. Because we always left the principal *P* in the bank, we could maintain this pattern forever.

- To understand the formula for a growing perpetuity intuitively, start with the formula for a perpetuity. In the earlier case, you had to put enough money in the bank to ensure that the interest earned matched the cash flows of the regular perpetuity.
- In the case of a growing perpetuity, you need to put more than that amount in the bank because you have to finance the growth in the cash flows. How much more? If the bank pays interest at a rate of 5%, then all that is left to take out if you want to make sure the principal grows 2% per year is the difference: 5% - 2% = 3%.
- **So instead of the present value of the perpetuity being the first cash flow divided by the interest rate, it is now the first cash flow divided by the difference between the interest rate and the growth rate.**

Annuities

- An **annuity** is a stream of *N* equal cash flows paid at regular intervals. The difference between an annuity and a perpetuity is that an annuity ends after some fixed number of payments. Most car loans, mortgages, and some bonds are annuities.
- Suppose you invest Rs 100 in a bank account paying 5% interest. At the end of one year, you will have Rs 105 in the bank—your original Rs 100 plus Rs 5 in interest. Now suppose you withdraw the Rs 5 interest and reinvest the Rs 100 for a second year. Further suppose that you repeat this process for 20 years.
- By the law of one price, because it took an initial investment of Rs 100 to create the cash flows on the timeline, the present value of these cash flows is Rs 100,

Rs 100 = *PV* (20 year annuity of Rs 5 per year) + *PV* (Rs 100 in 20 years)

- We can use the same idea to derive the general formula

***P(initial investment)* = *PV* (annuity of *C* for *N* periods) + *PV* (*P* in period *N*)**

$$PV(\text{annuity of } C \text{ for } N \text{ periods}) = P - PV(P \text{ in period } N)$$

$$= P - \frac{P}{(1+r)^N} = P\left(1 - \frac{1}{(1+r)^N}\right)$$

$$PV(\text{annuity of } C \text{ for } N \text{ periods with interest rate } r) = C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)$$

$$FV(\text{annuity}) = PV \times (1+r)^N$$

$$= \frac{C}{r}\left(1 - \frac{1}{(1+r)^N}\right) \times (1+r)^N$$

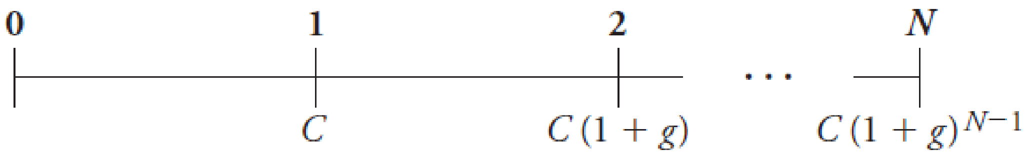
$$= C \times \frac{1}{r}\left((1+r)^N - 1\right)$$

Q. Suppose you are opening a business that requires an initial investment of Rs 100,000. Your bank manager has agreed to lend you this money. The terms of the loan state that you will make equal annual payments for the next 10 years and will pay an interest rate of 8% with the first payment due one year from today. What is your annual payment?

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	10	8.00%	100,000		0	
Solve for PMT				– 14,903		=PMT(0.08,10,100000,0)

Growing Annuity

- A **growing annuity** is a stream of *N* growing cash flows, paid at regular intervals. It is a growing perpetuity that eventually comes to an end. The following timeline shows a growing annuity with initial cash flow *C*, growing at rate *g* every period until period *N:*



- The conventions used earlier still apply: (1) The first cash flow arrives at the end of the first period, and (2) the first cash flow does not grow. The last cash flow therefore reflects only *N* - 1 periods of growth

- The present value of an *N*-period growing annuity with initial cash flow *C*, growth rate *g*, and interest rate *r* is given by. Because the annuity has only a finite number of terms, the formula also works when *g*>*r.*

Present Value of a Growing Annuity

$$PV = C \times \frac{1}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^N\right)$$

Q. Roshan, 30 years old considered saving Rs 10,000 per year for his retirement. Although Rs 10,000 is the most he can save in the first year, he expects his salary to increase each year so that he will be able to increase his savings by 5% per year. With this plan, if he earns 10% per year on his savings, how much will Roshan have saved at age 65?

Amortization

- **Amortization refers to the process of paying off a debt through scheduled, pre-determined installments that include principal and interest.** In almost every area where the term amortization is applicable, the payments are made in the form of principal and interest.
- Such usage of the term relates to debt or loans, but it is also used in the process of periodically lowering the value of intangible assets much like the concept of depreciation.
- The amortization of a loan is the process to pay back, in full, over time the outstanding balance. In most cases, when a loan is given, a series of fixed payments is established at the outset, and the individual who receives the loan is responsible for meeting each of the payments.
- The principal and interest amounts paid on the loan will vary from one month to the next; while the payment amount will be fixed each payment period.

Amortization of loan

Interest rate	2%

Year	Balance	Principal	Interest	Payment	Balance
1	10,000.00	913.27	200.00	1,113.27	9,086.73
2	9,086.73	931.53	181.73	1,113.27	8,155.20
3	8,155.20	950.16	163.10	1,113.27	7,205.04
4	7,205.04	969.16	144.10	1,113.27	6,235.88
5	6,235.88	988.55	124.72	1,113.27	5,247.33
6	5,247.33	1,008.32	104.95	1,113.27	4,239.01
7	4,239.01	1,028.49	84.78	1,113.27	3,210.53
8	3,210.53	1,049.05	64.21	1,113.27	2,161.47
9	2,161.47	1,070.04	43.23	1,113.27	1,091.44
10	1,091.44	1,091.44	21.83	1,113.27	0.00

- Interest costs are always highest at the beginning because the outstanding balance or principle outstanding is at its largest amount. It also serves as an incentive for the loan recipient to get the loan paid off in full. **As time progresses, more of each payment made goes toward the principal balance of the loan, meaning less and less goes toward interest.**
- Not all loans are designed in the same way, and much depends on who is receiving the loan, who is extending the loan, and what the loan is for. However, amortized loans are popular with both lenders and recipients because they are designed to be paid off entirely within a certain amount of time. It ensures that the recipient does not become weighed down with debt and the lender is paid back in a timely way.

Net Present Value

- When we compute the value of a cost or benefit in terms of cash today, we refer to it as the present value (PV). Similarly, we define the **net present value (NPV)** of a project or investment as the difference between the present value of its benefits and the present value of its costs:

Net Present Value(*NPV)* = *PV*(Benefits) - *PV*(Costs)

- As long as the NPV is positive, the decision increases the value of the firm and is a good decision regardless of your current cash needs or preferences regarding when to spend the money.

Internal Rate of Return

- The internal rate of return is defined as the interest rate that sets the net present value of cash flows equal to zero.
- When there are just two cash flows, it is easy to compute the IRR. Consider the general case in which you invest an amount *P* today, and receive *FV* in *N* years. Then the IRR satisfies the equation $P * (1 + IRR)^N = FV$, which implies

$$\textbf{\textit{IRR} with two cash flows} = (FV/P)^{1/N} - 1$$

Note in the formula that we take the total return of the investment over *N* years, *FV*/*P*, and convert it to an equivalent one-year return by raising it to the power 1/*N*. Because we are just comparing two cash flows, the IRR is equivalent to computing the **compound annual growth rate** (or **CAGR**) of the cash flow.

Excel also has a built-in function, IRR, that will calculate the IRR of a stream of cash flows. Excel's IRR function has the format, IRR (values, guess), where "values" is the range containing the cash flows, and "guess" is an optional starting guess where Excel begins its search for an IRR. See the example below:

	A	B	C	D	E
1	Period	0	1	2	3
2	Cash Flow C_t	(1,000.0)	300.0	400.0	500.0
3	IRR	8.9%	=IRR(B2:E2)		

Computing IRR for an annuity

Q. An investment bank has decided to fund Jaya's business. In return for providing the initial capital of Rs 10,00,000, Jaya has agreed to pay them Rs 1,25,000 at the end of each year for the next 30 years. What is the internal rate of return for the bank?

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	30		−1,000,000	125,000	0	
Solve for Rate		12.09%				=RATE(30,125000,−1000000,0)

Opportunity cost

- When economists speak of a firm’s cost of production, **they include all the opportunity costs** of making its output of goods and services. The opportunity cost of an item refers to all those things that must be forgone to acquire that item.

- The costs of inputs, wages etc require the firm to pay out some money. Hence they are called as explicit costs. By contrast, some of a firm’s opportunity costs, called **implicit costs**, do not require a cash outlay. Imagine if a proprietor of a firm could have made certain income by working elsewhere. This foregone income would be part of his implicit costs. Both are part of the firms total costs.

- The distinction between explicit and implicit costs highlights an important difference between how economists and accountants analyze a business. Economists are interested in studying how firms make production and pricing decisions. Because these decisions are based on both explicit and implicit costs, **economists include both when measuring a firm’s costs**.

- By contrast, accountants have the job of keeping track of the money that flows into and out of firms. As a result, they measure the explicit costs but usually ignore the implicit costs.

Economic profit vs accounting profit

- Because economists and accountants measure costs differently, they also measure profit differently. An economist measures a firm's **economic profit** as the firm's total revenue minus all the opportunity costs (explicit and implicit) of producing the goods and services sold.

- An accountant measures the firm's **accounting profit** as the firm's total revenue minus only the firm's explicit costs.

- Because the accountant ignores the implicit costs, **accounting profit is usually larger than economic profit**. For a business to be profitable from an economist's standpoint, total revenue must cover all the opportunity costs, both explicit and implicit.