Interest Rates

Introduction

- Interest rates are quoted in a variety of ways. While generally stated as an annual rate, the interest payments themselves may occur at different intervals, such as monthly or semiannually.
- The interest rates that are quoted by banks and other financial institutions, and that we have used for discounting cash flows are **nominal interest rates**, which indicate the rate at which your money will grow if invested for a certain period. Of course, if prices in the economy are also growing due to inflation, the nominal interest rate does not represent the increase in purchasing power that will result from investing. The rate of growth of your purchasing power, after adjusting for inflation, is determined by the **real interest rate.**
- When evaluating cash flows, however, **we must use a discount rate that matches the time period of our cash flows**; this discount rate should reflect the actual return we could earn over that time period.

The Effective Annual Rate

- Interest rates are often stated as an **effective annual rate (EAR)**, which indicates the actual amount of interest that will be earned at the end of one year.
- With an EAR of 5%, a Rs 1,00,000 investment grows to Rs 1,00,000 * (1 + r) = Rs 105,000 in one year. After two years it will grow to

$$\text{Rs } 1{,}00{,}000 * (1 + r)^2 = \text{Rs } 1{,}00{,}000 * (1.05)^2 = \text{Rs } 110{,}250$$

- **Thus, earning an effective annual rate of 5% for two years is equivalent to earning 10.25% in total interest over the entire period.**
- We can use the same method to find the equivalent interest rate for periods shorter than one year. In this case, we raise the interest rate factor (1 + r) to the appropriate fractional power. For example, earning 5% interest in one year is equivalent to receiving

$$(1 + r)^{1/2} = (1.05)^{1/2} = \text{Rs } 1.0247$$

for each Re 1 invested every half year, or equivalently, every six months.

- **That is, a 5% effective annual rate is equivalent to an interest rate of approximately 2.47% earned every six months.**

- In general, we can convert a discount rate of *r* for one period to an equivalent discount rate for *n* periods using the following formula:

Equivalent *n*-Period Discount Rate = $(1 + r)^n - 1$

- In this formula, *n* can be larger than 1 (to compute a rate over more than one period) or smaller than 1 (to compute a rate over a fraction of a period).
- When computing present or future values, it is convenient to adjust the discount rate to match the time period of the cash flows. **This adjustment is necessary to apply the perpetuity or annuity formulas.**

Q. Suppose your bank account pays interest monthly with the interest rate quoted as an effective annual rate (EAR) of 6%. What amount of interest will you earn each month? If you have no money in the bank today, how much will you need to save at the end of each month to accumulate Rs 1,00,000 in 10 years?

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A 6% EAR is equivalent to earning $(1.06)^{1/12} - 1 = 0.4868\%$ per month. We can view the savings plan as a monthly annuity with 10 * 12 = 120 monthly payments. The future value of this annuity is given as 1,00,000 and you have to solve for C

$$\boldsymbol{FV \text{ (annuity)} = C/r\ [(1 + r)^n - 1]}$$

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	120	0.4868%	0		100,000	
Solve for PMT				**−615.47**		=PMT(0.004868,120,0,100000)

Annual Percentage Rates

- Banks also quote interest rates in terms of an **annual percentage rate (APR)**, which indicates the amount of simple interest earned in one year, that is, **the amount of interest earned without the effect of compounding.**
- Because it does not include the effect of compounding, the APR quote is typically less than the actual amount of interest that you will earn. To compute the actual amount that you will earn in one year, we must first convert the APR to an effective annual rate.
- A savings accounts with an interest rate of "6% APR with monthly compounding" will earn 6%/12 = 0.5% every month. So **an APR with monthly compounding is actually a way of quoting a monthly interest rate**, rather than an annual interest rate.

- Because the interest compounds each month, you will earn $1 * (1.005)^{12}$ = Rs 1.061678 at the end of one year, for an effective annual rate of 6.1678%. The 6.1678% that you earn on your deposit is higher than the quoted 6% APR due to compounding; in later months, you earn interest on the interest paid in earlier months.
- It is important to remember that **because the APR does not reflect the true amount you will earn over one year, we cannot use the APR itself as a discount rate**. Instead, the APR with *k* compounding periods is a way of quoting the actual interest earned each compounding period.

Interest Rate per Compounding Period =*APR/k* periods per year

- Once we have computed the interest earned per compounding period, we can compute the effective annual rate.
- The effective annual rate corresponding to an APR with *k* compounding periods per year is given by the following conversion formula

$$\mathbf{EAR = (1+APR/k)^{k} - 1}$$

- The EAR increases with the frequency of compounding because of the ability to earn interest on interest sooner. Investments can compound even more frequently than daily.
- When working with APRs we must

➢ **Divide the APR by the number of compounding periods per year to determine the actual interest rate per compounding period.**

➢ **Then, if the cash flows occur at a different interval than the compounding period, compute the appropriate discount rate by compounding.**

Q. Your firm is purchasing a new telephone system, which will last for four years. You can purchase the system for an upfront cost of Rs 1,50,000, or you can lease the system from the manufacturer for Rs 4000 paid at the end of each month. Your firm can borrow at an interest rate of 5% APR with semiannual compounding. Should you purchase the system outright or pay Rs 4000 per month?

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- We can compute the present value of the lease cash flows using the annuity formula, but first we need to compute the discount rate that corresponds to a period length of one month. To do so, we convert the borrowing cost of 5% APR with semiannual compounding to a monthly discount rate
- The APR corresponds to a six-month discount rate of 5%/2 = 2.5%. To convert a six-month discount rate into a one-month discount rate, we compound the six month rate by 1/6

$$(1.025)^{1/6} - 1 = 0.4124\% \text{ per month}$$

- Given this discount rate, we can use the annuity formula to compute the present value of the 48 monthly payments:

$$PV = 4000 * 1/0.004124 \, (1 - (1/1.004124)^{48})$$

- Thus, paying Rs 4000 per month for 48 months is equivalent to paying a present value of Rs 173,867 today. This cost is Rs 173,867 - Rs 150,000 = Rs 23,867 higher than the cost of purchasing the system, so it is better to pay Rs 150,000 for the system rather than lease it.

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	48	0.4124%		−4,000	0	
Solve for PV			**173,867**			=PV(0.004124,48,−4000,0)

Application: Loans

- We can compute discount rate from interest rate quote and apply the concept to solve two common financial problems: **calculating a loan payment and calculating the remaining balance on a loan.**
- To calculate a loan payment, we equate the outstanding loan balance with the present value of the loan payments using the discount rate from the quoted interest rate of the loan, and then solve for the loan payment.
- Many loans, such as mortgages and car loans, are **amortizing loans**, which means that each month you pay interest on the loan plus some part of the loan balance. Usually, each monthly payment is the same, and the loan is fully repaid with the final payment.
- Typical terms for a loan might be "6.75% APR for 60 months." When the compounding interval for the APR is not stated explicitly, it is equal to the interval between the payments, or one month in this case. Thus, this quote means that the loan will be repaid with 60 equal monthly payments, computed using a 6.75% APR with monthly compounding.

- Consider a Rs 30,000 loan at 6.75% APR for 60 months. The payment, *C*, is set so that the present value of the cash flows, evaluated using the loan interest rate, equals the original principal amount of Rs 30,000.
- In this case, the 6.75% APR with monthly compounding corresponds to a one-month discount rate of 6.75%/12 = 0.5625%. Now using the annuity formula for the present value of the loan payments, we can find out the value of C, the monthly cash payments.

- The outstanding balance on a loan, also called the outstanding principal, is equal to the present value of the remaining future loan payments, again evaluated using the loan interest rate.

Q. Two years ago you took out a 30-year amortizing loan. The loan has a 4.80% APR with monthly payments of Rs 2623.33. How much do you owe on the loan today? How much interest did the firm pay on the loan in the past year?

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After 2 years, the loan has 28 years, or 336 months, remaining. The remaining balance on the loan (B_2) is the present value of these remaining payments, using the loan rate of 4.8%/12 = 0.4% per month.

During the past year, you made total payments of Rs 2623.33 * 12 = Rs 31,480 on the loan. To determine the amount that was interest, you need to first determine the amount that was used to repay the principal.

Calculate loan balance one year ago(B_1), with 29 years (348 months) remaining. Find B2-B1 which is the amount that was used to pay the principal out of the total payments made. The remaining is interest.

Determinants of Interest Rates

- Fundamentally, interest rates are determined in the market based on individuals' willingness to borrow and lend. Some of the factors that may influence interest rates are **inflation, government policy, and expectations of future growth**.
- Intuitively, individuals' willingness to save will depend on the growth in purchasing power they can expect (given by the real interest rate). Thus, when the inflation rate is high, a higher nominal interest rate is generally needed to induce individuals to save.

Real interest rate

- If *r* is the nominal interest rate and *i* is the rate of inflation, we can calculate the rate of growth of purchasing power as follows

$$\text{Growth in Purchasing Power} = 1 + r_r = \frac{1+r}{1+i} = \frac{\text{Growth of Money}}{\text{Growth of Prices}}$$

- We can rearrange the equation to find the following formula for the real interest rate, together with a convenient approximation for the real interest rate when inflation rates are low. That is the real interest rate is equal to the nominal interest rate minus inflation.

$$r_r = \frac{r-i}{1+i} \approx r - i$$

Inflation and present values

- One question that often comes up is the effect of inflation on present value calculations. The basic principle is simple: **Either discount nominal cash flows at a nominal rate or discount real cash flows at a real rate**
- To illustrate, suppose you want to withdraw money each year for the next three years, and you want each withdrawal to have Rs 25,000 worth of purchasing power as measured in current rupees. If the inflation rate is 4 percent per year, then the withdrawals will have to increase by 4 percent each year to compensate. The withdrawals each year will thus be:

$$C_1 = \text{Rs } 25{,}000(1.04) = \text{Rs } 26{,}000$$

$$C_2 = \text{Rs } 25{,}000(1.04)^2 = \text{Rs } 27{,}040$$

$$C_3 = \text{Rs } 25{,}000(1.04)^3 = \text{Rs } 28{,}121.60$$

- The present value of these cash flows if the appropriate nominal discount rate is 10 percent is given by:

$$PV = \text{Rs } 26{,}000/1.10 + \text{Rs } 27{,}040/1.102 + \text{Rs } 28{,}121.60/1.103 = \text{Rs } 67{,}111.65$$

Notice that we discounted the nominal cash flows at a nominal rate.

- To calculate the present value using real cash flows, we need the real discount rate. From the Fisher equation, we can get the real interest rate.
- By design, the real cash flows are an annuity of Rs 25,000 per year. So, the present value in real terms is:

$$PV = Rs\ 25,000[1 - (1/1.0577^3)]/.0577 = Rs\ 67,111.65$$

This is exactly the same present value we calculated before.

Interest rates and Investment

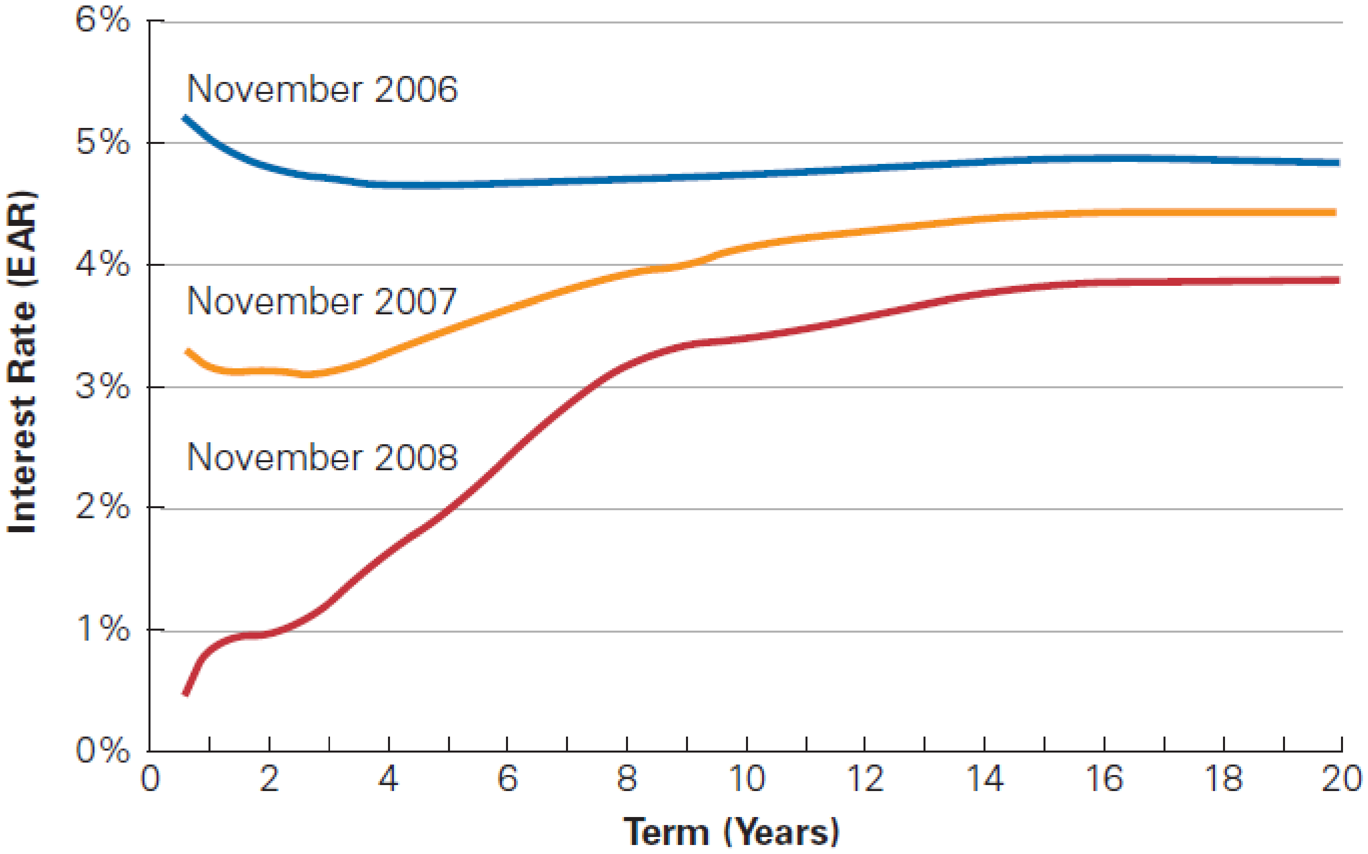
- Interest rates also affect firms' incentive to raise capital and invest. Consider a risk-free investment opportunity that requires an upfront investment of Rs 10 lakh and generates Rs 3 lakh as cash flow for four years. If the risk-free interest rate is 5%, the investment has a positive NPV. However, if the interest rate is 9%, the NPV is negative.
- The reason, of course, is that we are discounting the positive cash flows at a higher rate, which reduces their present value. The cost of Rs 10 lakh occurs today, however, so its present value is independent of the discount rate. More generally, when the costs of an investment precede the benefits, an increase in the interest rate will decrease the investment's NPV. **All else equal, higher interest rates will therefore tend to shrink the set of positive-NPV investments available to firms.**
- Central banks use this relationship between interest rates and investment to try to guide the economy. They can **raise interest rates to reduce investment if the economy is "overheating" and inflation is on the rise**, and they can lower interest rates to stimulate investment if the economy is slowing or in recession.

Yield curve and Discount Rate

- The interest rates that banks offer on investments or charge on loans depend on the horizon, or *term*, of the investment or loan. The relationship between the investment term and the interest rate is called the **term structure** of interest rates. We can plot this relationship on a graph called the **yield curve**.
- The following figure shows the term structure and corresponding yield curve of risk-free U.S. interest rates in November 2006, 2007, and 2008. In each case, note that the interest rate depends on the horizon, and that the difference between short-term and long-term interest rates was especially pronounced in 2008.

Term Structure of Risk-free U.S interest rates

Term (years)	Date Nov-06	Nov-07	Nov-08
0.5	5.23%	3.32%	0.47%
1	4.99%	3.16%	0.91%
2	4.80%	3.16%	0.98%
3	4.72%	3.12%	1.26%
4	4.63%	3.34%	1.69%
5	4.64%	3.48%	2.01%
6	4.65%	3.63%	2.49%
7	4.66%	3.79%	2.90%
8	4.69%	3.96%	3.21%
9	4.70%	4.00%	3.38%
10	4.73%	4.18%	3.41%
15	4.89%	4.44%	3.86%
20	4.87%	4.45%	3.87%



- **We can use the term structure to compute the present and future values of a risk-free cash flow over different investment horizons.**
- A risk-free cash flow received in two years should be discounted at the two-year interest rate, and a cash flow received in ten years should be discounted at the ten year interest rate. In general, a risk-free cash flow of C_n received in *n* years has present value $C_n/(1+r_n)^n$ where r_n is the risk-free interest rate (expressed as an EAR) for an *n*-year term.
- In other words, when computing a present value we must match the term of the cash flow and term of the discount rate. The present value of a cash flow stream using the term structure of discount rates is

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N} = \sum_{n=1}^{N} \frac{C_n}{(1 + r_n)^n}$$

Q. Compute the present value of a risk-free five-year annuity of Rs 1000 per year, given the yield curve for November 2008.

Yield curve and the Economy

- The central bank influences very short-term interest rates (call rates) through its influence on the **repo rate**, which is the rate at which banks can borrow cash reserves on an overnight basis. All other interest rates on the yield curve are set in the market and are adjusted until the supply of lending matches the demand for borrowing at each loan term.
- Expectations of future interest rate changes have a major effect on investors' willingness to lend or borrow for longer terms and, therefore, on the shape of the yield curve.

- Suppose short-term interest rates are equal to long-term interest rates. If investors expect interest rates to rise in the future, they would not want to make long-term investments. **Instead, they could do better by investing on a short-term basis and then reinvesting after interest rates rose**. Thus, if interest rates are expected to rise, long-term interest rates will tend to be higher than short-term rates to attract investors.
- Similarly, if interest rates are expected to fall in the future, then borrowers would not wish to borrow at long-term rates that are equal to short-term rates. They would do better by borrowing on a short-term basis, and then taking out a new loan after rates fall. So, if interest rates are expected to fall, long-term rates will tend to be lower than short-term rates to attract borrowers.
- These arguments imply that **the shape of the yield curve will be strongly influenced by interest rate expectations**. A sharply increasing (*steep*) yield curve, with long-term rates much higher than short-term rates, generally indicates that interest rates are expected to rise in the future (see the yield curve for November 2008 in the previous figure). A decreasing (*inverted)*yield curve, with long-term rates lower than short-term rates, generally signals an expected decline in future interest rates (see the yield curve for November 2006 in the previous figure). Because interest rates tend to drop in response to a slowdown in the economy, an inverted yield curve is often interpreted as a negative forecast for economic growth.

- Each of the last seven recessions in the U.S was preceded by a period in which the yield curve was inverted. Conversely, the yield curve tends to be steep as the economy comes out of a recession and interest rates are expected to rise.
- Clearly, the yield curve provides extremely important information for a business manager. **In addition to specifying the discount rates for risk-free cash flows that occur at different horizons, it is also a potential leading indicator of future economic growth**.

Q. Suppose the current one-year interest rate is 1%. If it is known with certainty that the one-year interest rate will be 2% next year and 4% the following year, what will the interest rates r_1, r_2, and r_3 of the yield curve be today? Is the yield curve flat, increasing, or inverted?

- The one-year rate r_1 = 1%. To find the two-year rate, note that if we invest Re 1 for one year at the current one-year rate and then reinvest next year at the new one-year rate, after two years we will earn

$$1 * (1.01) * (1.02) = \text{Rs } 1.0302$$

- We should earn the same payoff if we invest for two years at the current two-year rate r_2:

$$1 * (1 + r_2)^2 = \text{Rs } 1.0302$$

- Otherwise, there would be an arbitrage opportunity: If investing at the two-year rate led to a higher payoff, investors could invest for two years and borrow each year at the one-year rate. If investing at the two-year rate led to a lower payoff, investors could invest each year at the one year rate and borrow at the two-year rate.
- Solving for r_2, we find that

$$r_2 = (1.0302)^{1/2} - 1 = 1.499\%$$

- Similarly, investing for three years at the one-year rates should have the same payoff as investing at the current three-year rate:

$$(1.01) * (1.02) * (1.04) = 1.0714 = (1 + r_3)^3$$

$$r_3 = (1.0714)^{1/3} - 1 = 2.326\%.$$

- Therefore, the current yield curve has r_1=1%, r_2 = 1.499%, and r_3= 2.326%. The yield curve is increasing as a result of the anticipated higher interest rates in the future.

Interest rate and Taxes

- If the cash flows from an investment are taxed, the investor's actual cash flow will be reduced by the amount of the tax payments. Here, we consider the effect of taxes on the interest earned on savings (or paid on borrowing). Taxes reduce the amount of interest the investor can keep, and this reduced amount is referred to as the **after-tax interest rate**.
- In general, if the interest rate is *r* and the tax rate is t, then for each Re 1 invested you will earn interest equal to *r* and owe tax of t * *r* on the interest. The equivalent after-tax interest rate is therefore

$$\mathbf{r - (t*r) = r(1-t)}$$

Opportunity cost of capital

- The market interest rate provides the exchange rate that we need to compute present values and evaluate an investment opportunity. But with so many interest rates to choose from, the term market interest rate is inherently ambiguous.
- The discount rate that is typically used to evaluate cash flows is the investor’s **opportunity cost of capital** (or more simply, the **cost of capital**), which is *the best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted*.
- The cost of capital is clearly relevant for a firm seeking to raise capital from outside investors. In order to attract funds, **the firm must offer an expected return comparable to what investors could earn elsewhere with the same risk and horizon.** The same logic applies when a firm considers a project it can fund internally. Because any funds invested in a new project could be returned to shareholders to invest elsewhere, the new project should be taken only if it offers a better return than shareholders’ other opportunities.
- Thus, **the opportunity cost of capital provides the benchmark against which the cash flows of the new investment should be evaluated**. For a risk-free project, it will typically correspond to the interest rate on Treasury bills with a similar term. The cost of capital for risky projects will often exceed this amount, depending on the nature and magnitude of the risk.