Bond Valuation

Introduction

- When a corporation or government wishes to borrow money from the public on a long term basis, it usually does so by issuing or selling debt securities that are generically called *bonds.*
- A bond is a security sold by governments and corporations to raise money from investors today in exchange for promised future payments. The terms of the bond are described as part of the **bond certificate**, which indicates the amounts and dates of all payments to be made. These payments are made until a final repayment date, called the **maturity date** of the bond. The time remaining until the repayment date is known as the **term** of the bond.
- Bonds typically make two types of payments to their holders. **The promised interest payments of a bond are called coupons**. The bond certificate typically specifies that the coupons will be paid periodically (e.g., semiannually) until the maturity date of the bond. **The principal or face value of a bond is the notional amount we use to compute the interest payments**. Usually, the face value is repaid at maturity.
- The amount of each coupon payment is determined by the coupon rate of the bond. This coupon rate is set by the issuer and stated on the bond certificate.

- From a financial point of view, the main differences between debt and equity are the following:

➢ **Debt is not an ownership interest in the firm**. Creditors generally do not have voting power.

➢ The corporation’s payment of interest on debt is considered a cost of doing business and is **tax deductible** (up to certain limits). Dividends paid to stockholders are *not* tax deductible.

➢ Unpaid debt is a **liability of the firm**. If it is not paid, the creditors can legally claim the assets of the firm. This action can result in liquidation or reorganization, two of the possible consequences of bankruptcy. Thus, one of the costs of issuing debt is the possibility of financial failure. This possibility does not arise when equity is issued.

- The simplest type of bond is a **zero-coupon bond**, which does not make coupon payments. The only cash payment the investor receives is the face value of the bond on the maturity date. **Treasury bills** which are government bonds with a maturity of up to one year, are zero-coupon bonds.
- The present value of a future cash flow is less than the cash flow itself. As a result, prior to its maturity date, the price of a zero coupon bond is less than its face value. That is, zero-coupon bonds trade at a **discount**(a price lower than the face value), so they are also called **pure discount bonds.**

Types of bonds

- A **zero coupon bond** makes no coupon payments and is thus initially priced at a deep discount.
- A **convertible bond** can be swapped for a fixed number of shares of stock anytime before maturity at the holder's option.
- In conventional bonds, coupon rates are set as fixed percentages of the par values. Similarly, the principal amounts are set equal to the par values. With floating bonds, coupon payments are adjustable and are tied to an interest rate index (with a lag). The value of a **floating-rate bond** depends on exactly how the coupon payment adjustments are defined. A particularly interesting type of floating-rate bond is an *inflation-linked* bond.
- **Debenture** is unsecured debt usually with a maturity of more than 10 years.
- **Sovereign bonds** are bonds issued by national governments
- Catastrophe bonds

The Indenture

- The **indenture** is the written agreement between the corporation (the borrower) and its creditors. It is sometimes referred to as the *deed of trust*. Usually, a trustee (a bank) is appointed by the corporation to represent the bondholders. The trust company must (1) make sure the terms of the indenture are obeyed, (2) manage the sinking fund and (3) represent the bondholders in default—that is, if the company defaults on its payments to them.
- The bond indenture is a legal document. It can run several hundred pages and generally makes for tedious reading. It is an important document because it generally includes the following provisions:

➢ The basic terms of the bonds.

➢ The total amount of bonds issued.

➢ A description of property used as security.

➢ The repayment arrangements.

➢ The call provisions.

➢ Details of the protective covenants.

- Bonds can be repaid at maturity, at which time the bondholder will receive the stated, or face, value of the bond; or they may be repaid in part or in entirety before maturity. Early repayment in some form is more typical and is often handled through a sinking fund.
- A **sinking fund** is an account managed by the bond trustee for the purpose of repaying the bonds. The company makes annual payments to the trustee, who then uses the funds to retire a portion of the debt. The trustee does this by either buying up some of the bonds in the market or calling in a fraction of the outstanding bonds.

The Call Provision

- A **call provision** allows the company to repurchase or “call” part or all of the bond issue at stated prices over a specific period. Corporate bonds are usually callable.
- Generally, the call price is above the bond’s stated value (that is, the par value). The difference between the call price and the stated value is the **call premium**. The amount of the call premium may become smaller over time. One arrangement is to initially set the call premium equal to the annual coupon payment and then make it decline to zero as the call date moves closer to the time of maturity.
- Call provisions are often not operative during the first part of a bond’s life. This makes the call provision less of a worry for bondholders in the bond’s early years. For example, a company might be prohibited from calling its bonds for the first 10 years. This is a **deferred call provision**. During this period of prohibition, the bond is said to be **call-protected**.

Protective Covenants

- A **protective covenant** is that part of the indenture or loan agreement that limits certain actions a company might otherwise wish to take during the term of the loan. Protective covenants can be classified into two types: negative covenants and positive (or affirmative) covenants.
- A *negative covenant* is a "thou shalt not" type of covenant. It limits or prohibits actions the company might take. Here are some typical examples:

➢ The firm must limit the amount of dividends it pays according to some formula.

➢ The firm cannot pledge any assets to other lenders.

➢ The firm cannot merge with another firm.

➢ The firm cannot sell or lease any major assets without approval by the lender.

➢ The firm cannot issue additional long-term debt.

- A *positive covenant* is a "thou shalt" type of covenant. It specifies an action the company agrees to take or a condition the company must abide by. Here are some examples:

➢ The company must maintain its working capital at or above some specified minimum level.

➢ The company must periodically furnish audited financial statements to the lender.

➢ The firm must maintain any collateral or security in good condition.

Bond values and yields

- As time passes, interest rates change in the marketplace. The cash flows from a bond, however, stay the same. As a result, the value of the bond will fluctuate. When interest rates rise, the present value of the bond's remaining cash flows declines, and the bond is worth less. When interest rates fall, the bond is worth more.
- To determine the value of a bond at a particular point in time, we need to know the number of periods remaining until maturity, the face value, the coupon, and the market interest rate for bonds with similar features. This interest rate required in the market on a bond is called the bond's **yield to maturity (YTM)**. This rate is sometimes called the bond's *yield* for short. Given all this information, we can calculate the present value of the cash flows as an estimate of the bond's current market value.
- For example, XYZ bond will pay Rs 80 per year for the next 10 years in coupon interest Similar bonds have yield to maturity of 8 per cent. Thus company will pay coupon interest of Rs 80 and in 10 years Rs 1,000 to the owner of the bond. What would this bond sell for?
- The bond's cash flows have an annuity component (the coupons) and a lump sum (the face value paid at maturity). **We thus estimate the market value of the bond by calculating the present value of these two components separately and adding the results together.**

- First, at the going rate of 8 percent, the present value of the Rs 1,000 paid in 10 years is:

$$\text{Present value} = \text{Rs } 1{,}000/1.08^{10}$$
$$= \text{Rs } 1{,}000/2.1589 = \text{Rs } 463.19$$

- Second, the bond offers Rs 80 per year for 10 years; the present value of this annuity stream is:

$$\text{Annuity present value} = \text{Rs } 80 \times (1-1/1.08^{10})/.08$$
$$= \text{Rs } 536.81$$

- We can now add the values for the two parts together to get the bond's value: Total bond value = Rs 463.19 + Rs 536.81 = Rs 1,000
- This bond sells for exactly its face value. This is not a coincidence as the going interest rate in the market is 8 percent. Considered as an interest-only loan, what interest rate does this bond have? With an Rs 80 coupon, this bond pays exactly 8 percent interest only when it sells for Rs 1,000.

- To illustrate what happens as interest rates change, suppose a year has gone by. The XYZ bond now has nine years to maturity. If the interest rate in the market has risen to 10 percent, what will the bond be worth?
- To find out, we repeat the present value calculations with 9 years instead of 10, and a 10 percent yield instead of an 8 percent yield. First, the present value of the Rs 1,000 paid in nine years at 10 percent is:

$$\text{Present value} = \text{Rs } 1{,}000/1.10^9$$
$$= \text{Rs } 1{,}000/2.3579 = \text{Rs } 424.10$$

- Second, the bond now offers Rs 80 per year for nine years; the present value of this annuity stream at 10 percent is:

$$\text{Annuity present value} = \text{Rs } 80 \times (1-1/1.10^9)/.10$$
$$= \text{Rs } 80 \times (1-1/2.3579)/.10$$
$$= \text{Rs } 460.72$$

- We can now add the values for the two parts together to get the bond's value: Total bond value = Rs 424.10 + Rs 460.72 = Rs 884.82
- Therefore, the bond should sell for about Rs 885. **In the vernacular, we say that this bond, with its 8 percent coupon, is priced to yield 10 percent at Rs 885.**

- The XYZ bond now sells for less than its face value. Because this bond pays less than the going rate, investors are willing to lend only something less than the Rs 1,000 promised repayment. Because the bond sells for less than face value, it is said to be a **discount bond.**
- The only way to get the interest rate up to 10 percent is to lower the price to less than Rs 1,000 so that the purchaser, in effect, has a built-in gain. For the XYZ bond, the price of Rs 885 is Rs 115 less than the face value, so an investor who purchases and keeps the bond will get Rs 80 per year and will have a Rs 115 gain at maturity as well. This gain compensates the lender for the below-market coupon rate.

- Another way to see why the bond is discounted by Rs 115 is to note that the Rs 80 coupon is Rs 20 below the coupon on a newly issued par value bond, based on current market conditions. The bond would be worth Rs 1,000 only if it had a coupon of Rs 100 per year. In a sense, an investor who buys and keeps the bond gives up Rs 20 per year for nine years. At 10 percent, this annuity stream is worth:

$$\text{Annuity present value} = \text{Rs } 20 \times (1 - 1/1.10^9)/.10$$

$$= \text{Rs } 115.18$$

This is the amount of the discount.

- Now if the interest rates had dropped by 2 per cent instead of rising by 2 per cent. Then the bond would sell for more than Rs 1000. Such a bond is said to sell at a *premium* and is called a *premium bond*.
- The XYZ bond now has a coupon rate of 8 percent when the market rate is only 6 percent. Investors are willing to pay a premium to get this extra coupon amount. In this case, the relevant discount rate is 6 percent, and there are nine years remaining. The present value of the Rs 1,000 face amount is:

$$\text{Present value} = \text{Rs } 1{,}000/1.06^9 = \text{Rs } 1{,}000/1.6895 = \text{Rs } 591.90$$

- The present value of the coupon stream is:

$$\text{Annuity present value} = \text{Rs } 80 \times (1-1/1.06^9)/.06$$

$$= \text{Rs } 544.14$$

- We can now add the values for the two parts together to get the bond's value:

$$\text{Total bond value} = \text{Rs } 591.90 + \text{Rs } 544.14 = \text{Rs } 1{,}136.03$$

- Total bond value is therefore about Rs 136 in excess of par value. Once again, we can verify this amount by noting that the coupon is now Rs 20 too high, based on current market conditions.
- The present value of Rs 20 per year for nine years at 6 percent is:

$$\text{Annuity present value} = \text{Rs } 20 \times (1-1/1.06^9)/.06$$

$$= \text{Rs } 20 \times 6.8017$$

$$= \text{Rs } 136.03$$

- We can write the general expression for the value of bond as

$$\text{Bond value} = C \times [1 - 1/(1 + r)^n]/r + F/(1 + r)^n$$

i.e. Bond value = Present value of the coupons + Present value of the face amount

Yield to Maturity

- The IRR of an investment opportunity is the discount rate at which the NPV of the cash flows of the investment opportunity is equal to zero. So, the IRR of an investment in a zero-coupon bond is the rate of return that investors will earn on their money if they buy the bond at its current price and hold it to maturity.
- The IRR of an investment in a bond is given a special name, the **yield to maturity (YTM)** or just the *yield.* The yield to maturity of a bond is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.
- The yield to maturity for a zero-coupon bond with *n* periods to maturity, current price *P*, and face value *FV* solves $P = \frac{FV}{(1 + YTM_n)^n}$

- Intuitively, the yield to maturity for a zero-coupon bond is the return you will earn as an investor from holding the bond to maturity and receiving the promised face value payment

$$\mathbf{YTM= (FV/P)^{1/n} -1}$$

- The yield to maturity (YTM_n) is the per-period rate of return for holding the bond from today until maturity on date *n*.
- We can also compute the yield to maturity of a coupon bond. Because the **coupon payments represent an annuity**, the yield to maturity is the interest rate *y* that solves the following equation

Yield to Maturity of a Coupon Bond

$$P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N}$$

- Because we can convert any price into a yield, and vice versa, prices and yields are often used interchangeably. Indeed, **bond traders generally quote bond yields rather than bond prices**. **One advantage of quoting the yield to maturity rather than the price is that the yield is independent of the face value of the bond**. When prices are quoted in the bond market, they are conventionally quoted as a percentage of their face value.

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The price of a bond can be written as the sum of its annuity and lump sum components. Knowing that there is an Rs 80 coupon for six years and a Rs 1,000 face value, we can say that the price is:

$$\text{Rs } 955.14 = \text{Rs } 80 \times [1 - 1/(1 + r)^6]/r + 1{,}000/(1 + r)^6$$

where r is the unknown discount rate, or yield to maturity. We have one equation here and one unknown, but we cannot solve it for r explicitly. The only way to find the answer is to use trial and error.

We can speed up the trial-and-error process by using what we know about bond prices and yields. In this case, the bond has an Rs 80 coupon and is selling at a discount. So the yield is greater than 8 percent

- A bond’s yield to maturity should not be confused with its **current yield**, which is a bond’s annual coupon divided by its price. In the previous question, the bond’s annual coupon was Rs 80, and its price was Rs 955.14.
- Given these numbers, we see that the current yield is Rs 80/955.14 = 8.38 percent, which is less than the yield to maturity of 9 percent. **The reason the current yield is too low is that it considers only the coupon portion of your return**; it doesn’t consider the built-in gain from the price discount.

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Given the yield, we can compute the price. First, note that a 6.30% APR is equivalent to a semiannual rate of 3.15%.

Q. An ordinary bond has a coupon rate of 14 percent paid semiannually. The yield to maturity is quoted at 16 percent. With a 16 percent quoted yield and semiannual payments, the true yield is 8 percent per six months. The bond matures in seven years. What is the bond's price? What is the effective annual yield on this bond?

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We know the bond will sell at a discount because it has a coupon rate of 7 percent every six months when the market requires 8 percent every six months (with a 16 percent quoted yield and semiannual payments, the true yield is 8 percent per six months)

To get the exact price, we first calculate the present value of the bond's face value of Rs 1,000 paid in seven years. This seven-year period has 14 periods of six months each. The coupons can be viewed as a 14-period annuity of Rs 70 per period.

To calculate the effective yield on this bond, note that 8 percent every six months is equivalent to:

$$\text{Effective annual rate} = (1 + .08)^2 - 1 = 16.64\%$$

Q. Reliance Industries bond has a 10 percent coupon rate and a Rs 1,000 face value. Interest is paid semiannually, and the bond has 20 years to maturity. If investors require a 12 percent yield, what is the bond's value? What is the effective annual yield on the bond?

Dynamic behaviour of bond prices

- Coupon bonds may trade at a discount, at a **premium** (a price greater than their face value), or at **par** (a price equal to their face value).
- If the bond trades at a discount, an investor who buys the bond will earn a return both from receiving the coupons *and* from receiving a face value that exceeds the price paid for the bond. As a result, **if a bond trades at a discount, its yield to maturity will exceed its coupon rate**. Given the relationship between bond prices and yields, the reverse is clearly also true: If a coupon bond's yield to maturity exceeds its coupon rate, the present value of its cash flows at the yield to maturity will be less than its face value, and the bond will trade at a discount.
- A bond that pays a coupon can also trade at a premium to its face value. In this case, an investor's return from the coupons is diminished by receiving a face value less than the price paid for the bond. Thus, a bond trades at a premium whenever its yield to maturity is less than its coupon rate.
- When a bond trades at a price equal to its face value, it is said to trade at par. **A bond trades at par when its coupon rate is equal to its yield to maturity.** A bond that trades at a discount is also said to trade below par, and a bond that trades at a premium is said to trade above par.

Q. Consider three 30-year bonds with annual coupon payments. One bond has a 10% coupon rate, one has a 5% coupon rate, and one has a 3% coupon rate. If the yield to maturity of each bond is 5%, what is the price of each bond per Rs 100 face value? Which bond trades at a premium, which trades at a discount, and which trades at par?

Time and Bond Prices

- Most issuers of coupon bonds choose a coupon rate so that the bonds will *initially* trade at, or very close to, par (i.e., at face value). After the issue date, **the market price of a bond generally changes over time for two reasons**. **First, as time passes, the bond gets closer to its maturity date**. Holding fixed the bond's yield to maturity, the present value of the bond's remaining cash flows changes as the time to maturity decreases.
- **Second, at any point in time, changes in market interest rates affect the bond's yield to maturity** and its price (the present value of the remaining cash flows).
- Let's consider the effect of time on the price of a bond. Suppose you purchase a 30-year, zero-coupon bond with a yield to maturity of 5%. For a face value of Rs 100, the bond will initially trade for P(30 years to maturity) = Rs 23.14
- Now let's consider the price of this bond five years later, when it has 25 years remaining until maturity. If the bond's yield to maturity remains at 5%, the bond price in five years will be P(25 years to maturity) = Rs 29.53
- Note that the bond price is higher, and hence the discount from its face value is smaller, when there is less time to maturity. The discount shrinks because the yield has not changed, but there is less time until the face value will be received.

- If you purchased the bond for Rs 23.14 and then sold it after five years for Rs 29.53, the IRR of your investment would be 5 %.

$$IRR = (29.53/23.14)^{1/5} - 1$$

- That is, your return is the same as the yield to maturity of the bond. This example illustrates a more general property for bonds: **If a bond's yield to maturity has not changed, then the IRR of an investment in the bond equals its yield to maturity** even if you sell the bond early.
- These results also hold for coupon bonds. The pattern of price changes over time is a bit more complicated for coupon bonds, however, because as time passes, most of the cash flows get closer but some of the cash flows disappear as the coupons get paid.

Interest Rate changes and Bond Prices

- As interest rates in the economy fluctuate, the yields that investors demand to invest in bonds will also change. Consider a 30-year, zero-coupon bond with a yield to maturity of 5%. For a face value of Rs 100, the bond will initially trade for

$$P(5\%\text{ yield to maturity}) = 100/1.05^{30} = \text{Rs } 23.14$$

- But suppose interest rates suddenly rise so that investors now demand a 6% yield to maturity before they will invest in this bond. This change in yield implies that the bond price will fall to

$$P(6\%\text{ yield to maturity}) = 100/1.06^{30} = \text{Rs } 17.41$$

- A higher yield to maturity implies a higher discount rate for a bond's remaining cash flows, reducing their present value and hence the bond's price. Therefore, *as interest rates and bond yields rise, bond prices will fall, and vice versa.*

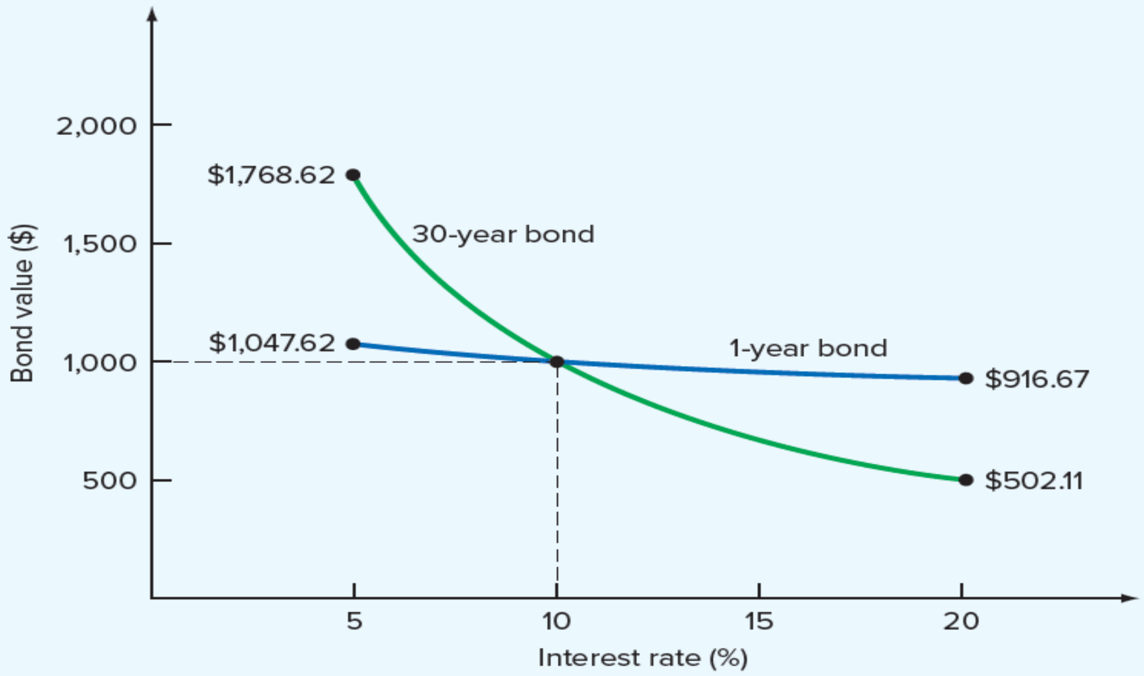
- The sensitivity of a bond’s price to changes in interest rates depends on the timing of its cash flows. Because it is discounted over a shorter period, **the present value of a cash flow that will be received in the near future is less dramatically affected by interest rates than a cash flow in the distant future**. Thus, shorter-maturity zero-coupon bonds are less sensitive to changes in interest rates than are longer-term zero-coupon bonds.
- Similarly, bonds with higher coupon rates—because they pay higher cash flows upfront—are less sensitive to interest rate changes than otherwise identical bonds with lower coupon rates. **The sensitivity of a bond’s price to changes in interest rates is measured by the bond’s duration**. Bonds with high durations are highly sensitive to interest rate changes.

Q. Consider a 15-year zero-coupon bond and a 30-year coupon bond with 10% annual coupons. By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%? Take face value of bond to be Rs 100

Interest rate risk

- The risk that arises for bond owners from fluctuating interest rates is called *interest rate risk*. How much interest rate risk a bond has depends on how sensitive its price is to interest rate changes. This sensitivity directly depends on two things: the time to maturity and the coupon rate.
- **All other things being equal, the longer the time to maturity, the greater the interest rate risk.** Because it is discounted over a shorter period, the present value of a cash flow that will be received in the near future is less dramatically affected by interest rates than a cash flow in the distant future. Thus, shorter-maturity zero-coupon bonds are less sensitive to changes in interest rates than are longer-term zero-coupon bonds.
- **All other things being equal, the lower the coupon rate, the greater the interest rate risk.** Bonds with higher coupon rates—because they pay higher cash flows upfront—are less sensitive to interest rate changes than otherwise identical bonds with lower coupon rates.

Interest rate risk and Time to Maturity



Value of a Bond with a 10 Percent Coupon Rate for Different Interest Rates and Maturities

	Time to Maturity	
Interest Rate	1 Year	30 Years
5%	$1,047.62	$1,768.62
10	1,000.00	1,000.00
15	956.52	671.70
20	916.67	502.11

- Notice how the slope of the line connecting the prices is much steeper for the 30-year maturity than it is for the 1-year maturity. This steepness tells us that a relatively small change in interest rates will lead to a substantial change in the bond’s value. In comparison, the 1-year bond’s price is relatively insensitive to interest rate changes.

- Intuitively, we can see that **longer-term bonds have greater interest rate sensitivity because a large portion of a bond’s value comes from the face amount**. The present value of this amount isn’t greatly affected by a small change in interest rates if the amount is to be received in one year. Even a small change in the interest rate, however, once it is compounded for 30 years, can have a significant effect on the present value.

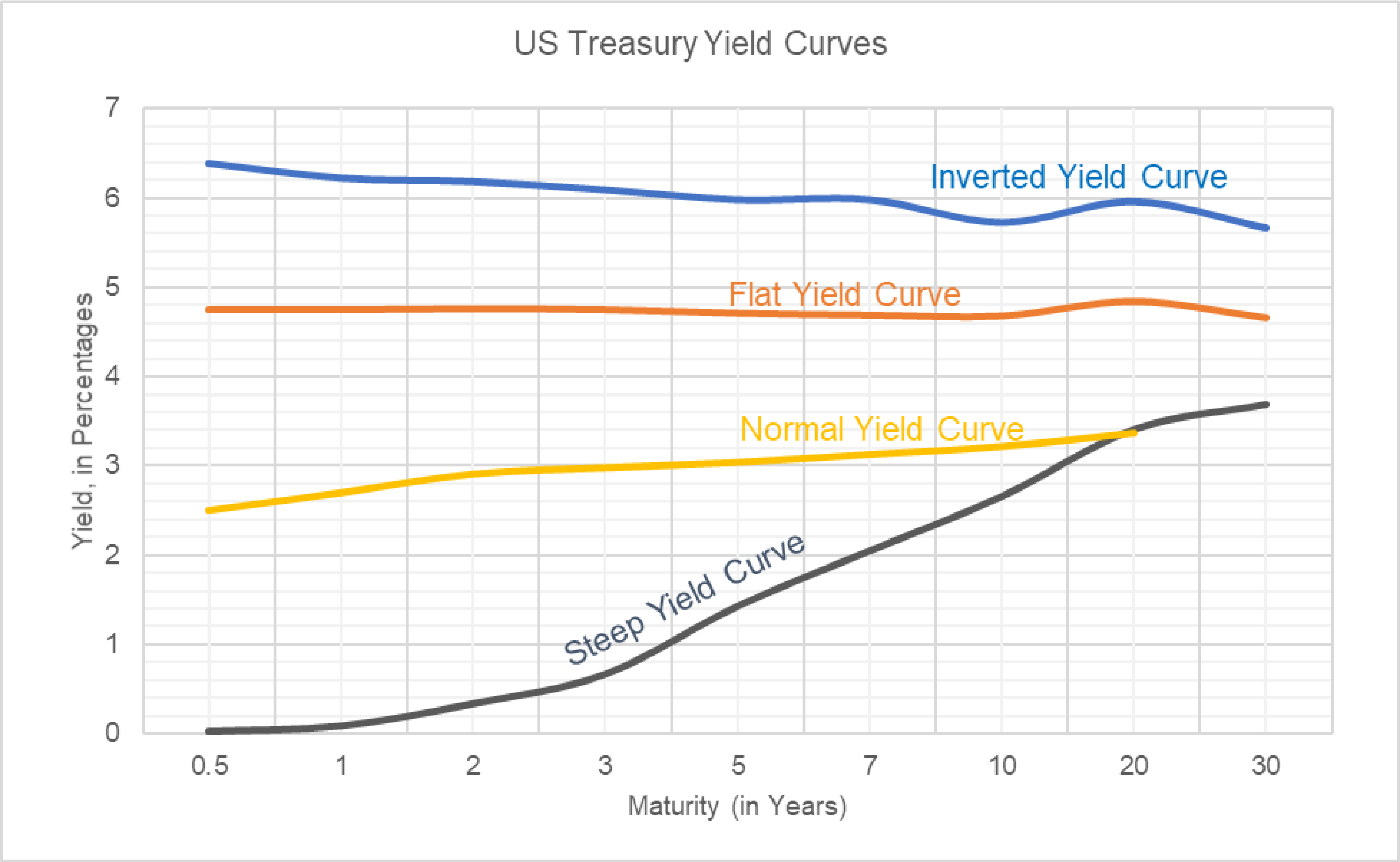
- The other thing to know about **interest rate risk** is that, like most things in finance and economics, it **increases at a decreasing rate**. In other words, if we compared a 10-year bond to a 1-year bond, we would see that the 10-year bond has much greater interest rate risk. If you were to compare a 20-year bond to a 30-year bond, you would find that the 30-year bond has somewhat greater interest rate risk because it has a longer maturity, but the difference in the risk would be fairly small.

- The reason that bonds with lower coupons have greater interest rate risk is essentially the same. The value of a bond depends on the present value of its coupons and the present value of the face amount. If two bonds with different coupon rate have the same maturity, then the value of the one with the lower coupon is proportionately more dependent on the face amount to be received at maturity. As a result, all other things being equal, its value will fluctuate more as interest rates change. Put another way, the bond with the higher coupon has a larger cash flow early in its life, so its value is less sensitive to changes in the discount rate.

The Yield Curve

- The relationship between the investment term and the yields (interest rates) is called the **term structure of interest rates**. To be a little more precise, the term structure of interest rates tells us what *nominal* interest rates are on *default-free*, *pure discount* bonds of all maturities. These rates are, in essence, “pure” interest rates because they involve no risk of default and a single, lump sum future payment. We can plot this relationship on a graph called the **yield curve.**
- The three key types of yield curves include normal, inverted, and flat. Upward sloping (also known as normal yield curves) is where longer-term bonds have higher yields than short-term ones. While normal curves point to economic expansion, downward sloping (inverted) curves point to economic recession.

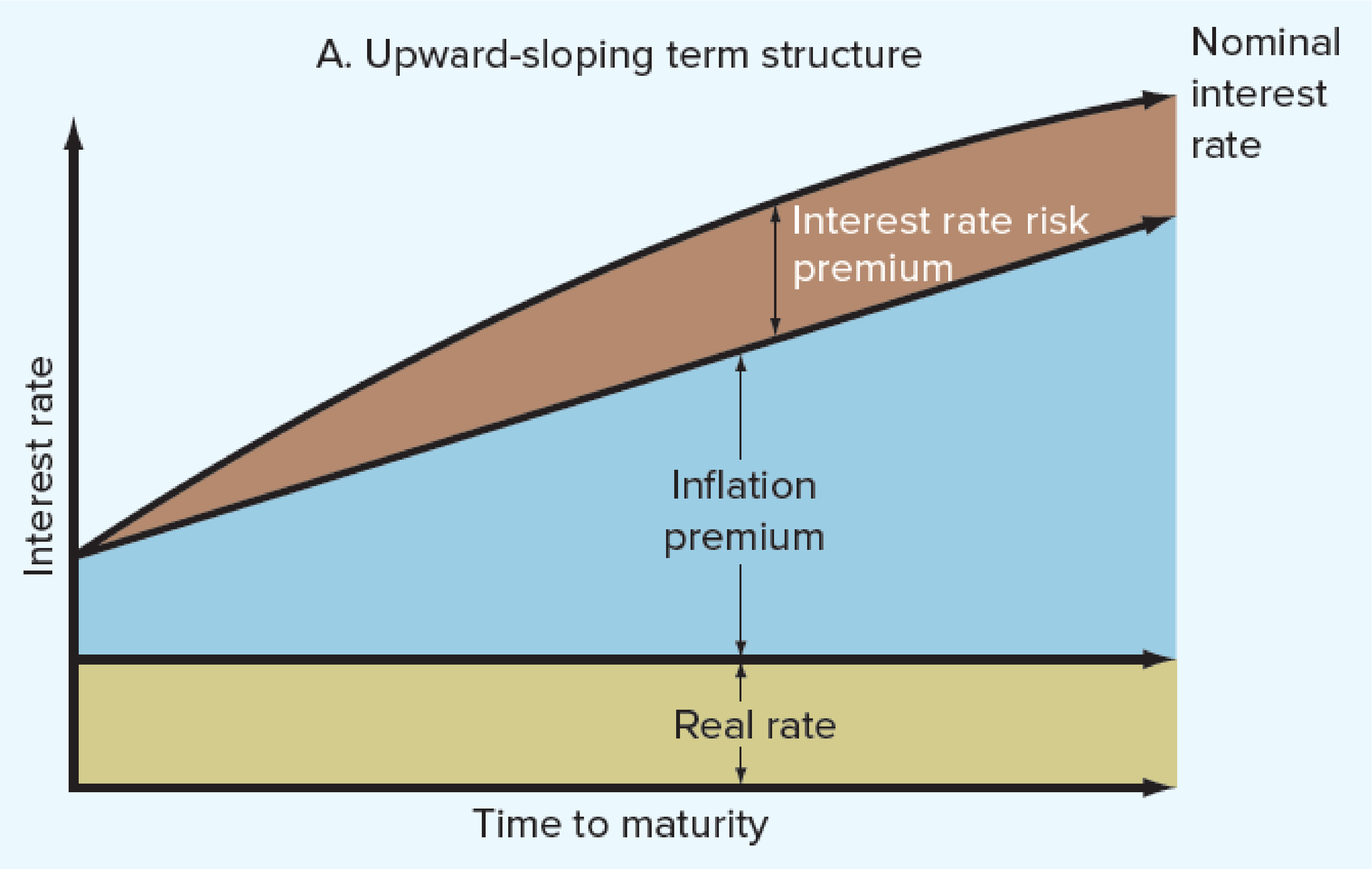
US Treasury yield curves



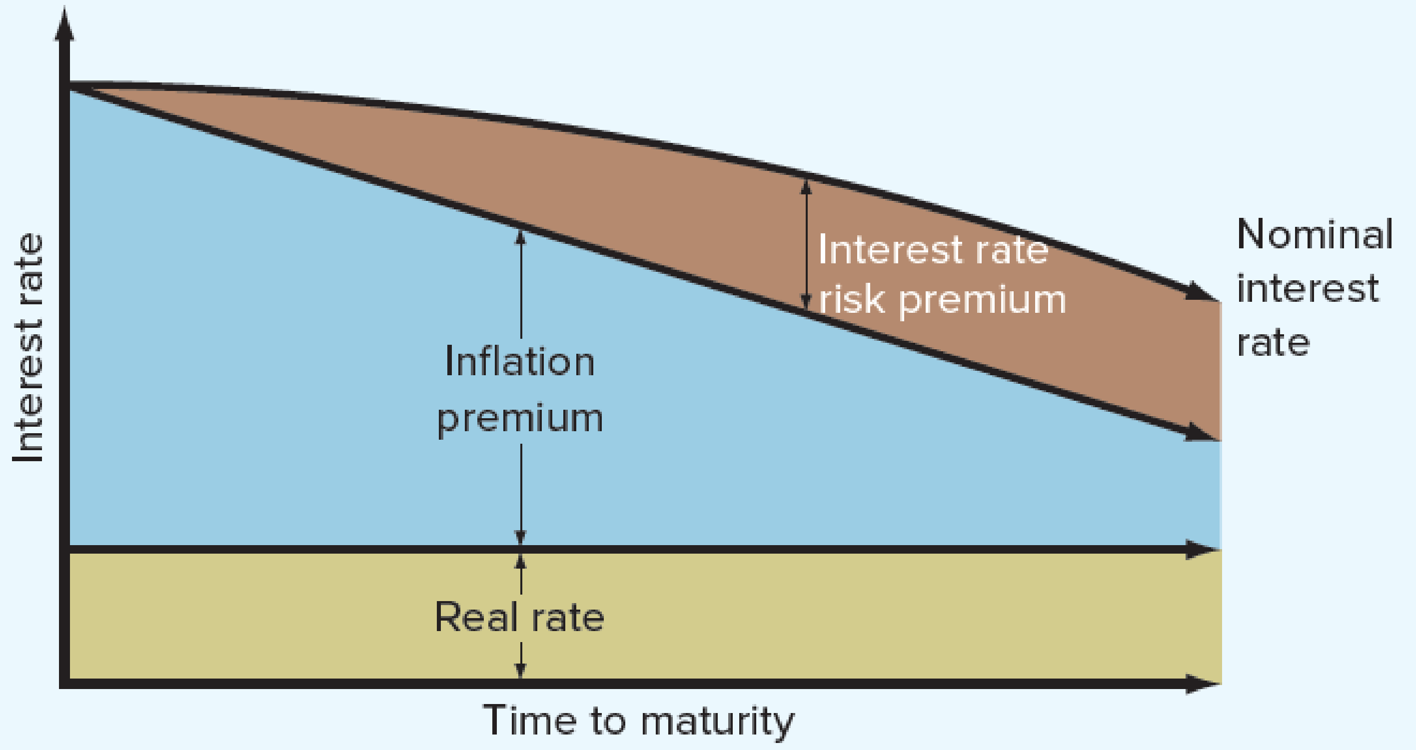
Determinants of bond yields

- **The term structure reflects the combined effect of the real rate of interest, the inflation premium, and the interest rate risk premium.**
- The real rate of interest is the compensation investors demand for forgoing the use of their money. You can think of it as the pure time value of money after adjusting for the effects of inflation.
- The real rate of interest is the basic component underlying every interest rate, regardless of the time to maturity. When the real rate is high, all interest rates will tend to be higher, and vice versa. Thus, the real rate doesn’t really determine the shape of the term structure; instead, it mostly influences the overall level of interest rates.
- In contrast, the prospect of future inflation strongly influences the shape of the term structure. Investors thinking about lending money for various lengths of time recognize that future inflation erodes the value of the rupees that will be returned. As a result, investors demand compensation for this loss in the form of higher nominal rates. This extra compensation is called the **inflation premium**.

- If investors believe the rate of inflation will be higher in the future, then long-term nominal interest rates will tend to be higher than short-term rates. Thus, an upward-sloping term structure may reflect anticipated increases in inflation. Similarly, a downward-sloping term structure probably reflects the belief that inflation will be falling in the future.
- The third, and last, component of the term structure has to do with interest rate risk. Longer-term bonds have much greater risk of loss resulting from changes in interest rates than do shorter-term bonds. Investors recognize this risk, and they demand extra compensation in the form of higher rates for bearing it. This extra compensation is called the **interest rate risk premium**. The longer the term to maturity, the greater the interest rate risk, so the interest rate risk premium increases with maturity. However, interest rate risk increases at a decreasing rate, so does the interest rate risk premium.


A. Upward-sloping term structure
Nominal interest rate
Interest rate risk premium
Inflation premium
Real rate
Interest rate
Time to maturity

B. Downward-sloping term structure



PV of cash flows based term structure of discount rates

- We can apply the same logic when computing the present value of cash flows with different maturities. A risk-free cash flow received in two years should be discounted at the two-year interest rate, and a cash flow received in ten years should be discounted at the ten year interest rate.

Present Value of a Cash Flow Stream Using a Term Structure of Discount Rate

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N} = \sum_{n=1}^{N} \frac{C_n}{(1 + r_n)^n} \qquad ($$

Replicating a coupon bond

- So far, we have focused on the relationship between the price of an individual bond and its yield to maturity. Here, we explore the **relationship between the prices and yields of different bonds**.
- Using the Law of One Price and given the spot interest rates, which are the yields of default-free zero-coupon bonds, we can determine the price and yield of any other default-free bond. As a result, the yield curve provides sufficient information to evaluate all such bonds.
- Because it is possible to replicate the cash flows of a coupon bond using zero-coupon bonds, we can use the Law of One Price to compute the price of a coupon bond from the prices of zero-coupon bonds. For example, **we can replicate a three-year, Rs 1000 bond that pays 10% annual coupons using a portfolio of three zero-coupon bonds**.

- We match each coupon payment to a zero-coupon bond with a face value equal to the coupon payment and a term equal to the time remaining to the coupon date. Similarly, we match the final bond payment (final coupon plus return of face value) in three years to a three-year, zero-coupon bond with a corresponding face value of Rs 1100.
- Because the coupon bond cash flows are identical to the cash flows of the portfolio of zero-coupon bonds, **the Law of One Price states that the price of the portfolio of zero-coupon bonds must be the same as the price of the coupon bond**.

Maturity	1 year	2 years	3 years	4 years
YTM	3.50%	4.00%	4.50%	4.75%
Price	Rs 96.62	Rs 92.45	Rs 87.63	Rs 83.06

- We can calculate the cost of the zero coupon bond portfolio that replicates the three-year coupon bond as follows:

Zero Coupon bond	Face Value required	Cost
1 year	100	Rs 96.62
2 years	100	Rs 92.45
3 years	1100	11*87.63 = 963.93

- By the Law of One Price, the three-year coupon bond must trade for a price of Rs 1153(total cost of the portfolio of zero coupon bonds). If the price of the coupon bond were higher, you could earn an arbitrage profit by selling the coupon bond and buying the zero-coupon bond portfolio. If the price of the coupon bond were lower, you could earn an arbitrage profit by buying the coupon bond and short selling the zero-coupon bonds.
- We have used the zero-coupon bond *prices* to derive the price of the coupon bond. Alternatively, we can use the zero-coupon bond *yields*. The yield to maturity of a zero-coupon bond is the competitive market interest rate for a risk-free investment with a term equal to the term of the zero-coupon bond. Therefore, the price of a coupon bond must equal the present value of its coupon payments and face value discounted at the competitive market interest rates.

$$P = PV(\text{Bond Cash Flows})$$

$$= \frac{CPN}{1 + YTM_1} + \frac{CPN}{(1 + YTM_2)^2} + \cdots + \frac{CPN + FV}{(1 + YTM_n)^n}$$

- where *CPN* is the bond coupon payment, YTM_n is the yield to maturity of a *zero-coupon* bond that matures at the same time as the *n*th coupon payment, and *FV* is the face value of the bond. For the three-year, Rs 1000 bond with 10% annual coupons considered earlier, we can calculate its price using the zero-coupon yields.
- This price is identical to the price we computed earlier by replicating the bond. Thus, we can determine the no-arbitrage price of a coupon bond by discounting its cash flows using the zero-coupon yields. In other words, **the information in the zero-coupon yield curve is sufficient to price all other risk-free bonds.**

Bond Ratings

- Firms frequently pay to have their debt rated. The two leading bond-rating firms are Moody’s and Standard & Poor’s (S&P). The debt ratings are an assessment of the creditworthiness of the corporate issuer. The definitions of creditworthiness used by Moody’s and S&P are based on how likely the firm is to default and the protection creditors have in the event of a default.
- It is important to recognize that bond ratings are concerned *only* with the possibility of default. Bond ratings do not address the issue of interest rate risk. As a result, the price of a highly rated bond can still be quite volatile.
- Debt rated Aaa (Moody’s)and AAA (S&P) has the highest rating. Capacity to pay interest and principal is extremely strong. Debt rated Aa and AA has a very strong capacity to pay interest and repay principal. Together with the highest rating, this group comprises the high-grade bond class.