CS401 -Computational finance
Properties of options

Jaideep Mulherkar

January 10, 2019

Factors determining European option price

S_0: Current stock price
K : Strike price
T : time to maturity
σ : Volatility

Variable	European call	European put
Current stock price S_0	+	-
Strike price K	-	+
Time to expiry T	+	+
Volatility	+	+

Table: Factors affecting option prices

Zero coupon bond

- A zero coupon bond pays K amount of cash at some future time T. (No intermediate payments are made.)
- The worth of a zero coupon bond at time $t = 0$ is Ke^{-rT}, where r is the risk free interest rate. (This is assuming that the interest rate r remains constant.)
- Examples are Govt. treasury bills.

Upper bounds on price of a European call option (non-dividend paying)

- c price of call option
  p price of put option

Upper bounds on price of a European call option (non-dividend paying)

- c price of call option
  p price of put option
- $c \leq S_0$

Upper bounds on price of a European call option (non-dividend paying)

- c price of call option
  p price of put option
- $c \leq S_0$
- if $c > S_0$
  $t = 0$: Sell call option for c and buy stock at S_0 and put $c - S_0$ in bank
  $t = T$: if $S_T > K$. Profit $= K + (c - S_0)e^{rT}$ (option ex)
  $t = T$: if $S_T \leq K$. Profit $= (c - S_0)e^{rT}$ (option not ex)

Upper bounds on price of a European put option (non-dividend paying)

- c price of call option
  p price of put option

Upper bounds on price of a European put option (non-dividend paying)

- c price of call option
  p price of put option
- $p \leq Ke^{-rT}$

Upper bounds on price of a European put option (non-dividend paying)

- c price of call option
  p price of put option
- $p \leq Ke^{-rT}$
- if $p > Ke^{-rT}$
  $t = 0$: Sell put option for p and put in bank
  $t = T$: if $S_T > K$. Profit $= pe^{rT}$ (option not ex.)
  $t = T$: if $S_T \leq K$. Profit $= S_T - K + pe^{rT}$ (option ex.)

Lower bounds on price of a European call option (Non dividend paying)

- $c \geq S_0 - Ke^{-rT}$

Lower bounds on price of a European call option (Non dividend paying)

- $c \geq S_0 - Ke^{-rT}$
- Consider the following portfolio
  (a): One European call option plus a zero coupon bond with payoff K at time T
  (b): one share of stock

Lower bounds on price of a European call option (Non dividend paying)

- $c \geq S_0 - Ke^{-rT}$
- Consider the following portfolio
  (a): One European call option plus a zero coupon bond with payoff K at time T
  (b): one share of stock
- Worth at time T:
  Portfolio (a) $\max(S_T, K)$ Portfolio (b) S_T. $((a) \geq (b))$
  Worth at time 0:
  Portfolio (a) $c + Ke^{-rT}$ Portfolio (b) S_0

  Hence no-arbitrage implies $c + Ke^{-rT} \geq S_0$.
  $c \geq \max(S_0 - Ke^{-rT}, 0)$.

Lower bounds on price of a European put option

- $p \geq Ke^{-rT} - S_0$

Lower bounds on price of a European put option

- $p \geq Ke^{-rT} - S_0$
- Consider the following portfolio
  (c): One European put option plus a share
  (d): one zero coupon bond paying K at time T.

Lower bounds on price of a European put option

- $p \geq Ke^{-rT} - S_0$
- Consider the following portfolio
  (c): One European put option plus a share
  (d): one zero coupon bond paying K at time T.
- Worth at time T
  Portfolio (c) $\max(S_T, K)$ Portfolio (b) K. $((c) \geq (d))$
  Worth at time 0
  Portfolio (a) $p + S_0$ Portfolio (b) Ke^{-rT}

  Hence no-arbitrage implies $p + S_0 \geq Ke^{-rT}$
  $p \geq \max(Ke^{-rT} - S_0, 0)$.

Put call parity for a European option

- Portfolio A : One European call option plus a zero coupon bond with payoff K at time T
  Portfolio B : One European put option plus a share
- Put-Call parity states that these two portfolios have the same value

$$c + Ke^{-rT} = p + S_0$$

- The value of a European call option with certain exercise price and date can be deduced from the put option with the same exercise price and date (and vice versa).

		$S_T > K$	$S_T < K$
Portfolio (A)	Call option	$S_T - K$	0
	ZC bond	K	K
Portfolio (B)	Put option	0	$K - S_T$
	Share	S_T	S_T

Table: Value of portfolio (A) and (B) at time T.

Therefore the present value of portfolio (A) and (B) must be the same

$$c + Ke^{-rT} = p + S_0$$

Further reading

Chap 11. *Properties of stock options, J.C. Hull* 9^{th} *edition.*