In Sem 1

Computational finance (CS401) Winter 2018

Time: 2hr

1. **Forward contracts arbitrage**
   The 2-month interest rate in Switzerland and United States are 1% and 2% per annum respectively with continuous compounding. The spot price of Swiss franc is \$1.05. The 2 month forward price is also \$1.05. Describe the arbitrage opportunity available.

2. **Options arbitrage**
   Consider the one period binomial model with $S_0 = 4, u = 2, d = \frac{1}{2}, r = \frac{1}{4}$. Consider a put option with strike price $K = \$5$. Show how an arbitrage can be done if the price of the put option is \$1.19 by starting with zero capital and building a portfolio to make a risk free profit.

3. **Options profit diagram**
   Consider the following option strategy. Suppose that a stock is currently worth \$61. The market prices of 6 month European call options is as follows: An inverstor buys

Strike price (\$)	Call price (\$)
55	10
60	7
65	5

   one call option with strike price \$55 and one call option with strike price \$65 and sells two call options with strike price \$60.
   i) Draw the diagram of Profit/Loss vs Stock price at maturity (S_T)
   ii)What is the investors strategy?

4. **Random walks:ballot theorem**
   i) Let $N_n(a, b)$ be the number of paths from $(0, a)$ to (n, b). Show that

$$N_n(a, b) = \binom{n}{\frac{n+b-a}{2}}$$

   ii)Let $N_n^0(a, b)$ be the number of paths from $(0, a)$ to (n, b) that either touch or cross the x-axis. Show that if $a, b > 0$ then $N_n^0(a, b) = N_n(-a, b)$ (Hint: Use reflection principle).
   iii) Show that if $b > 0$ then the number of paths from $(0, 0)$ to (n, b) which do not visit the x-axis is equal to $N_{n-1}(1, b) - N_{n-1}(-1, b)$. Simplify to show that $N_{n-1}(1, b) - N_{n-1}(-1, b) = \frac{b}{n} N_n(0, b)$.
   iv)Suppose that in a two-way ballot, Narendra Modi scores α votes and Rahul Gandhi scores β votes where $\alpha > \beta$. What is the probability that in the ballot Narendra Modi was always ahead of Rahul Gandhi?

5. **Option pricing in binomial model**
   Consider a two period model with volatility and random interest rates as shown in the figure. Both the up factor $u_n(\omega_1, \omega_1, ..., \omega_n)$, down factor $d_n(\omega_1, \omega_1, ..., \omega_n)$ and

interest rate $r_n(\omega_1, \omega_1, ..., \omega_n)$ are allowed to depend on n. Consider a European put option with pay off $(K - S_2)^+$ where $K = 9$ is the strike price. Compute the option price at time 0 under the conditions of arbitrage free markets.

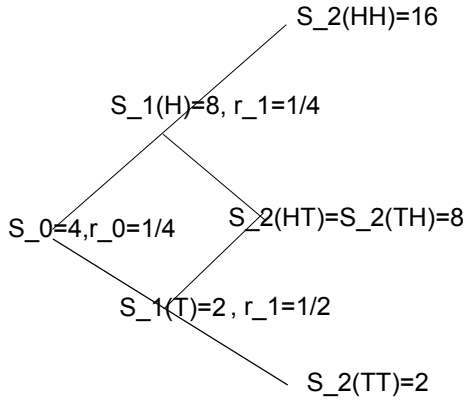

FIGURE 1. Binomial model with volatility and random interest rate