CS401 - Computational finance
Basics of forward contracts and options

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- Derivative is a financial instrument whose value depends on an underlying asset.
- Underlying asset can be stock, currency, commodity, interest rates etc.
- Derivatives are traded on exchange traded markets (e.g. Chicago Board Options Exchange) or Over The Counter (OTC) markets

- A forward contract is an agreement to buy or sell an asset at a certain future time at a certain price
- USD/GBP Exchange rate

	Rate USD per GBP
Spot	2.0562
1-month forward	2.0552
3-month forward	2.0531
6-month forward	2.0498

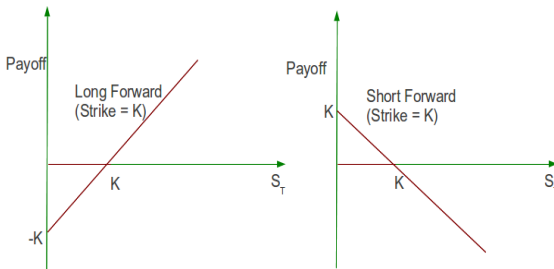Table: Spot and forward quotes for USD/GBP exchange rate, July 20th 2007.

Figure: Payoff from forward contract $S_T - K$ for buyer (long position) and $K - S_T$ for seller (short position).

- Suppose on July 20th 2007 a XYZ corporation in the US knows that it has to pay £1 million in six months to ABC corporation in Britain for services delivered and it wants to hedge against exchange rate movement.

- XYZ corporation will enter into a six month forward contract with a PQR bank to buy (long forward) £1 million. The bank will agree to sell (short forward) £1 million to XYZ corporation.

- Fast forward six months
  Scenario 1: Spot price is 2.1 USD/GBP then XYZ corporation will buy from £1 million from PQR bank at cost $2,049,800 and pay to ABC corporation. If the company had not hedged using the forward contract it would have to pay $2,100,000 at current spot price.

- <u>Scenario 2</u>: Spot price is 1.9 USD/GBP then XYZ corporation will buy from £1 million from PQR bank at cost $2, 049, 800 and pay to ABC corporation. If the company had not hedged using the forward contract it would have to pay only $1, 900, 000 at current spot price and the company will wish that it had not hedged.

A speculator can purchase of £250, 000 in the spot market (current rate is 2.0470) OR he/she can get into forward contract worth £250, 000 at a forward price of 2.0410.

	Buy £250, 000 Spot price = 2.0470	Buy Forward contracts Forward price 2.0410
Initial Investment	$511,750	small margin account
Profit in 3 months if Spot = 2.1	$13,250	$14,750
Profit in 3 months if Spot = 2.0	-$11,750	-$10,250

Table: Speculation using spot and futures contracts

Relationship between spot and forward rates

- The spot rate for a stock that pays no dividend is $60
  The one year risk free interest rate is 5%
  Then the 1-year forward rate should be $63, that is
  $60 \times 1.05 = 63$.
- If the one year forward rate is more than $63 say $66 then one can borrow $60 from the bank and use it to buy the stock. At the same time enter into a one year short forward contract. At the end of the year receive 66 from the forward contract, pay off the loan amount of $63 to make $3 of profit.
- If the forward rate is less than $63 say $58 then an investor owning a stock would sell it for $60 and put the proceeds in a bank. At the same time enter into a long forward contract to buy the stock back in a years time. In a years time investors money in the bank would grow to $63 from which he would buy the stock at $58 and still have $5 of profit while still owning the stock.

- Option is a contract between two parties (A and B) such that one of the party (Option holder A) has the right but no obligation to buy/sell an asset at a certain future date (maturity date T) at a fixed price (strike price K) while the other party (Option writer) has the obligation to sell/buy the asset.
- A call option gives the option holder the right (no obligation) to buy a certain quantity of an asset from the option writer (who has the obligation) at some future time T at some fixed price K.
- A put option gives the option holder the right (no obligation) to sell a certain quantity of an asset from the option writer (who has the obligation) at some future time T at some fixed price K.
- A European option can be exercised only at maturity whereas an American option can be exercised at anytime before maturity.
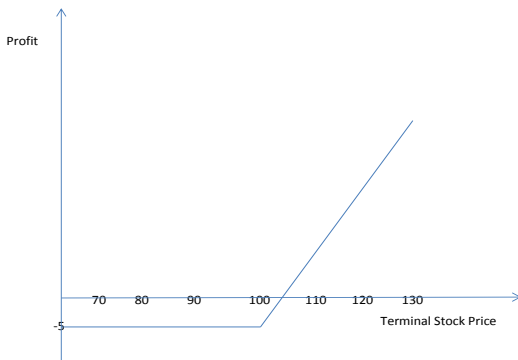
Types of positions in options and payoffs

1. Long position in put (Buyer/Holder of a put option)
2. Short position in call (Seller/Writer of a call option)
3. Short position in put (Seller/Writer of a put option)
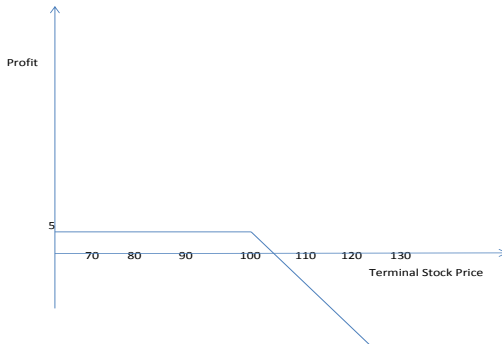4. Long position in call (Buyer/Holder of a call option)
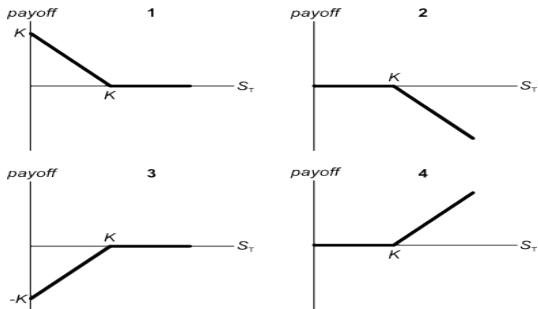
Figure: Option payoff for: long call position max($S_T - K, 0$); Short call max($S_T - K, 0$); Long Put max($K - S_T, 0$). Short Put = -max($K - S_T, 0$)
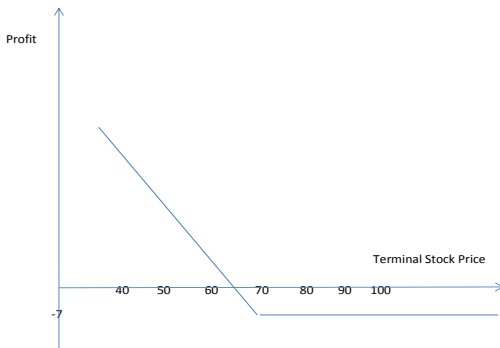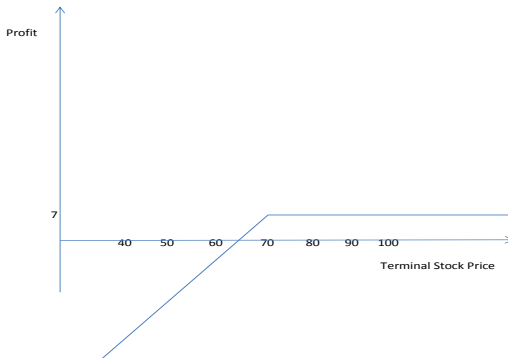
Figure: Profit from buying a European call option on one share of stock. Option price = $5, Strike price = $100

Options profits

Figure: Profit from writing a European call option on one share of stock.
Option price = $5, Strike price = $100

Figure: Profit from buying a European put option on one share of stock. Option price = \$7, Strike price = \$70

Figure: Profit from writing a European put option on one share of stock. Option price = $7, Strike price = $70

Investor owns $1,000$ Microsoft shares which are priced currently at $28. The investor is worried about a possible decline in stock in the next two months and wants some protection. Investor could buy 10 put option contracts with strike price 27.5 (each option contract gives the right but no obligation to sell 100 Microsoft shares). If the quoted option price is $1 then each options contract is worth $100 \times 1 = \$100$ and the total hedging strategy costs $1000. If in 2 months time Microsoft shares fall below 27.5 then the options are exercised giving $27,500$ and with the cost of the options the amount realized is $26,500$. If the shares are above 27.5 then the put options are not exercised.

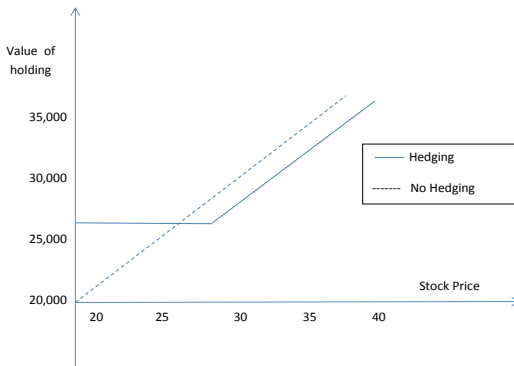Figure: Value of Microsoft holding with and without hedging.

ABC companies current stock price is $20. A two month call option with $22.5 as strike price is selling for $1. The table below shows two possible scenarios for a speculator willing to invest $2,000.

	Stock price in two months	
Speculator strategy	$15	$27
Buy 100 shares	($500)	$700
Buy 2,000 call options	($2000)	$7,000

Table: Comparison of profits (losses) from investing $2,000 in stocks or options.

An opportunity for a riskless profit is called arbitrage. Suppose a stock is trading at \$200 in New York and £100 in London at a time when the exchange rate is \$2.03 per pound. An arbitraguer could simultaneously buy 100 shares of the stock in New York and sell it in London to obtain a riskfree profit of

$$100 \times [(\$2.03 \times 100) - \$200] = \$300$$

- Short selling an asset (or simply shorting) involves selling an asset that is not owned.
- When an investor asks a broker to short shares, the broker borrows the the shares from another client and sells it in the open market giving the proceeds to the investor. At some later stage the investor will close out the position by buying the same volume of shares. These shares are used to replace the borrowed shares. The investor with the short position must pay any income from the shares like dividends or interest to the broker which gets passed on to the borrower.

Short sale of shares

April:	Borrow 500 shares and sell for $120	$60,000
May:	Pay dividend	-$500
June:	Buy 500 shares for $100 to close short position	-$50,000
		+$9,500

Chap 1. *Introduction*, *J.C. Hull* 9th *edition*.
Chap 10. *Mechanics of options markets*, *J.C. Hull* 9th *edition*.