## Design and Analysis of Algorithms

## CS575, Spring 2020

1. (16 points) Use the iteration method to solve the following recurrence equation.
   1. (8 points)

Answer:

T(n) = T(n-1) + n = T(n-2) + n + n-1 = T(n-3) + n + n-1 + n-2 = ... = T(1)

+ n + n-1 + n-2 + ... + 2 = 1 + 2 + ...+ n-1 + n = n(n+1)/2 = (n2).



* 1. (8 points)

Answer:

.

The iterative expansion process stops when for some *k*. When this happens,

*T*(*n*) = *T*(*2*) + *k* = *k*.

From and take logarithm on both sides, we have

(1/2*k*) lg *n* = 1, or 2*k* = lg *n*.

Take logarithm on both sides of 2*k* = lg *n*, we have *k* = lg lg *n*.

Therefore, *T*(*n*) = lg lg *n*.

1. (6 points) Use Master method to solve *T*(*n*) = 4*T*(*n*/2) + *n*2 and T(1)=1.

Answer:

a=4, b=2, k=2. As a=bk, T(n)=

1. Professor Caear wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen’s Algorithm. His algorithm will use divide-and conquer method, dividing each matrix into pieces of size n/4 n/4, and the divide and combine steps together will take time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen’s algorithm. If his algorithm creates a subproblems, then the recurrence for the running time T(n) becomes T(n) = aT(n/4)+ What is the largest integer value of a for which Professor Caesar’s algorithm would be asymptotically faster than Strassen’s algorithm? (10 points)

Answer:

Recall that the master theorem: T(n) = aT(n/b) + cnk .

* + - T(n) = Ѳ(nk) if a < bk
    -  T(n) = Ѳ(nk lgn) if a = bk
    - T(n)= if a > bk

We know that for than Strassen’s algorithm, T(n)=(nlg7).

For the algorithm to be developed by Professor Caear, b=4, k=2.

if a > bk, T(n)=

To make Professor Caear’s algorithm asymptotically faster than Strassen’s algorithm, we need:

log4a < lg7 a<4lg7=22lg7=2lg49=49.

Then the largest integer value for a is 48.