Homework 0: CS 436/ 580L: Introduction to Machine Learning:

Name : Shubham Patwa

B Number : B00818310

Solutions:

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1. The sample space S is
   S = {(1,1),(1,2),(1,3).....(1,6),
   (2,1),(2,2),(2,3)....(2,6),
   (3,1),(3,2),(3,3)....(3,6),
   (4,1),(4,2),(4,3),(4,4)...(4,6),
   (5,1),....(5,5),(5,6),
   (6,1)....(6,6)}
   Total possible outcomes are: 36
   $\therefore n(S) = 36$
   Let A be the event that both dice land on the same number.
   Favourable outcomes are: {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}
   $\therefore n(A) = 6$
   $\therefore P(A) = n(A) / n(S) = 6/36 = 1/6$

2. X and Y are independent random variables. Hence,
   $P(X \cap Y) = P(X) * P(Y)$
   $\therefore P(Y) = P(X \cap Y) / P(X)$
   $\therefore P(Y) = 0.2 / 0.5$
   $\therefore P(Y) = 0.4$

3. The person will require equal steps in both directions so that he will remain in his starting direction.
   Using Binomial Theorem,
   Probability that the man is at his starting position is:
   P =10C5 * $(0.6)^5$ * $(0.4)^5$

4. Given Data :
   $E[X]= 2$
   $Var(X) =1$
   $E[Y]=3$
   $Z = X^2Y$
   $E[Z] = ?$
   Sol^n:
   $\because$ X and Y are independent of each other
   $Var[X] = E[X^2] – E[X]^2$
   $\therefore E[X^2] = 1+ 2^2 = 1+ 4$
   $\therefore E[X^2] = 5$
   $\because Z = X^2Y$,
   $E[Z] = E[X^2Y]$
   $\therefore E[Z]= E[X^2] * E[Y]$ $\because$ X and Y are independent of each other

$\therefore E[Z] = 5 * 3$
$\therefore E[Z] = 15$

5. Numbers in Dataset/Sample Space : {1, 6, -1, 4, 10}
   $\therefore$Number of values , n = 5
   1. Mean $\bar{x} = (\Sigma x_i) / n$
      Mean = (1+6-1+4+10) / 5
      $\therefore$Mean=4
   2. Median is $\{(n + 1) / 2\}^{th}$ value
      $\therefore$Median is $((5+1)/2)=3^{rd}$ value in sample space in order i.e {-1,1,4,6,10}
      $\therefore$Median = 4
   3. Variance = $(\Sigma (x_i - \bar{x})^2) / n$
      Variance = $((1-4)^2 + (6-4)^2 + (-1-4)^2 + (4-4)^2 + (10-4)^2) / 5$
      = (9 + 4 + 25 + 0 + 36) / 5
      $\therefore$Variance= 14.8

6. The gambler wins $10 , 20% of the time and loses $5 , 80% of the time.
   Hence,
   Expected Gain = 0.2*10 + 0.8*(-5)
   = -2
   $\therefore$Expected Gain after n bets = (-2 * n)

7. Let A be the event that a card X is drawn and that it is spade.
   Let B be the event that a new 2^{nd} card Y is drawn without replacement and it is a spade.
   P(B) = ?
   A Card deck has 52 cards. Hence, sample space is 52.
   Once, event A takes place, there is one less card in spade. Total spades remaining =12
   Now card deck has 51 cards remaining.
   For event B , sample space would be 51 and favourable outcomes n(A) = 12, since a spade is already drawn and a deck has 13 spades.
   $\therefore P(B) = 12/51$

8. Let A be the event that outcome of the toss was head.
   Let B be the event that white ball was selected.
   P(A|B) = ?
   The first urn contains 2 white balls and 9 balls in total
   The 2^{nd} urn contains 5 white balls and 11 balls in total.
   If the toss gives head, first urn is selected, else the second urn is selected. Hence,
   P(B|A) = 2/9 = 0.22 (from the first urn since a head)
   P(A) = ½ = 0.5
   P(B) = ½ * (2/9 + 5/11) (from both urns since total whites)
   = 0.5 * (0.22 + 0.45)
   = 0.335
   Now,

$P(A|B) = [P(B|A) * P(A)] / P(B)$ (Bayes Rule)

$= [0.22 * 0.5] / 0.335$

$\therefore P(A|B) = 0.3283$

9. Let A be the event of getting more than 6 heads in 10 coin tosses
   The probability of getting a head in each toss is p
   Using Binomial Expression,
   $P(A) = 10C7 * p^7 * (1-p)^3 + 10C8 * p^8 * (1-p)^2 + 10C9 * p^9 * (1-p) + 10C10 * p^{10} * (1-p)^0$
   $\therefore P(A) = 10C7 * p^7 * (1-p)^3 + 10C8 * p^8 * (1-p)^2 + 10C9 * p^9 * (1-p) + p^{10}$

10. Let A be the event that the man wins for the first time after n bets.
    The probability of winning is given as p.
    Using properties of geometric random variable,
    $\therefore P(A) = (1-p)^n * p$
    The distribution is geometric.