

ProgRAG: Hallucination-Resistant Progressive Retrieval and Reasoning over Knowledge Graphs

Minbae Park^{*1}, Hyemin Yang^{*2}, Jeonghyun Kim², Kunsoo Park³, Hyunjoon Kim^{†1,2}

¹Department of Artificial Intelligence, Hanyang University, South Korea

²Department of Data Science, Hanyang University, South Korea

³Department of Computer Science and Engineering, Seoul National University, South Korea
{pmb0323, hmym7308, gemma1126, hyunjoonkim}@hanyang.ac.kr; kpark@thoery.snu.ac.kr

Abstract

Large Language Models (LLMs) demonstrate strong reasoning capabilities but struggle with hallucinations and limited transparency. Recently, KG-enhanced LLMs that integrate knowledge graphs (KGs) have been shown to improve reasoning performance, particularly for complex, knowledge-intensive tasks. However, these methods still face significant challenges, including inaccurate retrieval and reasoning failures, often exacerbated by long input contexts that obscure relevant information or by context constructions that struggle to capture the richer logical directions required by different question types. Furthermore, many of these approaches rely on LLMs to directly retrieve evidence from KGs, and to self-assess the sufficiency of this evidence, which often results in premature or incorrect reasoning. To address the retrieval and reasoning failures, we propose ProgRAG, a multi-hop knowledge graph question answering (KGQA) framework that decomposes complex questions into sub-questions, and progressively extends partial reasoning paths by answering each sub-question. At each step, external retrievers gather candidate evidence, which is then refined through uncertainty-aware pruning by the LLM. Finally, the context for LLM reasoning is optimized by organizing and rearranging the partial reasoning paths obtained from the sub-question answers. Experiments on three well-known datasets demonstrate that ProgRAG outperforms existing baselines in multi-hop KGQA, offering improved reliability and reasoning quality.

Code — <https://github.com/hyemin-yang/ProgRAG>

Introduction

Since the emergence of large language models (LLMs) like ChatGPT, their remarkable reasoning abilities have demonstrated impressive performance in natural language processing tasks, particularly in question answering (Wei et al. 2022; Brown et al. 2020; Wang et al. 2022; Besta et al. 2024; Yao et al. 2023). However, challenges such as hallucinations and limited performance on complex, knowledge-intensive tasks still persist (Ji et al. 2023). These limitations have fueled growing interest in incorporating external structured

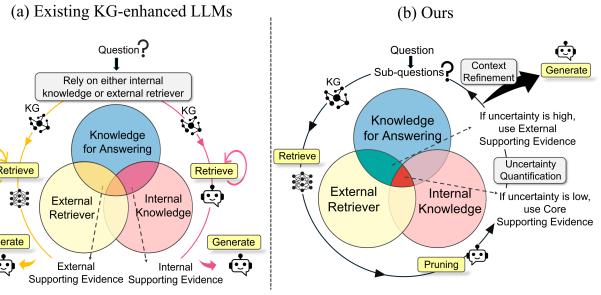


Figure 1: Comparison between existing KG-enhanced LLMs and the proposed framework.

knowledge sources, especially knowledge graphs (KGs), to enhance the reliability and accuracy of LLMs.

Recent advancements have led to the development of KG-enhanced LLMs (Pan et al. 2024), which integrate KGs in either the fine-tuning or inference phases. Despite these improvements, LLM fine-tuning methods (Mavromatis and Karypis 2024; Luo et al. 2023b,a) often require substantial computational resources and struggle to generalize effectively to unseen knowledge. A complementary line of methods using KGs only in the inference phase, i.e., KG-enhanced LLM inference methods, can be broadly categorized into two approaches in terms of retrieval, as illustrated in Figure 1(a): (1) LLM-as-retriever approach in which the LLM itself guides the retrieval process by leveraging its internal knowledge, and (2) external retriever-based approach in which an external retriever extracts relevant reasoning paths or subgraphs. Each approach relies on different sources of knowledge, i.e., internal or external, to retrieve supporting evidence for reasoning.

However, both approaches still face fundamental limitations. First, they often fail to retrieve accurate evidence. Second, even when correct supporting evidence is retrieved, the LLM frequently generates an incorrect answer. In Figure 2, we analyze representative failure cases on the CWQ dataset for LLM-as-retriever methods like ToG (Sun et al. 2023) and PoG (Chen et al. 2024), and the external retriever-based method SubgraphRAG (Li, Miao, and Li 2024). We define retrieval error as cases where the retrieved reasoning path or subgraph does not contain the answer entity, and reasoning

^{*}These authors contributed equally.

[†]Corresponding author.

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error as cases where the answer is included in the retrieved evidence but the LLM fails to generate the correct answer. ToG and PoG exhibit high rates of both retrieval and reasoning errors.

In contrast, SubgraphRAG extracts the top-100 question-relevant triples from the KG, thereby reducing retrieval error. Nonetheless, it still suffers from significant reasoning errors. These results can be attributed to four key factors:

1. Relying solely on the LLM to navigate the extensive search space of KGs is inherently ineffective, resulting in substantial retrieval errors.
2. The self-assessment used by LLM-as-retriever methods is prone to hallucination, often resulting in either premature termination or unnecessary continuation of the reasoning process.
3. Excessively long input contexts from numerous retrieved triples or paths dilutes relevant evidence and hinders answer identification (Liu et al. 2023). This affects SubgraphRAG, ToG, and PoG alike.
4. Existing KG-enhanced LLMs have limited ability to handle richer logical structures beyond simple linear hops (Zhu et al. 2025), leading to suboptimal contexts for LLM reasoning, as later validated through our experiments.

To address the challenges of retrieval and reasoning errors particularly those arising from the hallucinatory behavior of LLMs, we propose a progressive retrieval and reasoning framework, **ProgRAG**, as illustrated in Figure 1(b): (i) we decompose a complex question into sequential sub-questions, and answer each one iteratively, thereby progressively extending reasoning paths. For this, we treat the number of sub-questions as the exploration depth of its complete reasoning path, adapting it to balance sufficient reasoning with avoidance of over-exploration; (ii) for each sub-question, our external retrievers produce relevant evidence supporting the answer for that sub-question from the KG, which is then filtered by the LLM to improve precision. This retrieval-and-pruning process narrows the search space, allowing the LLM to focus on a more relevant candidate set. Moreover, we adaptively increase reliance on external knowledge when the uncertainty of the LLM’s predictions is high; (iii) to reduce hallucination during reasoning, we refine and enhance the LLM’s input context using effective prompting techniques such as prefix enumeration and repacking, which structure the input to better emphasize relevant evidence, thereby improving the model’s ability to generate accurate and grounded answers. We evaluate ProgRAG on three KGQA benchmarks—WebQSP, CWQ, and CR-LT—and observe state-of-the-art performance on all datasets. ProgRAG outperforms the best baseline by 3.3% on WebQSP, 4.9% on CWQ, and 10.9% on CR-LT in accuracy.

- We propose ProgRAG, a novel progressive retrieval and reasoning framework for multi-hop KGQA that iterates external retrieval and LLM-based pruning to address both retrieval and reasoning errors.
- We first propose to refine the LLM context with various reasoning paths, enabling ProgRAG to adaptively handle

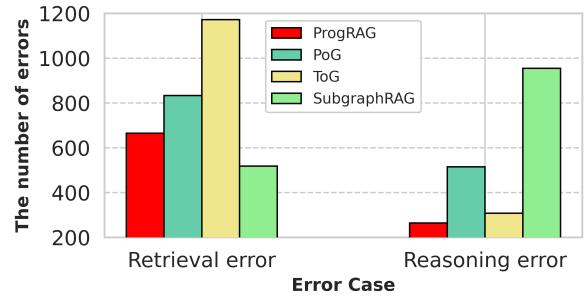


Figure 2: Comparison of typical error cases in existing methods versus the proposed framework.

different question types, improving the robustness and generalization of reasoning.

- We conduct comprehensive experiments on three multi-hop KGQA datasets, demonstrating that ProgRAG achieves state-of-the-art performance even with smaller LLMs, e.g., Gemma2-9B-it, GPT-4o-mini, without fine-tuning.

Related Work

Most KG-enhanced LLMs can be categorized into two approaches based on how LLMs are utilized. Several methods (Mavromatis and Karypis 2024; Long et al. 2025; Luo et al. 2024, 2023a; Xu et al. 2025; Luo et al. 2023b; Liu et al. 2025a; Ao et al. 2025) incorporate KGs during the fine-tuning of LLMs.

However, these approaches often generalize poorly to unseen knowledge and incur significant computational cost (Baek, Aji, and Saffari 2023; Axelsson and Skantze 2023; He et al. 2024). An alternative line of work retrieves structured evidence from KGs and feeds it into LLMs for reasoning without fine-tuning, using either LLM-internal knowledge or external retrievers.

LLM-as-retriever methods rely on the LLMs to directly explore the KG and iteratively accumulate requisite information. StructGPT (Jiang et al. 2023a) generates executable SQL queries to extract relevant KG evidence. ToG (Sun et al. 2023) explores multiple reasoning paths within a predefined exploration breadth by retrieving query-relevant relations, followed by the corresponding entities. PoG (Chen et al. 2024) decomposes the question into sub-tasks and prompts all sub-tasks, along with the question, to the LLM to guide answer prediction. Building on ToG’s framework, ReKnoS (Wang et al. 2025) and MFC (Zhang et al. 2025) further extend the search space using super-relations or meta-entities. However, due to unfiltered exploration, these methods often retrieve suboptimal paths and exhibit hallucination tendencies (Liu et al. 2023; Dhole 2025).

External retriever-based methods adopt a pipeline in which fine-tuned external retrieval models efficiently extract relevant subgraphs or reasoning paths from the KG (Jiang et al. 2022, 2023b; Liu et al. 2024). SubgraphRAG (Li, Miao, and Li 2024) integrates a lightweight MLP with a par-

allel triple-scoring mechanism for subgraph retrieval. KG-CoT (Zhao et al. 2024) explores KG stepwise from question entities, generates reasoning paths via transition matrices. However, these methods either retrieve excessive structural information or fail to leverage LLMs’ internal knowledge during retrieval, resulting in low reasoning accuracy due to both irrelevant volume and insufficient precision.

Preliminaries

Knowledge Graph (KG). A knowledge graph is a set of factual triples, denoted by $\mathcal{G} = \{(e, r, e') | e, e' \in \mathcal{E}, r \in \mathcal{R}\}$, where \mathcal{E} is a set of entities and \mathcal{R} is a set of relations. Each triple (e, r, e') represents a fact indicating that the head entity e is connected to the tail entity e' via relation r .

Knowledge Graph Question Answering (KGQA). Given a natural language question q , a knowledge graph \mathcal{G} , and a set of key entities $\mathcal{E}_q \subseteq \mathcal{E}$ extracted from q , the goal of KGQA is to predict the set $\mathcal{A}_q \subseteq \mathcal{E}$ of answers to the question q by performing reasoning over \mathcal{G} .

Reasoning Path. A reasoning path is a sequence of consecutive triples in a KG, denoted as $(e_0, r_1, e_1) \rightarrow (e_1, r_2, e_2) \rightarrow \dots \rightarrow (e_{d-1}, r_d, e_d)$, where each triple $(e_{i-1}, r_i, e_i) \in \mathcal{G}$. This sequence forms the reasoning trajectory from a key entity e_0 toward predicting the answer entity e_d . A prefix of a reasoning path is a subsequence of consecutive triples taken from the beginning of the path. It consists of the first k triples of the full reasoning path where $k \leq d$.

Method

Overview

Figure 3 illustrates the ProgRAG framework, which performs three stages. First, ProgRAG identifies key entities from a given question, and initializes a partial reasoning path as each key entity, and decomposes the question into multiple sub-questions in the question decomposition stage. Second, in the sub-question answering stage, ProgRAG iteratively answers each sub-question by extending the partial reasoning paths obtained from the previous iteration. Finally, in the prefix enumeration and repacking stage, ProgRAG reorganizes all complete reasoning paths and their partial reasoning paths to form a structured context. The LLM then infers the final answer based on this context.

Question Decomposition

In this stage, ProgRAG decomposes a question into simpler sub-questions. Specifically, we first identify key entities, i.e., the entities central to the semantics of the question. The LLM decomposes the question into sub-questions and associates each one with its corresponding key entity, which is called key entity mapping. For every key entity e_s , the LLM further decomposes its initial sub-question into more granular ones if possible; retains this sub-question as atomic otherwise. The full prompt is provided in Appendix K. Consequently, we obtain a chain $Q_{e_s} = \{q_1, \dots, q_d\}$ of sub-questions for every key entity e_s , where the depth d represents the number of reasoning steps, i.e., iterations, in the subsequent stage. In this manner, ProgRAG can dynamically

adjust the exploration depth depending on the question complexity, enabling more flexible and precise reasoning. Previous work such as Chain-of-Question (Yixing et al. 2024) also decomposes a complex question. Unlike our method, this decomposition does not take key entities into account.

Sub-question Answering

For simplicity, we focus on a question with a single key entity, but our method can be easily extended to a question with multiple key entities. From the key entity and its chain of sub-questions, the sub-question answering stage iteratively finds the answer to each sub-question by extending the partial reasoning paths one hop at a time. At the first iteration, the source entity for each sub-question is the key entity; in subsequent iterations, the source entity is the answer entity from the previous iteration. At each iteration i , ProgRAG takes the source entity and sub-question q_i as input, and performs four procedures: (1) relation retrieval, (2) relation pruning, (3) triple retrieval, and (4) triple pruning, as shown in Figure 3¹.

Relation Retrieval Seminal prior work in KGQA struggles with retrieving relevant relations from the KG, despite its significant impact on question answering quality. On the one hand, some methods pass a large number of relations linked to the final entities of partial reasoning paths to the LLM (Sun et al. 2023), leading to context overload. On the other hand, other methods retrieve relevant relations from the entire KG (Zhao et al. 2024), often neglecting the local context needed for step-wise reasoning.

For progressive relation retrieval, we select candidate relations $R_{re}(q_i)$ from the KG that are contextually relevant to a given sub-question q_i . To address the aforementioned limitations, we utilize a SentenceBERT-based cross-encoder (Reimers and Gurevych 2019, 2020) to score and rank one-hop triples $(s_i, r, e') \in \mathcal{G}$ connected to the source entity s_i based on their semantic relevance to q_i . The top- m unique relations with the highest scores are selected as $R_{re}(q_i)$.

Relation Pruning For source entity s_i , sub-question q_i , and $R_{re}(q_i)$, the LLM selects the top- n ($n \leq m$) relevant relations $R_{pr}(q_i) \subseteq R_{re}(q_i)$. All relations in $R_{re}(q_i)$ are sorted in descending order of their semantic similarity to q_i in this prompt. To encourage the LLM to articulate its step-by-step reasoning, we apply Chain-of-Thought prompting (CoT) (Wei et al. 2022) leveraging our key observations on the Freebase KG widely used in this study: this KG organizes each relation into a three-level hierarchy, i.e., *domain.source_type.target_type*. This hierarchy captures the semantic structure of the relation and its alignment with the associated triple, e.g., the relation of the triple (“*Lou Seal*”, “*sports.mascot.team*”, “*San Francisco Giants*”) consists of domain *sports*, source type *mascot*, and target type *team*.

Using CoT, the LLM infers three elements step by step: (1) the type of source (e.g., *mascot*), (2) the target type (e.g.,

¹For questions with multiple key entities, their corresponding sub-question chains are solved independently, and the resulting reasoning paths are fed into the next stage.

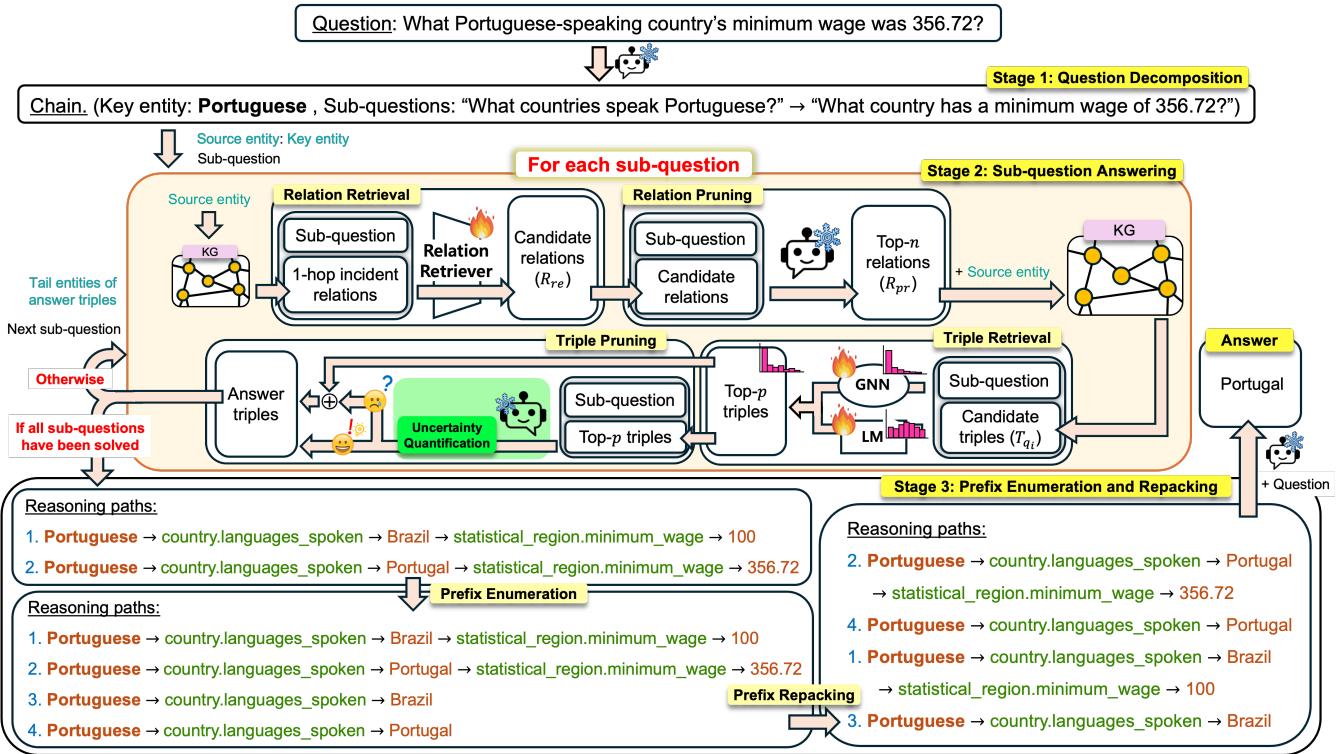


Figure 3: ProgRAG operates in three stages: (1) Question decomposition, where the question is split into sub-questions based on a key entity; (2) Sub-question answering, where partial reasoning paths are progressively extended through retrieval and pruning; and (3) Prefix enumeration and repacking, where all prefixes of the reasoning paths are enumerated and reordered. Finally, the LLM infers the answer based on these prefixes.

team), and (3) a relational phrase that links them semantically (e.g., “mascot for”). We observe that each sub-question typically corresponds to a single-hop triple, making the step-by-step reasoning process highly compatible with the question decomposition.

In contrast to the prior work (Sun et al. 2023) where the LLM directly evaluates the relevance of all retrieved relations to the question without the structure-aware step-by-step reasoning, we leverage hierarchical relation structure to enable the LLM to identify the relevant relations through reasoning. This process enhances interpretability and consistency.² The full prompt is provided in Appendix K.

Triple Retrieval As noted earlier, excessively long contexts can impair the focus of LLM on relevant information, increasing the risk of hallucinations and retrieval errors (Liu et al. 2023, 2025b). To construct a more compact context, we prune unpromising candidate triples unlikely to lead to the answer for q_i based on their relevance to q_i . The set T_{q_i} of candidate triples consists of all triples that include the source entity s_i and any relation in $R_{pr}(q_i)$. The relevance score between a triple and q_i is computed by summing two com-

ponents: (1) the textual semantic-based triple score, and (2) the structure-based entity score.

Textual Semantic-based Triple Scoring We employ a bi-encoder architecture composed of two MPNet-base models (Song et al. 2020), each generating embeddings for q_i and for every $t \in T_{q_i}$, respectively. The textual semantic-based triple score Φ_t is computed by applying a softmax to the cosine similarity between the sub-question embedding \mathbf{x}_{q_i} and the triple embedding \mathbf{x}_t , capturing their textual semantic relevance:

$$\Phi_t(q_i) = \text{softmax}(\{\cos(\mathbf{x}_{q_i}, \mathbf{x}_t)\}_{t \in T_{q_i}}) \quad (1)$$

The encoders are fine-tuned using the InfoNCE (Oord, Li, and Vinyals 2018) loss to learn semantic alignment between sub-questions and their corresponding golden triples.

Structure-based Entity Scoring In KGs, relations carry rich contextual information, making relation-centric approaches crucial for reasoning tasks (Wang, Ren, and Leskovec 2021). To leverage this, we employ a query-dependent graph neural network (GNN) (Luo et al. 2025), which is designed to focus on the semantic relevance of relations in the context of specific queries. We embed all relations in the graph and the sub-question q_i using the pre-trained text encoder (Li et al. 2023). Each entity is initialized with a zero vector, with the exception of the source entity s_i , which is assigned the sub-question embedding.

²Although this prompt is designed for hierarchical relations, it also performs effectively on CR-LT, a Wikidata-based dataset without hierarchical relation structure, where we observe consistently strong performance.

After passing through all GNN layers, we obtain the final representation for each entity in the graph. Let E_{q_i} be the set of all tail entities in T_{q_i} . The representations of all entities in E_{q_i} are concatenated and passed through an MLP followed by a softmax function, producing a probability that each $e \in E_{q_i}$ is the answer entity for q_i :

$$\Phi_e(q_i) = \text{softmax}(\text{MLP}(\text{concat}(\{\mathbf{h}_e\}_{e \in E_{q_i}}))) \quad (2)$$

For each $t \in T_{q_i}$, we compute the probability u_t that triple t is the answer triple for q_i :

$$u_t = \text{softmax}(\Phi_t(q_i) + \Phi_e(q_i)) \quad (3)$$

The above two components offer complementary perspectives, i.e., semantic relevance and structural context, enhancing both the accuracy of triple ranking. Finally, top- p sampling (Holtzman et al. 2019) is applied over the probabilities u_t , sequentially selecting triples in descending order of probability until the cumulative score exceeds a predefined threshold p .

Triple Pruning For the sub-question q_i , let $T_{re}(q_i)$ denote the set of triples remaining after top- p sampling. From $T_{re}(q_i)$, we aim to obtain answer triples, with the source entity s_i as the head and the predicted answer to q_i as the tail. Specifically, all triples $t \in T_{re}(q_i)$ are sorted in descending order of u_t and provided to the LLM along with q_i .³ ProgRAG then assesses the reliability of the LLM outputs to detect its potential hallucinations, preventing error propagation in multi-hop reasoning, which is called “Uncertainty Quantification”, i.e., ProgRAG measures Aleatoric Uncertainty (AU) of the LLM outputs based on evidential modeling (Sensoy, Kaplan, and Kandemir 2018; Ma et al. 2025). At the generation step c in which the LLM outputs the first token of answer entities, the top- K logits are selected and used as parameters of a Dirichlet distribution:

$$\alpha_k = \text{LLM}(\tau_k | q_i, \mathbf{a}_{c-1}), \alpha_0 = \sum_{k=1}^K \alpha_k \quad (4)$$

$$\text{AU}(a_c) = - \sum_{k=1}^K \frac{\alpha_k}{\alpha_0} (\Psi(\alpha_k + 1) - \Psi(\alpha_0 + 1)) \quad (5)$$

where \mathbf{a}_{c-1} is the token sequence generated up to step $c-1$, α_k ($1 \leq k \leq K$) represents the top- K logits, τ_k is the token with the k -th highest logit, α_0 is the total evidence of the Dirichlet distribution, and Ψ is the digamma function. The resulting $\text{AU}(a_c)$ serves as the uncertainty estimate for the LLM response. If it exceeds a predefined threshold, indicating low model confidence, the response is refined with the top- l triples from $T_{re}(q_i)$ as external evidence. Otherwise, the response is accepted as is. If q_i is the final sub-question, the model proceeds to the subsequent stage; otherwise, it continues to the next sub-question q_{i+1} .

³If the LLM identifies plausible answers in $T_{re}(q_i)$, it then outputs them; otherwise, it returns “None”, indicating insufficient evidence. In such cases, ProgRAG returns to the relation pruning step with the unused relations in $R_{re}(q_i) \setminus R_{pr}(q_i)$ to explore alternative paths, thereby enabling early termination of incorrect reasoning and reducing redundant inference.

Prefix Enumeration and Repacking

In the final stage, the LLM infers the answer to the question q based on all reasoning paths explored thus far. To enhance its reasoning ability, we enumerate all prefixes of every reasoning path, which are then included in the LLM context, as shown in Stage 3 of Figure 3. Next, we rank the prefixes by their semantic relevance to q , as scored by the triple retriever. Finally, the reordered prefixes and the question q are fed into the LLM, guiding it to focus on the most relevant evidence and improving the accuracy of its answer. By carefully manipulating partial reasoning paths, the LLM reasons from prefixes that capture intermediate states, better aligning even partial paths with the nuanced semantics of the question, particularly when answers lie in the middle of reasoning paths or when multiple constraints must be satisfied.

Experiments

To demonstrate the effectiveness and efficiency of ProgRAG on multi-hop KGQA, we conduct comprehensive experiments to address four research questions; **(RQ1)** How effective is ProgRAG in the multi-hop reasoning for the KGQA task?; **(RQ2)** How does each component in ProgRAG contribute to the overall performance?; **(RQ3)** How accurately does ProgRAG retrieve ground-truth reasoning paths?; **(RQ4)** How efficiently does ProgRAG perform multi-hop reasoning?

Experimental Setup

We adopt three publicly available multi-hop KGQA datasets: WebQuestionsSP (WebQSP) (Yih et al. 2016), ComplexWebQuestions (CWQ) (Talmor and Berant 2018), and CR-LT-KGQA (Guo, Toroghi, and Sanner 2024). Dataset statistics and details are provided in Appendix A.

As per prior works (Sun et al. 2023; Chen et al. 2024), we use exact match accuracy (Hit@1) as our primary evaluation metric. We use two small-scale LLMs for reasoning, i.e., Gemma 2-9b-it and GPT-4o-mini. Since black-box LLMs like GPT-4o-mini do not provide access to their logits, uncertainty quantification is only applied when using Gemma 2-9b, which is the main LLM for most experiments. Other implementation details are further documented in the Appendix B.

Baseline Methods

We evaluate the performance of ProgRAG against a diverse set of baselines spanning four major categories: (a) LLM-only prompting methods (Brown et al. 2020; Wei et al. 2022; Wang et al. 2022); (b) Fine-tuned LLM-based methods (Luo et al. 2023b; Liu et al. 2025a; Yu et al. 2022; Ao et al. 2025); (c) LLM-as-retriever methods (Sun et al. 2023; Chen et al. 2024; Jiang et al. 2023a; Wang et al. 2025; Zhang et al. 2025; Liang and Gu 2025); (d) External retriever-based methods (Zhao et al. 2024; Li, Miao, and Li 2024).⁴

⁴We do not extensively compare with fine-tuning-based (Mavromatis and Karypis 2024) or semantic parsing approaches (Luo et al. 2023a; Xu et al. 2025), as they follow fundamentally different paradigms from our retrieval-augmented setting.

Method	Category	WebQSP		CWQ	
		Hit@1	Hit@1	Hit@1	Hit@1
Zero-shot (GPT-3.5)	a	54.4	34.9		
Few-shot (GPT-3.5)	a	56.3	38.5		
CoT (GPT-3.5)	a	57.4	43.2		
Fine-tuned LLM					
DeCAF (FiD-3B)	b	82.1	70.4		
RoG (Llama2-7B-Chat)	b	85.7	62.6		
SymAgent (Qwen2-7B)	b	78.5	58.9		
LightPROF (Llama3-8B)	b	83.8	59.3		
Open source LLM or GPT-3.5					
StructGPT (GPT-3.5)	c	75.2	55.2		
ToG (GPT-3.5)	c	76.2	58.9		
PoG (GPT-3.5)	c	82.0	63.2		
ReKnoS (GPT-3.5)	c	81.1	58.5		
MFC (GPT-3.5)	c	78.9	62.8		
KG-CoT (GPT-3.5)	d	82.1	51.6		
SubgraphRAG (GPT-3.5)	d	83.1	56.3		
ProgRAG (Gemma2-9b)		88.5	73.7		
GPT-4o-mini					
StructGPT	c	79.5	64.7		
ToG	c	77.0	59.0		
PoG	c	83.2	63.5		
ReKnoS	c	83.8	68.8		
MFC	c	79.1	63.4		
FastToG	c	65.8	45.0		
KG-CoT	d	OOM	OOM		
SubgraphRAG	d	86.2	58.3		
ProgRAG*		90.4	73.3		

Table 1: Performance comparison on WebQSP and CWQ. Baselines are categorized into (a) LLM-only prompting methods, (b) methods using fine-tuned LLMs, (c) LLM-as-retriever methods, and (d) external retriever-based methods. ProgRAG* denotes ProgRAG without Uncertainty Quantification, due to the black-box nature of GPT.

RQ1: Main Results

As shown in Table 1 and Table 2, ProgRAG outperforms all baselines on WebQSP, CWQ, and CR-LT. Even without fine-tuning, ProgRAG with Gemma2-9b surpasses fine-tuned LLMs, outperforming RoG by 2.8% on WebQSP and 11.1% on CWQ. Compared to LLM-as-retriever methods, ProgRAG shows average improvements of 16.1% on WebQSP and 29.7% on CWQ, demonstrating the effectiveness of combining external retrieval with LLM-based pruning. ProgRAG outperforms external retriever-based methods by 7.1% on WebQSP and 17.0% on CWQ, with the large gain on CWQ highlighting the benefits of its progressive reasoning strategy for complex multi-hop queries requiring greater reasoning depth. Hop-wise performance details are provided in Appendix H. On CR-LT, ProgRAG outperforms all baselines, demonstrating its effectiveness in handling more complex queries.

Method	Hit@1
ToG (GPT-4o-mini)	50.0
PoG (GPT-4o-mini)	56.6
MFC (GPT-4o-mini)	61.7
ProgRAG (Gemma2-9b)	68.4

Table 2: Performance comparison on the CR-LT dataset.

Method	WebQSP	CWQ
ProgRAG	88.5	73.7
w/o Prefix Enumeration	88.5	63.9
w/o Relation Pruning	84.2	65.4
w/o Triple Retrieval	86.1	67.5
w/o Uncertainty Quantification	87.2	68.5
w/o Relation Retrieval	86.4	70.1
w/o Prefix Repacking	86.9	70.2
w/o Triple Pruning	87.8	72.8
w/o Question Decomposition	88.0	51.1
w/o Key Entity Mapping	88.5	70.0

Table 3: Ablation study of the proposed methods.

RQ2: Ablation Study

Effectiveness of Individual Techniques Table 3 reports the performance of ProgRAG and its ablated variants. The definition of each variant is provided in Appendix D. All variants show performance degradation, indicating that each component contributes positively to the overall system. On WebQSP, the most significant drops occur when relation pruning (3.8%) and triple retrieval (1.9%) are removed. These declines are even larger on CWQ, highlighting the importance of combining external retrieval with LLM-based filtering to accurately identify supporting evidence. The largest drop of 22.6% occurs when question decomposition is removed on CWQ, highlighting the importance of progressive reasoning through sub-question chains for complex queries. This suggests that using the number of sub-questions as exploration depth, rather than LLM self-assessment to decide whether to continue or stop exploration, helps avoid hallucinations and ensures more stable depth control. The second largest performance degradation in CWQ occurs when prefix enumeration is omitted, indicating that explicitly presenting diverse partial reasoning paths is essential to validate multiple constraints for complex multi-hop queries⁵. Experimental results comparing with existing question decomposition methods (Zhang et al. 2025) are presented in Appendix J.

Performance Analysis by Question Type Table 4 presents the Hit@1 scores of ProgRAG and several state-of-the-art or well-known KG-enhanced LLMs—ToG, PoG, MFC, and SubgraphRAG—across different CWQ question types. ProgRAG consistently outperforms all the baselines

⁵The impact of “prefix enumeration” and “key entity mapping” on WebQSP is minimal due to its predominance of 1-hop or simple composition questions.

Method	Question types (#Question)			
	Compo. (1546)	Conj. (1575)	Sup. (197)	Compa. (213)
Gemma2-9b				
ProgRAG	70.8	76.6	70.6	75.6
w/o P.E.	72.2	62.5	29.4	46.0
w/o P.R.	<u>71.7</u>	<u>69.2</u>	<u>63.5</u>	<u>72.3</u>
GPT-4o-mini				
MFC	66.0	62.8	39.6	55.0
PoG	67.4	60.2	36.0	57.3
ToG	62.4	57.8	35.0	49.3
SubgraphRAG	61.4	66.9	39.1	51.2

Table 4: Performance analysis for different question types. *Compo.*, *Conj.*, *Sup.*, and *Compa.* denote Composition, Conjunction, Superlative, and Comparative, respectively. w/o *P.E.* and w/o *P.R.* stand for without Prefix Enumeration and without Prefix Repacking, respectively.

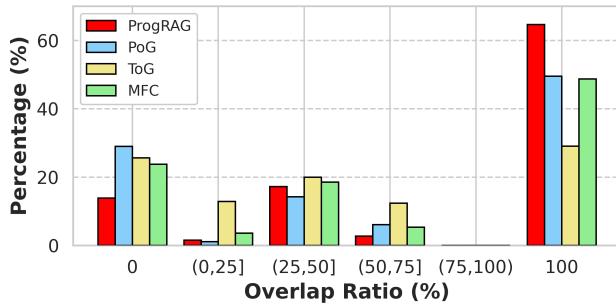


Figure 4: Explored path overlap ratio on CWQ.

across all question types. Notably, for superlative and comparative questions, where answers often reside at intermediate nodes of reasoning paths and thus require more structured reasoning, ProgRAG achieves scores of 70.6 and 75.6, respectively, while other methods show fall behind limited effectiveness. For conjunction questions, which demand satisfying multiple constraints across distinct paths, ProgRAG attains a Hit@1 of 76.6, substantially outperforming the baselines. The robustness of ProgRAG stems from prefix enumeration and repacking, which guide the LLM to focus on relevant partial reasoning paths and verify the constraints required by the question, leading to more accurate reasoning. Removing either component leads to substantial performance drops, particularly on complex questions⁶.

RQ3: Evaluation of Reasoning Path Retrieval

To address RQ3, the retrieval performance of ProgRAG is evaluated along two key dimensions. First, we assess how effectively the retrieved paths cover the correct answer enti-

⁶A slight decrease is observed for composition questions, where full-path reasoning is critical and partial path prioritization may hinder holistic understanding.

Method	# Call	Time (s)	# Path	# Token	F1
ToG	27.7	72.4	5.9	899	41.9
PoG	22.1	41.1	14.9	647	44.8
MFC	17.6	43.7	3.3	685	-
ProgRAG	9.0	26.1	8.3 (3.9)	312	53.3

Table 5: Efficiency analysis on CWQ. For ProgRAG, parentheses under # Path show the average number of paths before prefix enumeration.

ties. ProgRAG achieves 101.5% higher accuracy than ToG and 42% higher than PoG, with detailed results in Appendix E. Second, we conduct a relation-level comparison between the reasoning paths retrieved by each method and the relations in ground-truth SPARQL queries to evaluate their structural alignment. Figure 4 compares the overlap ratio (Sun et al. 2023), i.e., the proportion of overlapping relations to the total number of relations in the ground-truth SPARQL path, for ours and representative iterative retrieve-and-reason methods such as PoG, ToG, and MFC. ProgRAG achieves full relation overlap with ground-truth SPARQL queries in 65% of questions and only 13% with no overlap, outperforming all baselines. These results demonstrate the effectiveness of our progressive retrieval strategy, where relations are first retrieved by an external retriever and then pruned by the LLM at each iteration, yielding concise, semantically grounded reasoning paths.

RQ4: Efficiency Analysis

Table 5 compares the efficiency of ProgRAG and the state-of-the-art KG-enhanced LLM inference methods, i.e., ToG, PoG, and MFC, on CWQ. All methods are evaluated by using GPT-4o-mini to ensure fair comparison. The table reports the average number of LLM calls, query time, the number of reasoning paths, the number of input tokens during reasoning, and F1 score. The retrieve-then-prune strategy of ProgRAG reduces redundant LLM calls and shortens input length, leading to consistent performance gains over all baselines. MFC uses fewer paths via abstracting mid-path nodes into meta entities but still produces long inputs due to coarse retrieval. Overall, more compact inputs consistently correlate with better performance, as illustrated in Figure 2.

Conclusion

In this paper, we introduce ProgRAG, a novel progressive retrieval and reasoning framework for multi-hop KGQA. ProgRAG decomposes a complex question into sub-questions, and incrementally constructs reasoning paths by iteratively retrieving candidate evidence with external retrievers and refining it through LLM-based pruning with uncertainty estimation. This progressive strategy enables more accurate retrieval and reasoning by dynamically optimizing the input context and mitigating hallucinations. Extensive experiments on benchmark datasets demonstrate that ProgRAG consistently outperforms state-of-the-art baselines, achieving improved reasoning accuracy and reliability.

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References

- Ao, T.; Yu, Y.; Wang, Y.; Deng, Y.; Guo, Z.; Pang, L.; Wang, P.; Chua, T.-S.; Zhang, X.; and Cai, Z. 2025. Lightprof: A lightweight reasoning framework for large language model on knowledge graph. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, 23424–23432.
- Axelsson, A.; and Skantze, G. 2023. Using large language models for zero-shot natural language generation from knowledge graphs. *arXiv preprint arXiv:2307.07312*.
- Baek, J.; Aji, A. F.; and Saffari, A. 2023. Knowledge-augmented language model prompting for zero-shot knowledge graph question answering. *arXiv preprint arXiv:2306.04136*.
- Besta, M.; Blach, N.; Kubicek, A.; Gerstenberger, R.; Podstawska, M.; Gianinazzi, L.; Gajda, J.; Lehmann, T.; Niewiadomski, H.; Nyczyk, P.; et al. 2024. Graph of thoughts: Solving elaborate problems with large language models. In *Proceedings of the AAAI conference on artificial intelligence*, volume 38, 17682–17690.
- Bollacker, K.; Evans, C.; Paritosh, P.; Sturge, T.; and Taylor, J. 2008. Freebase: a collaboratively created graph database for structuring human knowledge. In *Proceedings of the 2008 ACM SIGMOD international conference on Management of data*, 1247–1250.
- Brown, T.; Mann, B.; Ryder, N.; Subbiah, M.; Kaplan, J. D.; Dhariwal, P.; Neelakantan, A.; Shyam, P.; Sastry, G.; Askell, A.; et al. 2020. Language models are few-shot learners. *Advances in neural information processing systems*, 33: 1877–1901.
- Chen, L.; Tong, P.; Jin, Z.; Sun, Y.; Ye, J.; and Xiong, H. 2024. Plan-on-graph: Self-correcting adaptive planning of large language model on knowledge graphs. *Advances in Neural Information Processing Systems*, 37: 37665–37691.
- Dhole, K. D. 2025. To retrieve or not to retrieve? uncertainty detection for dynamic retrieval augmented generation. *arXiv preprint arXiv:2501.09292*.
- Guo, W.; Toroghi, A.; and Sanner, S. 2024. Cr-lt-kgqa: A knowledge graph question answering dataset requiring commonsense reasoning and long-tail knowledge. *arXiv preprint arXiv:2403.01395*.
- He, X.; Tian, Y.; Sun, Y.; Chawla, N.; Laurent, T.; LeCun, Y.; Bresson, X.; and Hooi, B. 2024. G-retriever: Retrieval-augmented generation for textual graph understanding and question answering. *Advances in Neural Information Processing Systems*, 37: 132876–132907.
- Holtzman, A.; Buys, J.; Du, L.; Forbes, M.; and Choi, Y. 2019. The curious case of neural text degeneration. *arXiv preprint arXiv:1904.09751*.
- Ji, Z.; Lee, N.; Frieske, R.; Yu, T.; Su, D.; Xu, Y.; Ishii, E.; Bang, Y. J.; Madotto, A.; and Fung, P. 2023. Survey of hallucination in natural language generation. *ACM computing surveys*, 55(12): 1–38.
- Jiang, J.; Zhou, K.; Dong, Z.; Ye, K.; Zhao, W. X.; and Wen, J.-R. 2023a. Structgpt: A general framework for large language model to reason over structured data. *arXiv preprint arXiv:2305.09645*.
- Jiang, J.; Zhou, K.; Zhao, W. X.; Li, Y.; and Wen, J.-R. 2023b. Reasoninglm: Enabling structural subgraph reasoning in pre-trained language models for question answering over knowledge graph. *arXiv preprint arXiv:2401.00158*.
- Jiang, J.; Zhou, K.; Zhao, W. X.; and Wen, J.-R. 2022. Unikgqa: Unified retrieval and reasoning for solving multi-hop question answering over knowledge graph. *arXiv preprint arXiv:2212.00959*.
- Li, M.; Miao, S.; and Li, P. 2024. Simple is effective: The roles of graphs and large language models in knowledge-graph-based retrieval-augmented generation. *arXiv preprint arXiv:2410.20724*.
- Li, Z.; Zhang, X.; Zhang, Y.; Long, D.; Xie, P.; and Zhang, M. 2023. Towards general text embeddings with multi-stage contrastive learning. *arXiv preprint arXiv:2308.03281*.
- Liang, X.; and Gu, Z. 2025. Fast think-on-graph: Wider, deeper and faster reasoning of large language model on knowledge graph. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, 24558–24566.
- Liu, B.; Zhang, J.; Lin, F.; Yang, C.; Peng, M.; and Yin, W. 2025a. Symagent: A neural-symbolic self-learning agent framework for complex reasoning over knowledge graphs. In *Proceedings of the ACM on Web Conference 2025*, 98–108.
- Liu, G.; Zhang, Y.; Li, Y.; and Yao, Q. 2024. Explore then determine: A gnn-llm synergy framework for reasoning over knowledge graph. *arXiv e-prints*, arXiv–2406.
- Liu, N. F.; Lin, K.; Hewitt, J.; Paranjape, A.; Bevilacqua, M.; Petroni, F.; and Liang, P. 2023. Lost in the middle: How language models use long contexts. *arXiv preprint arXiv:2307.03172*.
- Liu, S.; Halder, K.; Qi, Z.; Xiao, W.; Pappas, N.; Htut, P. M.; John, N. A.; Benajiba, Y.; and Roth, D. 2025b. Towards long context hallucination detection. *arXiv preprint arXiv:2504.19457*.
- Long, X.; Zhuang, L.; Li, A.; Yao, M.; and Wang, S. 2025. Eperm: An evidence path enhanced reasoning model for knowledge graph question and answering. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, 12282–12290.
- Luo, H.; Tang, Z.; Peng, S.; Guo, Y.; Zhang, W.; Ma, C.; Dong, G.; Song, M.; Lin, W.; Zhu, Y.; et al. 2023a. Chatkbqa: A generate-then-retrieve framework for knowledge base question answering with fine-tuned large language models. *arXiv preprint arXiv:2310.08975*.

- Luo, L.; Li, Y.-F.; Haffari, G.; and Pan, S. 2023b. Reasoning on graphs: Faithful and interpretable large language model reasoning. *arXiv preprint arXiv:2310.01061*.
- Luo, L.; Zhao, Z.; Haffari, G.; Li, Y.-F.; Gong, C.; and Pan, S. 2024. Graph-constrained reasoning: Faithful reasoning on knowledge graphs with large language models. *arXiv preprint arXiv:2410.13080*.
- Luo, L.; Zhao, Z.; Haffari, G.; Phung, D.; Gong, C.; and Pan, S. 2025. GFM-RAG: graph foundation model for retrieval augmented generation. *arXiv preprint arXiv:2502.01113*.
- Ma, H.; Chen, J.; Zhou, J. T.; Wang, G.; and Zhang, C. 2025. Estimating LLM Uncertainty with Evidence. *arXiv preprint arXiv:2502.00290*.
- Mavromatis, C.; and Karypis, G. 2024. Gnn-rag: Graph neural retrieval for large language model reasoning. *arXiv preprint arXiv:2405.20139*.
- Oord, A. v. d.; Li, Y.; and Vinyals, O. 2018. Representation learning with contrastive predictive coding. *arXiv preprint arXiv:1807.03748*.
- Pan, S.; Luo, L.; Wang, Y.; Chen, C.; Wang, J.; and Wu, X. 2024. Unifying large language models and knowledge graphs: A roadmap. *IEEE Transactions on Knowledge and Data Engineering*, 36(7): 3580–3599.
- Reimers, N.; and Gurevych, I. 2019. Sentence-bert: Sentence embeddings using siamese bert-networks. *arXiv preprint arXiv:1908.10084*.
- Reimers, N.; and Gurevych, I. 2020. The curse of dense low-dimensional information retrieval for large index sizes. *arXiv preprint arXiv:2012.14210*.
- Sensoy, M.; Kaplan, L.; and Kandemir, M. 2018. Evidential deep learning to quantify classification uncertainty. *Advances in neural information processing systems*, 31.
- Song, K.; Tan, X.; Qin, T.; Lu, J.; and Liu, T.-Y. 2020. Mpnet: Masked and permuted pre-training for language understanding. *Advances in neural information processing systems*, 33: 16857–16867.
- Sun, J.; Xu, C.; Tang, L.; Wang, S.; Lin, C.; Gong, Y.; Ni, L. M.; Shum, H.-Y.; and Guo, J. 2023. Think-on-graph: Deep and responsible reasoning of large language model on knowledge graph. *arXiv preprint arXiv:2307.07697*.
- Talmor, A.; and Berant, J. 2018. The web as a knowledge-base for answering complex questions. *arXiv preprint arXiv:1803.06643*.
- Vrandečić, D.; and Krötzsch, M. 2014. Wikidata: a free collaborative knowledgebase. *Communications of the ACM*, 57(10): 78–85.
- Wang, H.; Ren, H.; and Leskovec, J. 2021. Relational message passing for knowledge graph completion. In *Proceedings of the 27th ACM SIGKDD conference on knowledge discovery & data mining*, 1697–1707.
- Wang, S.; Lin, J.; Guo, X.; Shun, J.; Li, J.; and Zhu, Y. 2025. Reasoning of large language models over knowledge graphs with super-relations. *arXiv preprint arXiv:2503.22166*.
- Wang, X.; Wei, J.; Schuurmans, D.; Le, Q.; Chi, E.; Narang, S.; Chowdhery, A.; and Zhou, D. 2022. Self-consistency improves chain of thought reasoning in language models. *arXiv preprint arXiv:2203.11171*.
- Wei, J.; Wang, X.; Schuurmans, D.; Bosma, M.; Xia, F.; Chi, E.; Le, Q. V.; Zhou, D.; et al. 2022. Chain-of-thought prompting elicits reasoning in large language models. *Advances in neural information processing systems*, 35: 24824–24837.
- Xu, D.; Li, X.; Zhang, Z.; Lin, Z.; Zhu, Z.; Zheng, Z.; Wu, X.; Zhao, X.; Xu, T.; and Chen, E. 2025. Harnessing large language models for knowledge graph question answering via adaptive multi-aspect retrieval-augmentation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, 25570–25578.
- Yao, S.; Yu, D.; Zhao, J.; Shafran, I.; Griffiths, T.; Cao, Y.; and Narasimhan, K. 2023. Tree of thoughts: Deliberate problem solving with large language models. *Advances in neural information processing systems*, 36: 11809–11822.
- Yih, W.-t.; Richardson, M.; Meek, C.; Chang, M.-W.; and Suh, J. 2016. The value of semantic parse labeling for knowledge base question answering. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 2: Short Papers)*, 201–206.
- Yixing, P.; Wang, Q.; Zhang, L.; Liu, Y.; and Mao, Z. 2024. Chain-of-question: A progressive question decomposition approach for complex knowledge base question answering. In *Findings of the Association for Computational Linguistics ACL 2024*, 4763–4776.
- Yu, D.; Zhang, S.; Ng, P.; Zhu, H.; Li, A. H.; Wang, J.; Hu, Y.; Wang, W.; Wang, Z.; and Xiang, B. 2022. Decaf: Joint decoding of answers and logical forms for question answering over knowledge bases. *arXiv preprint arXiv:2210.00063*.
- Zhang, B.; Zhu, J.; Li, C.; Yu, H.; Kong, L.; Wang, Z.; Miao, D.; Zhang, X.; and Zhou, J. 2025. What is a Good Question? Assessing Question Quality via Meta-Fact Checking. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, 15248–15256.
- Zhao, R.; Zhao, F.; Wang, L.; Wang, X.; and Xu, G. 2024. Kg-cot: Chain-of-thought prompting of large language models over knowledge graphs for knowledge-aware question answering. In *Proceedings of the Thirty-Third International Joint Conference on Artificial Intelligence (IJCAI-24)*, 6642–6650. International Joint Conferences on Artificial Intelligence.
- Zhu, Y.; Liu, Q.; Aizawa, A.; and Shimodaira, H. 2025. Beyond Chains: Bridging Large Language Models and Knowledge Bases in Complex Question Answering. *arXiv preprint arXiv:2505.14099*.

Appendix

A Datasets

We adopt three publicly available and challenging multi-hop KGQA datasets: WebQuestionsSP (WebQSP) (Yih et al. 2016), ComplexWebQuestions (CWQ) (Talmor and Berant 2018), and CR-LT-KGQA (Guo, Toroghi, and Sanner 2024). WebQSP and CWQ are constructed based on Freebase (Bollacker et al. 2008) and CR-LT-KGQA (CR-LT) is built on Wikidata (Vrandečić and Krötzsch 2014). Unlike Freebase, Wikidata lacks a strict relational hierarchy, leading to a more diverse and less structured relation space. In addition, CR-LT-KGQA includes questions about obscure or long-tail entities rarely seen in the training corpora of large language models, offering a more realistic evaluation setting that highlights the importance of external knowledge graphs when LLMs encounter entities beyond their internal knowledge. WebQSP comprises 2,826 training and 1,628 test instances with up to 2-hop reasoning, while CWQ includes 27,639 training and 3,531 test instances with up to 4-hop reasoning. CR-LT comprises 200 samples. For training the GNN used in our triple retrieval step, we utilized the question-specific subgraphs pre-extracted by RoG (Luo et al. 2023b). Specifically, for each key entity, we construct a single subgraph by merging the subgraphs of all training questions containing that entity. During inference, the subgraph corresponding to the key entity identified in the given question is used.

B Implementation Details

We conduct all experiments with PyTorch on two NVIDIA A6000 GPUs. Unless noted otherwise, hyperparameters are consistent across datasets. For the relation retrieval of our method, we set $m = 15$; for relation pruning, $n = 3$. Triple retrieval uses top- p sampling with p as 0.9 and a temperature of 0.07 during the fine-tuning of MPNet. The GNN for our structure-based entity scoring uses $L = 3$ layers for WebQSP and $L = 6$ for CWQ. For uncertainty quantification, we apply an AU threshold of 1.55 with $l = 4$, based on validation performance.

C Preliminary

Evidential modeling. Evidential modeling for uncertainty quantification represents class probabilities as a Dirichlet distribution parameterized by evidence values predicted by the model, indicating the amount of support for each class. The total evidence controls the confidence of a model, with higher evidence—arising when the model assigns strong evidence to one or a few classes—producing a sharper Dirichlet distribution and thus lower uncertainty. For more details, see (Sensoy, Kaplan, and Kandemir 2018)

D Definition of Variants for Ablation Study

To examine the individual contribution of each component in ProgRAG, we define the following ablated variants:

- **w/o Relation Retrieval:** Instead of retrieving relevant relations, this variant uses all 1-hop relations of each key entity as input for the relation pruning stage.

Method	entity recall	avg. # entity	entity hit
ProgRAG (Gemma2-9b)	81.0	7.0	85.3
PoG (GPT-4o-mini)	40.2	23.0	45.9
ToG (GPT-4o-mini)	57.0	20.0	72.2

Table 6: Performance of tail entity

- **w/o Relation Pruning:** This variant skips our relation pruning and directly uses the top-3 relations from the ranked candidate relation set $R_{re}(q_i)$ obtained from our triple retrieval.
- **w/o Triple Retrieval:** All candidate triples in T_{q_i} are used as input to the LLM without using external retrieval models.
- **w/o Triple Pruning:** The retrieved top- p triples are directly included in the partial reasoning paths without using any refinement in our triple pruning and uncertainty quantification.
- **w/o Uncertainty Quantification:** In this variant, the triple pruning stage relies solely on the predicted answers from the LLM without performing uncertainty quantification.
- **w/o Prefix Enumeration:** Only complete reasoning paths are provided as input to the LLM without enumerating their prefixes.
- **w/o Prefix Repacking:** All prefixes obtained by prefix enumeration are randomly shuffled before being passed to the LLM, preventing semantic relevance-based ordering from guiding the reasoning process.
- **w/o Question Decomposition:** Our retrieval and pruning techniques are performed on the original question without decomposition into sub-questions. To determine the exploration depth, we adopt a self-assessment prompting strategy commonly used in the iterative retrieve-and-reasoning approach (Sun et al. 2023; Chen et al. 2024; Zhang et al. 2025), i.e., after each hop, the LLM is prompted to assess whether the currently explored path sufficiently answers the original question. We terminate if the LLM responds “Yes.” or the number of hops reaches the predefined maximum depth; proceed to the next hop otherwise.
- **w/o Key Entity Mapping:** Following the Chain-of-Question method (Yixing et al. 2024), the question is decomposed into multiple sub-questions without associating them with their corresponding key entities.

E Evaluation of Reasoning Path Retrieval

Table 6 presents a comparison of reasoning path retrieval performance from an entity-level perspective on the CWQ dataset, evaluating ProgRAG against ToG and PoG. Specifically, *entity recall* refers to the proportion of all answer entities that are included in the retrieved reasoning paths; *entity hit* indicates the percentage of questions for which at least one correct answer entity appears in the reasoning path; *avg. # entity* denotes the average number of unique entities included in the reasoning paths. ProgRAG achieves

Method	# Call	Time (s)	# Path	# Token	F1
ToG	12.8	50.9	3.6	850	-
PoG	8.8	35.9	8.8	620	59.4
MFC	12.2	27.7	2.1	638	-
ProgRAG	5.1	7.8	3.9	171	75.9

Table 7: Efficiency analysis on the WebQSP dataset.

significantly higher accuracy, reaching 81.0% entity recall and 85.3% entity hit with only an average of 7 compact entities. In contrast, ToG and PoG show much lower performance (57.0% and 72.2% for entity recall and hit in ToG, and 40.2% and 45.9% in PoG) even though ToG includes an average of 20 entities and PoG includes an average of 23 entities in their reasoning paths. This performance gap is primarily attributed to differences in retrieval strategies. ToG employs a fixed exploration width in the knowledge graph, which results in low recall for questions with a lot of answer entities. PoG adjusts the path width adaptively, but unlike ProgRAG, it lacks a dedicated pruning step to filter the retrieved information. This leads to the accumulation of excessive and potentially irrelevant information, increasing the likelihood of hallucinations by the LLM. Consequently, PoG often makes incorrect termination decisions at each iteration, prematurely halting the reasoning process. In contrast, ProgRAG constructs precise reasoning paths by performing triple retrieval conditioned on sub-questions and applying LLM-based pruning. This approach ensures that only semantically relevant triples are selected at each iteration, enabling more accurate and efficient multi-hop reasoning.

F Efficiency Analysis on WebQSP

Table 7 presents the efficiency analysis on the WebQSP dataset. ProgRAG consistently outperforms all baselines in both efficiency and effectiveness. It requires 42% fewer LLM calls and is approximately 4.6 times faster than PoG. During the reasoning stage, ProgRAG uses 80% fewer input tokens than ToG and 72% fewer than PoG. Despite the reduced computational overhead, ProgRAG achieves significantly higher performance, with an F1 score of 75.9.

G Uncertainty Quantification

Figure 5 shows the relationship between the number of input triples fed into the our triple pruning component and the uncertainty (i.e., AU) of the LLM, categorized by whether the triple pruner successfully returned a triple containing the correct answer entity. We conduct experiments on the WebQSP dataset and use *Hit* metric to determine whether the output of the triple pruner includes any triples that contain the ground-truth answer entity: Hit = 1 if the triple pruner returns such a triple; Hit = 0 otherwise. As the number of input triples increases, the average AU also increases and eventually stabilizes, indicating that exposure to more candidate triples introduces more ambiguity. For each number of input triples, AU is generally higher in failure cases where Hit = 0 than in success cases where Hit = 1. These results demonstrate that our uncertainty quantification component

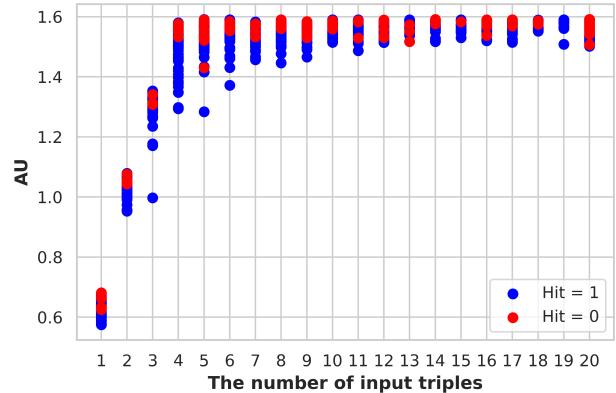


Figure 5: Uncertainty trends by triple input size on the WebQSP dataset.

Method	1 hop	2 hop	≥ 3 hop
ProgRAG (Gemma2-9b)	71.6	77.8	66.6
PoG (GPT-4o-mini)	60.5	62.5	62.8
ToG (GPT-4o-mini)	57.3	59.5	55.8

Table 8: Performance analysis based on the number of relational steps on the CWQ dataset.

effectively captures the reliability of LLM responses during the triple pruning step.

H Performance Analysis by Number of Relational Steps

Table 8 presents an analysis of the performance of ProgRAG and the two KG-enhanced LLMs using the two iterative retrieval-and-reasoning framework (i.e., PoG and ToG) on the CWQ dataset, across different number of relational steps. Here, the number of relational steps is defined as the number of relational hops needed to traverse the KG to find the answer from the key entity. This dataset consists of 1-hop (28.0%), 2-hop (65.9%), and 3-hop or more (6.1%) questions, with the majority requiring multi-hop reasoning. ProgRAG achieves the highest Hit@1 across all relational hops. It reaches 77.8 on 2-hop questions, representing a relative improvement of 24.5% over PoG and 30.7% over ToG. For questions with more than 2-hops, ProgRAG maintains stronger performance than the baselines, demonstrating robust stability under deeper reasoning demands. These results validate the effectiveness of our progressive retrieval and reasoning strategy, highlighting its strength for solving complex multi-hop reasoning tasks.

I Illustration of Question Decomposition

Figure 6 provides an illustrative example of the question decomposition process. Given the question “What country bordering France contains an airport that serves Nijmegen?”, the LLM identifies two key entities, i.e., France

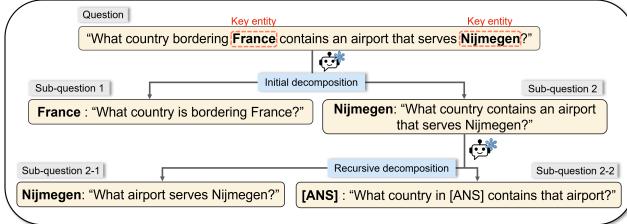


Figure 6: An example of our question decomposition method.

Method	WebQSP	CWQ
ProgRAG	88.5	73.7
w/ MFC-based Decomposition	-	61.8

Table 9: Experimental results comparing with existing question decomposition methods using Hit@1 as the evaluation metric.

and *Nijmegen*, and decomposes the question into two sub-question chains.

- For *France*: “*What country borders France?*”
- For *Nijmegen*: “*What airport serves Nijmegen?*” → “*What country contains that airport?*”

This decomposition allows the model to explore distinct reasoning paths rooted in different key entities, facilitating structured and interpretable multi-hop reasoning.

J Ablation Study

We further replace our question decomposition method with that of MFC to assess the effectiveness of our approach (denoted as w/ MFC-based Decomposition). Table 9 reports the performance, measured by Hit@1, when applying the question decomposition method adopted by MFC (Zhang et al. 2025). In contrast to our method that obtains in advance the full set of sub-questions, the MFC-based approach does not perform full decomposition upfront, resulting in a reasoning depth that is determined dynamically during inference. At each iteration, it decides whether to terminate based on the currently retrieved information. If not, it continues by prompting the LLM to decompose the next sub-question, using the original question, the current sub-question, and the retrieved information as inputs. In summary, while MFC determines whether to decompose further at each step through LLM self-assessment, often leading to premature termination or unnecessary continuation, our method avoids such assessments. Instead, we decompose the question into key entity-specific sub-question chains, where the reasoning depth is defined by the number of sub-questions in each chain. This approach is more robust to LLM hallucinations and results in superior performance.

K Prompts

Prompts used in our method are shown in Figures 7 and 8.

L Code

Our implementation of ProgRAG is available at <https://anonymous.4open.science/r/ProgRAG-F4C2>. The code is structured as follows:

main.py Main script for executing ProgRAG. You can execute the model using the command: `python3 main.py --dataset [Dataset_Name]`.

graph_preprocess.py This code constructs the partial KG used in our study, along with subgraphs corresponding to each key entity.

GNN Folders containing codes for structure-based triple retriever.

GNN/train.py It is used to train the GNN model employed as a triple retriever.

MPNet This folder contains the code for the Textual Semantic-based Triple Retriever. To fine-tune the retriever, run the corresponding bash script for each dataset (e.g., `webqsp.sh`).

prompts.py This file includes all prompts necessary for executing ProgRAG.

Question Decomposition Prompt

You are an expert in world knowledge with strong logical reasoning skills.

Your task is to identify the key entities and decompose the given question into sub-questions based on the key entities.

Follow a step-by-step reasoning process, using each previous answer to guide the next step.

Each step must be logically and semantically connected to the previous one.

Sub-questions must be constructed using the words and phrases found in the original question.

Use the following tags when formatting your answer:

[ANS]: marks an answer (or intermediate result) that is used in subsequent steps, and typically represents a set of multiple candidate answers.

Case:

Decompose the question: “What European Union country sharing borders with Germany contains the Lejre Municipality?” step by step:

The main entities are [“Germany”, “Lejre Municipality”, “Country”].

Step 1: Generate the first sub-question: “What European Union country shares borders with Germany?”

The answer is [ANS1].

Step 2:

Generate the second sub-question: “What country contains the Lejre Municipality?”

The answer is [ANS2].

Step 3:

The final answer is [ANS3], which is the intersection of [ANS1] and [ANS2].

Here, [ANS1] and [ANS2] each represent a set of multiple candidate answers, and the final result [ANS3] includes only those entities that appear in both sets.

Return:

SUB-QUESTION1: What European Union country shares borders with Germany?

ENTITY1: Germany

SUB-QUESTION2: What country contains the Lejre Municipality?

ENTITY2: Lejre Municipality

Figure 7: Prompt used for question decomposition.

Relation Pruning Prompt

You are an expert of world knowledge with strong logical skills.
You have to retrieve the top 3 relations that are most relevant to the question from the candidate relations.
You must select the answer only from the given candidate relations.
If there is no relevant relation to return, then return "None".

Case:
Question: What sports team's owners are Jerry Jones?
Topic entity: Jerry Jones
Candidate relations: [sports.pro_sports_played.athlete, sports.pro_athlete.teams, people.person.places_lived, people.person.profession, common.topic.notable_for, sports.sports_team_owner.teams_owned, sports.pro_athlete.sports_played_professionally, sports.sports_team_roster.player, people.person.employment_history, sports.professional_sports_team.owner_s]

Retrieve top 3 relations from question "What sports team's owners are Jerry Jones?" step by step:

First:
We can assume that the answer is a team, as the question explicitly asks "What sports team".
Therefore, we can infer that the answer type should be "sports team" or "team".

Second:
The question implies that "Jerry Jones", the topic entity, owns a sports team.

Third:
Therefore, appropriate relations would describe the ownership of a sports team by a person.

Fourth:
Based on the candidate relations, the most relevant ones are
sports.sports_team_owner.teams_owned, sports.professional_sports_team.owner_s, sports.pro_athlete.teams.

Return: sports.sports_team_owner.teams_owned, sports.professional_sports_team.owner_s, sports.pro_athlete.teams

Figure 8: Prompt used for relation pruning.