Simplification of context free grammar.

→ it require that L is same for the new grammar but has reduced set of productions

→ Reduced set of production can be obtained through

1. Removing useless production
2. Removing NULL production
3. Removing Unit production

order → NULL → Unit → Useless

Removing Useless Production

→ $A \in V$ is useful if and only if there is at least one $w \in L(G)$ such that

$$S \overset{*}{\Rightarrow} xAy \overset{*}{\Rightarrow} w$$

⇒ $A \in V$ is useless if

① No way of getting a terminal string from it (Non-generating)

② No way to reach to it from start symbol(S) (Non-reachable)

Ex.

$S \to A$
$A \to aA \mid \wedge$
$B \to bA \mid b$
(B is non reachable)

$S \to Aa \mid Bb$
$B \to aB \mid bC$
$A \to Aa \mid dA \mid d$
$C \to aC$
(C is Non-generating)

Step-1 Removing Non Reachable

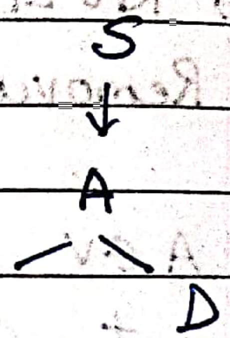
→ create a dependancy graph from start symbol from the variable that derivable from S and then from added variable

→ Repeat above step till no new variable can be added

All variable in dependancy graph are reachable

$S \rightarrow aS \mid A$
$A \rightarrow aB \mid cD$
$C \rightarrow ac \mid bD \mid e$
$D \rightarrow dD \mid d$

dependancy graph



So C (NT) does not exist therefore modified grammar is

$S \rightarrow aS \mid A$
$A \rightarrow aB \mid cD$
$D \rightarrow dD \mid d$

Step-2 Removing Non-Generating

- Let V_i be the set of NT generating string

1. $V_1 = \{A\}$ where $A \to \alpha \in P$ and $\alpha \in \Sigma^*$

2. $V_{i+1} = V_i \cup \{A\}$ where $A \to \alpha \in P$ and $\alpha \in (\Sigma \cup V_i)^*$

Repeat till $V_i = V_{i+1}$

P_i = contain all production in P whose symbol are in $V_i \cup T$

i.e $A \to \alpha \in P_i$ if $A \to \alpha \in P$ and $A, \alpha \in \{V_i \cup T\}$

From resulting grammar

$S \to aS \mid A$ $\quad V_1 = \{D\}$

$A \to aB \mid cD$

$D \to dD \mid d$ $\quad V_2 = \{D, A\}$ as $A \to cD$, $c \in T$ and $D \in V_1$

$V_3 = \{D, A, S\}$ as $S \to A$, $A \in V_2$

No more NT can be added so resulting grammar without useless production

$S \to aS \mid A$

$A \to cD$

$D \to dD \mid d$

Removing Λ-production

→ Any production of from $A \to \Lambda$ is called Λ-production

→ $A \overset{*}{\Rightarrow} \Lambda$ then A is Nullable variable

Method for removing NULL-production

1. let $A \in V_1$ if $A \to \Lambda \in P$

2. $V_{i+1} = V_i \cup \{A\}$ if $A \to \alpha \in P$

and $\alpha \in V_i$

repeat till $V_i = V_{i+1}$

3. Modified production P'

Rewrite all the production including and excluding NULLable variable.

Ex.

$S \to aS \mid Ad$

$A \to BC \mid dA$

$B \to \Lambda$

$C \to \Lambda$

$V_1 = \{B, C\}$

$V_2 = \{A, B, C\}$

P'

$S \to aS \mid d \mid Ad$

$A \to dD \mid B \mid C \mid BC$

Removing unit production

→ $A \to B$ are type of unit production where a single NT is generating another single NT.

Procedure for Removal

→ 1. Create a dependancy graph of unit production

→ 2. if $A \to B \in P$ then replace B by all its Non-unit production

→ 3. Apply step 2 from leaf to Root

Ex:

$S \to Aa \mid B$
$B \to A \mid bb$
$A \to a \mid bc \mid B$

dependancy graph

$$\begin{tikzcd} S \arrow[d] \\ B \arrow[d, bend left=0] \\ A \arrow[u, bend right] \end{tikzcd}$$

All Non-unit production

$S \to Aa$
$B \to bb$
$A \to a \mid bc$

B & A are having loop so their production will become same.

$B \to bb \mid a \mid bc$
$A \to a \mid bb \mid bc$
$S \to Aa \mid bb \mid a \mid bc$

Scanned with CamScanner

Ex. 1 Eliminate useless and NULL production

$S \to a \mid aA \mid BC$

$A \to aB \mid \wedge$

$B \to Aa$

$C \to cCD$

$D \to ddd$

Q2. Simplify following grammar

$S \to aA \mid aBB$

$A \to aaA \mid \wedge$

$B \to bB \mid bbC \mid \wedge$

$C \to B$

$D \to aD \mid cB$