

Context free grammar  $\rightarrow$  A grammar  $G = (V_N, T, S, P)$  is said to be CFG if all the production are of form

$A \rightarrow \alpha$   
where  $\alpha \in (V_N \cup T)^*$

Every regular grammar is CFG or family of of RG is proper subset of CFG

Ex.

$S \rightarrow aSa \mid bSb \mid \Lambda$

$S \rightarrow aSa \rightarrow aaSaa \rightarrow aabSbaa$   
 $\rightarrow aabbbaa$

or

$S \rightarrow bSb \rightarrow baSab \rightarrow baab$

from above derivation we can say that

$L(G) = ww^R : w \in (a,b)^*$

$S \rightarrow abB$

$A \rightarrow aaBb \mid \Lambda$

$B \rightarrow bbAa$

$S \rightarrow abB$

$\rightarrow abbbAa$

$\rightarrow abbbbaaBba$

$\rightarrow abbbbaaabbAaba$

$\rightarrow abbbbaaabbbaaba$

$L(G) \rightarrow ab(bbaa)^n bba(ba)^n$

## Generating CFG for any given CFL

1.  $L = a^n b^n \quad n \geq 0$

$$S \rightarrow a S b \mid \Lambda$$

2.  $L = a^n b^m \quad n \geq m, \quad n \geq 0$

$$S \rightarrow a S b \mid A$$

$$A \rightarrow a A \mid a$$

3.  $L = a^n b^m \quad n \leq m, \quad n \geq 0, m \geq 0$

$$S \rightarrow a S b \mid B$$

$$B \rightarrow B b \mid b$$

4.  $S \rightarrow a^n b^m c^n \quad n \geq 0, m \geq 0$

$$S \rightarrow a S c \mid A$$

$$A \rightarrow b A \mid \Lambda$$

5.  $S \rightarrow a^n b^n c^m d^m \quad n \geq 0, m \geq 0$

$$S \rightarrow A B$$

$$A \rightarrow a A b \mid a b$$

$$B \rightarrow c B d \mid \Lambda$$



$$6. L = a^n b^m c^m d^n \quad n \geq 0, m \geq 0$$

$$S \rightarrow a S d \mid A$$

$$A \rightarrow b A c \mid \Lambda$$

$$7. L = a^n b^m c^k \quad n = m \text{ or } m = k$$

$$\text{So } L = L_1 \cup L_2 \quad L_1 = a^n b^n c^k, L_2 = a^n b^m c^m$$

generate separate grammar for  $L_1$  and  $L_2$  by start symbol as  $S_1$  and  $S_2$

$$S_1 \rightarrow A C$$

$$S_2 \rightarrow B D$$

$$A \rightarrow a A b \mid \Lambda$$

$$B \rightarrow a B \mid \Lambda$$

$$C \rightarrow c C \mid \Lambda$$

$$D \rightarrow b D e \mid \Lambda$$

Now combine both as

$$S \rightarrow S_1 \mid S_2$$

$$8. L = w c w^R \quad \text{where } w, w^R \in (a, b)^*$$

Here the string are **palindrome** having same symbol as first and last and last end in c  
or grammar is

$$S \rightarrow a S a \mid b S b \mid c$$



9.  $L = ww^R$

$$S \rightarrow aSa \mid bSb \mid \Lambda$$

10.  $L = \text{Equal number of } a \text{ and } b$

→ to check for equality for each  $a$  there should exist corresponding  $b$  and vice versa so Grammar is

$$S \rightarrow aSb \mid bSa \mid \Lambda$$

but we are not able to generate string

$abba$  by this so add  $S \rightarrow SS$  therefore final grammar for same

$$S \rightarrow aSb \mid bSa \mid SS \mid \Lambda$$

or

$$S \rightarrow aSbS \mid bSaS \mid \Lambda$$

11.  $L = ww^x$  where  $w^x \neq w^R$

$$S \rightarrow aSa \mid bSb \mid A$$

$$A \rightarrow aAb \mid bAa \mid aA \mid bA \mid a \mid b$$

generate reverse then unbalance same.