COL 764 - Assignment 1

Name: Shubham Sarda Entry Number: 2018TT10958

Question 1

$$P(car\ passing) = p$$
$$P(car\ not\ passing) = 1 - p$$

Since the pedestrian waits for the first k+1 seconds before successfully crossing therefore no car should pass in the next k seconds.

$$P(no\ cars\ in\ last\ k\ seconds) = (1-p)^k \qquad (independent\ events\ \therefore\ multiply)$$

Also the pedestrian shouldn't be able to cross in the first k+1 seconds, this implies that in the first k seconds, a car should pass at least one time. Further in the (k+1)th second, a car should definitely pass.

$$P(at\,least\,1\,car\,passing\,in\,the\,k\,seconds) = 1-(1-p)^k\ (Using\,complement\,probability)$$
$$P(car\ passing\ in\ (k+1)^{th}\ second) = p$$

$$\implies P(pedestrian\,starts\,crossing\,exactly\,after\,k{+}1\,seconds) = (1-p)^k \cdot (1-(1-p)^k) \cdot p$$

(Since independent events $\therefore$ multiply)

Question 2

For random variable X and Y to be independent,

$$P_{XY}(x,y) = P_X(x) \cdot P_Y(y)\ \forall\ x, y$$

We'll find the marginal distributions of X and Y

$$P_X(x=a) = \sum_{y_i \in R_y} P_{XY}(x=a, y=y_i)$$

$$P_X(x) = \begin{cases} \frac{1}{4}, & x = 1 \\ \frac{1}{2}, & x = 2 \\ \frac{1}{4}, & x = 3 \end{cases}$$

Similarly,

$$P_Y(y=a) = \sum_{x_i \in R_x} P_{XY}(x=x_i, y=a)$$

$$P_Y(y) = \begin{cases} \frac{1}{3}, & y=2 \\ \frac{1}{3}, & y=3 \\ \frac{1}{3}, & y=4 \end{cases}$$

Let x=2 and y=3,

$$P_{XY}(x=2, y=3) \neq P_X(x=2) \cdot P_Y(y=3)$$
$$\implies X \textit{ and } Y \textit{ are dependent}$$

Let U and V be random variables with same marginal distributions as X and Y respectively. For U and V to be independent, we'll make sure that

$$P_{UV}(u,v) = P_U(u) \cdot P_V(v) \ \forall \ u, v$$

V U	1	2	3
2	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
3	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
4	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$

Question 3

We observe that,

$$X + Y = Max(X,Y) + Min(X,Y) \quad \forall \ X, Y$$

Taking Expectation on LHS & RHS since we have random variables on both sides

$$E[X+Y] = E[Max(X,Y) + Min(X,Y)]$$

Using Linearity of Expectation, we get

$$E[X] + E[Y] = E[Max(X,Y)] + E[Min(X,Y)]$$

$$\implies E[Max(X,Y)] = E[X] + E[Y] - E[Min(X,Y)]$$
$$\implies E[X \wedge Y)] = E[X] + E[Y] - E[X \vee Y]$$

Hence Proved