



# Fluctuation behavior analysis of international crude oil and gasoline price based on complex network perspective



Minggang Wang<sup>a,b</sup>, Ying Chen<sup>c</sup>, Lixin Tian<sup>a,c,\*</sup>, Shumin Jiang<sup>c</sup>, Zihao Tian<sup>d</sup>, Ruijin Du<sup>c</sup>

<sup>a</sup> School of Mathematical Science, Nanjing Normal University, Nanjing 210042, Jiangsu, China

<sup>b</sup> Department of Mathematics, Nanjing Normal University Taizhou College, Taizhou 225300, Jiangsu, China

<sup>c</sup> Energy Development and Environmental Protection Strategy Research Center, Jiangsu University, Zhenjiang, Jiangsu 212013, China

<sup>d</sup> College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, Jiangsu, China

## HIGHLIGHTS

- The directed and weighted networks of crude price and gasoline price were built.
- The evolution law of the new nodes appeared in the prices networks was obtained.
- The topological structure of price networks was analyzed in different periods.
- The dependency between the two type prices was analyzed based on network similarity.
- The core nodes and the distribution of the time these nodes appeared were identified.

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## ABSTRACT

The directed weighted networks of international crude oil and gasoline price were built in the different fluctuation periods. And then the evolution law of the new nodes in the prices networks was analyzed. The results indicated that the cumulative times of the new nodes that appeared in the crude oil and gasoline prices networks were not random but exhibited a high linear growth trend, which revealed the linear characteristics of the accumulation time of abnormal points that appeared in the process of oil price fluctuations. Based on the node strength, the calculation formula of the network similarity between the crude oil and gasoline price networks was designed, and the interdependence between the crude oil and gasoline price fluctuations was calculated, the results indicated that there was a strong interdependence between crude oil and gasoline prices in stable fluctuation periods, but the degree of dependence was significantly reduced in sharp fluctuation periods. The strength of nodes and their strength distribution, weighted clustering coefficient, and average shortest paths of the price network in different periods were calculated. The fluctuation characteristics in different periods were comparatively analyzed. The core fluctuation status and the conversion relationship between them in different periods were revealed. Finally, the important modes of price fluctuations of crude oil and gasoline were identified and the distribution characteristics of the time these modes appeared were studied.

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## 1. Introduction

The evolution of energy prices contains a variety of information in the energy market. Energy price plays an important role in national economic development, energy intensity, carbon emissions and so on. The volatility of energy prices not only affected

by the basic supply and demand in the market but also by many other factors, such as emergency, speculation, and market psychology, so the evolution processes of energy prices have great uncertainty. Therefore, how to explore the mechanism of fluctuations of energy prices is becoming the hot topic of academic research.

In recent years, the research of the issue of energy price has attracted a great deal of attention from various fields of researchers. Ji and Fan [1] measured the influence of the crude oil market on non-energy commodity markets before and after the 2008 financial crisis. Wang and Tian [2] proposed a novel dynamic system model of energy price–energy supply–economic growth based on the cau-

\* Corresponding author at: School of Mathematical Science, Nanjing Normal University, Nanjing 210042, Jiangsu, China.

E-mail addresses: [magic821204@sina.com](mailto:magic821204@sina.com) (M. Wang), [68345954@163.com](mailto:68345954@163.com) (Y. Chen), [tianlx@ujs.edu.cn](mailto:tianlx@ujs.edu.cn) (L. Tian), [jsm@ujs.edu.cn](mailto:jsm@ujs.edu.cn) (S. Jiang), [2277904843@qq.com](mailto:2277904843@qq.com) (Z. Tian), [dudo999@126.com](mailto:dudo999@126.com) (R. Du).

sal relationships among energy price, energy supply and economic growth during a given economic period. They established a network structure of mutual transmission between different elements and went on to establish a new dynamic systematic model of energy price–supply–economic growth. Koirala et al. [3] investigated the interdependence between agricultural commodity futures prices and energy futures prices, and the results showed that agricultural commodity and energy future prices were positively correlated. Dias et al. [4] made a joint analysis of the price of oil, natural gas, and electricity in U.S. markets using a multi-conversion model that captured the typical facts of energy prices. Nazlioglu et al. [5] examined whether there was a volatility transmission between oil prices and financial stress by means of the volatility spillover test. Atil et al. [6] used the nonlinear autoregressive distributed lags (NARDL) model to examine the pass-through of crude oil prices into gasoline and natural gas prices. Coleman [7] analyzed real oil prices during 1984–2007 using a monthly dataset of fundamental and market parameters. Chai et al. [8] obtained the core factors, built an oil price system VAR model, which used demand, supply, price, and inventory as endogenous variables, and China's net imports as well as dollar index as exogenous variables based on the PATH-ANALYSIS model. Zhang et al. [9] empirically investigated the functions of price discovery and risk transfer in crude oil and gasoline futures markets using multiple econometric models. Narayan et al. [10] considered five different forms of oil futures contracts and examined price fluctuation clustering. Chiroma et al. [11] proposed an alternative approach based on a genetic algorithm and neural network (GA-NN) for the prediction of the West Texas Intermediate (WTI) crude oil price. To sum up, a large number of scholars at home and abroad adopted a variety of methods for the systematic research of the related issues of energy price system. They established many practical models to research the conduction relationships between energy price and its related factors, and to predict of the trend and volatility of energy prices in the future, etc. However, the traditional approach to the problem of energy price volatility research mainly was to establish a variety of econometric model and explain the reason and mechanism of oil price fluctuations, but the energy price system is essentially a complex nonlinear and non-stationary system. Traditional approaches rarely touched on the questions of how to construct a complex network analysis model of energy price and how to explore the complex characteristics of energy price system deeply.

Complex network is a hot research topic in recent years, the main idea of which is to regard the link between the real parts of the system as a complex network, in order to better understand the essence of reality system. With the discoveries of the scale-free network model [12,13], the small-world network [14], the Newman and Watts network [15], and the random network model [16], complex network is applied in more and more fields, providing us a new perspective and approach to the studies of complexity problems.

Recently, complex network theory has been applied to solve problems in the fields of energy economy with considerable achievements. Chen et al. [17] proposed the international oil price network and analyzed the dynamic properties of the network. An et al. [18] studied the role of fluctuating autocorrelation modes in the crude oil price time series. An et al. [19] established a trading-based network model of international crude oil to study the relationship between countries with common trade partners. Hao et al. [20] built a global fossil energy exergy flow network with countries as nodes, the international exergy flows as edges and the exergy of each flow as the weight of the edges, and analyzed the distribution of countries, the overall structure, major countries and major exergy flow paths of the network from 1996 to 2012. Gao et al. [21] studied the transmission characteristics of fluctuant patterns of the forex burden based on international crude oil

prices. Zhong et al. [22] set up unweighted and weighted oil trade network models based on complex network theory using data from 2002 to 2011 to study the evolution of trade communities in the international oil trade network. Huang et al. [23] introduced an approach to the multiscale transmission characteristics of the correlation modes between bivariate time series. Gao et al. [24] built the international fossil energy trade multilayer network (ETMN), and studied the evolutionary characteristics of networks during 2002–2013. An et al. [25] designed a complex network approach to the dynamics of the co-movement between crude oil futures and spot prices.

Based on the above analysis, there are three main references to study the energy price issue based on the complex network theory in the previous literature, that is, Refs. [17,18,25]. Only Ref. [17] directly uses the energy price data to construct the network. The existing literature has provided a solid empirical investigation and a good reference for understanding the evolution of the energy price network, but some problems still need to be further examined. For instance, (1) the previous literature only used three states i.e. {increase, stable, decline} to describe the energy price fluctuations, which leads to the loss of complexity information of energy price fluctuations. (2) Previous studies lack of the research of network node evolution over time and considerations about such questions as whether the advent of the new node is regular, when an important node in network is to appear, and so on. (3) It is well known that the energy price fluctuations have different statuses at different times. Previous studies haven't done contrast analysis of what distinctions and connections the topology structure of the energy price network in different periods. (4) Different energy prices volatility have the strong dependency in the process. Previous studies haven't touched upon the question of whether the above-mentioned dependency can be measured through the similarity of the network.

In this paper, we construct the crude oil and gasoline price directed and weighted complex networks based on the novel framework in different periods, and explore the topological structure of the networks. Four main novel contributions in our studies are as follows: (1) we convert the crude oil and gasoline price volatility sequences into the characters composed by five symbols  $\{R, r, e, d, D\}$ , which is different from the previous literature converting the sequences into three symbols  $\{R, e, D\}$ . Our conversion can better reflect complexity of energy price fluctuations. (2) Based on the probability of different characters appearing in the price fluctuation sequences of crude oil futures and gasoline, we construct the crude oil and gasoline price network in different periods and conduct comparative analysis accordingly, which has never been achieved in previous studies. For the first time, we verified the crude oil and gasoline price networks are assortative networks. (3) We take the time factor into consideration in building the network and provide the evolution law of the nodes changing in time. Not only the important nodes are identified, but the distribution characteristics of the time when the important nodes appear are also studied. (4) We design the calculation formula to measure the similarity between the crude oil and gasoline price network based on the strength of same nodes. And we analyze the interdependence between the crude oil and gasoline price volatility based on the viewpoint of the similarity of the network.

The main idea of this paper is to build a directed weighted network of the prices of crude oil and gasoline in different periods, to comparatively analyze the topological structure of the network of the prices of crude oil and gasoline. The purpose is to find the evolution law of the network of the prices of crude oil and gasoline and the main characteristics in different periods. The research framework is shown in Fig. 1.

From the research framework of this paper, our research is divided into three levels. First of all, we convert crude oil and gaso-

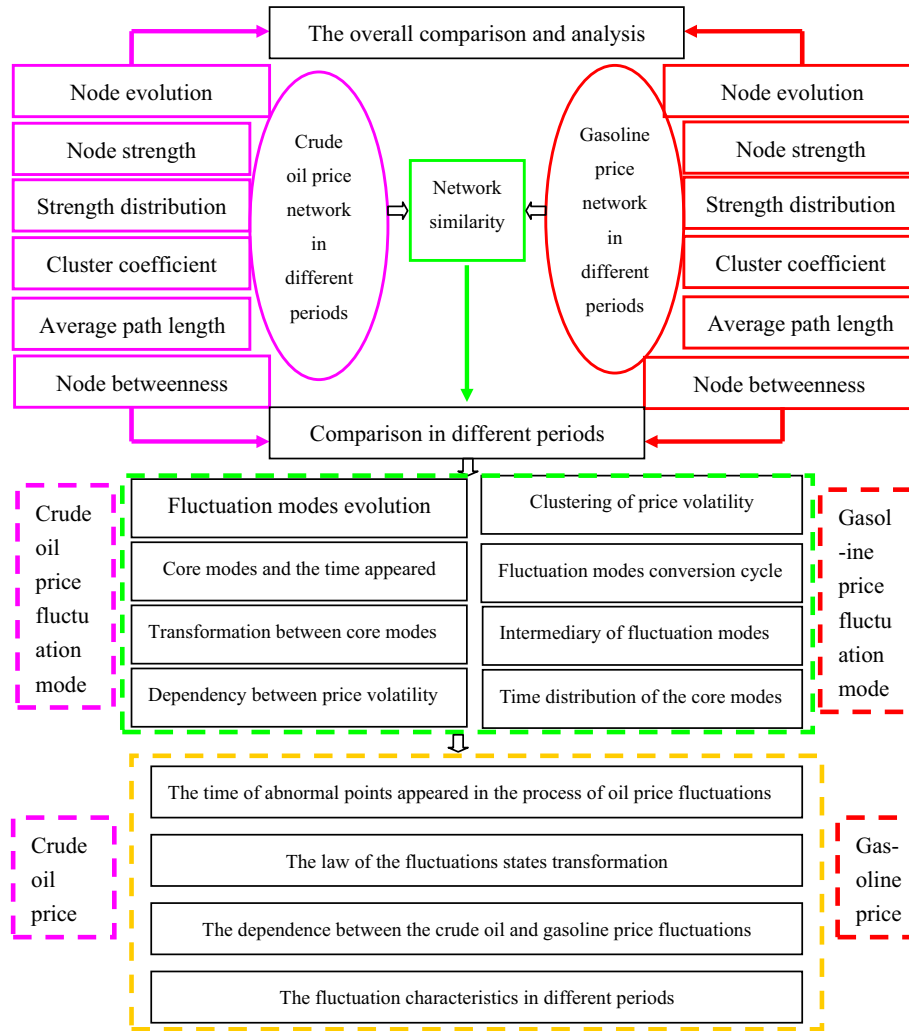


Fig. 1. The research framework of this paper.

line price volatility into different models, and then construct the complex networks in different periods based on the conversions between different models and discuss the topological properties of these complex networks. Then, we obtain the dynamic characteristics of fluctuation models of crude oil prices and gasoline prices from the topological properties of the complex networks. And then we obtain the fluctuation behavioral characteristics of international crude oil and gasoline prices from the dynamic characteristics of fluctuation model. Eventually we obtain the linear characteristics of the accumulated time when the abnormal points appear in the fluctuation processes of crude oil prices and gasoline oil prices, get the temporal distribution characteristics of the important modes that appear in the process of the crude oil and gasoline price fluctuations, obtain the calculation method to measure the interdependence between the crude oil and gasoline price fluctuations based on the network node angle, and comparatively analyze the typical fluctuations characteristics of crude oil and gasoline price in different periods.

## 2. Material and methods

### 2.1. Data

We have chosen Cushing, OK Crude Oil Future Contract 1 (Dollars per Barrel) and New York Harbor Regular Gasoline Future Contract 1 (Dollars per Gallon) from January 7, 1985 to August 11,

2015 as sample data. The datasets are derived from U.S. Energy Information Administration, as shown in Fig. 2(a and b).

### 2.2. Data processing

Denote crude oil future price series as  $T_{oil}(t)$ ,  $t = 1, 2, \dots, N$ ,  $N = 7681$ , and gasoline future price series as  $T_{gas}(t)$ ,  $t = 1, 2, \dots, M$ ,  $M = 7681$ . During the selected study period, the missing data we use linear interpolation method to supplement, and with this process, we obtain 7681 sample data of crude oil future price and gasoline future price, respectively. Then the crude oil future price volatility series is denoted as  $\Delta T_{oil}(t) = T_{oil}(t) - T_{oil}(t-1)$ , where  $T_{oil}(t)$  is the current price and  $T_{oil}(t-1)$  is the previous price. Similarly, the gasoline future price volatility series is denoted as  $\Delta T_{gas}(t) = T_{gas}(t) - T_{gas}(t-1)$ . In risk management analysis, the volatility is often described as the time-varying variance [26]. However, the volatility here we defined is the time-varying rate of price change, which is different from the traditional definition in risk management analysis.

Suppose  $E_{\Delta T_{oil}} = \frac{\sum_{t=1}^{N-1} |\Delta T_{oil}(t)|}{N-1}$ , when  $\Delta T_{oil} > E_{\Delta T_{oil}}$ , it means a sharp rise of the oil price; when  $0 < \Delta T_{oil} \leq E_{\Delta T_{oil}}$ , it means oil price rise; when  $\Delta T_{oil} = 0$ , it means the oil price is stable; when  $-E_{\Delta T_{oil}} \leq \Delta T_{oil} < 0$ , it means oil price decline; when  $\Delta T_{oil} < -E_{\Delta T_{oil}}$ , it means a sharp decline of oil price. The value of  $\Delta T_{oil}(t)$  reveals the intensity of the fluctuation of  $f_i$ .

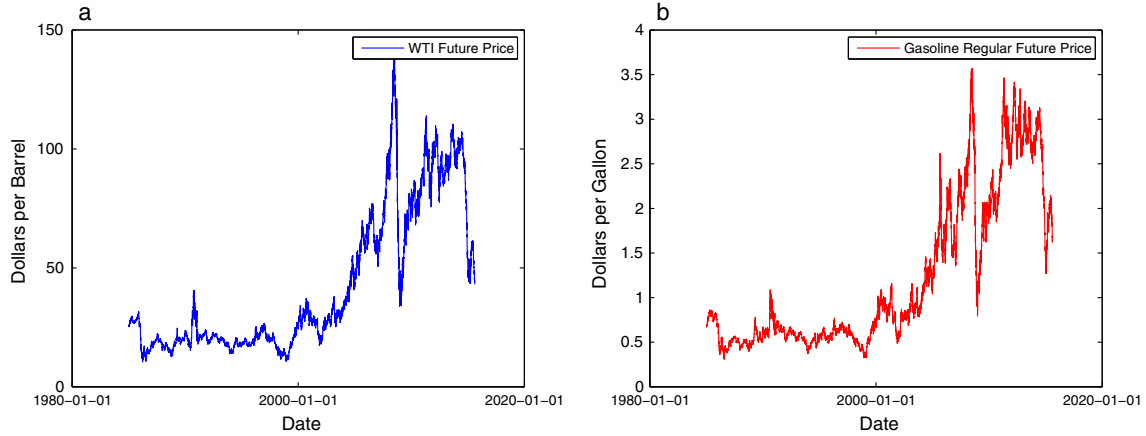


Fig. 2. (a) Crude oil future prices and (b) gasoline future prices.

$$of_i = \begin{cases} R, \Delta T_{oil} > E_{\Delta T_{oil}} \\ r, 0 < \Delta T_{oil} \leq E_{\Delta T_{oil}} \\ e, \Delta T_{oil} = 0 \\ d, -E_{\Delta T_{oil}} \leq \Delta T_{oil} < 0 \\ D, \Delta T_{oil} \leq -E_{\Delta T_{oil}} \end{cases} \quad (1)$$

The symbol 'R' denotes price sharp rise, 'r' denotes price rise, 'e' denotes price stable, 'd' denotes price decline and 'D' denotes price sharp decline. Therefore, the state of the fluctuation of crude oil price for the entire sample is denoted by a continuous sequence of symbols.

$$FT_{oil} = \{of_1, of_2, of_3, \dots\}, \quad of_i \in (R, r, e, d, D) \quad (2)$$

Similarly, the state of the fluctuation of gasoline price for the entire sample is denoted by a continuous sequence of symbols as following:

$$FT_{gas} = \{gf_1, gf_2, gf_3, \dots\}, \quad gf_i \in (R, r, e, d, D) \quad (3)$$

### 2.3. Period division

Based on the results of the data processing method in Section 2.2, we make 12 months a period, calculate the probability of the symbol 'e, r, R, d, D' in every period, and denote them as  $Pe, Pr, PR, Pd, PD$ . The evolution results of the probability of each character are shown in Fig. 3(a). From Fig. 3(a), we can see that in the process of crude oil price fluctuations, the probability of the symbol 'e' is minimum, almost zero. The probability of the symbol 'R', 'D' and the probability of the symbol 'r', 'd' appear alternately near 2014. In order to further distinguish the sharp fluctuation period of the crude oil price from the stable fluctuation period of the crude oil price, we have the following calculation:

$$Pr\ d = Pr + Pd, \quad PRD = PR + PD \quad (4)$$

where  $Pr\ d$  denotes the probability of the stable fluctuation,  $PRD$  denotes the probability of the sharp fluctuation. The evolution of  $Pr\ d$  and  $PRD$  is shown in Fig. 3(b). From Fig. 3(b), we can clearly see that the probability of the sharp fluctuation is higher than the probability of the stable fluctuation on February 10, 2004, which means that the fluctuation state of crude oil price is changed on February 10, 2004. Therefore, the period from January 7, 1985 to February 10, 2004 is categorized as the stable fluctuation period of crude oil price, and the period from February 11, 2004 to August 11, 2015 is categorized as the sharp fluctuation period. In order to further distinguish the sharp rise from the sharp decline in the

sharp fluctuation period, we continue to discuss in the sharp fluctuation period and calculate as following:

$$Pr\ R = Pr + PR, \quad PdD = Pd + PD \quad (5)$$

where  $Pr\ R$  denotes the probability of the sharp rise in the sharp fluctuation period, while  $PdD$  denotes the probability of the sharp decline in the sharp fluctuation period. The evolution of  $Pr\ R$  and  $PdD$  is shown in Fig. 3(c). Fig. 3(c) clearly shows that the sharp fluctuation period can be divided into four different periods according to the proportions of the rise and decline. To sum up, the crude oil price fluctuation period can be divided into five different periods, as shown in Table 1.

Similarly, the evolution of the fluctuation state of gasoline price is shown in Fig. 4(a)–(c).

Fig. 4 indicates that the gasoline price fluctuation period can be divided into four different periods, as shown in Table 2.

### 2.4. Analysis method of complex network

Based on the continuous symbol sequence  $FT_{oil} = \{of_i\}$ ,  $i = 1, 2, 3, \dots, 7680$  and  $FT_{gas} = \{gf_j\}$ ,  $j = 1, 2, 3, \dots, 7680$ , we utilize the sliding window method to divide the symbol sequence into modes, as the number of crude oil and gasoline trading days is 5 days per week. Accordingly, the fluctuation states of crude oil and gasoline price on these days are denoted by 5 symbols, so the length of the sliding window is 5 and then the symbol sequence  $FT_{oil} = \{of_i\}$  and  $FT_{gas} = \{gf_j\}$  are split into 7676 fragments. There is some overlap among these modes, and the former mode is the basis of the following mode. Therefore, the fluctuating modes have the feature of memory and transitivity (Table 3).

In Table 3, the fluctuating modes change with the sliding window, therefore, we obtain the sequence of the fluctuating modes  $\{ddrdd, drddd, rdddR, dddRr, \dots\}$ . We represent the fluctuating modes as nodes, and the transformations among modes as edges. That is, the fluctuating modes evolve into each other with time  $\{ddrdd \rightarrow drddd \rightarrow rdddR \rightarrow dddRr \rightarrow \dots\}$ . The weight of an edge is the frequency of the transformations. As the modes appear repeatedly, for crude oil price, we obtain only 1178 nodes from 7676 modes. Accordingly, the transformations among the 1178 nodes form the crude oil price network. For gasoline price, we obtain 1439 nodes from 7676 modes. Accordingly, the transformations among the 1439 nodes form the gasoline price network.

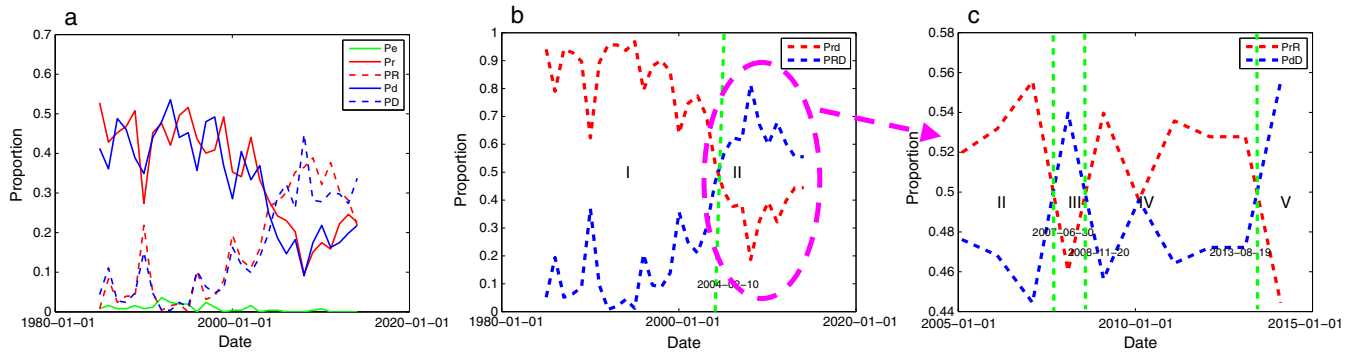


Fig. 3. (a) The probability evolution results of the five states of crude oil; (b) the evolution of  $Pr d$  and  $PRD$  and (c) the evolution of  $Pr R$  and  $PdD$  in the sharp period.

**Table 1**  
Period division of crude oil price.

NO.	Period	State
I	January 7, 1985–February 10, 2004	Stable fluctuation period
II	February 11, 2004–June 30, 2007	Sharp rise period
III	July 1, 2007–November 20, 2008	Sharp decline period
IV	November 20, 2008–June 19, 2013	Sharp rise period
V	June 20, 2013–August 11, 2015	Sharp decline period

**Table 2**  
Period division of gasoline price.

NO.	Period	State
I	January 7, 1985–June 10, 2003	Stable fluctuation period
II	June 11, 2003–August 14, 2006	Sharp decline period
III	August 15, 2006–August 20, 2013	Sharp rise period
IV	August 21, 2013–August 11, 2015	Sharp decline period

### 3. Results

#### 3.1. The crude oil and gasoline price network in different periods

In this paper, we establish different directed weighted network models of crude oil and gasoline price in order to examine both the overall and several partial evolutionary processes. The overall analysis includes all of the sample data (Figs. 5(a) and 6(a)). Then we build corresponding network models in different periods to comparatively analyze the evolution characteristics, as shown in (Figs. 5(b)–(g) and 6(b)–(f)). In Figs. 5 and 6, the solid line denotes the positive direction, whereas the dotted line denotes the negative direction and the size of the node denotes the weight.

Figs. 5(a) and 6(a) indicate that crude oil and gasoline price networks both have the same characteristics, that is, there are less big nodes and more small modes in the networks. From Figs. 5(b)–(g) and 6(b)–(f), we can clearly see that there are different network structures in different periods, for which we will give a contrastive analysis in our next paper. Firstly, we give the analysis of the statistical characteristics of the characters appeared in the crude oil and gasoline price networks.

According to the method of building network (see Section 2.4), the fluctuation mode of the crude oil and gasoline price is 5-string

mode constituted by five types of symbols. Hence, there will be  $5^5 = 3125$  different fluctuation modes in theory, but in fact, from Figs. 5(a) and 6(a), there are only 1178 and 1439 different fluctuation modes in crude oil and gasoline price networks respectively. From the number of different nodes, we can see that the different nodes in gasoline price fluctuation network are more than those in the crude oil price network, thus we can deduce that the volatility of gasoline price is more complex than that of crude oil price. The evolution relationship of the crude oil and gasoline price networks nodes is shown in Fig. 7(a) and (b). The accumulation time when the new nodes appear in crude oil and gasoline price networks is shown in Fig. 7(c), and the accumulation time when the new nodes appear in different periods is shown in Fig. 7(d).

From Fig. 7(a), we obtain that in the process of crude oil price fluctuations, the probabilities of symbols  $e$ ,  $r$ ,  $R$ ,  $d$ ,  $D$  are respectively 0.0303, 0.2593, 0.2284, 0.2576 and 0.2244, which means there are mainly transformations of rising and falling states in the process of crude oil price fluctuations. The probability of a stable state appearing very low, which means the crude oil price fluctuation is very complex. Further more, we can establish that the sum of the probability of rising state is 0.4877, which is larger than the sum of the probability of the falling state that is 0.4819. This indicates that although the crude oil price fluctuation state

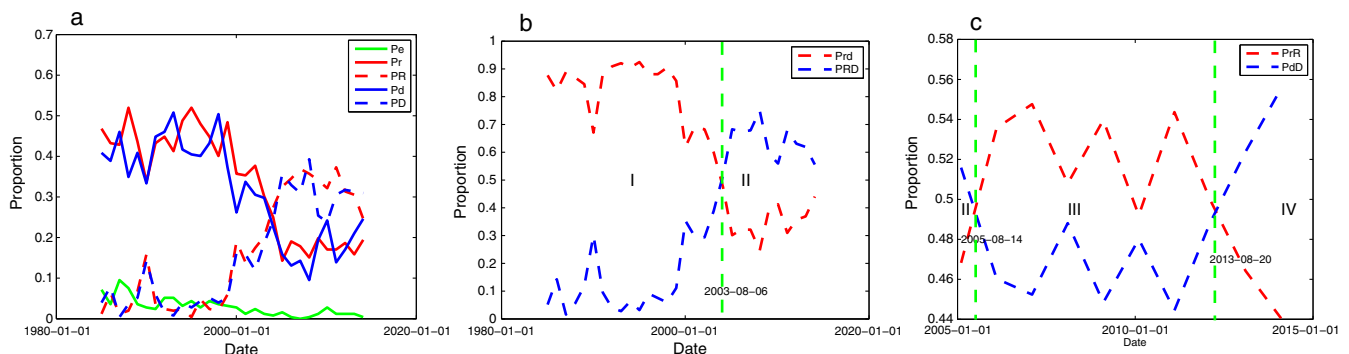


Fig. 4. (a) The probability evolution results of the five states of gasoline; (b) the evolution of  $Pr d$  and  $PRD$  and (c) the evolution of  $Pr R$  and  $PdD$  in the sharp period.



**Table 3**

The process of defining the fluctuating modes.

Date	$T_{oil}(t)$	$\Delta T_{oil}(t)$	$FT_{oil}$	mode
1986-06-02	13.8			
1986-06-03	13.35	-0.45	$d$	
1986-06-04	13.15	-0.2	$d$	
1986-06-05	13.21	0.06	$r$	
1986-06-06	12.73	-0.48	$d$	
1986-06-09	12.61	-0.12	$d$	ddrdd
1986-06-10	12.38	-0.23	$d$	drddd
1986-06-11	13.52	1.14	$R$	rdddR
1986-06-12	13.69	0.17	$r$	dddRr
...	...	...	...	...
1986-06-30	12.8	-0.58	$d$	dDdrd
1986-07-01	12.39	-0.41	$d$	Ddrdd
1986-07-02	12.04	-0.35	$d$	drddd
1986-07-03	11.7	-0.34	$d$	rddd
...	...	...	...	...

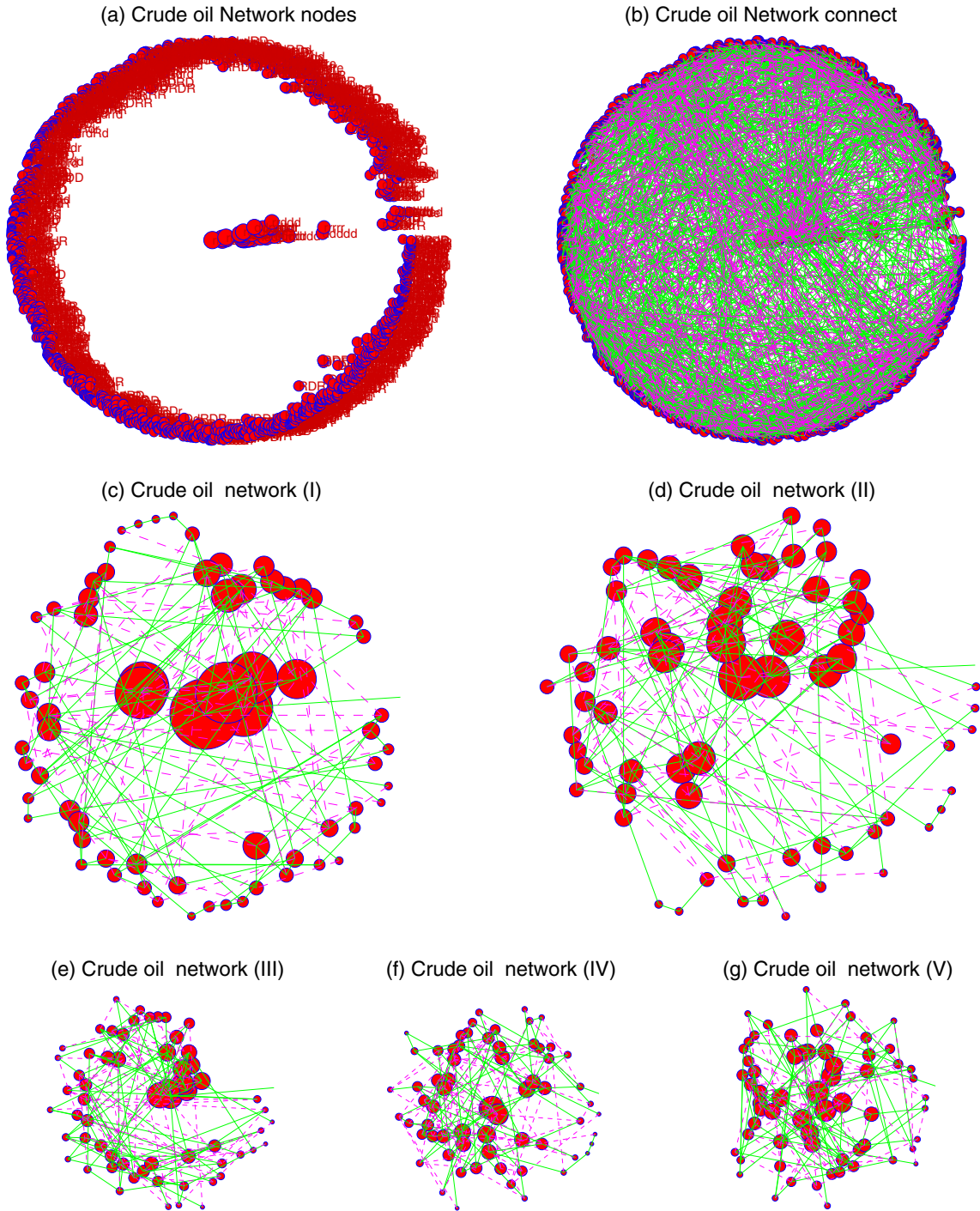
is irregular but there is still an overall rising trend. Similarly, from Fig. 7(b), we obtain that in the process of gasoline price fluctuations, the probability of symbol  $e$ ,  $r$ ,  $R$ ,  $d$ ,  $D$  is 0.0679, 0.2520, 0.2165, 0.2524, and 0.2112, respectively, which means there are mainly rising and falling states transformation in the process of gasoline price fluctuations, the probability of stable state appears very low, which means the gasoline price fluctuations is also very complex. Furthermore, we can obtain that the sum of the probability of rising state is 0.4685 which is larger than the sum of the probability of falling state 0.4636. This indicates that the gasoline price also has a rising trend.

Next, we discuss the law of the new nodes that appear in the crude oil and gasoline price networks. We look at the accumulation time of the new nodes that appear in crude oil and gasoline price network (see Fig. 7(c)). The red<sup>1</sup> solid line in Fig. 7(c) denotes the same time interval cumulative image. From Fig. 7(c), we can clearly see that the time intervals when new nodes appear in the process of crude oil and gasoline network evolution are not equality spaced, but grow bigger in time. The cumulative time interval presents a linearly tendency. We perform an ordinary least squares (OLS) linear regression [27] to estimate the tendency of the cumulative time interval, and get the regression equation  $y = 6.234x - 558.40$  and  $y = 5.262x - 520.83$  respectively. Since we only need to estimate the tendency of the cumulative time interval, thus the intercept values of the regression equations are not set equal to zero. The values of  $R^2$  are 0.980 and 0.981, thus the credibility of the results are high, which means that the cumulative time of the new nodes in the crude oil and gasoline prices networks are not random but exhibit a high linear growth trend. Will noise affect the results? To address this question, we add Gauss white noise to crude oil price and gasoline price to generate two new price time series whose SNR (signal-to-noise ratios) are both 10 dB. Using the same method to mapped the two new price time series into the networks, and the accumulation time of the new nodes appeared also presents a linearly tendency, the values of  $R^2$  are 0.960 and 0.951, respectively, which

are essentially the same with the previous, indicating the robustness of the results. Furthermore, from the contrast perspective of crude oil network versus gasoline price network, the time interval of the new nodes in crude oil price network is longer than gasoline price network. It indicates that if based on the same length data to construct the network, the nodes of the gasoline price network are more than the nodes of the crude oil price network; to a certain extent this shows the gasoline price network evolution is more complex than crude oil price network. In different periods, Fig. 7(d) shows the accumulation time of the new nodes appeared in the stable period and sharp period. For crude oil price network, the regression equation is  $y = 2.801x - 299.552$ ,  $R^2 = 0.9368$  in the stable period, and the regression equation is  $y = 2.468x - 200.211$ ,  $R^2 = 0.9341$  in the sharp period (see the upper part of Fig. 7(d)). The slope in the stable period is larger than in the sharp period, which means the time interval of the new nodes appeared in the stable period is longer than the sharp period. However, for gasoline price network, the regression equation is  $y = 1.885x - 147.309$ ,  $R^2 = 0.9842$  in the stable period, and the regression equation is  $y = 2.097x - 192.298$ ,  $R^2 = 0.9631$  in the sharp period (see the second part of Fig. 7(d)). The slope in the stable period is smaller than in the sharp period, which means the time interval when the new nodes appear in the stable period is shorter than in the sharp period. They are different.

Based on the above analysis, in both the stable fluctuation period and the sharp period, the cumulative times when the new nodes appear in the crude oil and gasoline prices networks are not random but exhibit a high linear growth trend. The new nodes in the price network mean the abnormal points in the process of the price fluctuation (it is different from the previous fluctuation state), which shows that although the crude oil and gasoline price volatility present complicated nonlinearity, the cumulative times when the abnormal points appear in the process of oil price fluctuations are linear. This rule can be used to effectively identify the times when the abnormal points appear in the process of oil price fluctuations, and to improve the accuracy of predicting energy prices. The advantages of this method to predict the energy prices are that it enables us to describe the price volatility more precisely

<sup>1</sup> For interpretation of color in Figs. 7, 9 and 12, the reader is referred to the web version of this article.



**Fig. 5.** (a) Crude oil price network nodes; (b) crude oil price network connect; (c) crude oil price network in period I; (d) crude oil price network in period II; (e) crude oil price network in period III; (f) crude oil price network in period IV and (g) crude oil price network in period V.

and reduce the computation complexity. For further study, we will propose the new prediction method in our research framework.

### 3.2. The dynamic characteristics of the crude oil and gasoline price networks

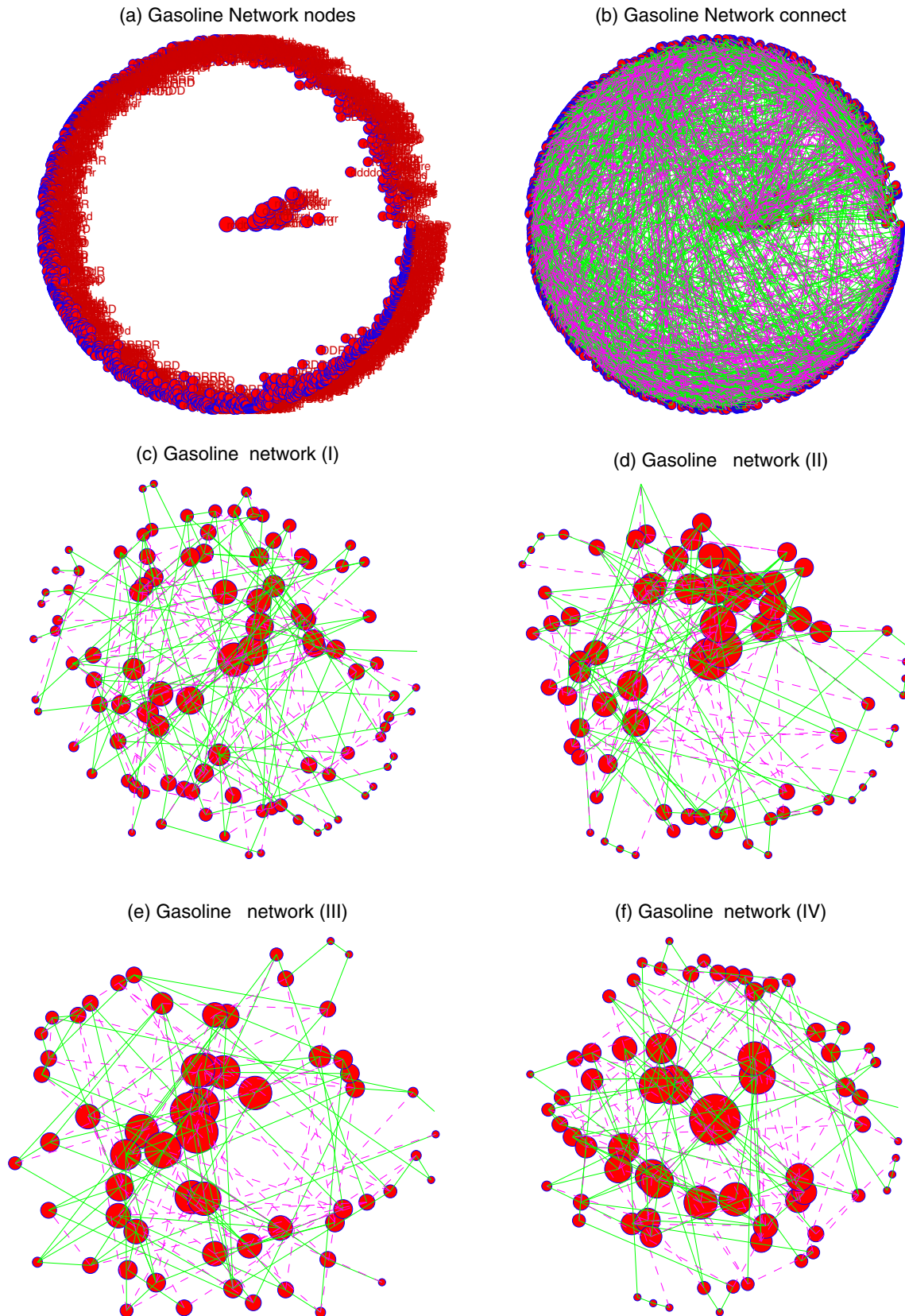
#### 3.2.1. The node strength and the distribution of the node strength

As the crude oil and gasoline price networks we have built are directed weighted networks, we discuss in this section the node strength and the distribution of the node strength [17,18,25]. For an unweighted complex network, the degree of a node represents the number of connections it has with other nodes. In weighted

networks, the node strength is used in a similar sense to the node degree in an unweighted network. The node strength not only accounts for the number of its neighboring nodes but also the weight of each edge. The strength of a node in a directed complex network can be divided into the in-strength and the out-strength, which are defined as follows:

$$s_i^{in} = \sum_{j=1}^N a_{ji} \omega_{ji}, \quad s_i^{out} = \sum_{j=1}^N a_{ij} \omega_{ij} \quad (6)$$

In the formulae,  $a_{ji}$  is the element of the adjacency matrix. If there is connection from  $v_j$  to  $v_i$ , then  $a_{ji} = 1$ . Otherwise,  $a_{ji} = 0$ , then  $\omega_{ji}$  is

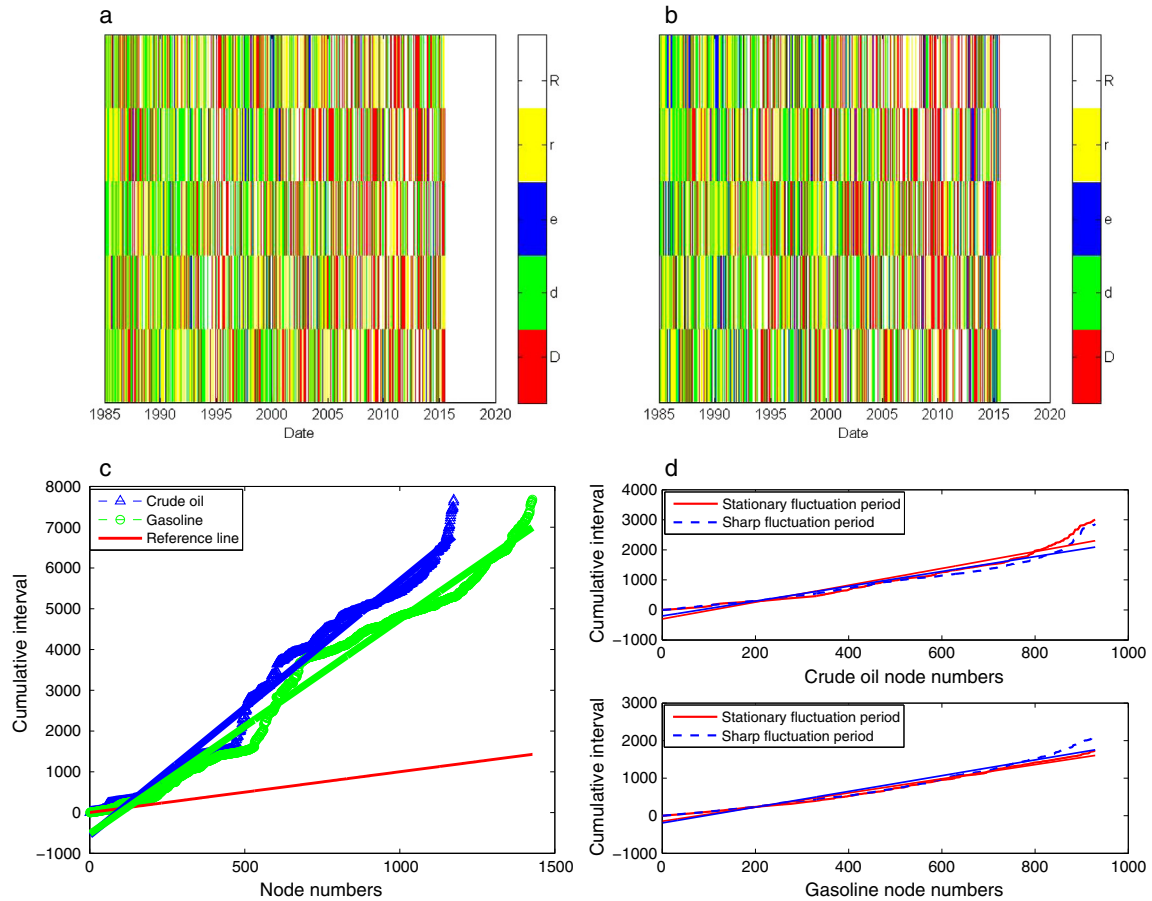


**Fig. 6.** (a) Gasoline price network nodes; (b) gasoline price network connect; (c) gasoline price network in period I; (d) gasoline price network in period II; (e) gasoline price network in period III and (f) gasoline price network in period IV.

the weight from node  $v_j$  to  $v_i$ . According to the method of network building in Section 2.4, we connect the nodes in time order, so the in-strength and the out-strength are the same except for the first

node and the last node. Therefore, we only select the out-strength for analysis. For the narration aspect, in the next paper we describe out-strength as strength.



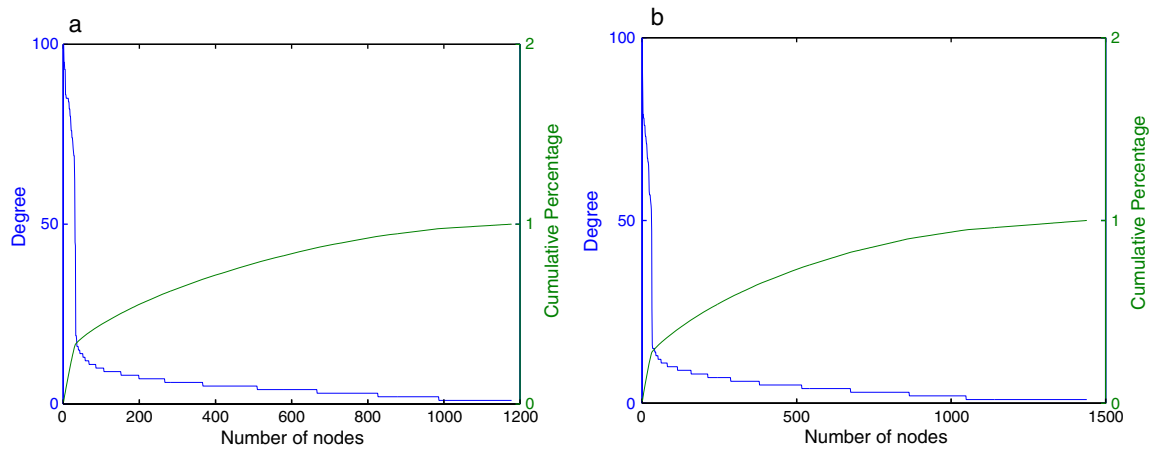


**Fig. 7.** (a) The evolution of the crude oil price networks nodes; (b) the evolution of the gasoline price networks nodes; (c) the accumulation time of the new nodes appeared and (d) the accumulation time of the new nodes appeared in different periods.

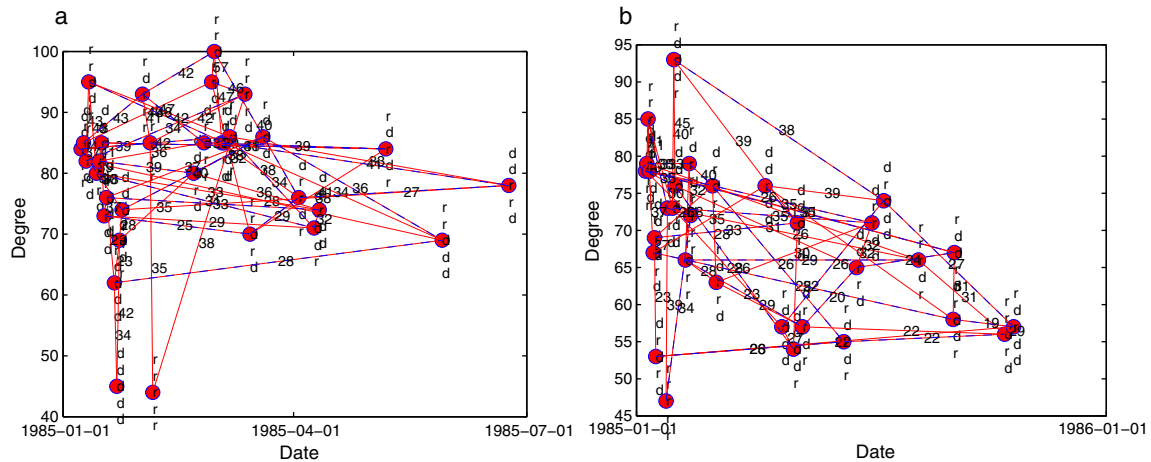
The node strength and the cumulative strength distribution of crude oil and gasoline price networks are obtained as shown in Fig. 8(a) and (b). The left coordinates in Fig. 8 denote the node strength, and the right coordinates denote the cumulative strength distribution of the nodes. Fig. 8(a) and (b) indicates that there are less nodes with big strength, and most nodes have small strength in both crude oil and gasoline price networks. For the crude oil price network (Fig. 8(a)), the total number of nodes is 1178, in which there are only 32 nodes with a strength higher than 40, accounting for 33.21% of the total strength, that is, 2.72% of nodes account for 33.21% of the total strength. For the gasoline price network (Fig. 8(b)), the total number of nodes is 1439, in which there are only 31 nodes with a strength higher than 40, accounting for 28.03% of the total strength, that is, 2.22% of nodes account for 28.03% of the total strength.

The node strength is a comprehensive reflection of the local information on a node. The greater the node strength, the more times it transforms to other modes, which means the higher probability of its presence in the network and the greater importance it has in the network. We calculate the nodes with strength higher than 40 of the crude oil and gasoline price networks respectively. The relationships among the nodes names, the nodes strength, the time when the nodes appear and the connections between nodes are obtained as shown in Fig. 9(a) and (b). The symbols in Fig. 9 denote the names of the nodes, the figures between the nodes representing the weights, the red solid line representing the positive direction and the blue line representing the opposite direction.

From the perspective of the time when the key nodes appear, Fig. 9(a) and (b) indicates that both in the crude oil and gasoline price networks, the big strength nodes often appear at an earlier time, but the nodes appear at earlier times must not be the big strength nodes. In the crude oil price network, the 32 nodes with a strength higher than 40 come from the 56 nodes in the front of the crude oil price network. The biggest strength node is *rrdr* which appeared on March 1, 1985, it is the 34th of the nodes in the network. There are two nodes with the strength of 1 in 56 nodes in the front of the crude oil price network. They are *rDrde* and *Drded* which appeared on February 11, 1985 and February 12, 1985, and they are the nodes on 26th and 27th in the network respectively. Therefore, the nodes appeared at earlier times must not be the big strength nodes in the crude oil price network. In the gasoline price network, the 31 nodes with a strength higher than 40 come from the 99 nodes in the front of the gasoline price network. The biggest strength node is *rdddr* which appeared on January 30, 1985, and it is the node on 17th in the network. There are nine nodes with the strength of 1 in 99 nodes in the front of the gasoline price network. At the earliest time it is *rrreR* which appeared on February 17, 1985, and it is the node on 37th in the network. Therefore, the nodes appeared at earlier times must not be the big strength nodes in the gasoline price network. From the perspective of the connections of the key nodes, the weights in Fig. 9(a) and (b) indicate that there are both very close links among the key nodes in crude oil and gasoline price networks. For the crude oil price network, the average contribution of the interconnections between key nodes to the node strength is



**Fig. 8.** (a) The node strength and the cumulative strength distribution of crude oil price network and (b) the node strength and the cumulative strength distribution of gasoline price network.



**Fig. 9.** (a) The key nodes and the connections of crude oil price network and (b) the key nodes and the connections of gasoline price network.

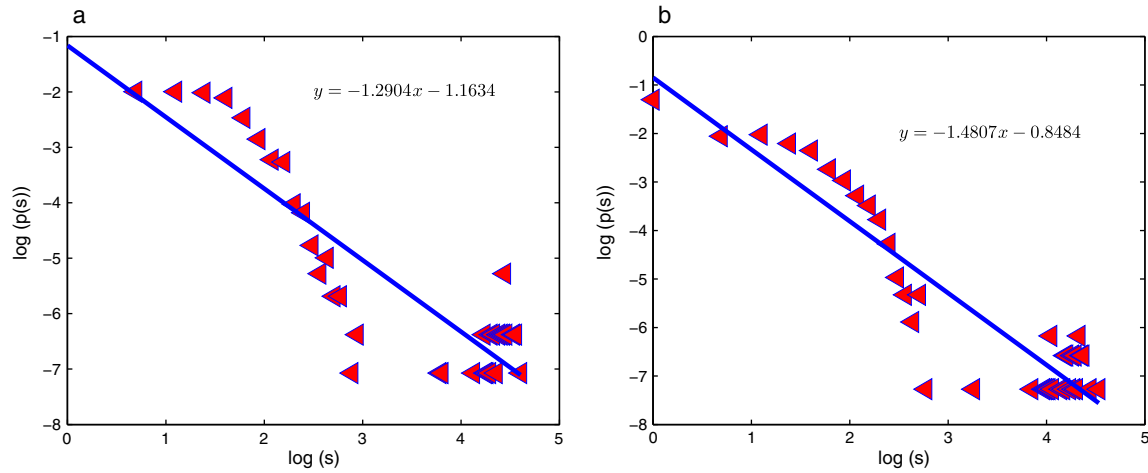
90.28%. And for the gasoline price network, the average contribution of the interconnections between key nodes to the node strength is 86.74%. Therefore, both in crude oil and gasoline price networks, the big strength nodes are inclined to connect the big strength nodes, and both the crude oil and gasoline price networks have obvious positive correlation characteristics. Based on the above analysis, we can see that, although the crude oil and gasoline price fluctuation states convert frequently and are complex, its core fluctuations states are in the top 3% of nodes. We can use the top 3% nodes to reflect the status and the conversion relationship between the fluctuation states and attempt approximate descriptions of the essential characteristics of crude oil, gasoline price fluctuations.

The double logarithmic plot of the distribution of the node strength of crude oil and gasoline price networks is shown in Fig. 10(a) and (b).

We calculate the fitting function of the strength distribution of crude oil and gasoline price networks. Fig. 10(a) and (b) indicates that the regression equation are  $y = -1.2904x - 1.1634$  and  $y = -1.4807x - 0.8484$ , respectively. The values of  $R^2$  are 0.8102 and 0.8629, thus the credibility of the results are both high, which means the strength distribution of crude oil and gasoline price networks are both follow power-law distribution. Therefore, the crude oil and gasoline price networks are both scale-free networks, whereas a few nodes have major connections, but most nodes only have a few. And the values of  $\gamma$  for crude oil and gasoline price net-

works are 1.2904 and 1.4807 respectively. What effect does data length has on the results? To address this question, we choose data lengths are 2000, 4000 and 6000 respectively, and calculate the fitting functions of the strength distribution of crude oil and gasoline price networks. The regression equations of crude oil price networks are  $y = -1.9074x - 0.8876$ ,  $y = -1.8088x - 0.4702$  and  $y = -1.4658x - 0.8853$ , the values of  $R^2$  are 0.8311, 0.8952 and 0.8740, respectively, which means the strength distribution of crude oil networks in these cases are all follow power-law distribution, and the values of  $\gamma$  are 1.9074, 1.8088 and 1.4658, respectively. The regression equations of gasoline price networks are  $y = 2.4048x - 0.1606$ ,  $y = -2.2215x - 0.0981$  and  $y = -1.5685x - 0.7977$ , the values of  $R^2$  are 0.9603, 0.9035 and 0.8682, respectively, which means the strength distribution of gasoline networks in these cases are also follow power-law distribution, and the values of  $\gamma$  are 2.4048, 2.2215 and 1.5685 respectively. As we know, in the scale-free network, the higher the value of  $\gamma$ , the better the power-law. Therefore, the gasoline price network has more heterogeneity than the crude oil price network.

From the perspective of different periods, the double logarithmic plots of the distribution of the node strength of the crude oil price network in five different periods are obtained as shown in Fig. 11(a). Meanwhile, the double logarithmic plots of the distribution of the node strength of gasoline price network in four different periods are also obtained as shown in Fig. 11(b).



**Fig. 10.** (a) The double logarithmic plot of the distribution of the node strength of crude oil price network and (b) the double logarithmic plot of the distribution of the node strength of gasoline price network.

We calculate the fitting function of the strength distribution of crude oil and gasoline price networks in different periods as shown in Table 4.

As shown in Fig. 11 and Table 4, the five crude oil price networks and four gasoline price networks in different periods display the scale-free characteristics. Furthermore, the value of  $R^2$  is more than 0.8. Therefore, it indicates that the crude oil and gasoline price networks are generally stable scale-free networks in different periods. It means that the role the fluctuation mode plays in each price network has obvious heterogeneity. Only a few fluctuation modes have high strength and dominate the structure of networks, while other secondary fluctuation modes have low strength. However, the value of  $\gamma$  is different in different periods. From an overall perspective, for both crude oil and gasoline price networks, the values of  $\gamma$  in sharp fluctuation periods are higher than in the stable fluctuation period, which means that the crude oil and gasoline price networks have better heterogeneity in the sharp fluctuation periods than in the stable fluctuation periods. From the perspective of the sharp fluctuation periods, both for crude oil and gasoline price networks, the values of  $\gamma$  in sharp decline periods are higher than in the sharp rise period, which means that the crude oil and gasoline price networks have better heterogeneity in the sharp decline periods than in the sharp rise periods. Based on the above analysis, we obtain that both for the crude oil and gasoline price networks, the networks in the sharp decline periods have the best heterogeneity, followed by the networks in the sharp rise periods. The strength distribution of the networks in the stable periods is the lowest. These results show the complex intrinsic kinetic characteristics of the crude oil and gasoline prices.

### 3.2.2. The similarity measure between the crude oil and gasoline price networks

As is known to all, there is close relationship between crude oil prices and gasoline prices. In recent years, numerous scholars have studied the relationship between them. In this section, we will give the measurement of the correlation between crude oil prices and gasoline prices based on the perspective of the network nodes. Based on the above analysis, there are 1178 and 1439 nodes in crude oil and gasoline price networks respectively. We calculate their same nodes and the strength of the same nodes, the results of which are shown in Fig. 12(a) and (b). Fig. 12(a) shows there are 1093 nodes that are the same nodes. The red “○” represent the nodes of crude oil price network, the green “○” the nodes of gasoline price network, the sizes of the “○” the strength of the

nodes, the red symbols the nodes where the strength of the crude oil price network is higher than the gasoline price network, the green symbols represent the nodes where the strength of the gasoline price network is higher than the crude oil price network. Fig. 12(b) shows there are close relationship of the strength of the same nodes of crude oil and gasoline price network.

Based on the above results, we design the following formula to calculate the similarity between the crude oil and gasoline price networks:

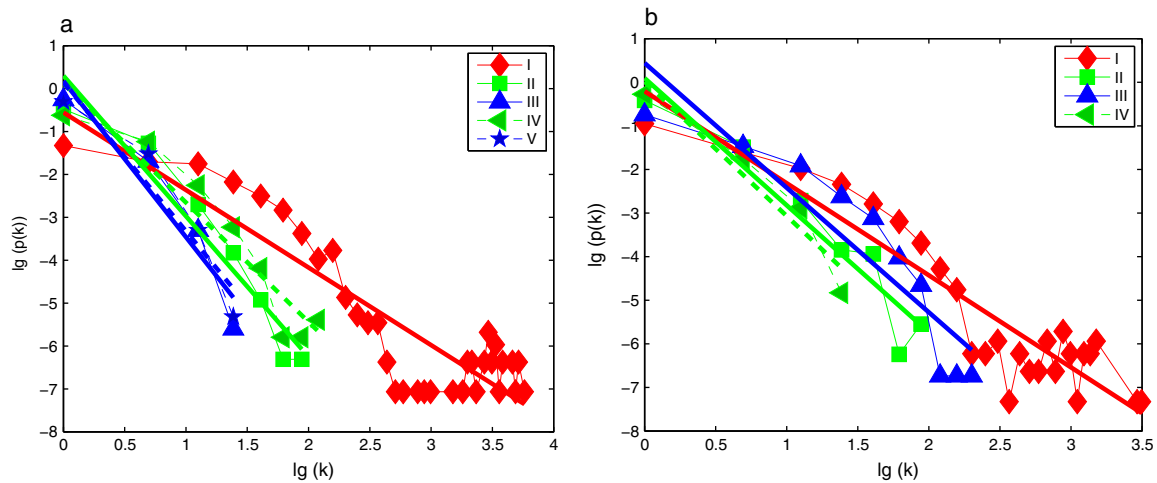
$$r = \frac{\sum_{i=1}^n (s_i^{oil} - \bar{s}^{oil})(s_i^{gasoil\ln e} - \bar{s}^{gasoil\ln e})}{\sqrt{\sum_{i=1}^n (s_i^{oil} - \bar{s}^{oil})^2 \sum_{i=1}^n (s_i^{gasoil\ln e} - \bar{s}^{gasoil\ln e})^2}} \frac{2N_{same}}{N_{oil} + N_{gasoil\ln e}} \quad (7)$$

where  $N_{same}$  denotes the number of the same nodes,  $N_{oil}$  the total of the crude oil price network nodes, and  $N_{gasoil\ln e}$  the total of the gasoline price network nodes. Obviously,  $0 \leq r \leq 1$ , the closer  $r$  is to 1, the higher the degree of the similarity is between the networks. Using formula (7), we calculate the similarity between the crude oil and gasoline price networks, which is 0.7970. The result means that the similarity between the crude oil and gasoline price networks is high, that is, there is a close relationship between crude oil prices and gasoline prices.

From the perspective of different periods, base on the results of Section 2.3, we divide the overall periods into stable fluctuation period and sharp fluctuation period, and the relationships of the nodes between the crude oil and gasoline prices in stable fluctuation periods and sharp fluctuation periods are obtained respectively. The results are shown in Fig. 13(a)–(d).

Fig. 13(a) and (b) shows that there are 1029 same nodes in the stable fluctuation periods, but only 801 same nodes in the sharp fluctuation periods (Fig. 13(c) and (d)). The network similarity in stable fluctuation periods is 0.8349, but only 0.6322 in the sharp fluctuation periods. These results indicate that there is a higher dependency between the crude oil prices and gasoline prices in the stable fluctuation periods than in the sharp fluctuation periods.

Based on the above analysis, the interdependence between the crude oil and gasoline price fluctuations can be described based on the network similarity. If we use the Pearson correlation coefficient calculation formula to calculate the correlation of the crude oil and gasoline prices in the stable and sharp fluctuation periods respectively, we can obtain that the correlation coefficient is 0.9431 in the stable fluctuation periods, and the correlation coefficient is 0.9368 in the sharp fluctuation periods. The results also show that the interdependence is reduced in the sharp fluctuation periods,



**Fig. 11.** (a) The double logarithmic plots of the distribution of the node strength of crude oil price network in five different periods and (b) the double logarithmic plots of the distribution of the node strength of gasoline price network in four different periods.

**Table 4**

The fitting function of the strength distribution of crude oil and gasoline price networks in different periods.

	Period	$\gamma$	$R^2$
Crude oil price network	I	1.8070	0.8154
	II	3.2749	0.9312
	III	3.6383	0.9096
	IV	2.7795	0.9046
	V	3.4640	0.9089
Gasoline price network	I	2.1108	0.8674
	II	2.9142	0.9126
	III	2.8582	0.8454
	IV	3.0814	0.9294

which is consistent with our result. However, our result has a higher degree of differentiation. At the same time, we use the network similarity measure. Not only we can measure the degree of the interdependence of the crude oil and gasoline price fluctuations, but we can also identify the same nodes and the quantity.

### 3.2.3. Grouping of the fluctuation modes

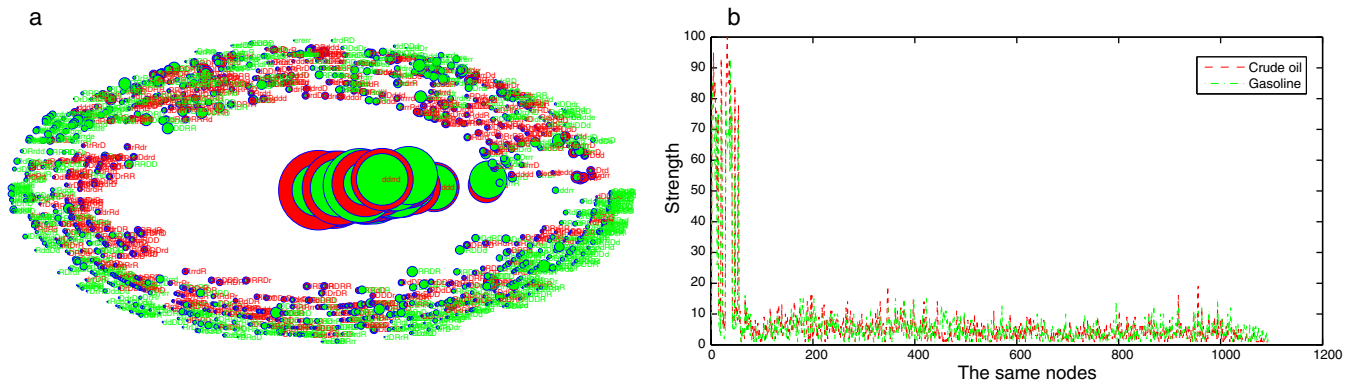
Clustering coefficient is used to describe the level of polymerization among nodes in a network. It reflects the grouping feature of the network. The higher the clustering coefficient of the node is, the greater the possibility is for the grouping of modes. The formula to calculate the clustering coefficients is shown below [17,18,25,28]:

$$C_i = \frac{1}{s_i(k_i - 1)} \sum_{j,k} \frac{\omega_{ij} + \omega_{ik}}{2} a_{ij} a_{jk} a_{ik} \quad (8)$$

where  $\omega_{ij}$  is the weight from node  $v_i$  to  $v_j$ ,  $\mu_i$  is the strength of node  $v_i$ ,  $\mu_i = \sum_j \omega_{ij}$ ,  $k_i$  the degree of node  $v_i$ ,  $\sum_{k>j} a_{ij} a_{jk} a_{ik}$  the total number of triangles contained node  $v_i$ . Obviously,  $0 \leq C \leq 1$ , when  $C = 0$ , all nodes in the network are isolated nodes. When  $C = 1$ , the network is globally coupled, that is, any two nodes are connected in the network.

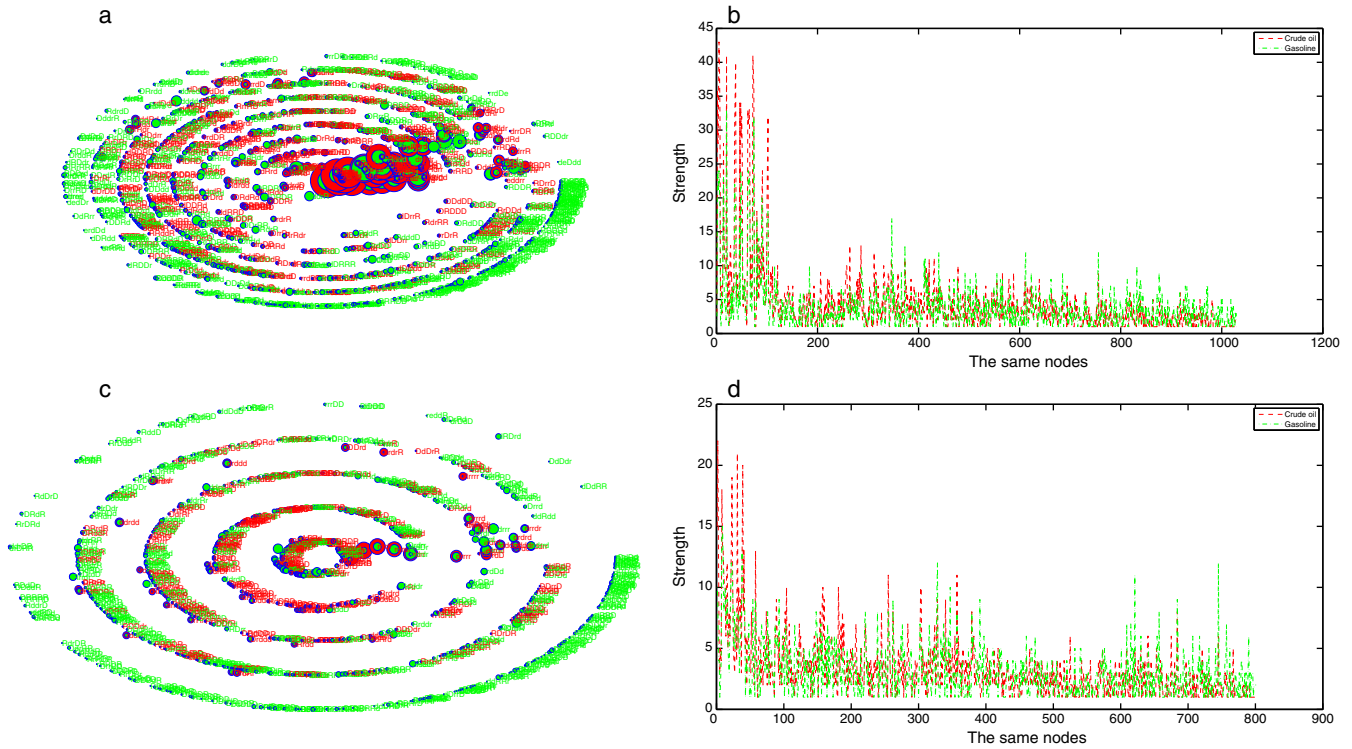
The evolution relationships between the clustering coefficient and time, the node strength of the crude oil and gasoline price network are shown in Fig. 14(a) and (b).

For the crude oil price network, Fig. 14(a) indicates that the average clustering coefficient is 0.0017. Only 9 nodes' clustering coefficients are not 0. According to the time sequence, the nine nodes are rddddd, dddddd, drrrrr, rrrrrr, edddd, Drrrr, errrr, Rrrrr, Rddddd. The corresponding clustering coefficients are 0.0941, 0.0056, 0.0529, 0.1439, 0.5, 0.25, 0.5, 0.1515, 0.25, and the nodes strength are 62, 45, 85, 44, 8, 8, 4, 11, 7 respectively. For the gasoline price network, Fig. 14(b) indicates that the average clustering coefficient is 0.001. Only 10 nodes' clustering coefficients are not 0. According to the time sequence, the ten nodes are errrr, rrrrr, drrrr, Drrrr, Rrrrr, RDDDD, Dddddd, DDDDD, DRRRR, RRRRR. The corresponding clustering coefficients are 0.2727, 0.0071, 0.088, 0.1458, 0.0741, 0.1071, 0.375, 0.1071, 0.2188, 0.0833, and the nodes strength are 11, 47, 72, 8, 9, 14, 4, 7, 8, 3 respectively. By contrast, the average clustering coefficient of the crude oil price network is slightly lar-

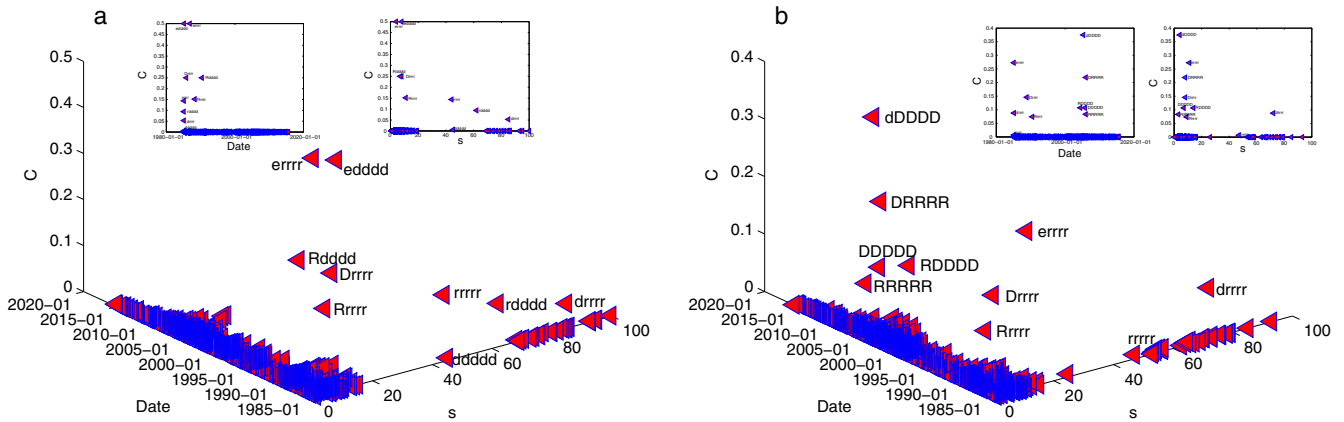


**Fig. 12.** (a) The characteristics of the same nodes of crude oil and gasoline price network and (b) the strength of the same nodes of crude oil and gasoline price network.





**Fig. 13.** (a) The characteristics of the same nodes of crude oil and gasoline price network in stable period; (b) the strength of the same nodes of crude oil and gasoline price network in stable period; (c) the characteristics of the same nodes of crude oil and gasoline price network in sharp period and (d) the strength of the same nodes of crude oil and gasoline price network in sharp period.



**Fig. 14.** (a) 3D plot of the clustering coefficient of crude oil price network (s-Date-C), the plot of C versus Date (left subplot), the plot of C versus s (right subplot) and (b) 3D plot of the clustering coefficient of gasoline price network, the plot of C versus Date (left subplot), the plot of C versus s (right subplot).

ger than the gasoline price network, which means that the crude oil price network is closer than the gasoline price network. However, both the crude oil price network and the gasoline price network exhibit the characteristics that the clustering coefficient varies significantly with the size of the change range of node strength, which means that the fluctuations in crude oil and gasoline network are not completely random, but to some extent are similar to the characteristics of social network, “birds of a feather flock together” [29]. But they are not like social network in that they don’t have small nodes with a high clustering coefficient, or strong nodes with low clustering coefficient. That is to say, noticeable volatility clustering network characteristics of crude oil and gasoline may occur in a small group and also in a large group. In

time the clustered price fluctuations of crude oil and gasoline are sometimes reflected on the small time scale and sometimes also on the large time scale. Studying the network clustering coefficients of the fluctuations in crude oil and gasoline can provide reference and help for our future studies of clustering volatility of crude oil and gasoline prices.

From the perspective of different periods, the clustering coefficients of the crude oil price and gasoline price networks in different periods are calculated, the evolution images of which are shown in Fig. 15(a)–(d).

In the stable periods, for the crude oil price network, Fig. 15(a) indicates that the average clustering coefficient is 0.00132. Only 7 nodes’ clustering coefficients are not 0. According to the time

sequence, the 7 nodes are *drrrr*, *rddddd*, *rrrrr*, *errrr*, *ddddd*, *rRRRR*, *RRRRR*. The corresponding clustering coefficients are 0.06, 0.0606, 0.0303, 0.3333, 0.05, 0.5, 0.25, and the nodes strength are 25, 11, 11, 3, 5, 2, 2, respectively. For the gasoline price network, Fig. 15(b) indicates that the average clustering coefficient is 0.0013. Only 9 nodes' clustering coefficients are not 0. According to the time sequence, the 9 nodes are *Rdddd*, *rddddd*, *drrrr*, *Rrrrr*, *rrrrr*, *ddddd*, *Dddddd*, *errrr*, *Drrrr*. The corresponding clustering coefficients are 0.1250, 0.0865, 0.1444, 0.1875, 0.1071, 0.1667, 0.3750, 0.1667, 0.2500, and the nodes strength are 4, 13, 15, 4, 14, 9, 4, 3, 3 respectively.

In the sharp period, for the crude oil price network, Fig. 15(c) indicates that the average clustering coefficient is 0.00038. Only 4 nodes' clustering coefficients are not 0. According to the time sequence, the 4 nodes are *rrrrr*, *Rrrrr*, *Drrrr*, *rddddd*. The corresponding clustering coefficients are 0.0385, 0.1167, 0.125, 0.0741, and the nodes strength are 13, 10, 4, 9 respectively. For gasoline price network, Fig. 15(d) indicates that the average clustering coefficient is 0.0012. Only 5 nodes' clustering coefficients are not 0. According to the time sequence, the 5 nodes are *drrrr*, *rrrrr*, *RDDDD*, *DDDDD*, *Drrrr*. The corresponding clustering coefficients are 0.175, 0.0278, 0.3333, 0.5, 0.3, and the nodes strength are 10, 9, 3, 2, 5 respectively.

By contrast, for both the crude oil price network and the gasoline price network, the number of nodes where cluster coefficient is not zero in the sharp periods is less than that in the stable periods, and the average cluster coefficient in the sharp periods is less than that in the stable period. The above results indicate that the crude oil and gasoline price networks are closer in the stable periods. They are more complexity in the sharp periods.

### 3.2.4. Analysis of conversion cycle of the modes

Let  $d_{ij}$  be the distance between the nodes  $v_i$  and  $v_j$ . The diameter  $D$  of the network is defined as the maximum distance between all the nodes, that is

$$D = \max d_{ij} \quad (9)$$

The average path length  $L$  is defined as the average of the distance between any two nodes, the computation formula is as follows [17,28]:

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij} \quad (10)$$

where  $N$  is the total number of network nodes.

Based on the Floyd algorithm [30], we calculate the distances between any two nodes in the crude oil and gasoline price networks respectively, the results of which are shown in Fig. 16(a) and (b). The different path length distribution is shown in Fig. 16(c) and (d).

Fig. 16(a) and (c) indicates that the diameter of the crude oil price network is 20. The distance between the nodes is 5, 6, 7, 8, which accounts for 74.98% of the total. The average shortest path is 6.9708, which means the conversion between the fluctuation modes of crude oil price presents a short-range correlation. The conversion between the fluctuation modes is frequent, requiring about an average of 6–7. Fig. 16(b) and (d) indicates that the diameter of gasoline price network is 23. The distance between the nodes is 6, 7, 8, 9, which accounts for 66.25% of the total. The average shortest path is 7.6902, which means the conversion between the fluctuation modes of gasoline price also presents a short-range correlation. The conversion between the fluctuation modes is frequent, requiring about an average of 7–8. By contrast, the diameter of the crude oil price network is less than that of the gasoline price network, and the conversion cycle between the fluctuation modes of crude oil prices is less than that of gasoline prices. These prop-

erties provide the basis for the prediction of crude oil and gasoline price fluctuations.

From the perspective of different periods, the shortest path length distribution of crude oil and gasoline price networks in the stable fluctuation periods and sharp fluctuation periods are calculated respectively, as shown in Fig. 17(a) and (b).

Fig. 17(a) indicates that in the stable fluctuation periods, the diameter of crude oil price network is 25, and the distance between the nodes is 7, 8, 9, 10, 11, which accounts for 65.91% of the total. The average shortest path is 9.1520, which means the conversion between the fluctuation modes requires an average of 9–10. However, in the sharp fluctuation periods, the diameter of crude oil price network is 19, and the distance between the nodes is 6, 7, 8, 9, which accounts for 67.09% of the total. The average shortest path is 7.8717, which means the conversion between the fluctuation modes requires an average of 7–8. Therefore, the diameter of crude oil price network in the sharp fluctuation periods is less than in the stable fluctuation periods, and the conversion cycle between the fluctuation modes in the sharp fluctuation periods is less than in the stable fluctuation periods.

Fig. 17(b) indicates that in the stable fluctuation period, the diameter of gasoline price network is 33, and the distance between the nodes is 7, 8, 9, 10, 11, which accounts for 67.6% of the total. The average shortest path is 10.2408, which means the conversion between the fluctuation modes requires an average of 10–11. However, in the sharp fluctuation periods, the diameter of crude oil price network is 28, and the distance between the nodes is 6, 7, 8, 9, which accounts for 72.6% of the total. The average shortest path is 8.3715, which means the conversion between the fluctuation modes requires an average of 8–9. Therefore, the diameter of the gasoline price network in the sharp fluctuation periods is less than in the stable fluctuation periods, and the conversion cycle between the fluctuation modes in the sharp fluctuation periods is less than in the stable fluctuation periods.

### 3.2.5. Analysis of the transition between the fluctuation modes

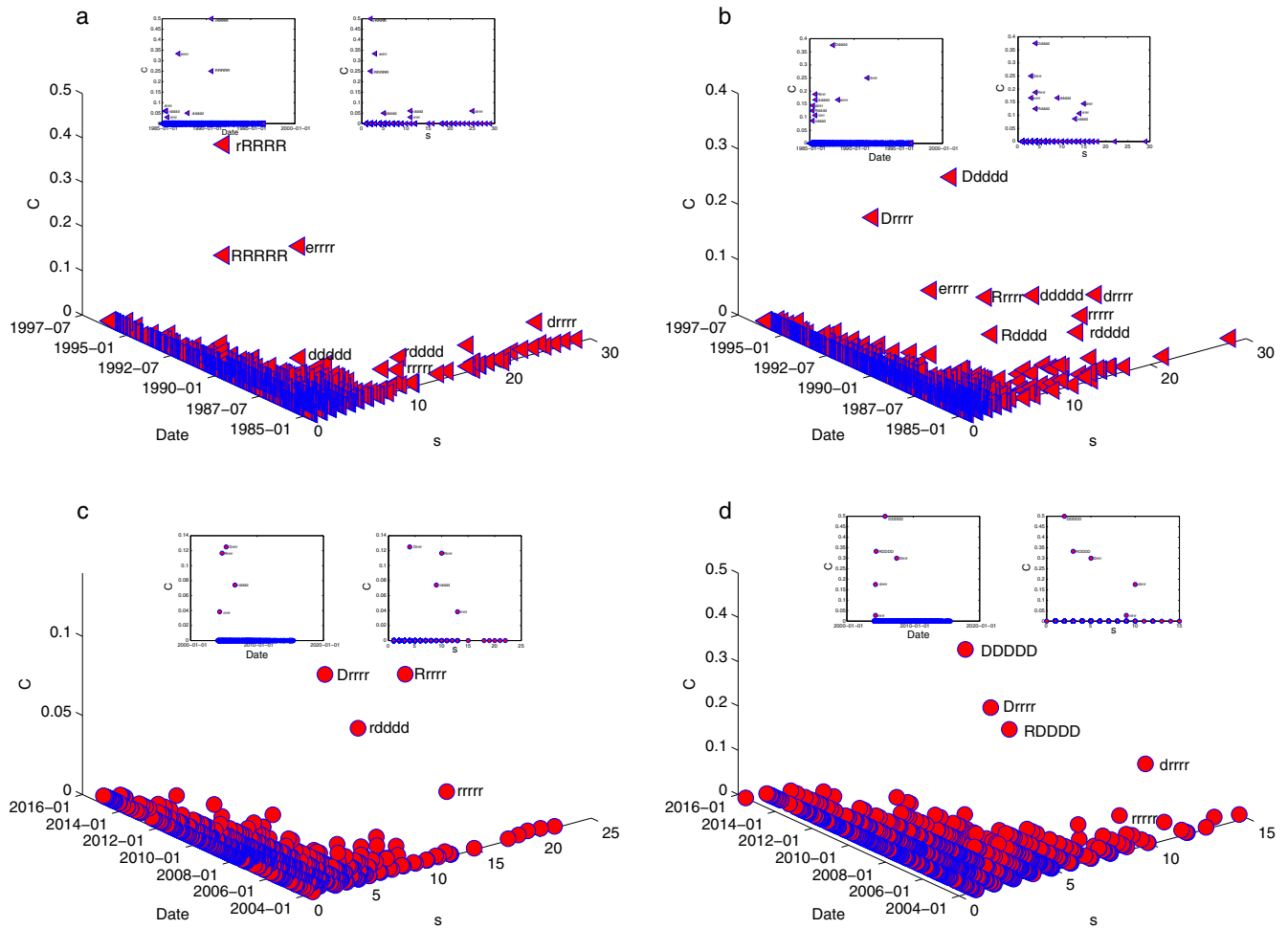
Betweenness centrality is often used to study the hub level of each node in the network and can provide a better understanding of the intermediate process of network evolution. The more such positions a network node occupies, the higher the level of betweenness centrality it possesses [28]. To further uncover the transformational form of the fluctuation modes, we introduce betweenness centrality to analyze the network. If a mode stands in the shortest path between two modes, it plays a medium role in the fluctuation process. The node betweenness is defined as  $B_i$

$$B_i = \sum_{j \neq l \neq i} [N_{jl}(i)/N_{jl}] \quad (11)$$

where  $N_{jl}$  is the total number of the shortest path of node  $v_j$  to  $v_l$ ,  $N_{jl}(i)$  the total number of the shortest path of node  $v_j$  to  $v_l$  and passing node  $v_i$ .

The evolution relationship of the node betweenness and strength of crude oil and gasoline price network is shown in Fig. 18(a) and (b).

For the crude oil price network, Fig. 18(a) indicates that the nodes with the node betweenness in the top three are *RdRdr*, *ddDDr*, *DDRRr*, and their betweenness numbers are 0.0357, 0.0342, 0.0333, with strengths of 6, 7, 5. The first time they appeared are December 3, 1999, April 5, 1990, August 17, 2006 respectively. For the gasoline price network, Fig. 18(b) indicates that the nodes with the node betweenness in the top three are *dDdrD*, *rdDdr*, *RrrDR*, and their betweenness numbers are 0.0435, 0.0359, 0.0317, with strengths of 4, 7, 4. The first time they appeared are February 11, 1986, June 29, 1989, September 24, 2003 respectively.



**Fig. 15.** (a) 3D plot of the clustering coefficient of crude oil price network (s-Date-C) in stable period, the plot of C versus Date (left subplot), the plot of C versus s (right subplot); (b) 3D plot of the clustering coefficient of gasoline price network (s-Date-C) in stable period, the plot of C versus Date (left subplot), the plot of C versus s (right subplot); (c) 3D plot of the clustering coefficient of crude oil price network (s-Date-C) in sharp period, the plot of C versus Date (left subplot), the plot of C versus s (right subplot) and (d) 3D plot of the clustering coefficient of gasoline price network (s-Date-C) in sharp period, the plot of C versus Date (left subplot), the plot of C versus s (right subplot).

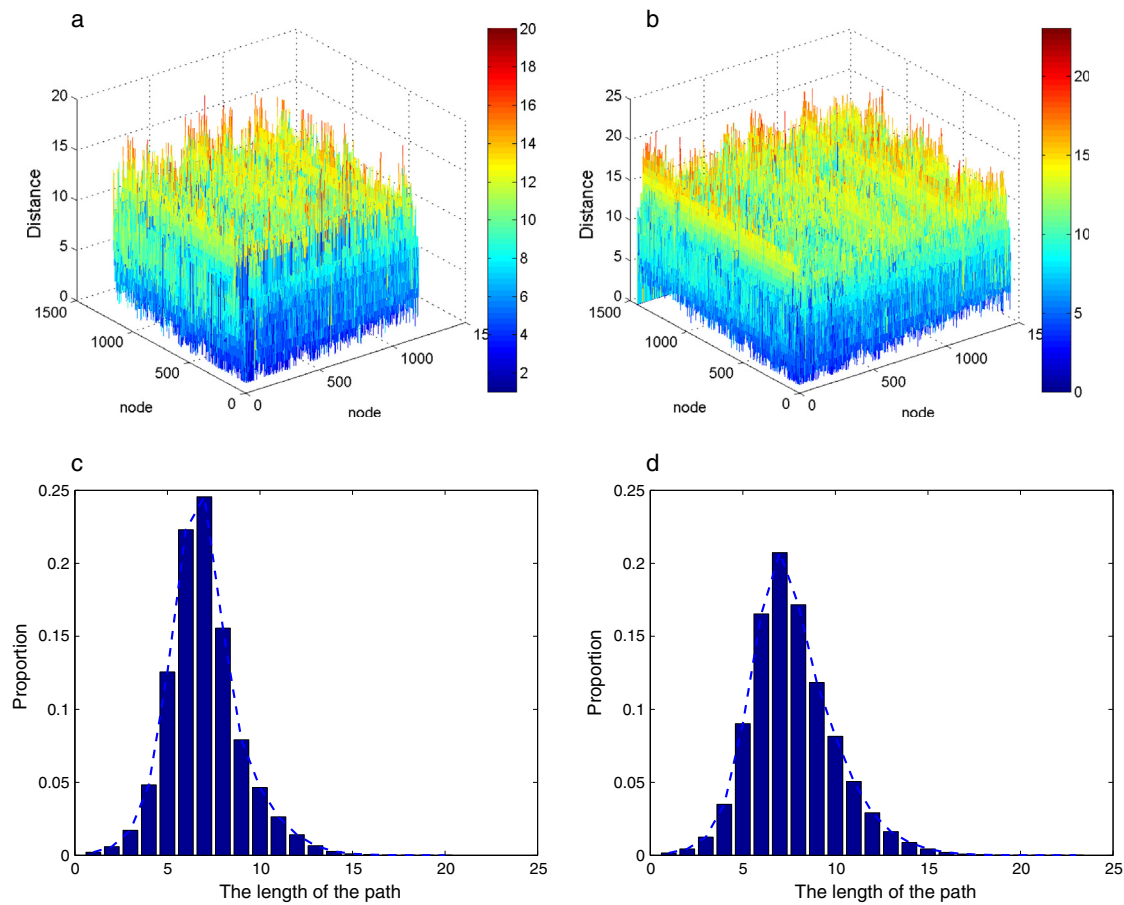
From the perspective of different periods, the evolution relationship of the node betweenness and strength of crude oil and gasoline price networks in different periods are shown in Fig. 19 (a)–(d).

In the stable fluctuation period, for the crude oil price network, Fig. 19(a) indicates that the nodes with betweenness in the top three are  $dRrrr$ ,  $rDrdd$ ,  $dRdr$ , and their betweenness numbers are 0.0520, 0.0502, 0.0476, the strength of them are 5, 4, 5 respectively. For the gasoline price network, Fig. 19(b) indicates that the nodes with betweenness in the top three are  $DrRdd$ ,  $ddrdd$ ,  $rRddr$ , and their betweenness numbers are 0.0511, 0.0505, 0.0468, with strengths of 5, 6, 7 respectively. In the sharp fluctuation periods, for the crude oil price network, Fig. 19(c) indicates that the nodes with betweenness in the top three are  $dRdDR$ ,  $RdrDd$ ,  $DDRdr$ , and their betweenness numbers are 0.0494, 0.0481, 0.0466, the strength of them are all 4, respectively. For the gasoline price network, Fig. 19(d) indicates that the nodes with betweenness in the top three are:  $DDdr$ ,  $rdrDr$ ,  $ddRrD$ , and their betweenness numbers are 0.0480, 0.0440, 0.0418, with strengths of 4, 5, 3, respectively. Based on the above analysis, the nodes with small strength have the main intermediary function, whereas the large strong nodes transit through some small strong nodes. When the nodes with high betweenness appear, which means the period in a transitional period, identifying these nodes can effectively predict the state of oil price fluctuations in the next period.

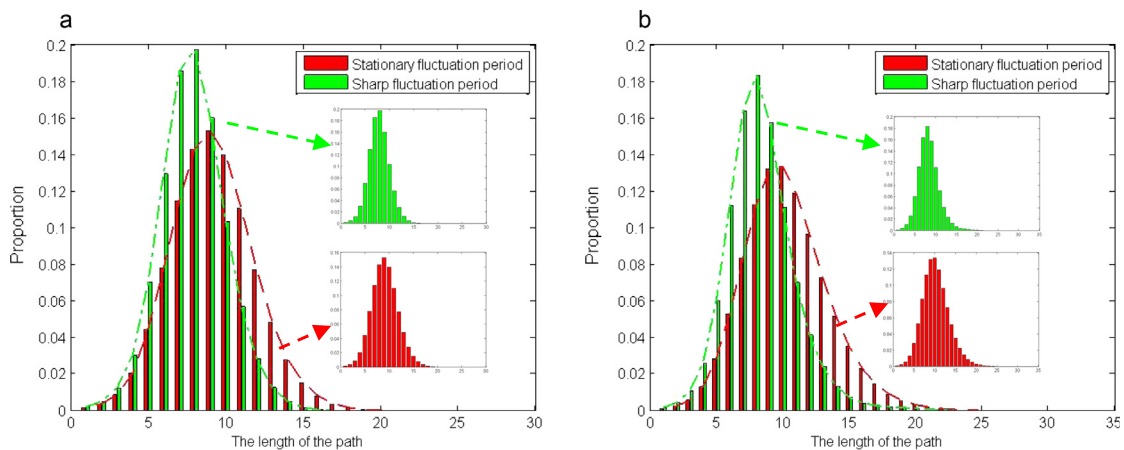
From the results discussed above, to further uncover the time distribution of the core nodes, we choose the core nodes which have the large nodes strength or the large clustering coefficients or the large nodes betweenness, as shown in Fig. 20(a) and (c). The distribution of time when the core nodes first appear is shown in Fig. 20(b) and (d).

Fig. 20(a) and (c) indicates that for both crude oil and gasoline network, when nodes strength are high, the clustering coefficients and nodes betweenness are both small. When the clustering coefficients are big, nodes strength and nodes betweenness are both small. When nodes betweenness are large, nodes strength and the clustering coefficients are both small. Synthesize the above analysis and we find that crude oil and gasoline price networks have small clustering coefficients, small the average path lengths, and small node betweenness. This is different from random networks and chaotic networks.

From the perspective of the distribution of time when the core nodes first appear in crude oil and gasoline price networks, Fig. 20 (b) and (d) indicates that the nodes with large strength (see ‘○’) first appear at an earlier time, which means the nodes with large strength in crude oil and gasoline price networks are nodes that appear at an earlier time. The nodes with large clustering coefficients (see ‘◇’) first appear in a relatively scattered way, but still at an earlier time. The nodes with large betweenness (see ‘△’) first appear in the most decentralized time and may occur at any time.



**Fig. 16.** (a) The distances between any two nodes in crude oil price network; (b) the distances between any two nodes in gasoline price network; (c) the different path length distribution of crude oil price network and (d) the different path length distribution of gasoline price network.



**Fig. 17.** (a) The shortest path length distribution of crude oil price network in different periods and (b) the shortest path length distribution of gasoline price network in different periods.

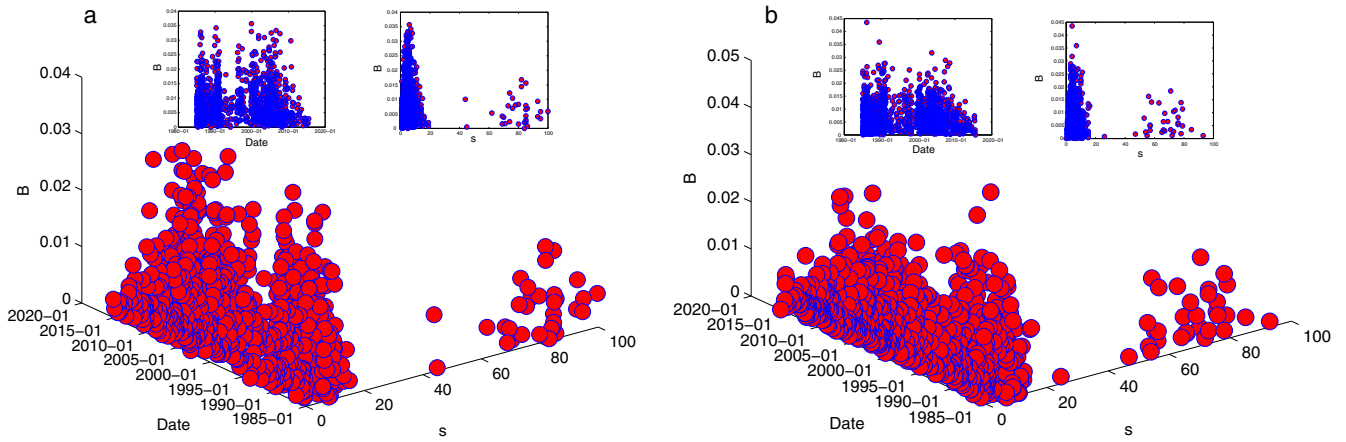
Once they appear, it means that the crude oil and gasoline prices fluctuate in a transitional stage. Carrying out in-depth research on them will help to better grasp the regularity of oil price changes.

#### 4. Conclusions

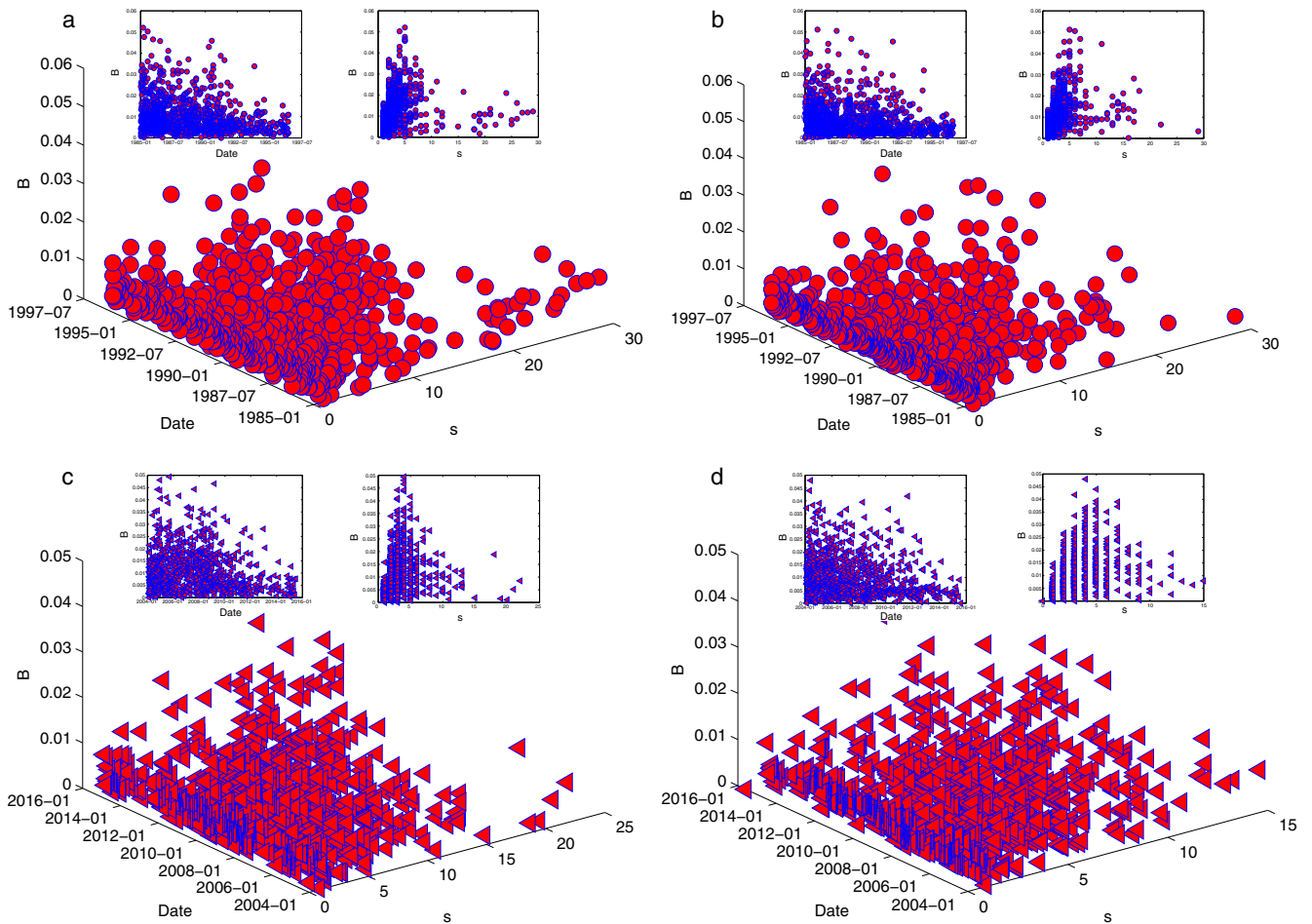
In this paper, we have defined the fluctuation modes by coarse-graining procedure, using the international crude oil and gasoline prices from January 7, 1985 to August 11, 2015 as sample data.

We have converted the crude oil and gasoline price volatility sequences into the characters composed by five symbols  $\{R, r, e, d, D\}$ , 5 days for a model, 1 day for sliding step, and built the sliding window. The periods of the sample data are divided into stable fluctuation periods and sharp fluctuation periods according to the different fluctuation states. In different periods, let the modes be nodes according to the time sequence, let the transformation between the modal be the edge, and let the times of the transformation be weigh. The directed weighted networks of international





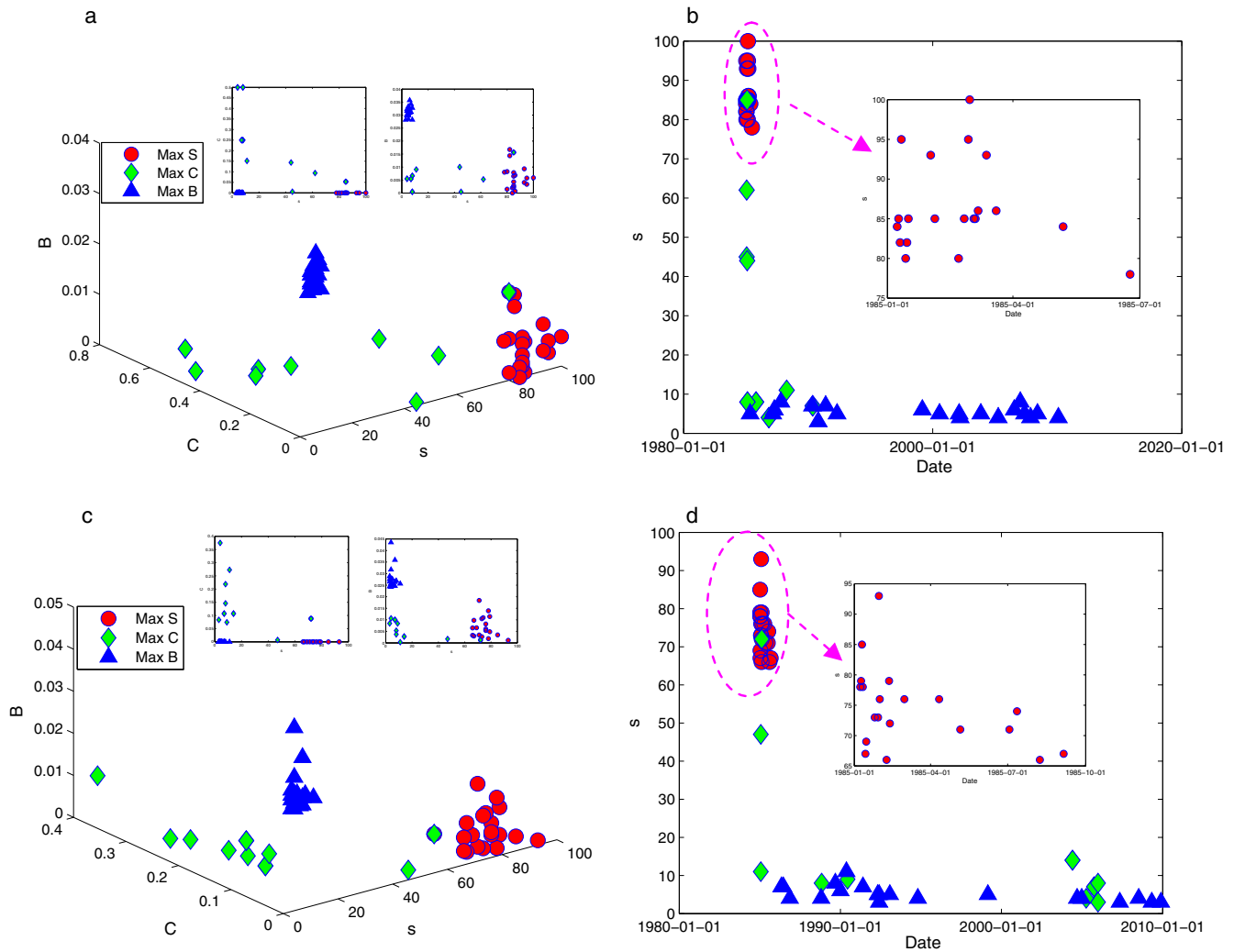
**Fig. 18.** (a) 3D plot of the betweenness of crude oil price network (s-Date-B), the plot of B versus Date (left subplot), the plot of B versus s (right subplot) and (b) 3D plot of the betweenness of gasoline price network, the plot of B versus Date (left subplot), the plot of B versus s (right subplot).



**Fig. 19.** (a) 3D plot of the betweenness of crude oil price network (s-Date-B) in stable period, the plot of B versus Date (left subplot), the plot of B versus s (right subplot); (b) 3D plot of the betweenness of gasoline price network (s-Date-B) in stable period, the plot of B versus Date (left subplot), the plot of B versus s (right subplot); (c) 3D plot of the betweenness of crude oil price network (s-Date-B) in sharp period, the plot of B versus Date (left subplot), the plot of B versus s (right subplot) and (d) 3D plot of the betweenness of gasoline price network (s-Date-B) in sharp period, the plot of B versus Date (left subplot), the plot of B versus s (right subplot).

crude oil and gasoline prices are built in the different fluctuation periods. And then the evolution law of the new nodes in the prices networks is analyzed. The point of strength, strength distribution, weighted clustering coefficient, average shortest paths of the price

network are calculated in different periods. The important nodes and the time they appeared are identified. The calculation formula of the similarity between the crude oil and gasoline price network is given based on the node strength, and the similarity between the



**Fig. 20.** (a) The core nodes of crude oil price network (s-C-B), the plot of C versus s (left subplot), the plot of B versus s (right subplot); (b) the time distribution of the core nodes first appeared of crude oil price network; (c) the core nodes of gasoline price network (s-C-B), the plot of C versus s (left subplot), the plot of B versus s (right subplot) and (d) the time distribution of the core nodes first appeared of gasoline price network.

crude oil and gasoline price network is discussed. The results are as follows:

- (1) Although crude oil and gasoline price fluctuations have complex nonlinearity, the cumulative times when the new nodes appear in the crude oil and gasoline prices networks are not nonlinear but exhibit a high linear growth trend. In different periods, the cumulative time when the new nodes appear in the crude oil and gasoline price networks are different. For the crude oil price network, the cumulative time when the new nodes appear in stable fluctuation periods is longer than the sharp fluctuation periods. However, for gasoline price network it is contrary.
- (2) For the crude oil price network, the average contribution of the interconnections between key nodes to the node strength is 90.28%. And for the gasoline price network the average contribution of the interconnections between key nodes to the node strength is 86.74%. Therefore, in both crude oil and gasoline price networks, the big strength nodes are inclined to connect the big strength nodes, both the crude oil and gasoline price networks have obvious positive correlation characteristics.
- (3) The crude oil and gasoline price networks are both scale-free networks, and the gasoline price network has more heterogeneity than the crude oil price network. The value of  $\gamma$  is different in different periods. For both the crude oil and gasoline price networks, the networks in the sharp decline periods have the best heterogeneity, followed by the networks in the sharp rise periods. The strength distribution of the networks in the stable periods is the lowest.
- (4) The similarity between the crude oil and gasoline price network in the overall sample period is 0.7970. However, the network similarity in stable fluctuation periods is 0.8349, but only 0.6322 in the sharp fluctuation periods. These results indicate that there is higher interdependence between the crude oil price and gasoline price in the stable fluctuation periods than in the sharp fluctuation periods.
- (5) The average clustering coefficient of the crude oil price network is 0.0017, and that of the gasoline price network is 0.001. The crude oil price network is closer than the gasoline price network. However, both the crude oil price network and the gasoline price network exhibit the characteristics that the clustering coefficient varies significantly with the size of the change range of node strength. For both the crude

oil price network and the gasoline price network, the average cluster coefficient in sharp periods is less than the stable periods. The above results indicate that the crude oil and gasoline price networks are closer in the stable periods, and more complex in the sharp periods.

- (6) The diameter of the crude oil price network is less than that of the gasoline price network, and the conversion cycle between the fluctuation modes of crude oil price is less than gasoline price. However, the conversion between the fluctuation modes of crude oil and gasoline price both present a short-range correlation. The conversion between the fluctuation modes requires an average of 6–7 and 7–8, respectively. For both the crude oil and gasoline price networks, the network diameter in the sharp fluctuation periods is less than that in the stable fluctuation periods, and the conversion cycle between the fluctuation modes in the sharp fluctuation periods is less than those in the stable fluctuation periods.
- (7) The nodes with small strength have the main intermediary function and large strength nodes transit through some small strong nodes. For both the crude oil and gasoline price networks, the nodes betweenness in the sharp fluctuation periods is less than that in the stable fluctuation periods. Therefore, in the sharp fluctuation periods, the ability of mediation is on the decline.

Research of this paper is based on the analysis of the collected crude oil, gasoline prices data, and the results that we have obtained are based on the data sample. The characteristics of crude oil and gasoline prices volatility that we have obtained are based on complex network theory are completely new. These results can help us to understand the essential characteristics of energy price fluctuations, and lay a foundation for us to study the stability of the oil price fluctuations and to build new oil price prediction system.

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