2018130050
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classmate
Date
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Tutorial-4

Q1. mixing statistically independent sources.

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$$var(x) = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$= \langle (\sum_i w_i s_i)^2 \rangle - \langle \sum_i w_i s_i \rangle^2$$

$$= \langle (\sum_i w_i s_i)^2 \rangle - \langle \sum_i w_i \langle s_i \rangle)^2$$

$$= \langle (\sum_i w_i s_i)(\sum_j w_j s_j) \rangle - (\sum_i w_i \langle s_i \rangle)(\sum_j w_j \langle s_j \rangle)$$

$$= \langle \sum_{i,j} w_i w_j s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) +$$

$$\sum_{i,j : i \neq j} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle)$$

$$= \sum_i w_i^2 (\langle s_i s_i \rangle - \langle s_i \rangle^2) + \sum_{i,j : i \neq j} w_i w_j (\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle)$$

s_i & s_j are statistically independent for $i \neq j \Rightarrow \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0$

Also, $var(s_i) = 1$

$$\therefore var(x) = \sum_i w_i^2$$

To guarantee unit variance,

$$var(x) = 1$$

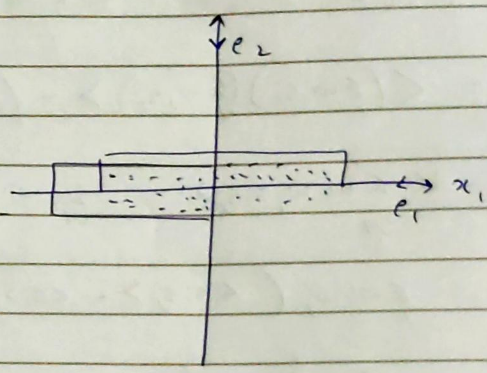
$\therefore \sum_i w_i^2 = 1$

$\therefore$ Following constraint has to be imposed on w_i for mixture to have var as 1.

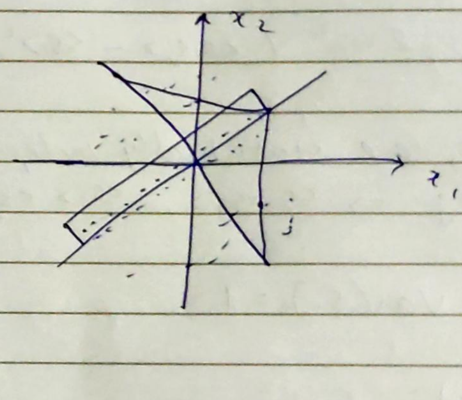
$$\boxed{\sum_i w_i^2 = 1}$$

Q2.

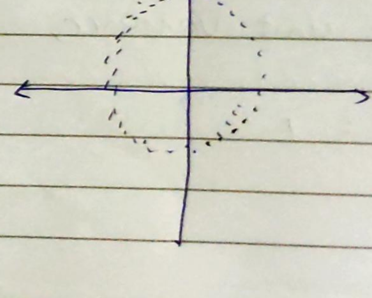
(a)



(b)



(c)



$\Rightarrow$ cant seperate into independent comp.