

Tutorial-5

Hypothesis testing

Q1. $H_0 : P = 0.7$
 $H_1 : P \neq 0.7$
 $\alpha = 0.10$

Test stat : Binomial var X with $p = 0.7$ & $N = 15$

$$X = 8 \quad \& \quad nP_0 = 15 \times 0.7 = 10.5$$

$$P = 2P(X \leq 8 \quad \& \quad p = 0.7)$$

$$= 2 \sum_{x=0}^8 b(x; 15, 0.7)$$

$$= 2 \times 0.1311$$

$$= 0.2622$$

$$P > 0.10$$

$$\therefore P > \alpha$$

Do not reject H_0 .

\therefore There is ~~is~~ not enough reason to doubt the builders claim.

Q2. $H_0 : p = 0.8$

$H_1 : p \geq 0.6$

$\alpha = 0.05$

Given, $x = 70, n = 100, p = 0.6$

$$Z = \frac{x - np_0}{\sqrt{np_0q_0}}$$

$Z = 2.04$

$P = P(Z > 2.04)$

$P = 0.0207$

$P < \alpha$

\therefore New drug is superior.

Q3. $P_1 \rightarrow$ proportion of Mumbai voters

$P_2 \rightarrow$ " " surrounding residents

$P_1 = \frac{120}{200} = 0.6$

$P_2 = \frac{240}{500} = 0.48$

$P_p = \frac{120 + 240}{200 + 500} = 0.514$

$\alpha = 0.05$

$H_0 : P_1 \leq P_2$

$H_1 : P_1 > P_2$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_p(1-\hat{P}_p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$z = 2.869$$

$$P = P(z > 2.869) = 0.0044$$

$P < \alpha \rightarrow \text{Reject } H_0$

ie

Proportion of Mumbai favouring proposal is higher.

Q4 (a) $H_0: p = 0.20$
 $H_1: p > 0.20$

Critical region is right tail.

(b) $H_0: \mu = 3$
 $H_1: \mu \neq 3$

Critical region is both tails.

(c) $H_0: p = 0.15$
 $H_1: p < 0.15$

Critical region is left tail.

(d) $H_0: \mu = 500$
 $H_1: \mu > 500$

Critical region is in right tail.

(e) $H_0: \mu = 15$
 $H_1: \mu \neq 15$

Critical region is in both tails.

Q5. $\mu_1 \rightarrow$ Company A
 $\mu_2 \rightarrow$ Company B

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

$$\alpha = 0.05$$

$$\bar{x}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{x}_1 = 7.95$$

||y

$$\bar{x}_2 = 10.26$$

$$S_1^2 = \frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - n_1 \bar{x}_1^2 \right]$$

$$= \frac{10.865}{9} = 1.207$$

||y $S_2^2 = 0.325$

\therefore Sample variances are different.

Cannot assume that popⁿ variances are equal.

\therefore we use unpooled t-test

Deg of freedom are,

$$V = \frac{\left(\frac{S_1^2}{n_1} \right) + \left(\frac{S_2^2}{n_2} \right)}{\frac{1}{n-1} \left(\frac{S_1^2}{n_1} \right)^2 + \frac{1}{n_2-1} \left(\frac{S_2^2}{n_2} \right)^2}$$

$$V \leq 10.3$$

$$|V| \leq 10$$

Now,

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Also, under null hypothesis $\mu_1 - \mu_2 = 0$.

$$\therefore T = \frac{7.95 - 10.26}{\sqrt{\frac{1.807}{10} + \frac{0.325}{10}}} = -5.90$$

$$|t| = 5.90.$$

Now,

$$p = 2 \cdot P(T \geq |t|) = 2 \cdot P(T \geq 5.90)$$

$$p\text{-val} < 0.001$$

ie $p < \alpha$ \therefore we reject null hypothesis.