SHINDE SHUBHAM SUNIL
email: `me18b183@smail.iitm.ac.in`
Course: BT6270 - Computational Neuroscience
Instructor: Prof. V. Srinivasa Chakravarthy

ASSIGNMENT #2
Roll No.: ME18B183
Semester: JUL - NOV 2021
Due Date: 12^{th} October, 2021

Assignment Description:
Simulate the two variable **FitzHugh-Nagumo** neuron model using the following equations:

$$\frac{dv}{dt} = f(v) - w + I_{ext}, \qquad \frac{dw}{dt} = bv - rw; \qquad where, f(v) = v(a-v)(v-1)$$

The parameters can be initialised as $a = 0.5, b = 0.1$ and $r = 0.1$. To solve the above differential equations *Euler Integration method* can be used.

Case 1: I_{ext}=0 μA

(a) Draw a Phase Plot.

(b) Plot v(t) vs t, w(t) vs t and also show the trajectory on the phase plane for the both sub-cases:

(i) v(0) < a and w(0)=0

(ii) v(0) > a and w(0)=0

Case 2: Choose some current value between I and I_2 where it exhibits oscillations. Find the values if I_1 and I_2.

(a) Draw a Phase Plot for some sample value of I_{ext}.

(b) Show that the fixed point is unstable i.e., for a small perturbation there is no return to the fixed point (show the trajectory on the phase plane) – also show limit cycle on the phase plane.

(c) Plot v(t) vs t and w(t) vs t.

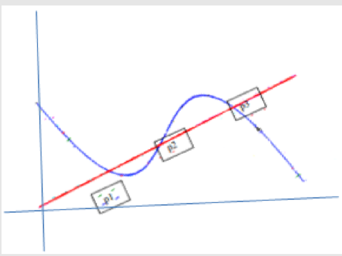
Case 3: Choose some $I_{ext} > I_2$.

(a) Draw a Phase Plot for some sample value of I_{ext}.

(b) Show that the fixed point is stable i.e., for a small perturbation there is a return to the fixed point (show the trajectory on the phase plane).

(c) Plot v(t) vs t and w(t) vs t.

Case 4: Find suitable values of I_{ext} and (b/r) such that the graph looks as phase plot shown as below.



(a) Redraw the Phase plot.

(b) Show stability of P1, P2, P3.

(c) Plot v(t) vs t and w(t) vs t.

Solution:

Case 1: I_{ext}=0 μA **[Excitability]**

(a) The phase plot for this case:

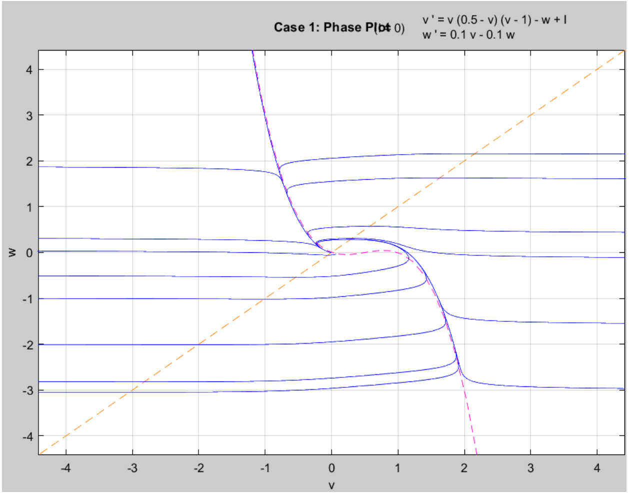
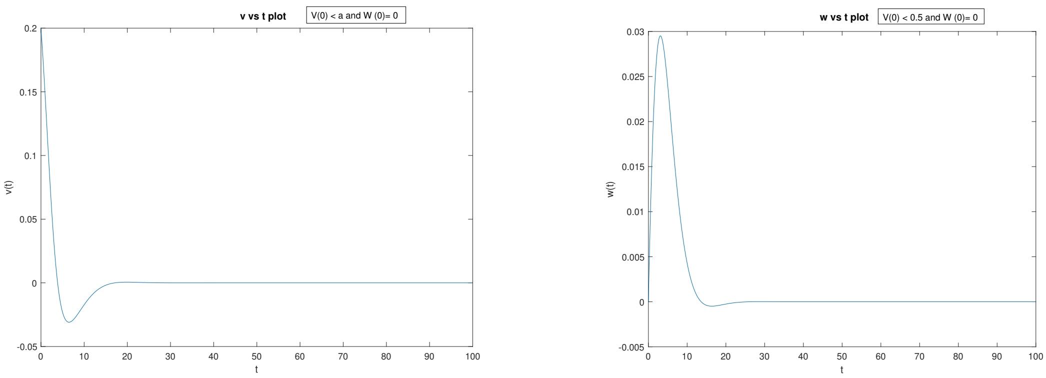


Figure 1: Phase plot for I_{ext}=0 μA

(b) Plots of v(t) vs t and w(t) vs t. Also the trajectory on the phase plane for the both sub-cases:

(i) v(0)<a and w(0)=0



(a) (b)

Figure 2: (a) Plot of v(t) vs t; (b) Plot of w(t) vs t for v(0)=0.2 and w(0)=0

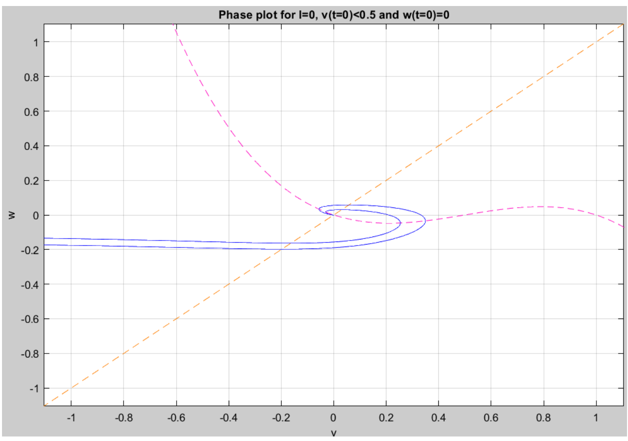
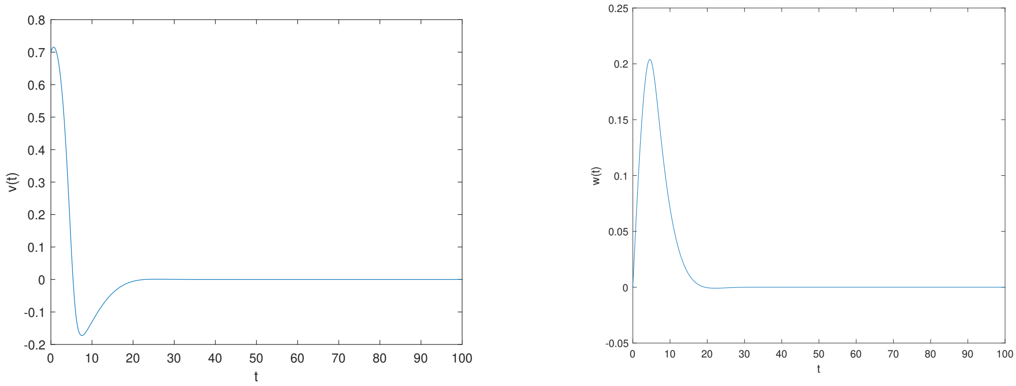


Figure 3: Phase plane for I_{ext}=0, v(0)=0.2<a and w(0)=0

(ii) v(0)>a and w(0)=0



(a) (b)

Figure 4: (a) Plot of v(t) vs t; (b) Plot of w(t) vs t for v(0)=0.7 and w(0)=0

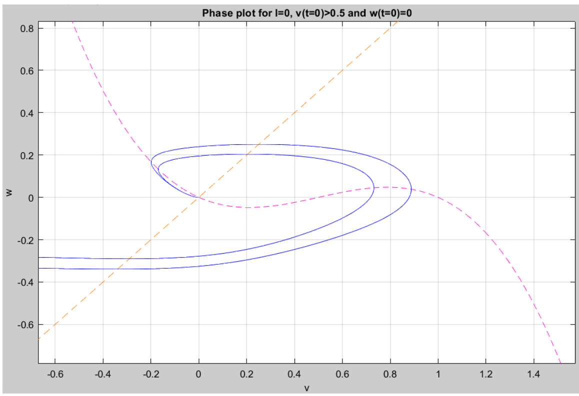


Figure 5: Phase plane for I_{ext}=0, v(0)=0.7>a and w(0)=0

<u>Case 2:</u> I_{ext}=0.5 μA **[Limit Cycles]**

First, we find the currents I_1 and I_2 for which limit cycles are observed. To do this, we need to find the maxima and minima of $f(v)$. Hence, we differentiate $f(v)$ with respect to v. On calculation, it is found that minima is observed for v=**0.2113** and maxima for v=**0.7887**. It can be argued that the intersection of both the nullclines will have same abscissa and ordinate. Which means:

$$v = v(0.5 - v)(v - 1) + I_{ext} \tag{1}$$

By substituting v_{min} and v_{max} values in equation (1), we get $\mathbf{I_1}$**=0.2594** $\boldsymbol{\mu}$**A** and $\mathbf{I_2}$**=0.7405** $\boldsymbol{\mu}$**A**.

(a) The phase plot for I_{ext}=0.5 μA:

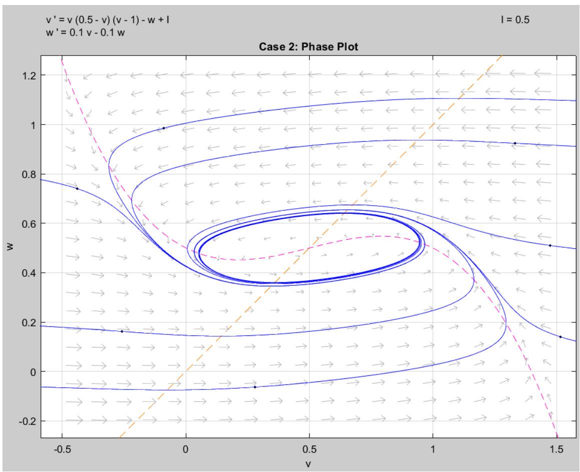


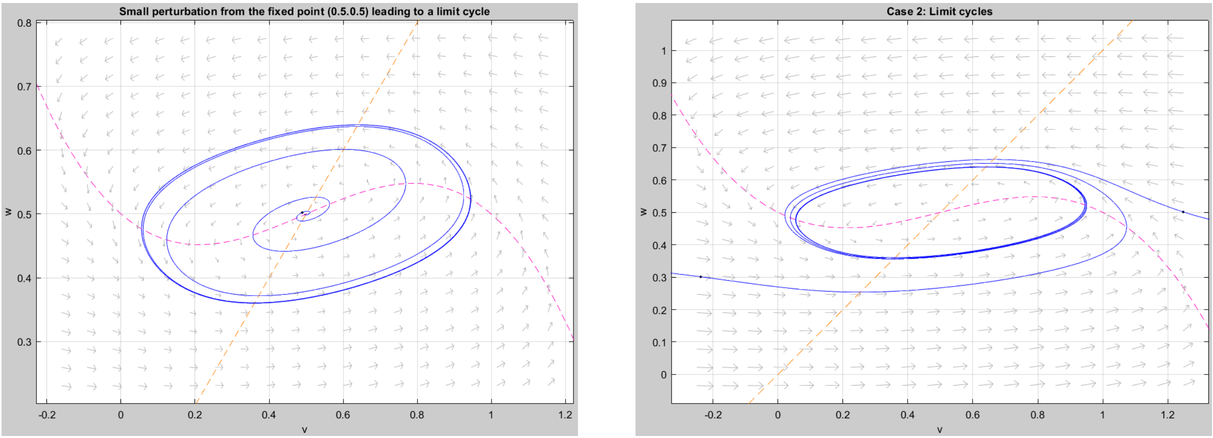
Figure 6: Phase plot for I_{ext}=0.5 μA

(b) As I_{ext} increases and falls in the range of I_1 and I_2, the w-nullcline intersects the v-nullcline in the middle branch where the v-nullcline has a positive slope. For this case, there is only a single fixed point- (0.5,0.5) for which the below relations hold true:

$$f'(v) < \frac{b}{r} = 1, \Delta > 0$$

$$\tau = f'(v) > 0$$

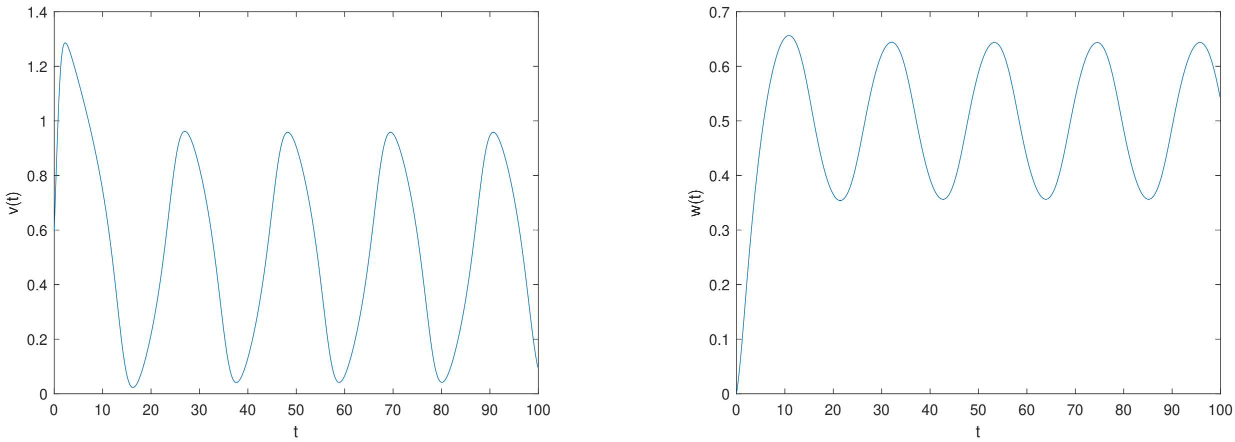
Hence, we conclude that the fixed point is **unstable**. From Fig. 7(a) we can see that for a small perturbations there is no return to the fixed point.



(a) (b)

Figure 7: (a) Small perturbations leading to a limit cycle; (b) Limit Cycle on the Phase plane

(c) v(0)=0.6 and w(0)=0



(a) (b)

Figure 8: (a) Plot of v(t) vs t; (b) Plot of w(t) vs t for v(0)=0.6 and w(0)=0

<u>**Case 3:**</u> I_{ext}=1.0 μA **[Depolarisation]**

As I_{ext} increases further, the two nullclines intersect at the right branch of the v-nullcline, where its slope is negative i.e., $f'(v) < 0$. For this case, there is only a single fixed point- (1.0,1.0) for which the below relations hold true:

$$f'(v) < 0 < \frac{b}{r}, \Delta > 0$$

$$\tau = f'(v) < 0$$

Hence, we conclude that the fixed point is **stable**. From Fig. 10 we can see that for a small perturbations there is a return to the fixed point.

(a) The phase plot for I_{ext}=1.0 μA:

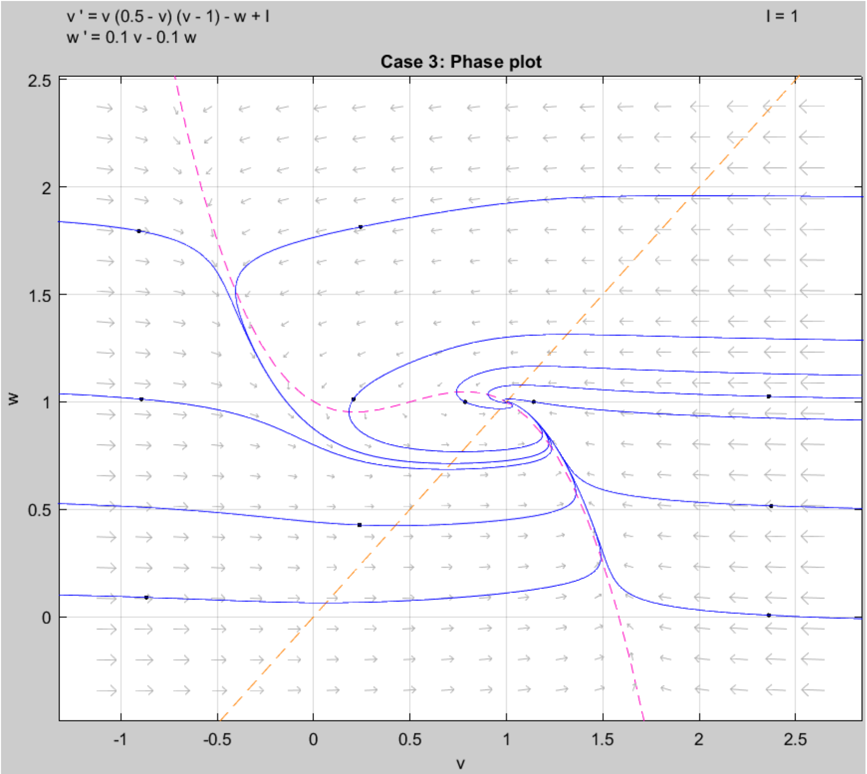


Figure 9: Phase plot for I_{ext}=1.0 μA

(b) Trajectories traced due to small perturbations that leads to depolarisation.

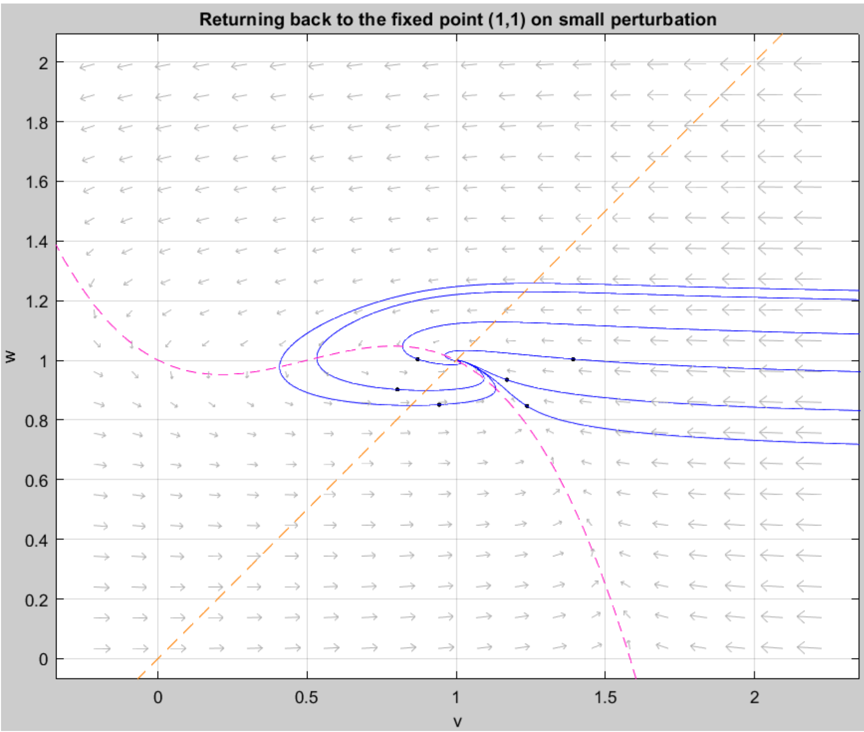
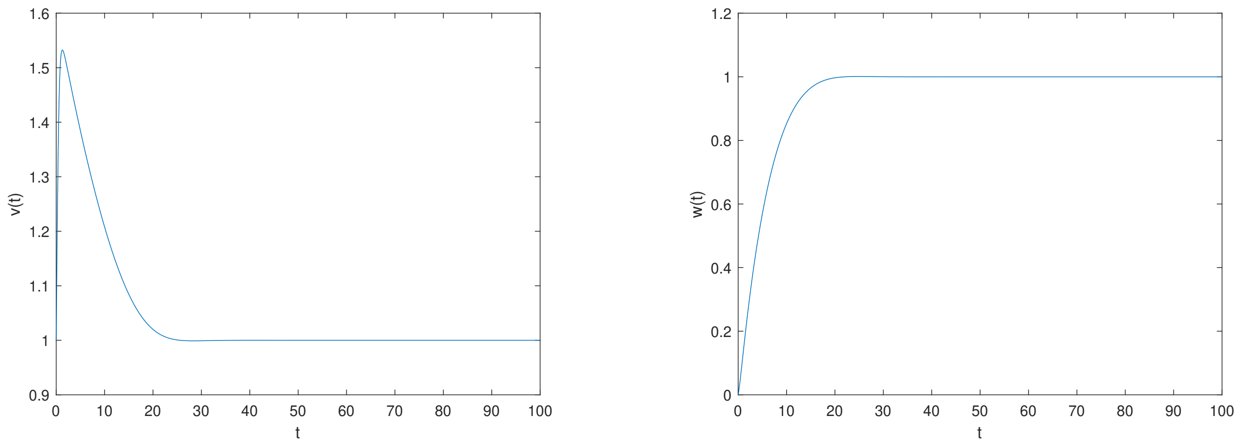


Figure 10: Small perturbations leading to depolarisation

(c) v(0)=1.2 and w(0)=0



(a) (b)

Figure 11: (a) Plot of v(t) vs t; (b) Plot of w(t) vs t for v(0)=1.2 and w(0)=0

Case 4: I_{ext}=0 μA **[Bi-stability]**

For this case specifically we **assume:** I_{ext}=0 μA and $a = 0.5$. On plotting both the nullclines, we can confidently say that slope of w-nullcline should be less than **0.06098**. Mathematically,

$$\frac{b}{r} < 0.06098$$

For simplicity, in this case I have taken $r = 1.0$ and $b = 0.02$. Therefore, the parameters for this case are: I_{ext}=0 μA, $a = 0.5$, $r = 1.0$ and $b = 0.02$.

(a) The Phase plot for this case:

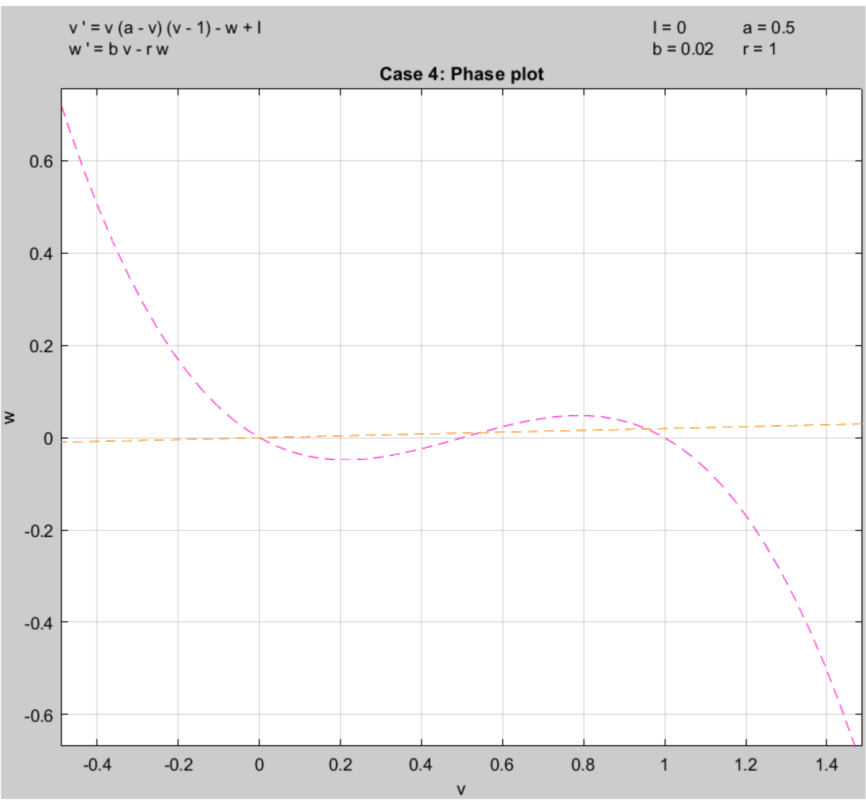


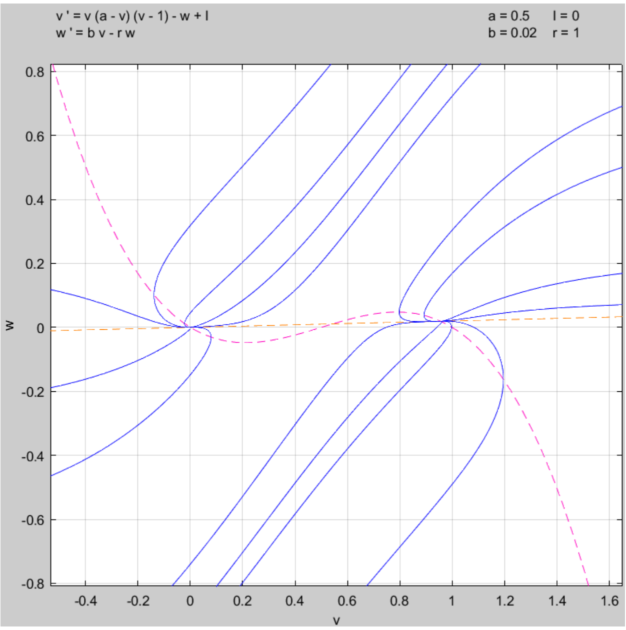
Figure 12: Phase plot for I_{ext}=0, $a = 0.5$, $r = 1.0$ and $b = 0.02$.

(b) Stability of points P1, P2 and P3.

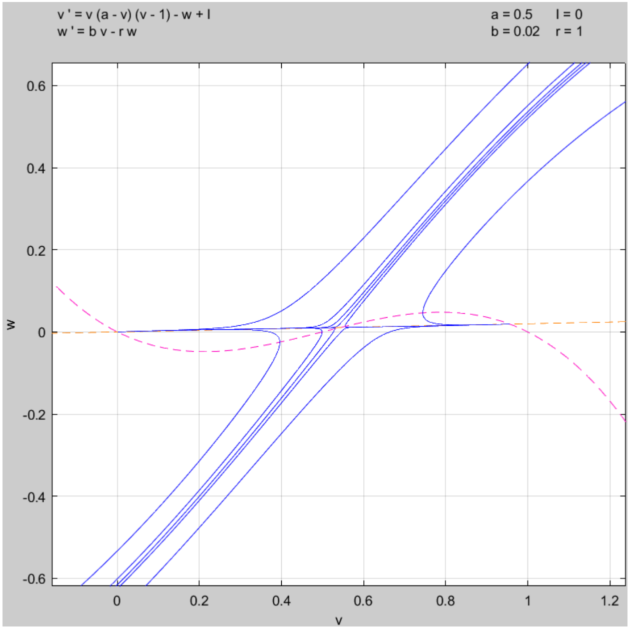
1. **Point P1** (fixed point on the left branch of v-nullcline)
   $f'(v) < 0 < \frac{b}{r}$
   $\Delta > 0$
   $\tau = f'(v) < 0$
   $\implies$ point P1 is a **stable** point.

2. **Point P2** (fixed point on the middle branch of v-nullcline)
   $f'(v) > \frac{b}{r}$
   $\Delta < 0$
   $\tau = f'(v) > 0$
   $\implies$ point P2 is a **saddle** point.

3. **Point P3** (fixed point on the right branch of v-nullcline)
   $f'(v) < 0 < \frac{b}{r}$
   $\Delta > 0$
   $\tau = f'(v) < 0$
   $\implies$ point P3 is a **stable** point.



(a)



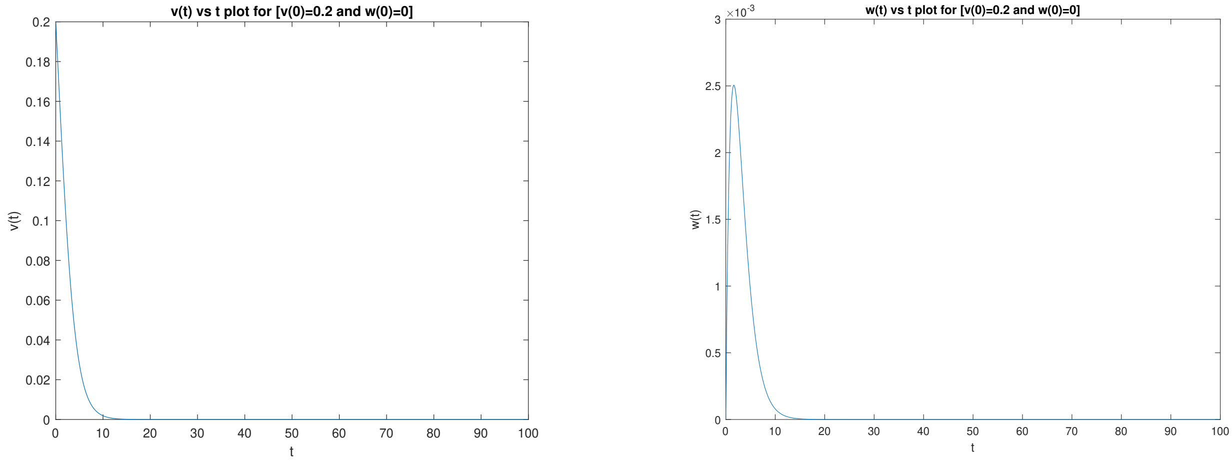
(b)

Figure 13: (a) Small perturbations near P1 and P3; (b) Small perturbations near P2

From Fig.13(a) it can be clearly seen that points P1 and P2 are stable, whereas, Fig.13(b) depicts that point P2 is a saddle.

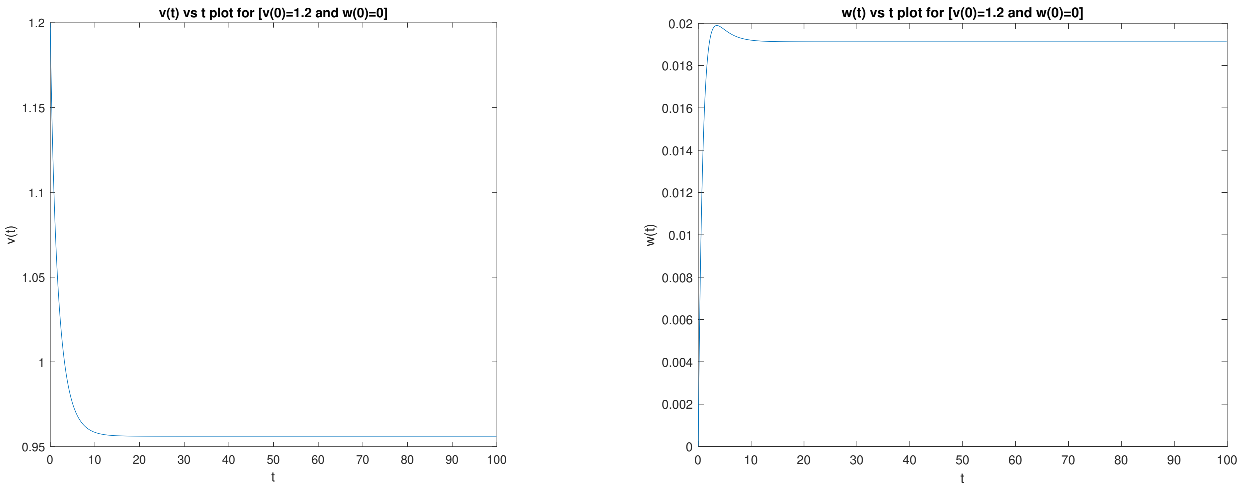
(c) v(t) vs t and w(t) vs t plots for **(v(0)=0.2,w(0)=0)** and **(v(0)=1.2,w(0)=0)**.

From the stability analysis of points P1, P2 and P3, we know that there are two fixed points, namely P1 and P3, that are stable and hence the case of **Bi-stability**. We then plot the v(t) vs t and w(t) vs t plot for an initial v value as 0.2 and w value as 0. In Fig.14 we confirm the stability behaviour around the point P1. Finally, we also follow the same procedure for confirming the results of our stability analysis for point P3, with initial v value as 1.2 and w value as 0. In Fig.15 we confirm the stability behaviour around the point P3.



(a) (b)

Figure 14: (a) Plot of v(t) vs t; (b) Plot of w(t) vs t for v(0)=0.2 and w(0)=0



(a) (b)

Figure 15: (a) Plot of v(t) vs t; (b) Plot of w(t) vs t for v(0)=1.2 and w(0)=0

$**$ End of Assignment $**$