2021-2022

Q-1 a) Karl's perason's cofficient of correlation.

$$r_{(x,y)} = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma y^2}} = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma x^2-(\Sigma x)^2}\sqrt{n\Sigma y^2-(\Sigma y)^2}}$$

b) t-test :-

It is used to test the significance of :

(i) Mean of small sample

(ii) Compare two small sample

(iii) Cofficient correlation

$$t = \frac{x-\mu}{s/\sqrt{n}} , \quad s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{(n-1)}}$$

c) UCL of p chart

$$U.C.L = \bar{p} + 3\sigma_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})}$$

d) Relative errors :-

A measure of the accuracy of a measurement or calc- expressed as a ratio of the absolute error to the actual val.

$$\%\ \text{Relative Error} = \frac{|\text{actual} - \text{measured}|}{\text{actual}} \times 100$$

e) Regula-falsi formula :-

It is also known as false position maithod. because each step we will decided that this point is our root.

~~Calcul~~ Calculate two point a & b such that

$$f(a) \times f(b) < 0$$

~~Check f(a) & f(b) which is near~~ 0.

$$x_n = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

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a) Order of convergence of Newton's raphson formula?

$$x_n = \alpha + h_n$$
$$x_{n+1} = \alpha + h_{n+1}$$
$$h_{n+1} = h_n^k$$

k is called the order of convergance.

if $k=1$ → Linear convergence.

if $k=2$ → Quadratic.

c) Lagrange's interpolation formula :-

$$f(x) = \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} f(x_1) + \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)} f(x_2) \cdots$$

$$+ \frac{(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} f(x_n)$$

b) Runge-Kutta method of order four

$$Y_{n+1} = Y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$
$$K_1 = h \times f(x_n, y_n)$$
$$K_2 = h \times f\left(x_n + \frac{h}{2}, y_n + \frac{K_1}{2}\right)$$
$$K_3 = h \times f\left(x_n + \frac{h}{2}, y_n + \frac{K_2}{2}\right)$$
$$K_4 = h \times f(x_n + h + y_n + K_3)$$

Section-B

Q.2. Solve using Karl pearson's

a)

x	y	$X = x-\bar{x}$	$Y = y-\bar{y}$	XY	X^2	Y^2
3	7	-2	-3	6	4	9
7	12	+2	+2	4	4	04
5	8	0	+2	0	0	4
4	8	-1	-2	2	1	4
6	10	+1	0	0	1	0
8	13	+3	+3	9	9	9
2	5	-3	-5	15	9	25
7	10	+2	0	0	4	0
		2	-7	36	32	55

Let mean of $\bar{x} = 5$ $\bar{y} = 10$

$$r_{(x,y)} = \frac{n\sum XY - \sum X \sum Y}{\sqrt{n\sum X^2 - (\sum X)^2}\sqrt{n\sum Y^2 - (\sum Y)^2}}$$

$$\Rightarrow \frac{8 \times 36 - 2 \times (-7)}{\sqrt{8 \times 32 - 4}\sqrt{8 \times 55 - 49}} \Rightarrow \frac{288 + 14}{\sqrt{252 \times 391}} \Rightarrow \frac{302}{\sqrt{252 \times 391}}$$

$r = 0.962$

b) $f(x) = x^3 - 5x - 7 = 0$

root lie b/w 2 & 3

$f(2) = -9$ $f(3) = 5$

1st a b

$$x_n = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

$$x_1 = \frac{2 \times 5 - 3 \times (-9)}{5 - (-9)} \Rightarrow \frac{37}{14} = 2.6428$$

$f(x_1) = -1.7556$ -(ve)

$f(x_1) \times f(b) < 0$

IInd $x_2 = \dfrac{2.6420 \times 5 - 3 \times (-1.7556)}{5 + 1.7556} \Rightarrow 2.7356$

$f(x_2) = -0.2061$ (-ve)

$f(x_2) f(b) < 0$

IIIrd $x_3 = \dfrac{2.7356 \times 5 - 3 \times (-0.2061)}{5 + 0.2061} \Rightarrow 2.8475$

$f(x_3) = 1.8507$ (+ve)

$f(x_2) \times f(x_3) < 0$

(IV) $x_4 = \dfrac{2.7356 \times 1.8507 - 2.8475 \times (-0.2061)}{1.8507 + 0.2061} = 2.7468$

$f(x_4) = -0.0096$

$f(x_4) \times f(x_3) < 0$

(V) $x_5 = \dfrac{2.7468 \times 1.8507 + 2.8475 \times 0.0096}{1.8507 + 0.0096} = 2.7473$

Since x_4 & x_5 value are correct at two decimal place

So root

$x_5 = 2.7473$

c)

x	y	Δ_{x_0}	Δ_{x_1,x_2}	Δ_{x_1,x_2,x_3}
0	6			
		1		
1	7			
		1	0	
2	8			1
		7	3	
3	15			

$y(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3)$

$y(x) = 6 + (x - 0) \times 1 + (x - 0)(x - 1) \times 0 + (x - 0)(x - 1)(x - 2) \times 1$

$y(x) = 6 + x + x^3 - x^2 - 2x^2 + 2x$

$y(x) = x^3 - 3x^2 + 3x + 6$

$y(5) = 125 - 3 \times 25 + 3 \times 5 + 6 = 125 - 75 + 15 + 6 = 71$

$y(5) = 71$

d) $4x + y + 3z = 17$; $x + 5y + z = 14$; $2x + y + 8z = 12$

$x = \dfrac{17 - y - 3z}{4}$; $y = \dfrac{14 - x - z}{5}$; $z = \dfrac{12 - 2x - y}{8}$

Ist but $x, y, z = 0$ in all the eq except L.H.S

$x_1 = \dfrac{17}{4} = 4.25$; $y = \dfrac{14}{5} = 2.8$; $z = \dfrac{12}{8} = 1.5$

IInd $x_2 = \dfrac{17 - 2.8 - 3 \times 1.5}{4}$; $y_2 = \dfrac{14 - 4.25 - 1.5}{5}$; $z_2 = \dfrac{12 - 2 \times 4.25 + 2.8}{8}$

$x_2 = 2.425$; $y_2 = 1.65$; $z_2 = 0.7875$

IIIrd $x_3 = \dfrac{17 - 1.65 - 3 \times 0.7875}{4}$; $y_3 = \dfrac{14 - 2.425 - 0.7875}{5}$; $z_3 = \dfrac{12 - 2 \times 2.425 + 1.65}{8}$

$x_3 = 3.25$; $y_3 = 2.16$; $z_3 = 1.13$

(IV) $x_4 = \dfrac{17 - 2.16 - 3 \times 1.13}{4}$; $y_4 = \dfrac{14 - 3.25 - 1.13}{5}$; $z_4 = \dfrac{12 - 2 \times 3.25 + 2.16}{8}$

$x_4 = 2.885$; $y_4 = 1.93$; $z_4 = 0.96$

(V) $x_5 = \dfrac{17 - 1.93 - 3 \times 0.96}{4}$; $y_5 = \dfrac{14 - 2.885 - 0.96}{5}$; $z_5 = \dfrac{12 - 2 \times 2.885 + 1.93}{8}$

$x_5 = 3.05$; $y_5 = 2.15$; $z_5 = 1.02$

$x \approx 3$; $y \approx 2$; $z \approx 1$

Section - C

3) a) $Y = 0.516X + 33.78$; $x = 0.512y + 32.52$

Compare with original eqn

Y on x

$Y = b_{yx}X + c$

$b_{yx} = 0.516$

x on y

$x = b_{xy}Y + c$

$b_{xy} = 0.512$

$r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{0.516 \times 0.512}$

$r = 0.5140$

(iii) Mean $\bar{X}$ & $\bar{Y}$

Intercept point of both the line of regression is ~~called~~ at mean value.

$\bar{y} = 0.516\bar{x} + 33.78$ $\qquad \bar{x} = 0.512\bar{y} + 32.52$

$\bar{X} = -67.67$ $\qquad \bar{Y} = -68.65$

b)

I.Q	High	Low	Total.
Rich	100	300	400
Poor	350	250	600
Total	450	550	1000

Ei

	180	220.	
	270	330	

Let H0: There is any association b/w economic condition at home and I.Q. of the student.

H_1: There is no association.

$Ei = \dfrac{\text{row total} \times \text{col total}}{\text{Grand total}}$

Oi	Ei	Oi - Ei	$(Oi - Ei)^2$	$(Oi-Ei)^2/Ei$
100	180	-80	6400	35.55
300	220	+80	6400	29.09
350	270	+80	6400	23.70
250	330	-80	6400	19.39
				107.79

1 d.f. χ^2_{tab} at 0.05 = 3.85

$\chi^2_{cal} = 107.79$

$\because \chi^2_{cal} > \chi^2_{tab}$

so H0 - rejected H_1 - accepted

Question - no - 4

b) $f(x) = x^3 - 5x + 1 = 0$ using bisection method for five iteration

$f(1) = -3$ $\quad f(2) = -1$ $\quad f(3) = 13$

$\therefore$ root lie b/w 2 & 3

(1st) $x_1 = \dfrac{2+3}{2} = 2.5$

$f(x_1) = 4.125$ +ve

root lie b/w 2, 2.5

(2) $x_2 = \dfrac{2+2.5}{2} = 2.25$

$f(x_2) = 1.140625$ +ve $\qquad f(2.25) f(2) < 0$

(3) $x_3 = \dfrac{2+2.25}{2} = 2.125$

$f(x_3) = -0.029$ - (ive)

(4) $x_4 = \dfrac{2.125+2.25}{2} = 2.1875$

$f(x_4) = 0.53$ +ve

(5) $x_5 = \dfrac{2.1875+2.125}{2} = 2.15625$

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$f(x_5) = 0.244$ +ve

after five iteration smallest positive root

$x \approx 2.15625$

Q.

$x+y+z=9$; $2x-3y+4z=13$; $3x+4y+5z=40$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 13 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ -5 \end{bmatrix}$$

$R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 12 \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 60 \end{bmatrix}$$

$R_2 \to R_2 - \frac{R_3}{6}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 12 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 60 \end{bmatrix}$$

$R_3 \to R_3/12$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 5 \end{bmatrix}$$

$R_1 \to R_1 - R_2$; $R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$x=1$ $y=3$ $z=5$

Question - 5

a) Prove that

$$\left[E^{1/2} + E^{-1/2}\right)(1+\Delta)^{1/2} = 2+\Delta$$

L.H.S

$$\left[E^{1/2} + E^{-1/2}\right]\left[1+\Delta\right)^{1/2}$$

$$= \left[(1+\Delta)^{1/2} + (1+\Delta)^{-1/2}\right]\left[(1+\Delta)^{1/2}\right)$$

$$\Rightarrow \left[(1+\Delta) + 1\right)$$

$$\Rightarrow \Delta + 2 = \text{R.H.S}$$

Proved.

b) Third divided differences

$f(x) = x^3 - 2x$

x	$f(x)$	Δ_{x_1}	Δ_{x_1,x_2}	Δ_{x_1,x_2,x_3}
2	4	26	15	1
4	56	131	23	
9	711	269		
10	980			

So third divided differences = 1

Q-6

b) find root to three decimal places by Newton's Raphson

$f(x) = x^3 - 2x + 5 = 0$

~~$f(-1) = -1+2+5 = 6$~~

$f(-2) = +1$ $f(-3) = -16$

Since $f(-2)$ & $f(-3)$ are opposite sign so root lie b/w -2 & -3 and $f(-2) \to 0$

$x_0 = -2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(m) = m^3 - 2m - 5$ $\quad f'(m) = 3m^2 - 2$

$m_1 = -2 - \frac{(1)}{10} = -\cancel{2.0909}\ -2-0.1 = -2.1$

~~$m_2 = -2.0909$~~ $\cancel{\frac{(-9.9593)}{}}$

$m_2 = -2.1 - \frac{(-10.061)}{11.23} =$

Q-6 - b) Using newton's raphson method solve

$f(m) = m^3 - 2m - 5 \qquad f'(m) = 3m^2 - 2$

$f(2) = -1 \quad f(3) = 16$

Since $f(2)$ & $f(3)$ are opposite sign ~~so~~ & and $f(2)$ is near to zero.

So $\quad x_0 = 2$

Newton's raphson's formu

$$m_{n+1} = m_n - \frac{f(m_n)}{f'(m_n)}$$

$$m_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{(-1)}{10} \Rightarrow 2.1$$

$$m_2 = 2.1 - \frac{f(2.1)}{f'(2.1)} = 2.1 - \frac{0.061}{11.23} = 2.0945$$

$$m_3 = 2.0945 - \frac{f(2.0945)}{f'(2.0945)} = 2.0945 - \frac{(-0.00057)}{11.160} = 2.0945$$

Since m_2 & m_3 value are same at correct three decimal places so root

$m = 2.0945$

c)

	$x - \bar{x}$	$(x - \bar{x})^2$
63	−4	16
63	−4	16
64	−3	9
65	−2	4
66	−1	1
69	+2	4
69	+2	4
70	+3	9
70	+3	9
71	+4	16
670		88

Let $H_0: \mu = 65$

$H_1: \mu \neq 65$

$\bar{x} = 67$

$$s = \sqrt{\frac{\Sigma (m - \bar{m})^2}{n-1}}$$

$$s \Rightarrow \sqrt{\frac{88}{9}} = 3.13$$

acc to t-test $t = \frac{\bar{m} - \mu}{s/\sqrt{n}} \Rightarrow \frac{67-65}{3.13} \times \sqrt{10}$

$t_{cal} = 2.020$

for 9 d.f 't' at 5% = 2.262

$\because t_{cal} < t_{tab}$ so H_0 accepted

H_1 rejected