

# **HARMONIC DRIVER USING PD CONTROLLER**

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## **Declaration :**

We, the undersigned students of B. Tech. of Electrical and Electronics Engineering Department hereby declare that we own the full responsibility for the information, results etc. provided in this project titled “HARMONIC DRIVER USING PD CONTROLLER” submitted to **Siksha ‘O’ Anusandhan University, Bhubaneswar** for the partial fulfilment of the subject Control Systems (EET3071). We have taken care in all respect to honour the intellectual property right and have acknowledged the contribution of others for using them in academic purpose and further declare that in case of any violation of intellectual property right or copyright we, as the candidates, will be fully responsible for the same.

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## **ABET outcome /Student outcome**

There are eleven student outcomes (A-K) for the Electrical Engineering B.Tech program.

Abet Outcomes	Description of Outcome
A	An Ability to apply knowledge of mathematics, science, and engineering.
B	An Ability to design and conduct experiments ,as well as to analyse and interpret data.
C	An ability to design a system, component, or process to meet desired needs within realistic constraints such as economic ,environmental, social, political, ethical, health and safety ability ,and sustainability.
D	An ability to function on multidisciplinary teams.
E	An ability to identify, formulate, and solve engineering problems.
F	An understanding of professional and ethical responsibility.
G	An ability to communicate effectively.
H	The broad education necessary to understand the impact of engineering solutions
I	A recognition of the need for and an ability to engage in life-long learning.
J	A knowledge of contemporary issues.
K	An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

The students will be able to satisfy ABET outcomes A ,E ,B, C,G and K in this subject. They will satisfy outcome A, E through mid-semester, end semester, Quizzes and assignments, where as they will satisfy B,C,G and K through lab tests, projects, reports and viva-voce.

## **Abstract**

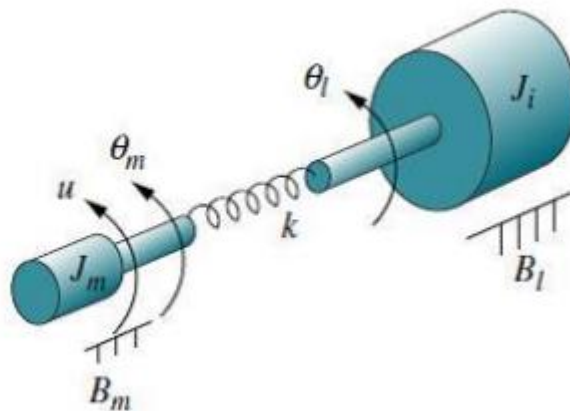
This paper deals with a simulation and modeling of a harmonic drive mechanism with DC motor, couplers and mechanical load. The harmonic drive mechanism is used for a precise motion of several mechatronic systems (robots, medical devices, military, aeronautics, etc.). The harmonic drive provides precise and specific gear systems with high transmission ration and very low backlash. This paper analyzed a multi-body model of the harmonic drive and the nonlinear model of transmission dynamic was created and the harmonic drive operation was simulated. This non-linear model will be used for dynamic analyses of developed precise mechatronic systems. Control loops could be adapted onto the non-linear model.

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## 1. Problem Statement :

Harmonic drives are very popular for use in robotic manipulators due to their low back-lash, high torque transmission, and compact size. The problem of joint flexibility is sometimes a limiting factor in achieving good performance. Consider that the idealized model representing joint flexibility in Figure.4. The input to the drive is from an actuator and is applied at  $\theta_m$ . The output is connected to a load at  $\theta_i$ . The spring represents the joint flexibility and  $B_m$  and  $B_i$  represent the viscous damping of the actuator and load, respectively. Use PD controller to improve the transient performance of the system. Design the controller such that the maximum transient error will be approximately 5% Overshoot. Parameters:  $J_m = 2$ ;  $B_m = 0.5$ ;  $J_i = 10$ ;  $B_i = 1$ ;  $K = 100$ .



## 2. Background theory and analysis:

### 1. Controller

Controllers are some additional sub-systems which can be employed either when the system is having steady state error or if the system response is sluggish in nature. These controllers can be either placed in the forward path or may be placed in the feedback path in order to improve the system's overall performance.

Here in this system we have used a PD controller in order to reduce the steady state error.

The command `linmod()` performs the linear simulation of a system, for a particular type of I/P signal.

It can also help to simulate the responses of more than one system at a time on the same plot area.

### 2. Actuator

An actuator is a component of a machine that is responsible for moving and controlling a mechanism or system, for example by opening a valve. In simple terms, it is a "mover". An actuator requires a control signal and a source of energy. The control signal is relatively low energy and may be electric voltage or current, pneumatic or hydraulic pressure, or even human power. Its main

energy source may be an electric current, hydraulic fluid pressure, or pneumatic pressure. When it receives a control signal, an actuator responds by converting the signal's energy into mechanical motion. An actuator is the mechanism by which a control system acts upon an environment. The control system can be simple (a fixed mechanical or electronic system), software-based (e.g. a printer driver, robot control system), a human, or any other input.[1]

### 3. Mathematical Modelling:

At node  $\theta_m$

$$u(t) = J_m \frac{d^2 \theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + k(\theta_m - \theta_i)$$

$$= J_m \frac{d^2 \theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + k\theta_m - k\theta_i$$

Using the Laplace transformation

$$u = J_m s^2 \theta_m(s) + B_m s \theta_m(s) + K \theta_m(s) - K \theta_i(s) \text{ ----- (1)}$$

Now, at node  $\theta_i$

$$0 = J_i \frac{d^2 \theta_i}{dt^2} + B_i \frac{d\theta_i}{dt} + k(\theta_i - \theta_m)$$

Using Laplace transformation

$$0 = J_i s^2 \theta_i(s) + B_i s \theta_i(s) + k \theta_i(s) - k \theta_m(s) \text{ -----2}$$

$$k \theta_m(s) = J_i s^2 \theta_i(s) + B_i s \theta_i(s) + k \theta_i(s)$$

$$\theta_m(s) = \frac{(J_i s^2 \theta_i(s) + B_i s \theta_i(s) + k \theta_i(s))}{k}$$

Putting 3 in 1, we get

$$u = (J_m s^2 + B_m s + k) \theta_m(s) - k \theta_i(s)$$

$$= \frac{(J_m s^2 + B_m s + k) * (J_i s^2 + B_i s + k) \theta_i(s) - k \theta_i(s)}{k}$$

$$= [(J_m s^2 + B_m s + k) * (J_i s^2 + B_i s + k) - k^2] \theta_i(s)$$

$$\frac{\theta_i(s)}{u} = \left[ \frac{k}{(J_m s^2 + B_m s + k) * (J_i s^2 + B_i s + k) - k^2} \right]$$

Now putting the value of  $J_m, J_i, B_m, B_i$  and  $k$  we get

$$\frac{\theta_i(s)}{u} = \frac{100}{20 s^4 + 7 s^2 + 1200 .5s + 150} \quad \text{is the transfer putting } \theta_i$$

$$\frac{x_i(s)}{u} = \frac{1}{20 s^3 + 7 s^2 + 1200 .5s + 150}$$

$$u = \theta_i(s) / x_i(s) = 100/s$$

$$u = (20 s^3 + 7 s^2 + 1200 .5s + 150) x_i(s) \text{ -----4}$$

$$s \theta_i(s) = (100)xi(s) \text{ -----}5$$

In equation 4

$$u = (20 s^3 xi(s) + 7 s^2 xi(s) + 1200 .5s xi(s) + 150 xi(s))$$

Apply inverse laplace of the above equation

$$u(t) = 20 Xi'''(t) + 7xi''(t) + 1200.5xi'(t) + 150 Xi(t)$$

Let Xi be the first variable

$$\left. \begin{aligned} X_1(t) &= X(t) \\ X_2(t) &= \dot{X}_1(t) \\ X_3(t) &= \ddot{X}_1(t) \end{aligned} \right\} \text{-----}(6)$$

From equation (6)

The state equation is:

$$X' = AX + BU$$

$$\dot{x}_3(t) = -7.5x_1(t) - 60.025x_2(t) - 0.35x_3(t) + u(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -7.5 & -60.025 & -0.35 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [u(t)]$$

From the output equation of the state equation:

$$S\Theta_i(S) = 100X_1(S)$$

$$\Theta_i(t) = 100X_1(t)$$

$$Y = CX + DU$$

$$\Theta_i(t) = [100 \ 25 \ 0 \ 0] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + [0](u(t))$$

From the calculation of the equation when the

$$Eq = \frac{(\ln(0.05))/100}{\sqrt{(\pi^2 + \ln^2(0.05/100))}}$$

$$= \frac{\ln(0.05)}{\sqrt{(\pi^2 + \ln^2(0.05))^2}} = 0.6901$$



$$\omega_n = \frac{2}{el} = \frac{2}{0.6901} = 2.8981$$

$$K_p = \lim_{s \rightarrow 0} s G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{s \cdot 100}{20s^4 + 7s^3 + 1200.5s^2 + 150s}$$

$$= \lim_{s \rightarrow 0} \frac{100s}{s(20s^3 + 7s^2 + 1200.5s + 150)}$$

$$= \frac{100}{150} = 0.667$$

That is,  $K_p = 0.667$

#### 4. Implementation and Verification of Model in MATLAB:

##### MATLAB CODE FOR TRANSFER FUNCTION FROM STATE MODEL

```
clc;
close all;
clear all;
[A,B,C,D]=linmod('untitled1');
[n,d]=ss2tf(A,B,C,D);
g=tf(n,d);
disp(g);
```

##### OUTPUT OF THE CODE

A =

-0.3500	-60.0250	-7.5000	0
1.0000	0	0	0
0	1.0000	0	0
0	0	1.0000	0

B =

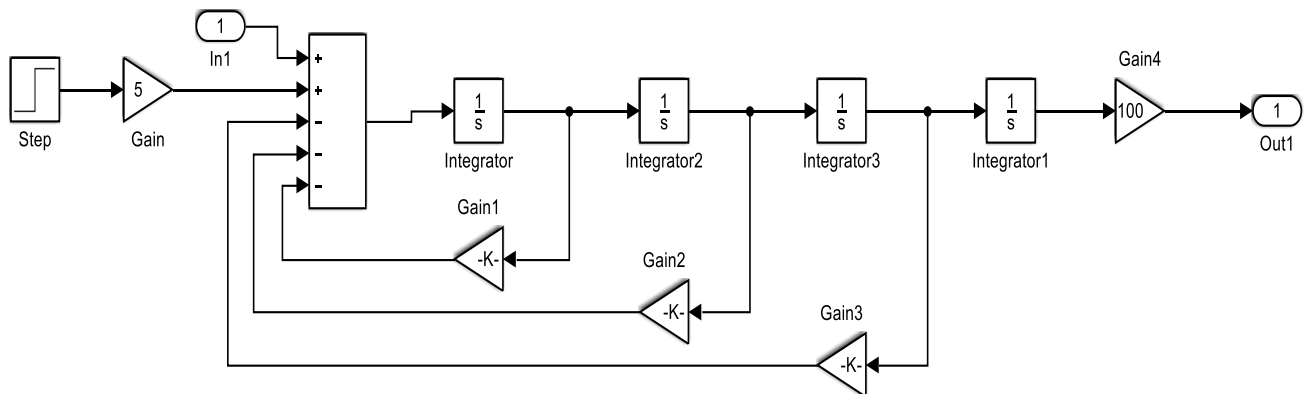
1  
0  
0  
0

C =

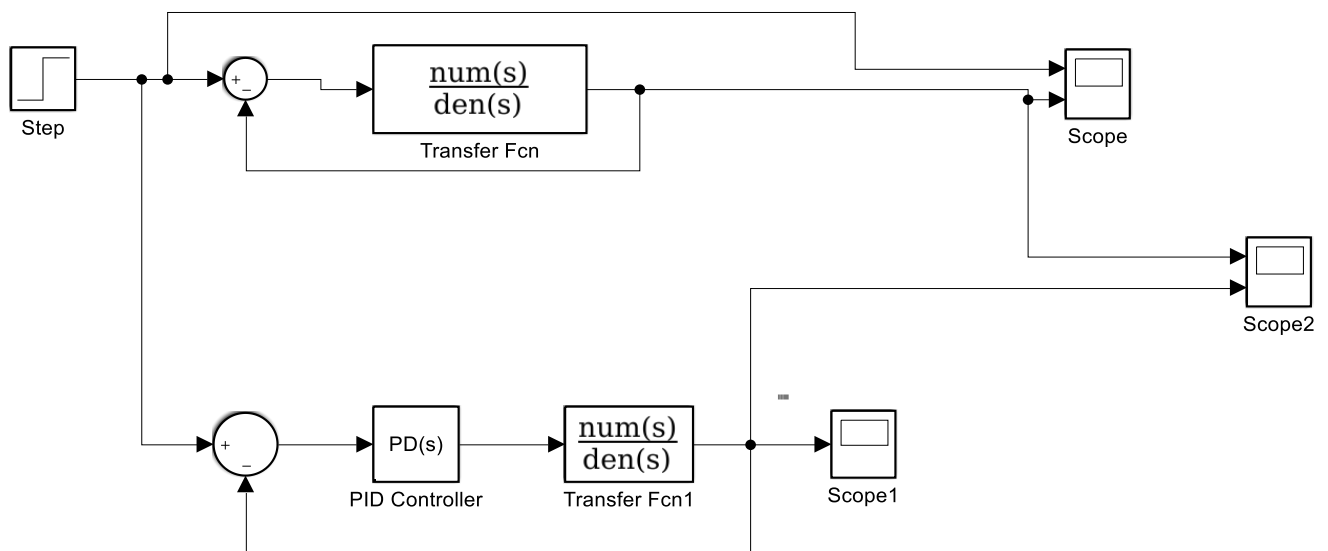
0      0    5.0000    1.2500

$D = 0$

Mathematical model of the given mechanical system:



Mathematical model with PD Controller:



## 5. Result Analysis:

Output of the mathematical model system

$A =$

-0.3500	-60.0250	-7.5000	0
1.0000	0	0	0
0	1.0000	0	0
0	0	1.0000	0

B =

1  
0  
0  
0

C =

0      0   5.0000   1.2500

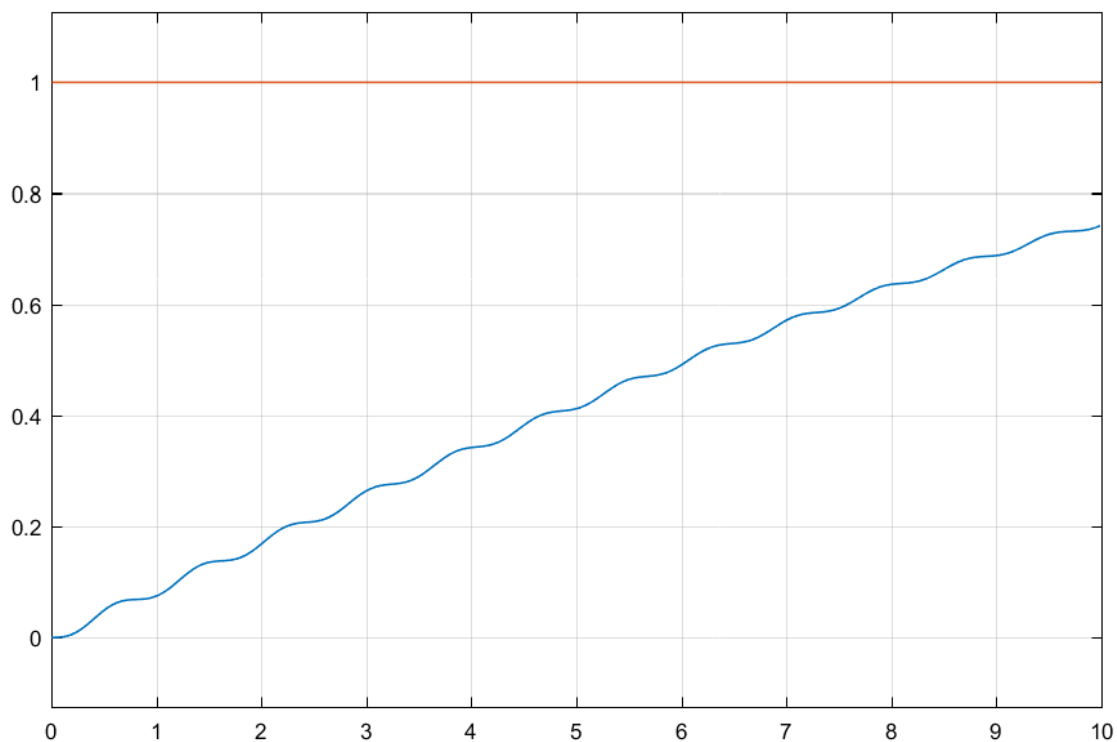
D =

0

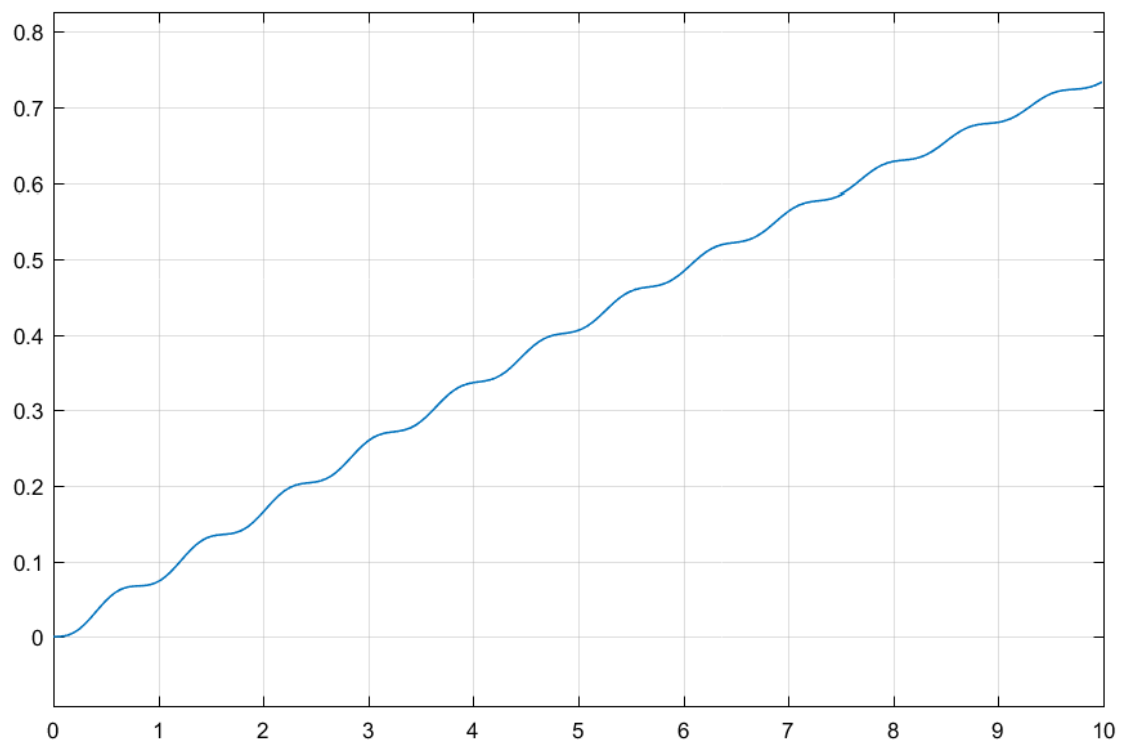
### Analysis:

- 1) The values of A B C D are obtained from the mathematical model, after calculating the differential equation from the mechanical system.
- 2) The values of A B C D were also calculated from the transfer function of the mechanical system.

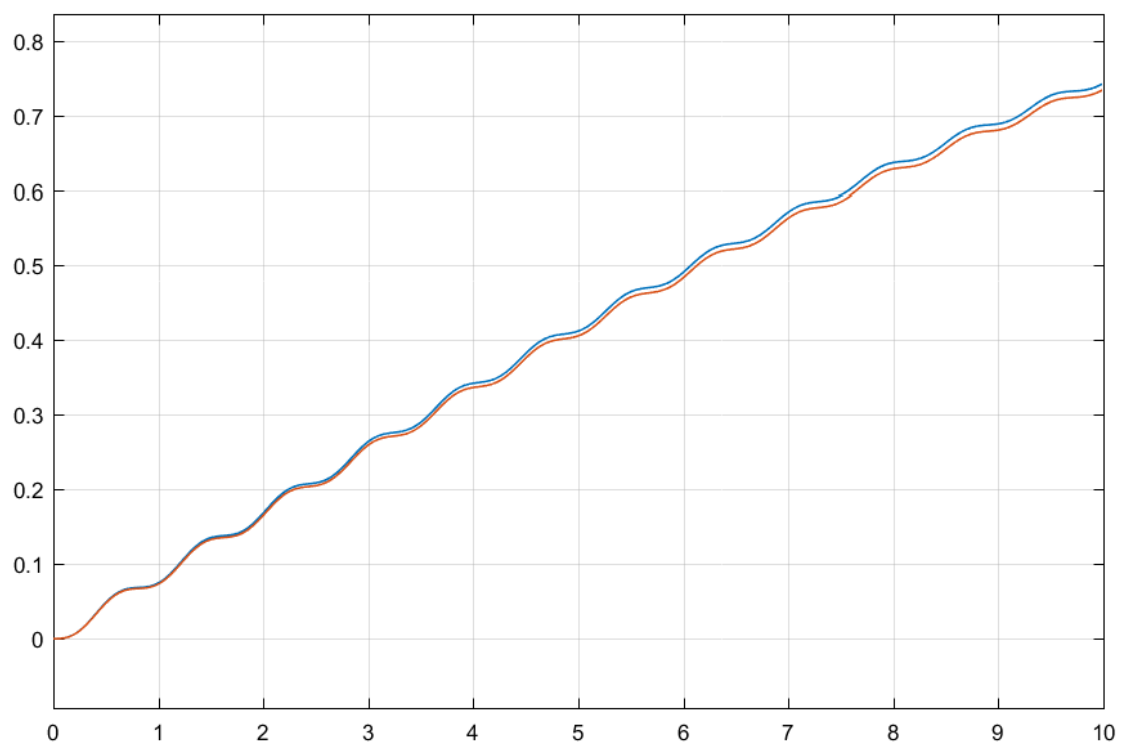
Output from the scope obtained with error



Output from scope using PD controller

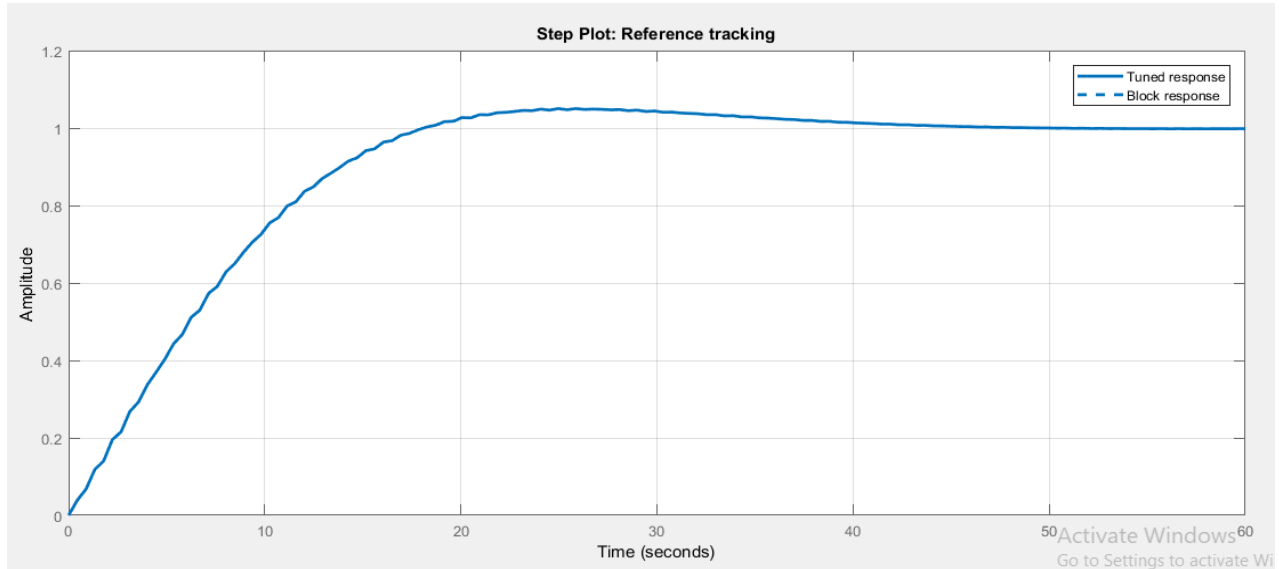


Final output from scope of the mechanical system



## Analysis:

- 1) The output of the scope was obtained from the transfer function when provided a step input.
- 2) The transient performance of the mechanical system was improved using PD controller such that the maximum error was approximated to 5% overshoot.



Controller Parameters		
	Tuned	Block
P	0.97855	0.97855
I	n/a	n/a
D	0	0
N	100	100
Performance and Robustness		
Tuned		Block
12.7 seconds	Rise time	12.7 seconds
37.4 seconds	Settling time	37.4 seconds
5.03 %	Overshoot	5.03 %
1.05	Peak	1.05
8.81 dB @ 7.74 rad/s	Gain margin	8.81 dB @ 7.74 rad/s
71.4 deg @ 0.128 rad/s	Phase margin	71.4 deg @ 0.128 rad/s
Stable	Closed-loop stability	Stable

## 6. Conclusion:

The harmonic drive provides precise and specific gear systems with high transmission ratio and very low backlash. However, for a military purpose the actuated mechanism has to provide very smooth positioning without any source of mechanical vibrations. This project analyzed a multi-body model of the harmonic drive and the nonlinear model of transmission dynamic was created and the harmonic drive operation was simulated. This non-linear model will be used for the design of control loops which avoid output mechanical vibrations of the mechanical load precise mechatronic or military applications. By using a PD controller to improve the transient performance of the system, such that the maximum transient error will be approximately 5%.

## 7. References:

- 1) <http://ctms.engin.umich.edu/CTMS/index.php?example=InvertedPendulum&section=SystemModeling>
- 2) [https://en.wikipedia.org/wiki/Inverted\\_pendulum](https://en.wikipedia.org/wiki/Inverted_pendulum)
- 3) N. Tan, I. Kaya, C. Yeroglu, and D. P. Atherton, "Computation of stabilizing PI and PID controllers using the stability boundary locus," Energy Conversion and Manageme