

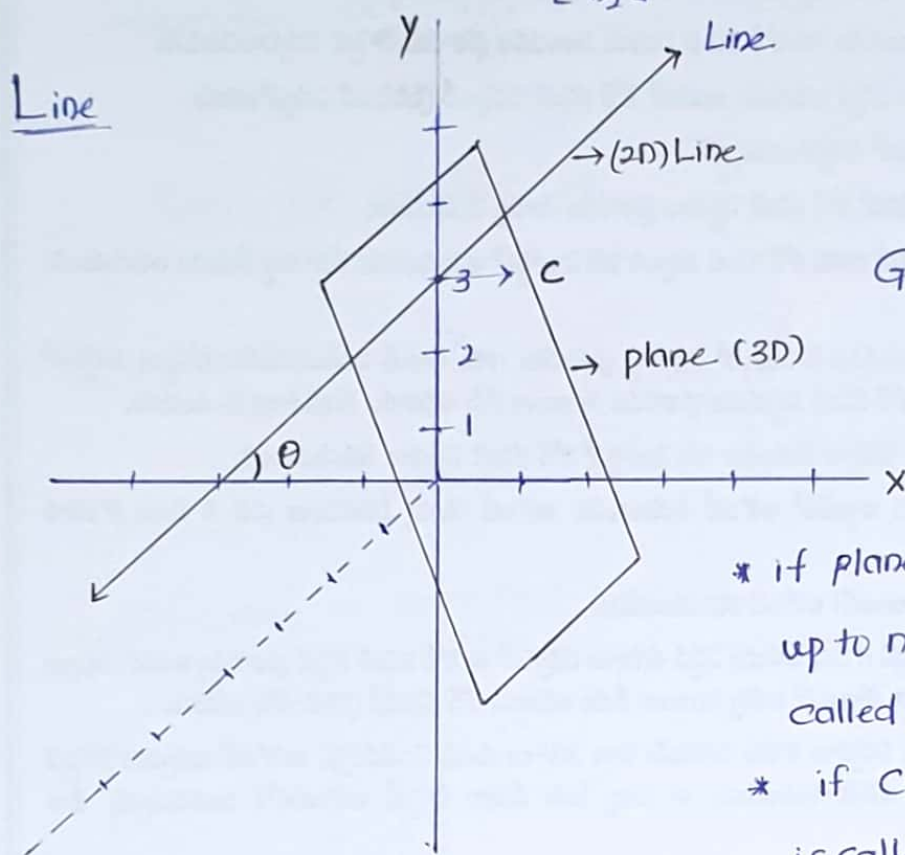
Line Equation :-  $y = mx + c$   
 $\xrightarrow{\text{Intercept}}$   
 $\xrightarrow{\text{Slope}}$

DOT PRODUCT :-  $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$   $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos \theta_{a,b} \quad (\text{or})$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 = \sum_{i=1}^d a_i b_i \quad (\text{or})$$

$$\vec{a} \cdot \vec{b} = [a_1, a_2] \times \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \vec{a}^T \times \vec{b}$$



$c \rightarrow$  Intercept at y axis

$m \rightarrow$  Slope with x axis

$$m = \tan \theta$$

General line eqn =  $ax + by + c = 0$

$$by = -ax - c$$

$$y = \left[ \frac{-a}{b} \right] x + \left[ \frac{-c}{b} \right]$$

$m \quad c$

\* if plane more than 3D (Dimensionality up to  $n$  (4D, 5D, 6D, ...,  $n$ D)) is called as Hyperplane.

\* if Circle (2D) in 3-Dimensionality is called as 'Sphere'

\* Above 3D upto  $n$ -D is called as Hyper Sphere

if we have line in 2D and plane in 3D

Hyper plane (4D) Equation :-

General Eq -  $ax + by + c = 0 \Rightarrow m = -\frac{a}{b} ; c = -\frac{c}{b}$

(4D)-Hyper plane -  $w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_0 = 0$

G. Eq. can be written as -  $w_1 x_1 + w_2 x_2 + w_0 = 0$

$$m = -\frac{w_1}{w_2} ; c = -\frac{w_0}{w_2}$$

if Hyperplane (3D) -  $w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$

In this plane we have two Slopes (m) and one Intercept

$$\left. \begin{aligned} m_1 &= \frac{-w_1}{w_2} \text{ wrt to } x_1 \text{ (plane)} \\ m_2 &= \frac{-w_3}{w_2} \text{ wrt to } x_3 \text{ plane} \\ c &= \frac{-w_0}{w_2} \text{ (Intercept with 'y' axis)} \end{aligned} \right\} y = m_1x_1 + m_2x_2 + c$$

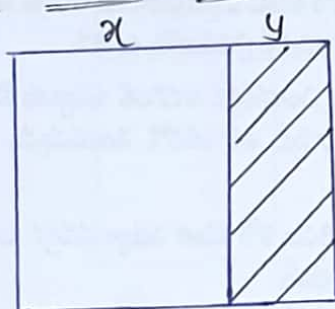
from this we can write as -

$$\sum_{i=1}^d w_i x_i + w_0 = 0$$

Simply we can say  $\rightarrow \pi_d : w^T \cdot x + w_0 = 0$   
 $\hookrightarrow$  No of Dimensions

if we have 5D  $\rightarrow \pi_5 : w^T$   
 $\left[ \begin{array}{c} \text{Coefficient} \end{array} \right] \left[ \begin{array}{c} x \\ \text{Dimensions} \end{array} \right] + w_0 = 0$

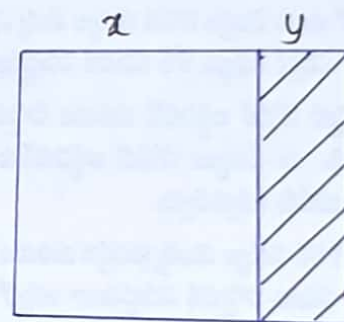
Classification vs regression :-



$y \rightarrow$  discrete  
 (or)  
 Categorical variable

$$y_i \in \{0, 1\}$$

it is Countable - finite set



$y \rightarrow$  Continuous  
 Variable

$$y_i \in \text{Any Real NO. } (\mathbb{R})$$

it is infinite set

NOTE  $\rightarrow$  After E.D.A we have to do Data preparation.

Ex:

x	y
	Red
	Green
	Blue
	Black
	White

if we observe this  $\rightarrow$  Data is categorical  
 So we can say classification problem

$$y \in \{ \text{Red, Green, Blue, Black, White} \}$$

$\downarrow$     $\downarrow$     $\downarrow$     $\downarrow$     $\downarrow$   
 0   1   2   3   4

Linear Regression :- it is a machine learning algorithm on Supervised learning, it performs the task to predict a dependent variable value (y) based on a given independent variable value (x) this technique finds out a linear relationship between x (input) and y (output)

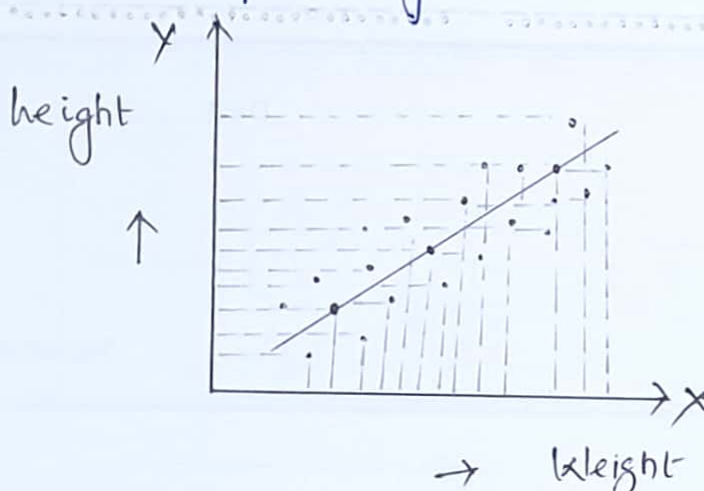
Ex: if we take a heights & weight of a Students

weight	height
x	y

$\rightarrow$  Continuous variable

So i need to do regression task

plot a Scatterplot diagram :-

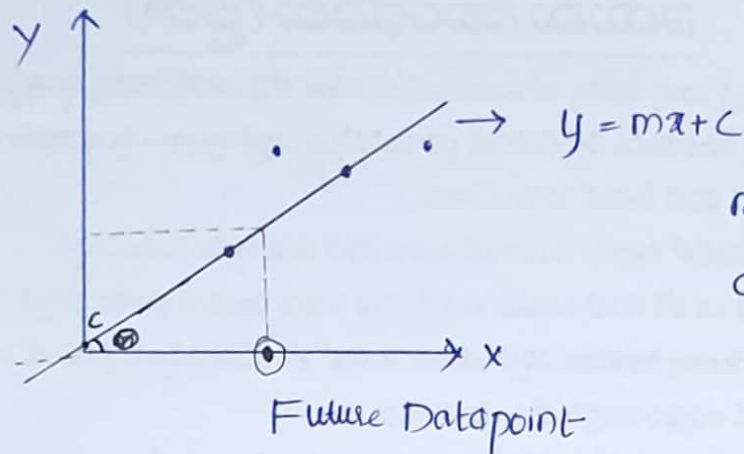


$H(\uparrow) \Rightarrow W(\uparrow)$

positive Correlation



Suppose if i fit a line b/w the datapoints, i can easily predict height of a person by given weight. am able to predict the future data point because of that best fit Line.

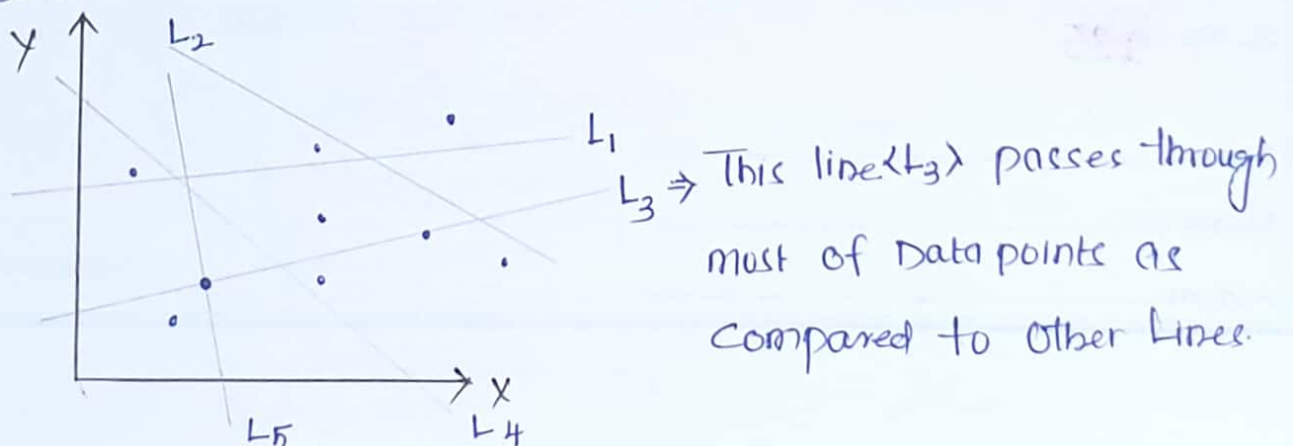


We know that line is  $(y = mx + c)$ , if i want the line i want to know the  $m^*$ ,  $c^*$  values otherwise we can't that line.

Suppose if i have  $m$  &  $c$  values i can draw a line  $(y = mx + c)$  by using this line. if any future data point comes simply we can predict it by using this line as above shown.

How to find a line that best-fit the given data ?

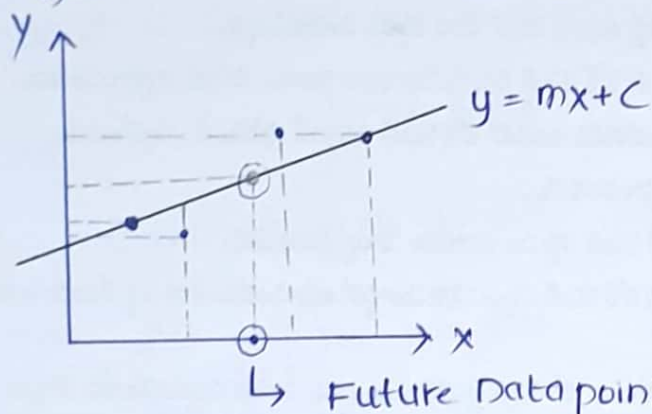
A line passes through most of datapoints in a given dataset Generally we consider it as a best-fit line of that dataset.



The above shown figure we draw a line based on data points  
there are so many lines that pass through data.

if we observe line 3 that contains maximum no. of points lying with that line as compared to rest of line. So simply we can

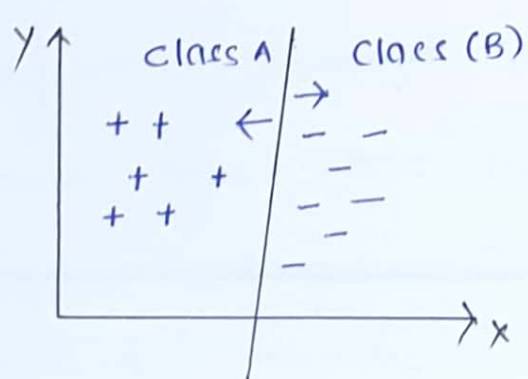
Say that the line 3 is Best-fit line.



The above shown figure says line passes through points, all which in data set, by this we can predict the values of 'y' by using this line. ~~if we~~ In case if we have future data point on x axis we have to find  $y_{\text{predict}}$  value by using this line we simply predict. if in case future data point is Real number (Continuous)

Note — i) In Regression we have to fit the line with data points

ii) In classification we have to separate the data by line



\* class (A), (B) separated by a line

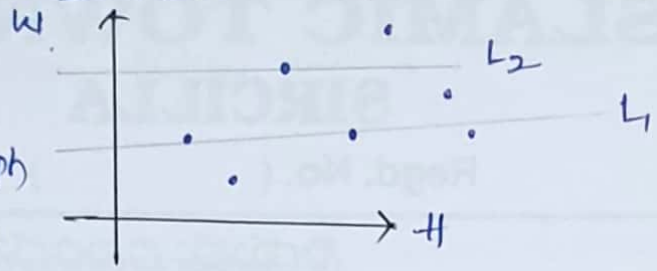


\* Process - to find a line that BEST FIT the Given data. —

+1	W

↙ Data set

plot a graph  
(scatter)

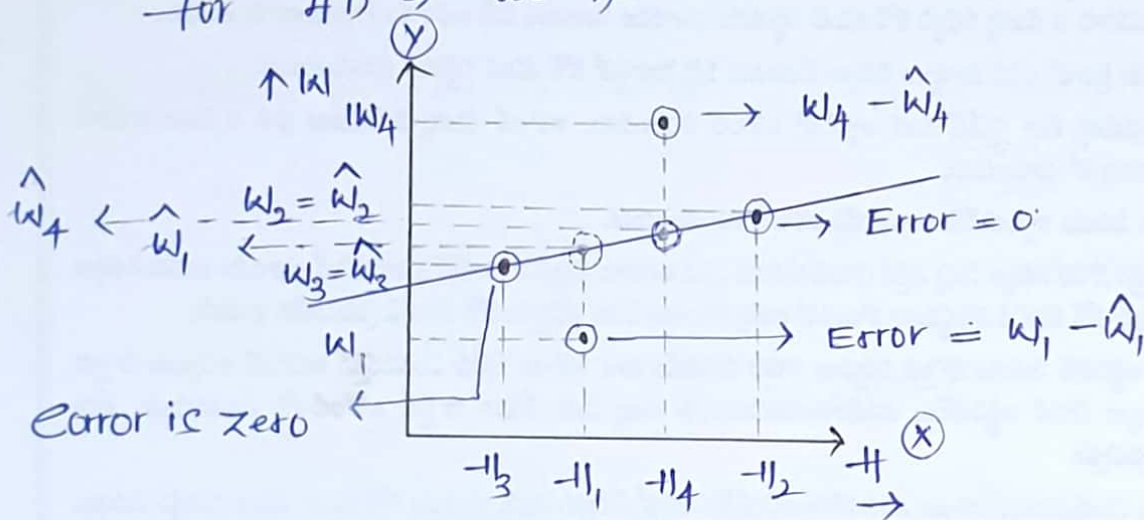


from the above graph we can say line ( $L_1$ ) is Best fit line as compared to  $L_2$  but how-to practically do you come up with relationship that help us which line is better

So here we are trying to fit a line as good as possible. —

for 2D  $\rightarrow$  we use line \$

for 4D  $\rightarrow$  we have to use Hyperplane.



For the point  $th$  the actual value of weight is  $w$ , Suppose if  $i$

want to predict  $\theta_1$  by using line I have to scale the point on line  $y = mx + c$  and predict the weight on y-axis as  $\hat{w}_1$

if we observe there is error associated with the point  $+1$ ,

When Compared to Actual with predicted value are -

$$w_1 - \hat{w}_1$$

\* For the point  $+1_2$  there is no error because the point is lying on line ( $y=mx+c$ ), So here - Actual value & predicted value is same, Error = 0.

\* For point  $+1_4$  there is an error because the actual & predicted value is not same.

$$\text{point } +1_1 \rightarrow w_1 - \hat{w}_1 = -ve$$

$$\text{point } +1_2 \rightarrow w_2 - \hat{w}_2 = 0$$

$$+1_3 \rightarrow w_3 - \hat{w}_3 = 0$$

$$+1_4 \rightarrow w_4 - \hat{w}_4 = +ve$$

In Order to get total Error made by this line we have to add

all points.	$\rightarrow +1_1 \rightarrow -ve$	* +ve magnitude is equal to -ve magnitude get cancelled.
	$+1_2 \rightarrow 0$	
	$+1_3 \rightarrow 0$	
	$+1_4 \rightarrow +ve$	
	$\frac{0}{0} \rightarrow \text{Total Error}$	

Hence the total error made by this line is zero, but if we

See the line there is an error associated with the line

In order to avoid it, we have to do the square for all errors and we have to add it.

$$(w_1 - \hat{w}_1)^2 + (w_2 - \hat{w}_2)^2 + (w_3 - \hat{w}_3)^2 + (w_4 - \hat{w}_4)^2$$

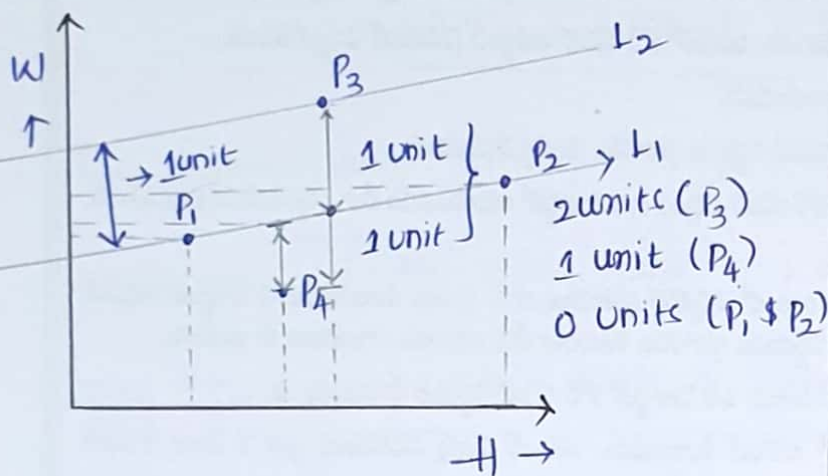
In Simply we can write as —  $\sum_{i=1}^n (w_i - \hat{w}_i)^2$

Where 'n' total NO of points, we have to generalize this as —

$$y \text{ then Eqn} = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

↳ Total Error made by line

Mathematically :-



$L_1 \Rightarrow$

$$\Rightarrow P_1 \Rightarrow (y_1 - \hat{y}_1) = 0$$

$$P_2 \Rightarrow (y_2 - \hat{y}_2) = 0$$

$$P_3 \Rightarrow (y_3 - \hat{y}_3) = 2$$

$$P_4 \Rightarrow (y_4 - \hat{y}_4) = 1$$

$$\text{Eqn} \Rightarrow \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$\Rightarrow (0)^2 + (0)^2 + (2)^2 + (1)^2$$

$$= 4 + 1 = \underline{5}$$

According to  $L_2 \Rightarrow$

$$P_1 = (y_1 - \hat{y}_1)^2 = (1)^2$$

$$P_2 = (y_2 - \hat{y}_2)^2 = (1)^2$$

$$P_3 = (y_3 - \hat{y}_3)^2 = 0$$

$$P_4 = (y_4 - \hat{y}_4)^2 = (2)^2$$

$$\left. \begin{array}{l} P_1 = (y_1 - \hat{y}_1)^2 = (1)^2 \\ P_2 = (y_2 - \hat{y}_2)^2 = (1)^2 \\ P_3 = (y_3 - \hat{y}_3)^2 = 0 \\ P_4 = (y_4 - \hat{y}_4)^2 = (2)^2 \end{array} \right\} \text{Eqn} = (1)^2 + (1)^2 + (0)^2 + (2)^2$$

$$= 4 + 1 + 1$$

$$= 6$$

$\therefore$  Total Error made by line ( $L_1$ ) is 5 and ( $L_2$ ) is 6 so  $L_1$  has min-error as compared to  $L_2$ , Hence we will go through  $L_1$  as Best-fit Line for this dataset.

$$\text{Min} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$



So we want to minimize the total Error (or) Sum Squared Error

$\therefore$  Minimum Error is the Best Fit line.

The line which gives minimum Error is the BEST FIT LINE

$$\min \sum \text{Error}^2$$

$$\min \sum (\text{actual}_i - \text{predicted}_i)^2$$

$$\min \sum |\text{actual}_i - \text{predicted}_i|$$

The Square value will get magnitized, the absolute value will not get magnitized; By doing Square we can magnified value and by doing absolute we can't magnify value.

$$m^*, c^* = \min \sum_{i=1}^n (y_i - \hat{y}_i)^2 \rightarrow \begin{array}{l} * \text{ Represent} \\ \text{Optimal value} \end{array}$$

$\rightarrow$  A. value  $\rightarrow$  p. value.

So here predicted value of  $y \rightarrow (\hat{y}_i)$  is given by the line  $(y = mx + c)$

$$m^*, c^* = \arg \min_{m, c} \left\{ \sum_{i=1}^n (y_i - \hat{y}_i(m, c))^2 \right\}$$

if we know  $(m, c)$  values, then any future data point comes.

We can easily predict the value by pointing all values in above eqn.

Note :- We Select the line in such way which gives the minimum

Error.

By using Gradient Descent  $\rightarrow$  Used in ML, DL

- \* By Using linear Regression we will get the BEST FIT Line
- \* While using linear Regression our data doesn't contain Outliers
- \* In above eqn if we find argument  $m, c$  that gives best fit line with minimum Error.

$$\text{Eqn} - \boxed{m^*, c^* = \arg_{m, c} \min \left\{ \sum_{i=1}^n (y_i - (mx_i + c))^2 \right\}}$$

is also called as Ordinary Least Square (OLS), ~~only for continuous data~~

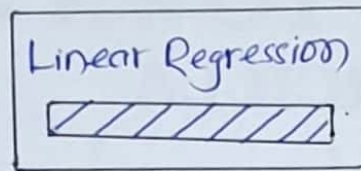
$$\begin{aligned} & \min \left\{ \sum_{i=1}^n (y_i - (mx_i + c))^2 \right\} \\ & \text{or} \\ & \min \left\{ \sum_{i=1}^n (y_i - \hat{y}_i)^2 \right\} \end{aligned} \rightarrow \text{Error / Residual Error}$$

- \* We have to do regression task only when input & output both are mapping and output is Continuous in Supervised Learning.
- \* All the points which are lying on line ( $y = mx + c$ ) that gives error as zero, and Best-fit line gives minimum Error.
- \* The points which are not lying on line will give Error and we call it as (Training Error)  $(y_i - \hat{y}_i)$
- \* if we don't square the error, the total Error made by the line is zero, but by visualizing the points there is an error.
- \* In Eqn  $x_i, y_i$  is data given in training data.



When Data given to Linear Regression :-

x	y
DATA	



→  $m^*, c^*$

This gives the line with minimum error by optimizing  $m, c$ . and there is a technique (Algorithm) to Optimizing  $m$  &  $c$  values.

When Data Given to Logistic Regression :-

x	y
DATA	



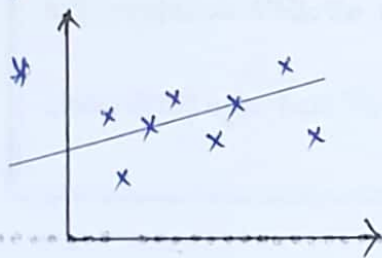
→  $m^*, c^*$

\* Logistic Regression is used for classification (Discrete / ~~Continuous~~ <sup>categorical</sup>)

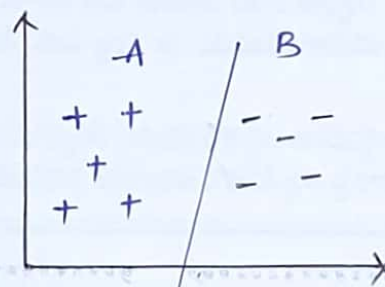
Type of tasks.

\* there is an algorithm technique for Optimizing  $m, c$  values in Logistic R.

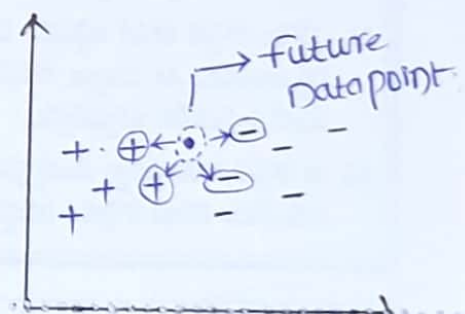
\* In this we draw a line that separates the classes.



Linear Regression



Logistic Regression



KNN Neighbour

→ In case of KNN Neighbour, if any future Data point come, then we finding the  $k$ -nearest neighbour to that point by using the Euclidean distance.



→ In KNN there is no Separation of the classes.

→ In logistic regression we draw a line that separate class, if in case the point lying on line we can't say anything.

## Linear Regression vs Logistic Regression

1) Supervised Learning Technique Supervised Learning.

2) Regression Task

3) O/p variable is Continuous

4)

X	Y
$x_1$	$y_1$
$x_2$	$y_2$
$\vdots$	$\vdots$
$x_n$	$y_n$

$$\mathcal{D}_n = \{(x_i, y_i)_{i=1}^n \mid y_i \in \mathbb{R}\}$$

5) Task - Find a line  $(m^*, c^*)$  that

BEST FIT the training data.

→ Minimizing the Sum of Squared Errors.

$$\min \sum_{i=1}^n (y_i - (mx_i + c))^2$$

Classification Tasks

O/p variable is Discrete

$$\mathcal{D}_n = \{(x_i, y_i)_{i=1}^n \mid y_i \in \{+1, -1\}\}$$

Where  $\mathcal{D}$  = Dataset

$n$  = no of Data points.

→ Find a line that BEST SEPARATE +ve's from -ve's.

$$\{(x_i, y_i)_{i=1}^n \mid y_i \in \{+1, -1\}\}$$