MCA UT Solution -2019

MCA·II year.

Sub: -CBOT

Code : RCA-304.

Session-2019-20
Max Marks -70
Section- MCA-1

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Section-A

Question-1(a) Write the algorithm of graphical solution for LP models.

Solution:-

Step1 → Formulate the LPP with given variables

Step 2 → Convert the inequalities of given constraints to equalities

Step3 → Solve the two equatn to find the value of the variables of constraints.

Step4 → Represent (x, y) on the x axis & y axis

Step 5 → Calculate the co-ordinates of the intersection point of the two equation of constraint if available

Step6 → find the value of the objective function. and intersection point.

Question1 (b) Define following terms used in LPP

(i) Basic feasible Solution

(ii) Optimum feasible Solution.

Solution:- i) It is defined as solution with min. No. of non-zero variables

→ This solutn gives the Satisfiable response to the LPP.

(ii) It is defined as the Solution where the objective function satisfies the maximum or min value for which it is given.

→ It always gives the best optimum response.

<u>Question 1(c)</u> Formulate transpartation problem as an LPP.

<u>Solution:-</u>

$$\text{Min. } Z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}\, x_{ij}$$

St

$$\sum_{j=1}^{m} x_{ij} = a_i \quad \{ i = 1, 2, \ldots n \}$$

$$\sum_{i=1}^{n} x_{ij} = b_j \quad \{ j = 1, 2 \ldots m \}$$

$$\boxed{\sum_{j=1}^{m} a_i = \sum_{i=1}^{n} b_j} \quad \forall \; x_{ij} \geq 0$$

<u>Question 1(d)</u> Define Poisson distribution

<u>Solution</u> Poission arrival in time t is given by

$$P_n(T) = \frac{\lambda t^n e^{-\lambda T}}{\underline{|n}}$$

then $n = 0$

$$\boxed{P_0(T) = \lambda e^{-\lambda T}}$$

Question 1(e) List the five applications of inventory model.

Solution:-

1. Demand
2. Shortage Cost
3. Holding Cost
4. Lead time
5. Accounting.

Question 1 (f) Define characteristics of NLPP

Solution:- i) It contains atleast one non-linear function.

ii) It is applicable to the problem where the constraint & objective function may be non-linear in nature

iii) It form a necessary and sufficient condition

iv) Various techniques are:-

→ Lagrange Function

→ Kuhn-tucker Problem

→ Wolfe's method.

Question 1 (g) Define transient state & steady state.

Solution:-

Transient State:- It is the state in which the probability of the customer is dependent on time t

Steady state:→ It is the state in which the probability of the customer is independent of time t

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Section - B

Question 2 (a) What is an inventory System? Explain clearly the different costs that are involved in inventory problems with suitable examples.

Solution:- Inventory can be define as a stock of goods which is held for the purpose of future production or Sale.

The word inventory says that any kind of resource that has economic value and is maintained to fulfill the present & future needs of the organization.

Different Cost that are involved in inventory problem are as follows.

1. Ordering Cost / Setup Cost:-

The Cost associated with placing an order for purchasing the goods or it is the cost of setting a machine before it starts production.

Example:-

Cost of finding supplies and expediting orders, cost of moving goods 2 warehouse or store.

2. Holding Costs / Storage Cost

Also Known as Carrying costs, cost associated with the storage of inventory until it is sold of used.

example:-

Inventory finacing costs, storage spacecosts.

3. Shortage Cost | Penalty cost:-

- Also known as stock-out costs.
- The unsatisfied demand or shortage penalty cost occur when the stock quantity in inadequate the demand of the customer.

Example:-

Emergency shipments for retailers, customer loyalty & reputation from loss of business from customers.

Question 2 (b) Solve the following LPP by dual simplex method

Minimize $z = 2x_1 + x_2$

s.t.

$$3x_1 + x_2 \geq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Solution:-

Step 1:- First we write problem in maximize form

$$\text{Max } z' = -2x_1 - x_2$$

St.

$$-3x_1 - x_2 \leq -3$$
$$-4x_1 - 3x_2 \leq -6$$
$$-x_1 - 2x_2 \leq -3$$
$$x_1, x_2 \geq 0$$

Step 2 Now adding slack variables x_3, x_4, x_5 in each constraint

$$-3x_1 - x_2 + x_3 = -3$$
$$-4x_1 - 3x_2 + x_4 = -6$$
$$-x_1 - 2x_2 + x_5 = -3$$

Consequently, in matrix form equations

$$\begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array}$$
$$\begin{bmatrix} -3 & -1 & 1 & 0 & 0 \\ -4 & -3 & 0 & 1 & 0 \\ -1 & -2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ -3 \end{bmatrix}$$

we construct a simplex method

B	C_B	X_B	y_1	y_2	y_3	y_4	y_5	min ratio
x_3	0	−3	$\boxed{-3}$	−1	1	0	0	$-3/-3 = 1 \rightarrow$
x_4	0	−6	−4	−3	0	1	0	$-6/-4 = 3/2$
x_5	0	−3	−1	−2	0	0	0	$-3/-1 = 3$
		ΔJ	2 ↑	1	0	0	0	

								Negative
y_1	2	1	1	1/3	−1/3	0	0	
y_4	0	−2	0	−5/3	−4/3	1	0	$3/2 - 1.5$
y_5	0	−2	0	−5/3	$\boxed{-1/3}$	0	1	$6/5 - 1.2 \rightarrow$
			0	1/3	2/3 ↑	0	0	

both constraints become positive so no further solution can be further improved.

Question 2(c) A steel company has three open earth furnaces & five rolling mills. Transportation cost (rupees per quintal) for shipping steel from furnaces to rolling mills are shown.

	M_1	M_2	M_3	M_4	M_5	Capacity
F1	4	2	3	2	6	8
F2	5	4	5	2	1	12
F3	6	5	4	7	3	14
requirement →	4	4	6	8	8	

Solution:-

	M_1	M_2	M_3	M_4	M_5	Capacity (a_i)
F1	4	2	3	2	6	8
F2	5	4	5	2	1	12
F3	6	5	4	7	3	14
Req. (b_j)	4	4	6	8	8	34 | 30

$$\Sigma a_i \neq \Sigma b_j$$

	M1	M2	M3	M4	M5	M6	Capacity	Penalty
F1	4	2	3	2	6	0	8	2
F2	5	4	5	2	1	0	12	1
F3	6	5	4	7	3 (8)	0 (4)	~~14~~ 60	3=
Req.	4	4	6	8	8	4	$\Sigma a_i = \Sigma b_j$	
Penalty	1	2	1	0	2	0		

	M1	M2	M3	M4	M5	Capacity	Penalty
F1	4	2	3	2	6	8	0
F2	5	4	5	2	1 (8)	~~12~~ 4	1
F3	6	5	4	7	3	10	1
Req →	4	4	6	8	~~8~~		
Penalty	1	2	1	0	2 ↑		

	M1	M2	M3	M4	Capacity	Penalty
F1	4	2 (4)	3	2	4 ~~8~~	0
F2	5	4	5	2	4	2
F3	6	5	4	7	10	1
req.	4	~~4~~	6	8		
	1	2 ↑	1	0		

	M1	M3	M4	Capacity	Penalty
F1	4	3	2	4	1
F2	5	5	2 (4)	~~4~~	3 ←
F3	6	4	7	10	2
req.	4	6	~~8~~ 4		
	1	1	0		

	M1	M3	M4	Capacity	Penalty
F1	4	3	2 (4)	~~4~~	1
F3	6	4	7	10	2
req.	4	6	~~4~~		
	2	1	5 ↑		

	M1	M3	Capacity	Penalty
F3	6 (4)	4 (6)	~~10~~	2 ←
req →	~~4~~	~~6~~		

Total cost = $0 \times 4 + 1 \times 8 + 2 \times 4 + 2 \times 4 + 2 \times 4 + 6 \times 4 + 6 \times 4$

$\Rightarrow 0 + 8 + 8 + 8 + 8 + 24 + 24$

$= 80$

Total allocation = 7 which is not equal to $(m+n-1)$

	M1	M2	M3	M4	M5	M6	u_i
F1	4 (Δ)	2 (4)	3 $\boxed{3}$ /0	2 (4)	6 $\boxed{1}$ /5	0 $\boxed{-1}$ /1	−1
F2	5 $\boxed{5}$ /0	4 $\boxed{2}$ /2	5 $\boxed{3}$ /2	2 (4)	1 (8)	0 $\boxed{-1}$ /1	−1
F3	6 (4)	5 $\boxed{3}$ /2	4 (6)	7 $\boxed{3}$ /4	3 $\boxed{2}$ /1	0 (4)	0
v_j	6	3	4	3	2	0	

No. of allocating = $m+n-1 = 8$

Hence solution is optimal.

Question 2 (d) Use the Wolfe's method to solve the quadratic programming problem

$$\text{Max. } Z = 2x_1 + x_2 - x_1^2$$

s.t.

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Solution:- Write the equation in standard form.

$$\text{Max } z = f(x) = 2x_1 + x_2 - x_1^2$$

S.t.

$$2x_1 + 3x_2 + S_1^2 = 6$$
$$2x_1 + x_2 + S_2^2 = 4$$
$$-x_1 + r_1^2 = 0$$
$$-x_2 + r_2^2 = 0$$

Lagrange function.

$$L(x_1, x_2, \lambda_1, \lambda_2, S_1, S_2, \mu_1, \mu_2) = (2x_1 + x_2 - x_1^2) - \lambda_1(2x_1 + 3x_2 + S_1^2 - 6) - \lambda_2(2x_1 + x_2 + S_2^2 - 4) - \mu_1(-x_1 + r_1^2) - \mu_2(-x_2 + r_2^2)$$

Necessary & Sufficient Condition for function L is to be maximum that.

$$\frac{\delta L}{\delta x_1} = 2 - 2x_1 - 2\lambda_1 - 2\lambda_2 + \mu_1$$

$$\frac{\delta L}{\delta x_2} = 1 - 3\lambda_1 - \lambda_2 + \mu_2$$

$$\frac{\delta L}{\delta \lambda_1} = (2x_1 + 3x_2 + S_1^2 - 6)$$

$$\frac{\delta L}{\delta \lambda_2} = (2x_1 + x_2 + S_2^2 - 4)$$

$$\frac{\delta L}{\delta S_1} = -2\lambda_1 S_1 \quad , \quad \frac{\delta L}{\delta S_2} = -2\lambda_2 S_2$$

$$\frac{\delta L}{\delta \mu_1} = -x_1 + r_1^2 \quad , \quad \frac{\delta L}{\delta \mu_2} = -x_2 + r_2^2$$

Simplified equation

$$2x_1 + 2\lambda_2 + 2\lambda_2 - \mu_1 = 2 \quad \text{——} \ (1)$$

$$3\lambda_1 + \lambda_2 - \mu_2 = 1 \quad \text{——} \ (2)$$

$$2x_1 + 3x_2 + S_1^2 = 6 \quad \text{——} \ (3)$$

$$2x_1 + 3x_2 + S_2^2 - 4 \quad \text{——} \ (4)$$

$$\lambda_1 S_1 = 0 \ , \ \lambda_2 S_2 = 0$$

$$\mu_1 x_1 = \mu_2 x_2 = 0$$

Let us introduce 2 artifical variable A_1 & A_2 with cost -1 such that

$$\text{Min } z^* = A_1 + A_2$$

St.

$$2x_1 + 2\lambda_1 + 2\lambda_2 - \mu_1 + A_1 = 2$$

$$3\lambda_1 + \lambda_2 - \mu_2 + A_2 = 1$$

$$2x_1 + 3x_2 + S_1^2 = 6$$

$$2x_1 + x_2 + S_2^2 = 4$$

B	CB	XB	x_1	x_2	λ_1	λ_2	S_1	S_2	A_1	A_2	Min ratio
A_1	1	2	$\boxed{2}$	0	2	2	0	0	1	0	1 →
A_2	1	1	0	0	3	1	0	0	0	1	∞
S_1	0	6	2	3	0	0	1	0	0	0	3
S_2	0	4	2	1	0	0	0	1	0	0	2
		ΔJ	↑2	0	5	3	0	0	0	0	

x_1	0	1	1	0	1	1	0	0	~~1/2~~	0	0
A_2	1	1	0	0	3	1	0	0		1	0
S_1	0	4	0	$\boxed{3}$	−2	−2	1	0		0	4/3 →
S_2	0	2	0	1	−2	−2	0	1		0	2
		ΔJ	0	↑0	3	1	0	0		0	

B	C_B	X_B	x_1	x_2	λ_1	λ_2	S_1	S_2	Min ratio
x_1	0	1	1	0	1	1	0	0	1
A_2	0	1	0	0	3	1	0	0	$1/3 \longrightarrow$
x_2	0	$4/3$	0	1	$-2/3$	$-2/3$	$1/3$	0	-2
S_2	0	$2/3$	0	0	$-4/3$	$-4/3$	$-1/3$	1	Neg
		Δ_J	0	0	$3\uparrow$	1	0	0	

x_1	0	$2/3$	1	0	0	$2/3$	0	0
λ_1	0	$1/3$	0	0	1	$1/3$	0	0
x_2	0	$14/9$	0	1	0	$-4/9$	$1/3$	0
S_2	0	$10/9$	0	0	0	$-8/9$	$-1/3$	1
		Δ_J	0	0	0	0		

$x_1 = 2/3, \quad \lambda_1 = 1/3, \quad x_2 = 14/9, \quad S_2 = 10/9$

$$\text{Max } z = 2x_1 + x_2 - x_1^2$$

$$= 2 \times \frac{2}{3} + \frac{14}{9} - \frac{4}{9}$$

$$= \frac{22}{9} \quad \text{Ans}$$

Question-2(e) Explain Exponential distribution and Erlang distribution

Solution:-

The Sum of n mutually independent exponential destribution raddom variables, each with common population mean $\mu > 0$ is an Erlang (μ, λ) random variable in a queueing theory

$$P_n = \rho^n (1-\rho) \qquad \left[\rho = \frac{\lambda}{\mu}\right]$$

$$= \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

E[LS] = Expected line of length of System
E[Lq] = Expected line of length of queue
W[LS] = waiting time of length of system
W[Lq] = waiting time of length of queue

$$E[LS] = \sum_{n=1}^{\infty} n P_n$$

$$= \sum_{n=1}^{\infty} n \rho^n (1-\rho)$$

$$= \rho(1-\rho) \sum_{n=1}^{\infty} n \rho^{n-1}$$

$$= \rho(1-\rho) \times (1-\rho)^{-2}$$

$$= \frac{\rho}{1-\rho}$$

$$= \frac{\lambda/\mu}{1-\lambda/\mu}$$

$$= \frac{\lambda}{\mu - \lambda}$$

$$w[L_s] = \frac{E[L_q]}{\lambda}$$

$$= \frac{1}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\mu-\lambda}$$

$$= \frac{1}{\mu-\lambda}$$

$$E[L_q] = \sum_{n=1}^{\infty} (n-1) P_n$$

$$= \sum_{n=1}^{\infty} nP_n - P_n$$

$$= \sum_{n=1}^{\infty} nP_n - \sum_{n=1}^{\infty} P_n$$

$$= \frac{\rho}{1-\rho} - (1-\rho)$$

$$= \frac{\rho^2}{1-\rho} \Rightarrow \frac{\lambda^2/\mu^2}{\frac{\mu-\lambda}{\mu}} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$w[L_q] = \frac{E[L_q]}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)}$$

Erlang Distribution :- It is same as the model of Birth & death.

λ_n = mean arrival rate when there are n customer in the system.

μ_n = there are n customer who depart from the system

$$P_n(t+\Delta t) = P_{n-1}(t)(\lambda_{n-1}\Delta t)(1-\mu_{n-1}\Delta t) + P_n(t)(1-\lambda_n\Delta t)(1-\mu_n\Delta t) + P_{n+1}(t)(1-\lambda_{n+1}\Delta t))(\mu_{n+1}\Delta t)$$

$$P_1 = \left(\frac{\lambda_0}{\mu_0}\right) P_0$$

$$P_2 = \left(\frac{\lambda_0\lambda_1}{\mu_0\mu_1}\right) P_0$$

$$\vdots$$

$$P_n = \left(\frac{\lambda_0\lambda_1\lambda_2 \cdots \lambda_n}{\mu_0\mu_1\mu_2 \cdots \mu}\right) P_0$$

Section-C

Question 3(a) what are various assumptions of EOQ formula. what are limitations of EOQ

Solution:

There are three assumptions of EOQ

1. Economic lot size with uniform rate of demand infinite production rate & having no shortages

Here,

$$q^* = \sqrt{\frac{2C_3 r}{C_1}}$$

$$t^* = \sqrt{\frac{2C_3}{rC_1}}$$

$$C_{min} = \sqrt{2C_1C_3 r}$$

2. Economic lot size model with different rate of demand in different production in finite production rate having no shortage.

Here

$$q^* = \sqrt{\frac{2RC_3}{tC_1}} \qquad t = \frac{q}{r}$$

$$C_{min} = \sqrt{\frac{2C_1C_3R}{t}}$$

3. Economic lot size model with uniform rate of demand, finite rate of replenishment having no shortage

$$q^* = \sqrt{\frac{2C_3(rk)}{C_1(k-r)}}$$

$$C_{min} = \sqrt{2C_1C_3 r\left(1-\frac{k}{r}\right)}$$

where C_1 = holding cost , r = demand rate
C_3 = setup cost , t = time interval b/w two consecutive
q = lot of size.

Limitations:-

1) Demand is uniform at the rate r per unit time
2) Lead time is zero
3) C_1 is holding cost per unit time
4) C_3 is setup cost
5) Production rate is infinite
6) Shortages are not allowed

Question 3 (b) A pipeline is due for repairs. It will cost Rs 10,000 & lasts for 3 yrs, Alternatively, a new pipeline can be laid down at a cost of Rs 30,000 and lasts for 10 yrs. Assuming cost of capital to be 10% and ignoring salvage value, which alternative should be chosen?

Solution :- Let us consider two types of pipeline for infinite replacement cycles of 10 yr for the new pipline & 3 yr for a pipline

Amt. rate of money per yr is 10%

$$v = \frac{100}{100+r}$$

$$= \frac{100}{100+10} = \frac{100}{110} = 0.9091$$

To be discounted value of all future costs associated with a policy of replacing the equipment after n yrs

So if we design

$$d_n = C + Cv^n + Cv^{2n} + \cdots$$

$$= C[1 + v^n + v^{2n} - - -]$$

$$= \frac{C}{1-v^n}$$

making use of value of C, v, and n for two types of pipelines

$$d_3 = \frac{10000}{1-(0.9091)^3} = \underline{\underline{4021}}$$

$$d_{10} = \frac{30,000}{1-(0.9091)^{10}} = \frac{30,000}{1-0.3855}$$

$$= \underline{\underline{48,820}}$$

$d_3 < d_{10}$ the present pipeline should remain continued, otherwise the comparsion may be done over $3 \times 10 = 30$ years

Question 4 (a) Use Big M method

$$\min Z = x_1 - 3x_2 + 2x_3$$

Constraint

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Solution $\longrightarrow$

Big M method

Sol. :- $\text{Min } z = x_1 - 3x_2 + 2x_3$

St

$$3x_1 - x_2 + 2x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2 \, x_3 \ge 0$$

Step1 Problem is not max. So convert

$$\max z' = -\text{Min} z = -x_1 + 3x_2 - 2x_3$$

Step2 All bis are already positive

Step3 change the inequality into equality by using slack / surplus variable.

$$3x_1 - x_2 + 2x_3 + x_4 = 7$$
$$-2x_1 + 4x_2 + x_5 = 12$$
$$-4x_1 + 3x_2 + 8x_3 = 10$$

Here x_4, x_5, x_6 are slack variable

Step4 Matrix form

$$AX = B$$

$$\begin{array}{cccccc} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\ \end{array}$$
$$\begin{bmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ -2 & 4 & 0 & 0 & 1 & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 10 \end{bmatrix}$$

$$\text{Max } z' = -x_1 + 3x_2 - 2x_3 + 0x_4 + 0x_5 + 0x_6$$

Construct the Simplex table

B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Min ratio
Y_4	0	7	3	-1	2	1	0	0	Neg
Y_5	0	12	-2	$\boxed{4}$	0	0	1	0	3 →
Y_6	0	10	-4	3	8	0	0	1	3.3
		ΔJ	-1	↑3	-2	0	0	0	

y_4	0	10	5/2	0	2	1	1/4	0	4 →
y_2	3	3	-1/2	1	0	0	1/4	0	Neg
y_6	0	1	-5/2	0	8	0	-3/4	1	Neg
		ΔJ_r	1/2	0	-2	0	-3/4	0	

y_1	-1	4	1	0	4/5	2/5	1/10	0
y_2	3	5	0	1	2/5	1/5	3/10	0
y_6	0	11	0	0	10	1	-1/2	1
		Δj	0	0	-12/5	-1/5	-4/5	0

All $\Delta j \leq 0$

$y_1 = x_1 = 4$, $y_2 = x_2 = 5$, $x_3 = 0$

Max $Z = -4 \times 3(5)$

$= -4 + 15$

$\boxed{\text{Max } Z = 11}$

Question-4(b) Prove that dual of dual is given primal itself.

Solution:-

Show that dual of a dual of a given primal is primal itself

$$\left.\begin{array}{l} \text{Max } Z_p = CX \\ \text{St} \\ \quad AX \le b \\ \quad X \ge 0 \end{array}\right\} \text{primal form} \longrightarrow (1)$$

Then dual of the given problem

$$\text{Min} Z_D = b'W$$

St

$$A'W \ge C'$$

$$W \ge 0$$

Considering the dual problem eq(2) is a primal

$$\text{Max } Z_D' = (C')'V$$

St

$$(A')'V \le (b')'$$

$$V \ge 0$$

Since $(C')' = C$, $(b')' = b$, $(A')' = A$

Such that

$$\left.\begin{array}{l} \text{Max } Z_D = CV \\ AV \le b \\ V \ge 0 \end{array}\right\} \longrightarrow (2)$$

eq(4) is identical to eq(1)

Hence proved

Question 5(a) Solve Problem by Gomary's algorithm

$$\text{Max } Z = 3x_1 + 4x_2$$

S.t.

$$x_1 + x_2 < 4$$

$$0.6x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0 \;\&\; x_1 \,\&\, x_2 \text{ are integer}$$

Solution:-

Since X_1 & X_2 are required as Integer. and Constraints given are not in Integer form. So problem is not Salvable with these Integral value. hence using the Gomory's algorithm, Constraints should not be in form of (.6)X_1, hence problem is not Salvable in this Case.

Question 5(b) Five men are available to do five different job. From past record the time that each man takes to do each job is known and given in the following.

	I	II	III	IV	V
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Solution:-

Subtract each row by smallest one

	I	II	III	IV	V
A	1	8	1	6	0
B	5	7	6	5	0
C	3	5	4	2	0
D	3	1	6	2	0
E	4	2	8	4	0

Subtract each column by smallest one

	I	II	III	IV	V	
A	$\boxed{0}$	7	~~0~~	4	~~0~~	
B	4	6	5	3	$\boxed{0}$	✓
C	2	4	3	$\boxed{0}$	~~0~~	
D	2	$\boxed{0}$	5	~~0~~	~~0~~	
E	3	1	7	4	~~0~~	✓

Here No of job is not equal to No. of men then we use hungarian method.

	I	II	III	IV	V
A	$\boxed{0}$	7	$\cancel{0}$	4	1
B	3	5	4	2	$\boxed{0}$
C	2	4	3	$\boxed{0}$	1
D	2	$\boxed{0}$	5	$\cancel{0}$	1
E	2	$\cancel{0}$	6	3	$\cancel{0}$

	I	II	III	IV	V
A	$\boxed{0}$	7	$\cancel{0}$	4	3
B	1	3	2	$\cancel{0}$	$\boxed{0}$
C	2	4	3	$\boxed{0}$	3
D	2	$\boxed{0}$	5	$\cancel{0}$	3
E	$\cancel{0}$	$\cancel{0}$	4	1	0

	I	II	III	IV	V
A	$\cancel{0}$	7	$\boxed{0}$	4	4
B	$\boxed{0}$	2	1	$\cancel{0}$	$\cancel{0}$
C	2	4	3	$\boxed{0}$	4
D	2	$\boxed{0}$	5	$\cancel{0}$	4
E	$\cancel{0}$	$\cancel{0}$	3	$\cancel{0}$	$\boxed{0}$

A – III → 2

B – I → 6

C – IV → 3

D → II → 2

E → V → 1

14 – Total cost

Question 6 (a) Use DP to Solve LPP

$$\text{Max } z = 3x_1 + 5x_2$$

$$x_1 \le 4$$
$$x_2 \le 6$$
$$3x_1 + 2x_2 \le 18$$
$$x_1, x_2 \ge 0$$

Solution:-

Step1 All problem are maximize

Step2 All bis are already positive

Step3 Change the inequality into equality by using slack/ surplus method

$$x_1 + x_3 = 4$$
$$x_2 + x_4 = 6$$
$$3x_1 + 2x_2 + x_5 = 18$$

Here x_3, x_4, x_5 are the variable write a equation

$$\text{Max } z' = 3x_1 + 5x_2 + 0x_3 + 0x_4 + 0x_5$$

Form a matrix

$$\begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array}$$
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 18 \end{bmatrix}$$

Using a Simplex table

B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	min. ratio
Y_3	0	4	1	0	1	0	0	0
Y_4	0	6	0	$\boxed{1}$	0	1	0	6 →
Y_5	0	18	3	2	0	0	1	9
		ΔJ	3	↑5	0	0	0	

Y_4 out & Y_2 in

B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	min.. ratio
Y_3	0	4	1	0	1	0	0	4
Y_2	5	6	0	1	0	1	0	∞
Y_5	0	6	$\boxed{3}$	0	0	-2	1.	2 →
		ΔJ	3↑	0	0	-5	0	

Y_1 in & Y_5 out

B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	min ratio
Y_3	0	2	0	0	1	2/3	-1/3	
Y_2	5	6	0	1	0	1	0	
Y_1	3	2	1	0	0	-2/3	1/3	
		ΔJ	0	0	0	-3	-1	

All $\Delta J \leq 0$

Solution is optimal $\begin{cases} Y_1 = x_1 = 2 \\ Y_2 = x_2 = 6 \end{cases}$

$\max Z = 3x_1 + 5x_2 + 0 + 0 + 0$

$\Rightarrow 3 \times 2 + 5 \times 6$

$\Rightarrow 6 + 30$

$\boxed{\max Z = 36}$

Question-6(b) Derive Kuhn-Tucker necessary condition for an optimum Solution to a quadratic programming problem.

Solution:-

The Necessary and Sufficient Kuhn-tucker conditions to get an optimal Solution to the problem of maximizing the given quadratic Objective function subject to linear constraints.

Step1 Introducing Slack variables S_i^2 & r_j^2 to constraints (3) and (4) the problem becomes

$$\text{Max } f(x) = \sum_{j=1}^{n} c_j x_j - \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} x_j d_{jk} x_k$$

Subject to the Constraints.

$$\sum_{j=1}^{n} a_{ij} x_j + S_i^2 = b_i; \quad i = 1, 2 --- m$$

$$-x_j + r_j^2 = 0 \; ; \; j = 1, 2 --- n$$

Step2 Forming the Lagrange function as follows:-

$$L(x, S, r, \lambda, \mu) = f(x) - \sum_{j=1}^{n} \lambda_i \{a_{ij} x_j + S_i^2 - b_i\}$$

$$- \sum_{j=1}^{n} \mu_j \{-x_j + r_j^2\}$$

Step3 Differentiate $L(x, S, r, \lambda, \mu)$ partially with respect to the components of x, S, r, λ and μ. Then equate these derivates to zero to get the required Kuhn-Tucker necessary conditions.

(i) $c - \frac{1}{2}(2x^T D) - \lambda A + \mu = 0$

or $c_j - \sum_{k=1}^{n} x_k d_{jk} - \sum_{i=1}^{m} \lambda_i a_{ij} + \mu_j = 0 \; ; \; j = 1, 2 \ldots n$

(ii) $-2\lambda S = 0$ or $\lambda_i S_i^2 = 0$

or $\lambda_i \left\{ \sum_{j=1}^{n} a_{ij} x_j - b_i \right\} = 0 \; ; \; i = 1, 2 \ldots n$

(iii) $-2\mu r = 0$ or $\mu_j r_j = 0 \; ; \; j = 1, 2 \ldots n$

$\mu_j x_j = 0 \, , \; j = 1, 2 \ldots n$

(iv) $Ax + S^2 - b = 0$, i.e $Ax \leq b$

or $\sum_{j=1}^{n} a_{ij} x_j \leq b_i \, , \; i = 1, 2 \ldots m$

(v) $-x + r^2 = 0$ i.e $x \geq 0$

or $x_j \geq 0 \quad j = 1, 2 \ldots n$

(vi) $\lambda_i , \mu_j , x_j , S_i , r_j \geq 0$

These conditions are linear constraints involving $2(n+m)$ variables.

Question 7 (a) State the assumptions under which a arrival process is Poission process. Using these assumptions derive the distribution.

Solution:-

If the arrival pattern in queuing problem follows the poission process then the random variable T representing the inter arrival time follows the exponential distribution & vice versa

Poission arrival in Time (T) is given by

$$P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{\lfloor n}$$

If random variable (T) represent if no arrival during the time T is given by

$$P_0(T) = (\lambda T)^0 e^{-\lambda T}$$

$$P_0(T) = e^{-\lambda T}$$

If the time period become T to T+ΔT then

$$P_0(T+\Delta T) = e^{-\lambda(T+\Delta T)}$$

$$= e^{-\lambda T} e^{-\lambda \Delta T}$$

$$= P_0(T)\left[1 - \lambda\Delta T + (\lambda\Delta T)^2 - -\right]$$

ΔT is smallest time is rejected

$$P_0(T+\Delta T) - P_0(T) = -\lambda P_0(T)\,\Delta T$$

$$\frac{P_0(T+\Delta T) - P_0(T)}{\Delta T} = -\lambda P_0(T)$$

· taken $\lim \Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{P_0(T+\Delta T) - P_0(T)}{\Delta T} = -\lambda P_0(T)$$

$$\frac{d}{dt} P_0(T) = -\lambda P_0(T)$$

$$\frac{d}{dt} P_0(T) = -\lambda e^{-\lambda t}$$; negative sign neglected

$$\boxed{P_0'(T) = \lambda e^{-\lambda T}}$$

Question 7(b) A TV repairman find that the spend on his job has an exponential distribution with mean 30 minutes if the repair set in order in which they come in and if arrival of set is approximately poision with an average rate of 10 per 8 hour. What is repairman expected ideal time each day. How many jobs are ahead of average set just brought.

Solution :-

It is given

$$\lambda = \frac{10}{8 \times 60} = \frac{1}{48} \text{ sets/min}$$

$$\mu = 1/30 \text{ sets/min}$$

If there is no unit in the system

$$e = \frac{\lambda}{\mu}$$

$$P_0 = 1 - e$$

$$= 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{1/48}{1/30}$$

$$= 3/8$$

Therefore repairman idle time expected in 8 hr

$$= \frac{3}{\cancel{8}} \times \cancel{8}$$

$$= 3 \text{ hours.}$$

Expected line length

$$E[L_S] = \frac{e}{1-e} = \frac{\lambda/\mu}{1-\lambda/\mu} = \frac{\lambda}{\mu-\lambda} = \frac{1/48}{1/30 - 1/48}$$

$$= \frac{5}{3} \text{ job}$$

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