Fundamentals of Data Science

Practical session 2: Sets; Vector operations

Problem 1. Let A = {0, {1, 2, 3}, 4, {4, 5}, {0, 4}, 1, {7}}. Decide whether each of the following statements are true or false, providing justification for each:
(a) $0 \in A$,
(b) $7 \in A$,
(c) {7} $\subset A$,
(d) {{1, 2, 3}} $\subset A$,
(e) {{0, 4}, 5} $\subset A$,
(f) {1, 2, 3} $\in A$,
(g) {1, 4} $\subset A$,
(h) {1, 2, 3, 4, 5} $\in A$,
(i) $\emptyset \subset A$.

Problem 2. Consider the following tables involving some students and a tutor on two modules:

Module 1	
ID	Role
1001	Student
3291	Student
1194	Student
4003	Tutor

Module 2	
ID	Role
2001	Student
4003	Tutor
1101	Student
3291	Student

(a) Form a new table that is the union of the two tables above.
(b) Produce a new table consisting of those in Module 1 that are *not* in Module 2
(c) Produce a new table consisting of those in both Module 1 and Module 2.

Problem 3. Given U = {x : 1 ≤ x ≤ 16, x is an integer} is the Universal set. A, B and C are further sets in the universe U, and are defined as follows:
A is the set of elements defined as 2^n, where $n \in \mathbb{N}_0$, i.e. n = 0, 1, 2, 3...,
B is the set of elements defined as 3^n-1, where $n \in \mathbb{N}_0$,
and C = {5, 7, 9}.

Find cardinalities of the following sets:
a) V = A ∩ B,
b) W = A ∪ C,
c) X = B ∩ C.

Problem 4. Consider the following subsets of the real numbers:
- $A = \{0, 1\}$,
- $B = (0, 1)$,
- $C = [0, 1]$,
- $D = [0, 1]^c$,
- $E = (0, 1]$,
- $F = [0, 1)$.

Find among them all pairs X, Y such that $X \not\subseteq Y$.

Problem 5. Given vectors

$$\vec{s} = (0,2,2)$$

$$\vec{u} = (1,0,3)$$

$$\vec{v} = (1,0,0)$$

$$\vec{w} = (2,0,6)$$

find

a) $\vec{s} + 3\vec{v}$

b) $\vec{u} \cdot \vec{v}$

c) $\vec{u} \cdot \vec{w}$

d) $2\vec{u} - \vec{w}$

e) $|\vec{s} \times \vec{v}|$

f) $\vec{u} \times \vec{w}$

Problem 6. Find the angles between following pairs of vectors

a) (2, 1, 4) and (2, 0, 1)
b) (1, 3, 2) and (0,2,-3)