Weighting: why?

- Some data entries may be more or less reliable than others – use of different measurement tools, a large dataset created from smaller datasets obtained in different ways etc etc

- We may want to make some data entries more or less important based on their properties

Weighting

Example: weighting in regression

Simple linear regression with weighting

Find the values of a and b so that

$$\frac{\partial E}{\partial a} = \frac{\partial}{\partial a}\sum_i (Y_i - a - bXi)^2 = -2\sum_i \boxed{(Y_i - a - bXi) = 0}$$

and

$$\frac{\partial E}{\partial b} = \frac{\partial}{\partial b}\sum_i (Y_i - a - bXi)^2 = -2\sum_i \boxed{X_i (Y_i - a - bXi) = 0}$$

$$\boxed{\begin{aligned} b &= \frac{\sum_i X_i Y_i - \frac{1}{N}\sum_i X_i \sum_i Y_i}{\sum_i {X_i}^2 - \frac{1}{N}\left(\sum_i X_i\right)^2} \\ a &= \frac{1}{N}\sum_i Y_i - b\frac{1}{N}\sum_{Ni} X_i \end{aligned}}$$

Example: weighting in regression

Simple linear regression with weighting

Find the values of a and b so that

$$\frac{\partial E}{\partial a} = \frac{\partial}{\partial a}\sum_i W_i(Y_i - a - bXi)^2 = -2\boxed{\sum_i W_i(Y_i - a - bXi) = 0}$$

and

$$\frac{\partial E}{\partial b} = \frac{\partial}{\partial b}\sum_i W_i(Y_i - a - bXi)^2 = -2\boxed{\sum_i W_i X_i(Y_i - a - bXi) = 0}$$

$$b = \frac{\sum_i W_i X_i Y_i - \frac{1}{\sum_i W_i}\sum_i W_i X_i \sum_i W_i Y_i}{\sum_i W_i {X_i}^2 - \frac{1}{\sum_i W_i}\left(\sum_i W_i X_i\right)^2}$$

$$a = \frac{1}{\sum_i W_i}\sum_i W_i Y_i - b\frac{1}{\sum_i W_i}\sum_{Ni} W_i X_i$$

Example: weighting in regression

Linear regression with weighting

$$b = \frac{\sum_i W_i X_i Y_i - \frac{1}{\sum_i W_i}\sum_i W_i X_i \sum_i W_i Y_i}{\sum_i W_i X_i{}^2 - \frac{1}{\sum_i W_i}\left(\sum_i W_i X_i\right)^2}$$

$$a = \frac{1}{\sum_i W_i}\sum_i W_i Y_i - b\frac{1}{\sum_i W_i}\sum_{Ni} W_i X_i$$

W_i=1

$$b = \frac{\sum_i X_i Y_i - \frac{1}{N}\sum_i X_i \sum_i Y_i}{\sum_i X_i{}^2 - \frac{1}{N}\left(\sum_i X_i\right)^2}$$

$$a = \frac{1}{N}\sum_i Y_i - b\frac{1}{N}\sum_{Ni} X_i$$

Weighting in kNN

Based on the properties of training data entries

- E.g. when some training data entries can be “trusted” more than others. Most often, because they are obtained from different sources with different characteristic error etc
- Another possibility is that training data has other labels/properties which we want to take into account
- Static weights assigned to each training data points

Based on the properties of “training-test” pairs

- E.g. when some training data entries can be “trusted” more than others, but this depends on the test data entry, which needs to be labelled. Most often, to take into account “distances” in pairs (each pair includes a training data entry and test data entry).
- Another possibility is that training data has other labels/properties which we want to take into account, depending on the test data entry.
- Dynamic weights, different for each pair

Could be both

Weighting in kNN

- The effect of neighbours may change with distance. We might want neighbours which are further away to have lower significance. How do we do this? – Weighting!

- $P_t = \text{sign}(\sum_{j=0}^{k-1} W_j P_{i(j)})$

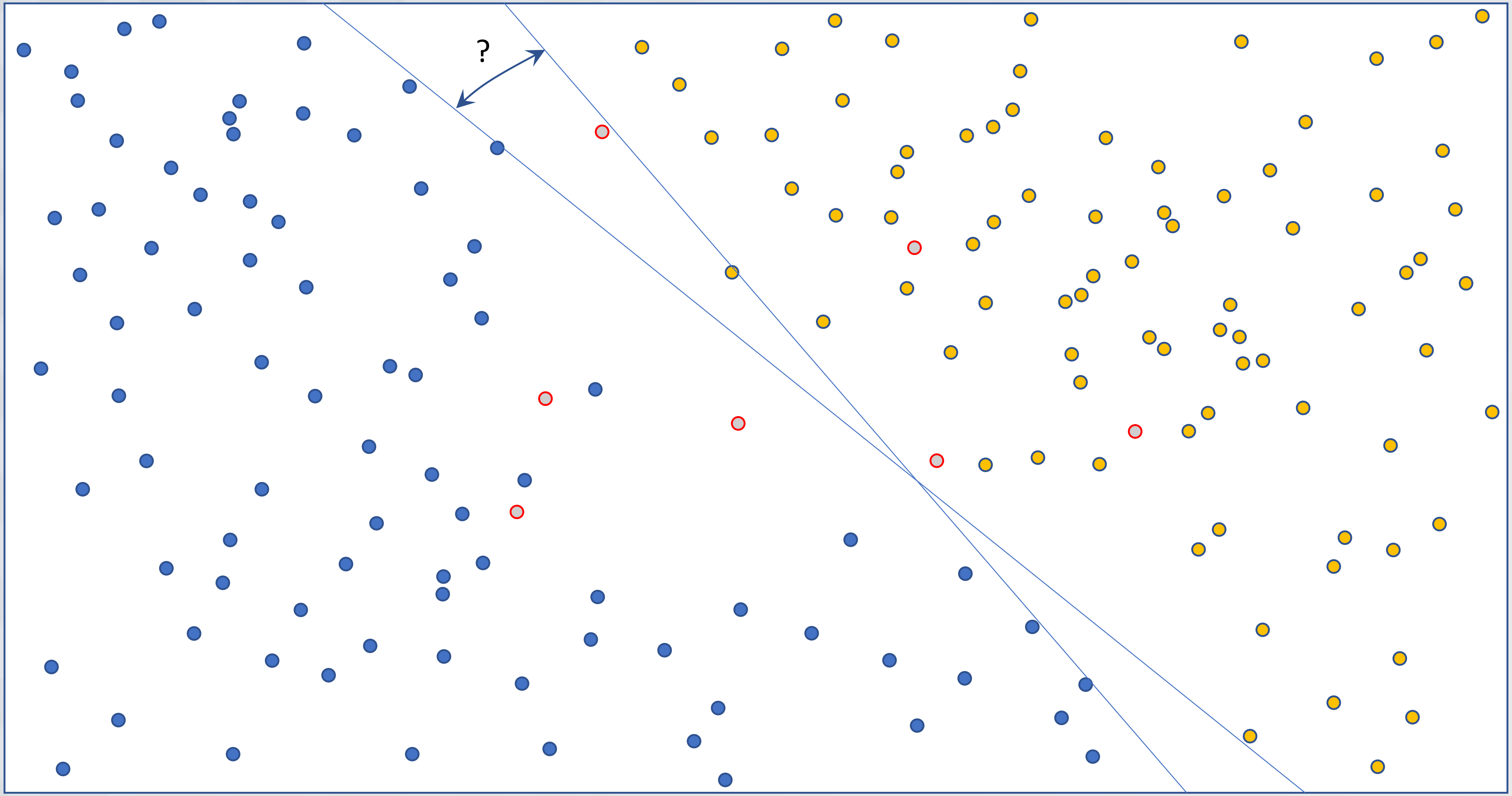
- E.g. $P_t = \text{sign}(\sum_{j=0}^{k-1} e^{-L_j} P_{i(j)})$ - in this case weights exponentially decrease with distance to the corresponding neighbour

Weighting in kNN

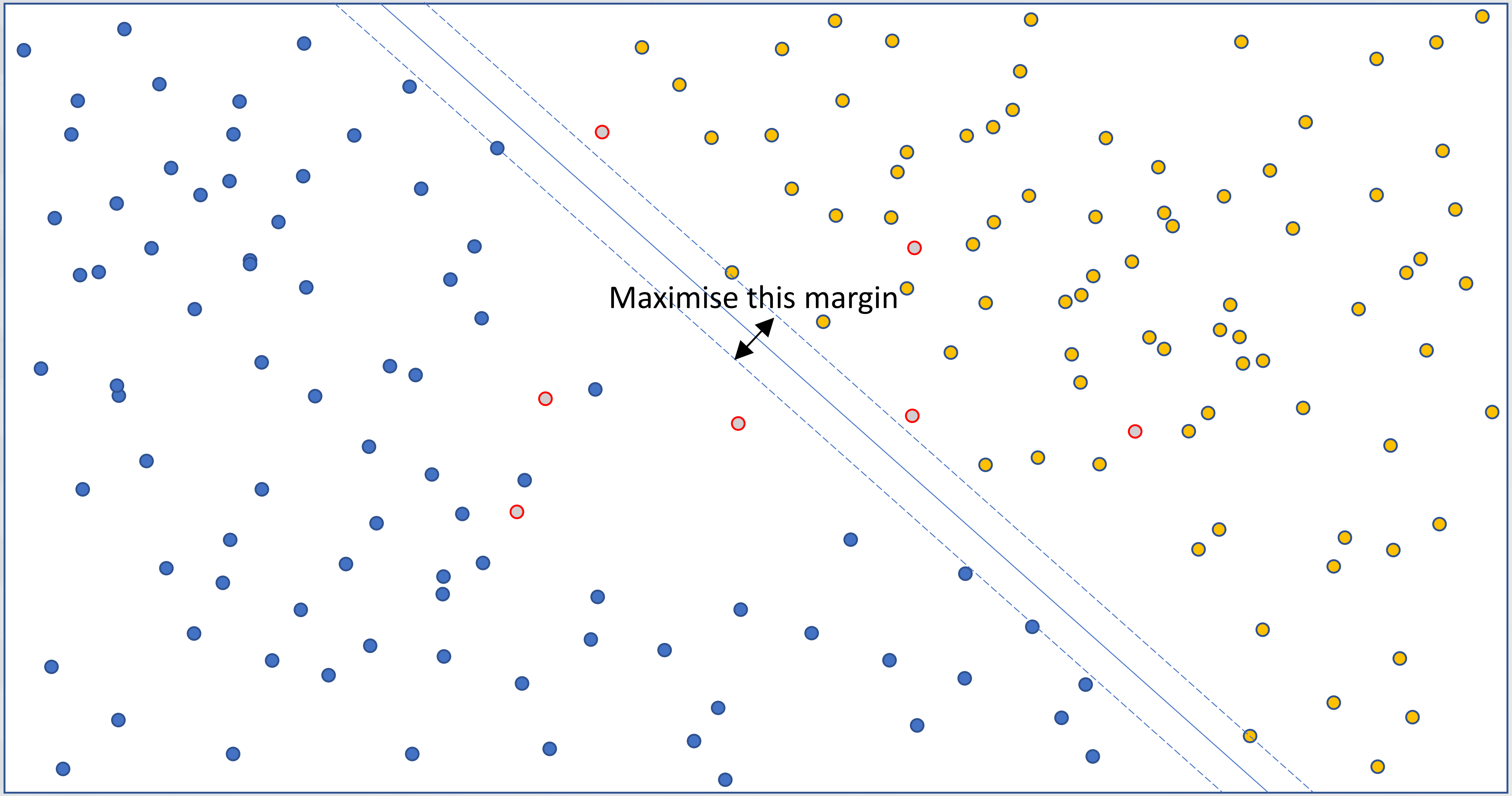
- The effect of neighbours may depend on their reliability. We can give different ‘voting weight’ to each training data entry

- $P_t = \text{sign}(\sum_{j=0}^{k-1} W_{i(j)} P_{i(j)})$ – in this case each entry in the training data has its own weight

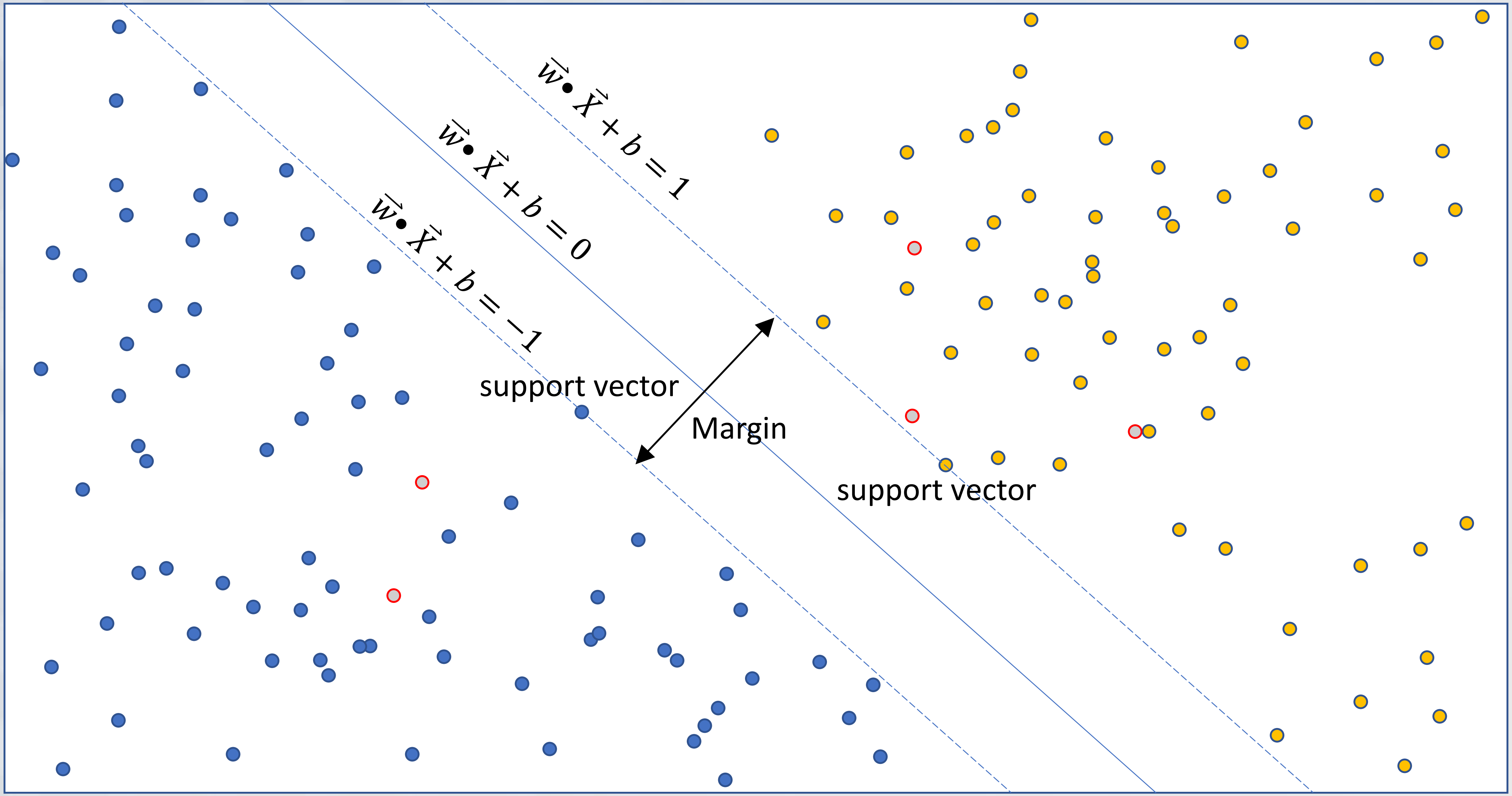
Example



Example



Support Vector Machine (SVM)



Support Vector Machine (SVM)

$$S = \begin{cases} +1 & \text{if } \overrightarrow{X}.\overrightarrow{w} + \mathrm{b} \geq 0 \\ -1 & \text{if } \overrightarrow{X}.\overrightarrow{w} + \mathrm{b} < 0 \end{cases}$$

- What is $\vec{w} \bullet \vec{X} + b = 0$?

  - this is another way of representing a line

$$\vec{X} = [x, y]$$
$$\vec{w} = [w_1, w_2]$$

$$xw_1 + yw_2 + b = 0$$

$$y = -\frac{w_1}{w_2}x - \frac{b}{w_2}$$

Support Vector Machine (SVM)

- We have two lines

$$xw_1 + yw_2 + b + 1 = 0$$

$$xw_1 + yw_2 + b - 1 = 0$$

- The distance Δ between them is

$$\Delta = \frac{2}{\sqrt{w_1^2 + w_1^2}} = \frac{2}{|\vec{w}|}$$

- **Maximise Δ = minimise absolute value of $\vec{w}$**

Support Vector Machine (SVM)

- **Maximise Δ = minimise absolute value of $\vec{w}$**

- Taking into account that

$$\vec{w}\bullet \vec{X} + b \geq 1 \quad \text{when S =1}$$
$$\vec{w}\bullet \vec{X} + b \leq 1 \quad \text{when S =-1}$$

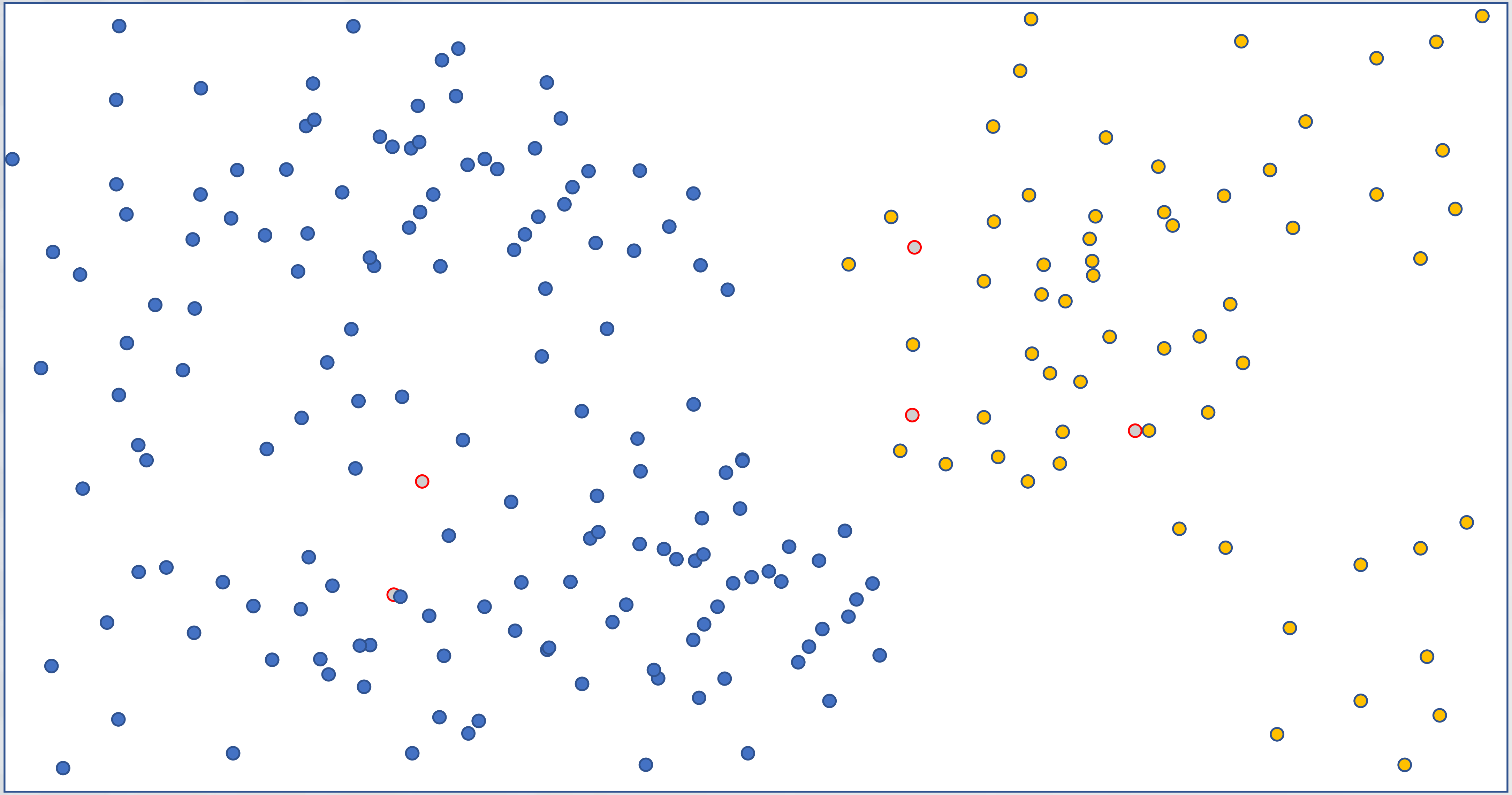
we can write

$$S_i(\vec{w}\bullet \vec{X_i}) \geq 1$$

Support Vector Machine (SVM)

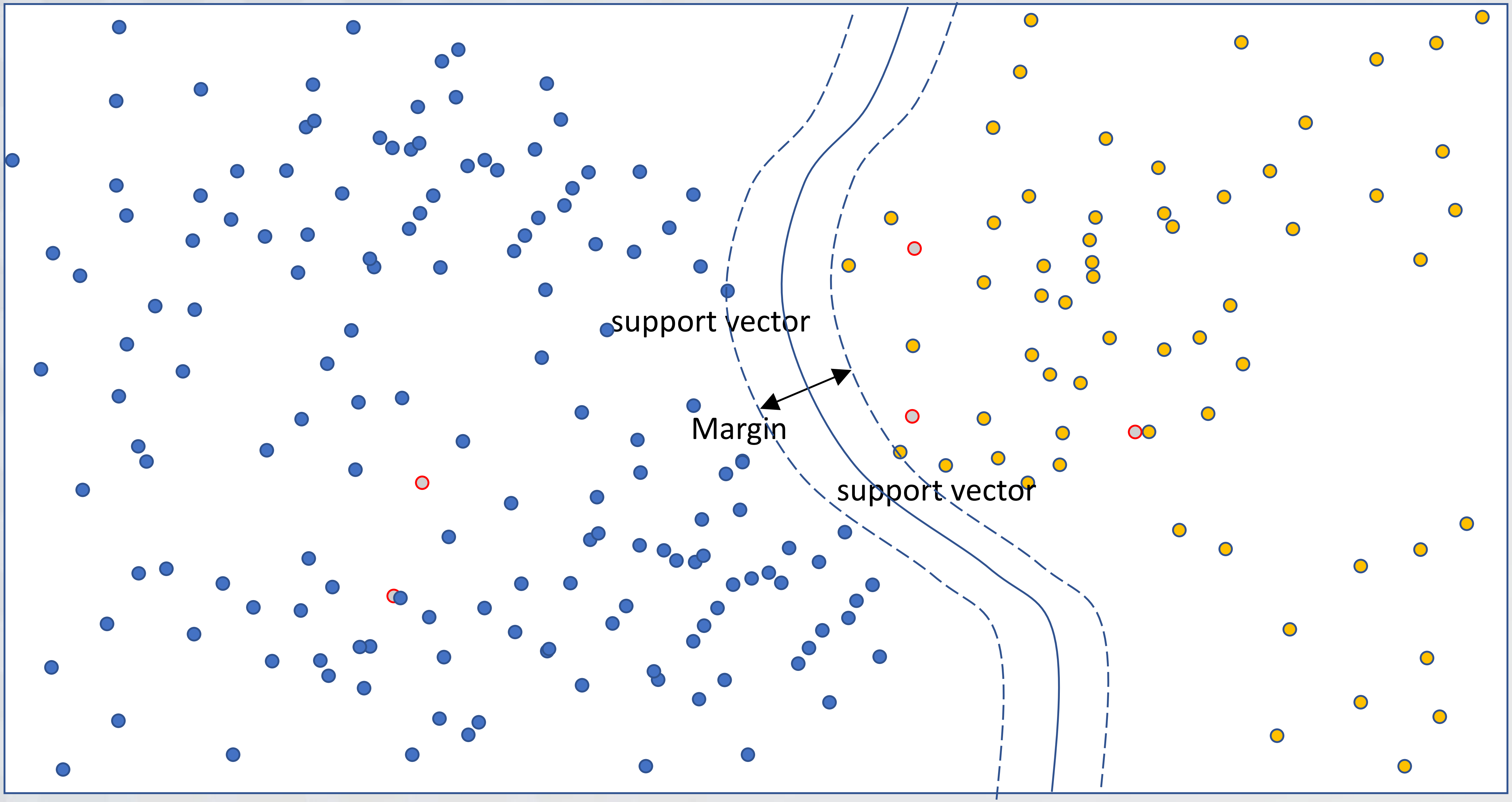
- Linear = separated by line
- Non-linear = separated by curve

Support Vector Machine (SVM)



Support Vector Machine (SVM)

$$F(\vec{w}\bullet \vec{X} + \vec{w}\bullet \overrightarrow{X^2} + \cdots) \geq 1$$



Support Vector Machine (SVM)

- Hard margin = clear separation between classes
- Soft margin = no clear separation, some entries will be in “the band”