1. Block diagram algebra

Friday, April 22, 2022 12:47 AM

(a) Feed forward

$$T_{FF}(s) = \frac{1}{s} \cdot \frac{k_m}{1+sT_m} \cdot K_p \frac{1+sT_p}{s}$$

$$\boxed{T_{FF}(s) = \frac{K_p k_m (1+sT_p)}{s^2(1+sT_m)}}$$

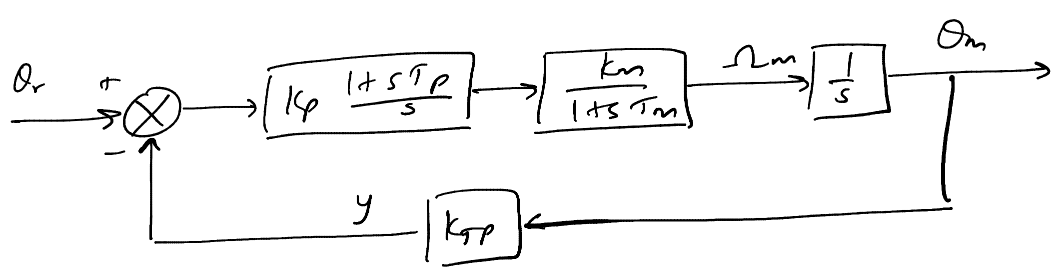
(b) Feedback

$$\boxed{T_{FB}(s) = k_{TP}}$$

(c) Closed-loop input/output T.F

Consider disturbance, $D=0$ (superposition principle)

Block diagram:



let feedback be, $y = k_{TP}\,\theta_m$

$$\rightarrow \theta_m = \frac{1}{s} \cdot \Omega_m$$

$$= \frac{1}{s} \cdot \frac{k_m}{1+sT_m} \cdot K_p \frac{1+sT_p}{s} (\theta_r - y)$$

$$= \frac{K_m K_p (1+sT_p)}{s^2(1+sT_m)} (\theta_r - K_{TP}\theta_m)$$

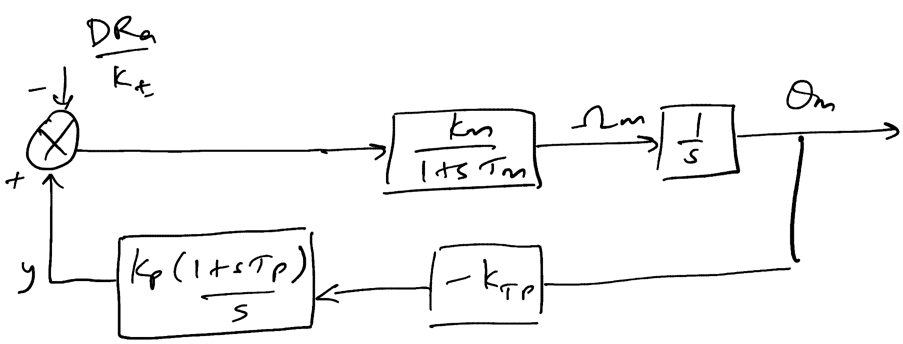
$$\Rightarrow \theta_m \left[1 + \frac{K_m K_p (1+sT_p)}{s^2(1+sT_m)} K_{TP} \right] = \frac{K_m K_p (1+sT_p)}{s^2(1+sT_m)} \theta_r$$

$$\therefore \boxed{\frac{\theta_m(s)}{\theta_r(s)} = \frac{K_m K_p (1+sT_p)}{s^2(1+sT_m) + K_m K_p K_{TP}(1+sT_m)}}$$

(d) Closed-loop disturbance/output T.F

Consider required input, $\theta_r = 0$ (superposition ppl.)

Block diagram:



Let feedback term, $y = \frac{K_p(1+sT_p)}{s} \cdot (-K_{TP})\, \theta_m$

$$\theta_m = \frac{1}{s} \cdot \Omega_m$$

$$= \frac{1}{s}\left(\frac{K_m}{1+sT_m}\right)\left[y - \frac{DR_a}{K_t}\right]$$

$$= \frac{K_m}{s(1+sT_m)}\left[-\frac{K_p K_{TP}(1+sT_p)}{s}\theta_m - \frac{DR_a}{K_t}\right]$$

$$\Rightarrow \theta_m \left[1 + \frac{k_m k_p k_{TP} (1 + sT_p)}{s^2 (1 + sT_m)} \right] = \frac{-D R_a k_m}{k_t s (1 + sT_m)}$$

$$\therefore \quad \boxed{\frac{\theta_m(s)}{D(s)} = \frac{-R_a k_m s}{k_t \left[s^3 T_m + s^2 + k_m k_p k_{TP} (1 + sT_p) \right]}}$$

2. Independent joint control

Friday, April 22, 2022 1:35 AM

Given: $I_m = 6\ kgm^2$, $R_a = 0.3\ \Omega$, $k_t = 0.5\ Nm/A$, $k_v = 0.5\ Vs/rad$

$F_m = 0.001\ Nms/rad$, $k_{TP} = k_{TV} = 1$

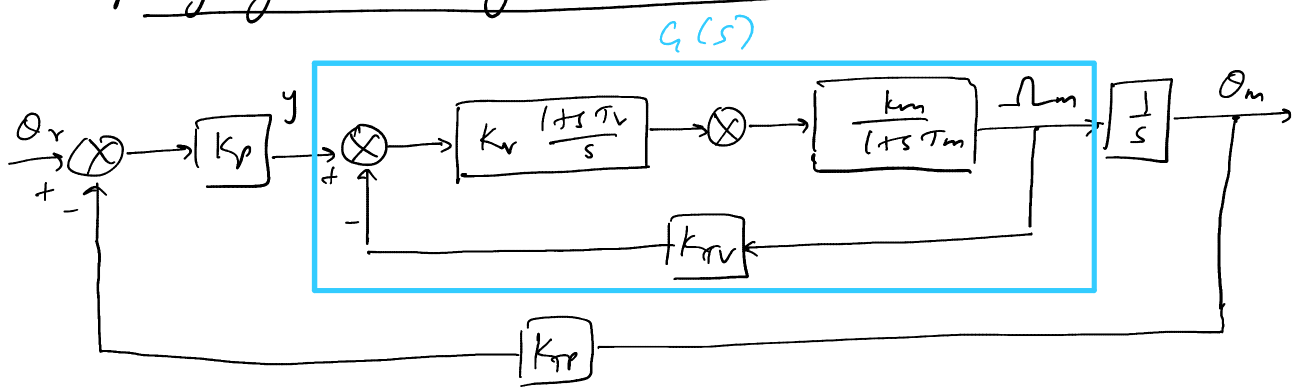
→ Also, from Siciliano et al., Sec. 5.2.1

- $k_m = \frac{1}{k_v} = \frac{1}{0.5} = 2$
- $T_m = \frac{R_a I_m}{k_v k_t} = \frac{(0.3)(6)}{(0.5)(0.5)} = 7.2$
- $T_v = T_m = 7.2$

a) Closed-loop input/output T.F

Consider $D(s) = 0$ (superposition principle)

→ Simplifying the given block diagram:



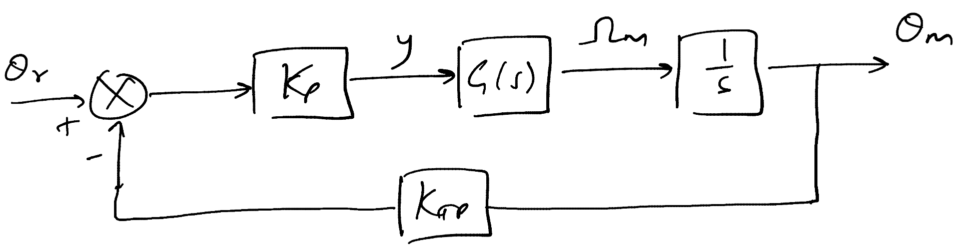
Let the blue block be $G(s)$

$$\Omega_m = \frac{k_m}{1+sT_m} \cdot K_v \frac{1+sT_v}{s} \left[y - k_{TV} \Omega_m \right]$$

$$\Rightarrow \Omega_m \left[1 + \frac{K_v k_m (1+sT_v)}{s(1+sT_m)} k_{TV}\right] = \frac{K_v k_m (1+sT_v)}{s(1+sT_m)} y$$

$$\therefore \frac{\Omega_m}{y} = \frac{K_v k_m (1+sT_v)}{s(1+sT_m) + K_v k_m k_{TV}(1+sT_v)} = G(s)$$

→ Simplified block diagram:



$$\theta_m = \frac{1}{s} \cdot \Omega_m$$

$$= \frac{1}{s}\left[G y\right]$$

$$= \frac{1}{s}\left[G K_p (\theta_r - K_{TP}\theta_m)\right]$$

$$= \frac{1}{s} G K_p \theta_r - \frac{1}{s} G K_p k_{TP} \theta_m$$

$$\Rightarrow \theta_m \left[1 + \frac{G K_p k_{TP}}{s}\right] = \frac{G K_p \theta_r}{s}$$

$$\therefore \frac{\theta_m(s)}{\theta_r(s)} = \frac{K_p G}{s + K_p k_{TP} G}$$

Here $$G(s) = \frac{K_v (2)(1+7.2s)}{s(1+7.2s) + 2K_v(1+7.2s)}$$

$$\Rightarrow \quad G(s) = \frac{K_v(2+14.4s)}{7.2s^2 + s + K_v(2+14.4s)}$$

$$\therefore \quad \frac{\theta_m(s)}{\theta_r(s)} = \frac{K_p \cdot \dfrac{K_v(2+14.4s)}{7.2s^2+s+K_v(2+14.4s)}}{s + K_p(1) \cdot \dfrac{K_v(2+14.4s)}{7.2s^2+s+K_v(2+14.4s)}}$$

$$= \frac{K_v K_p (2+14.4s)}{7.2s^3 + s^2 + K_v s(2+14.4s) + K_v K_p(2+14.4s)}$$

$$= \frac{K_v K_p (2+14.4s)}{7.2s^3 + s^2 + K_v(2+14.4s)[s+K_p]}$$

$$= \frac{2K_v K_p (1\cancel{+7.2s})}{s^2(1+\cancel{7.2s}) + 2K_v(1+\cancel{7.2s})(s+K_p)}$$

$$= \frac{2K_v K_p}{s^2 + 2K_v(s+K_p)}$$

$$\therefore \quad \boxed{\frac{\theta_m(s)}{\theta_r(s)} = \frac{2K_v K_p}{s^2 + s(2K_v) + 2K_v K_v}}$$

b) Feedback controller design

A typical 2^{nd} order system can be represented as:

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad : \quad \omega_n = 20 \text{ rad/s}, \quad \xi = 0.4$$

Comparing this with the transfer function from part (a),

$$\omega_n^2 = 2 K_v K_p \quad \& \quad 2\xi\omega_n = 2K_v$$

$$\Rightarrow K_p = \frac{\omega_n^2}{2K_v} = \frac{20^2}{2(8)} = 25$$

$$\Rightarrow K_v = \xi\omega_n = (0.4)(20) = 8$$

$$\therefore \boxed{K_p = 25, \quad K_v = 8}$$

Finding poles :

Denominator = 0

$$\Rightarrow s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\Rightarrow s^2 + 2(0.4)(20)s + 20^2 = 0$$

$$\Rightarrow s^2 + 16s + 400 = 0$$

$\therefore$ Poles are $\boxed{-8 \pm 18.33\,i}$

c) → Disturbance rejection factor,

$$x_R = K_p K_{op} K_v = (25)(1)(8) = 200$$

→ Output recovery time,

$$T_R = \max\left\{T_m, \frac{1}{\xi\omega_n}\right\} = \max\left\{7.2, \frac{1}{8}\right\} = 7.2\,s$$

$$\boxed{X_R = 200 \quad , \quad T_R = 7.2\,s}$$

→ To improve (decrease) T_R,

- Choose a motor with lower T_m
- $\frac{1}{\xi \omega_n} = \frac{1}{K_v}$ ⇒ Increase K_v

MAE C263C - HW #3.3 - RUBIN JACOB (105498838)

```
% clear all;
close all; clc;

%***************************************************************************
% VERONICA J. SANTOS
% 4/15/22
% HW3_main.m
%
% This script file was originally created by L. Villani, G. Oriolo, and
% B. Siciliano in Feb. 2009.  It has been modified for MAE C163C / C263C
% HW #3.
%***************************************************************************

% Variable initialization

global a k_r1 k_r2 pi_m pi_l

% load manipulator dynamic parameters without load mass
  param;
  pi_l = pi_m;
  F_m1 = 0.01; F_m2 = 0.01;

% gravity acceleration
  g = 9.81;

% friction matrix
  K_r = [k_r1 0; 0 k_r2];
  F_v = K_r * [F_m1 0; 0 F_m2] * K_r;

% sample time of controller
  Tc = 0.001;

% controller gains
  K_p = [1250 0; 0 1250];
  K_d = [680 0; 0 680];

% desired position
  q_d = [pi/4; -pi/2];
% q_d = [-pi; -3*pi/4];

% initial position
  q_i = q_d - 0.1;

% duration of simulation
  t_d = 2.5;

% sample time for plots
```

```
Ts = Tc;
```

a) Kp and Kd matrices for minimal error

```
K_p
```

```
K_p = 2×2
        1250           0
           0        1250
```

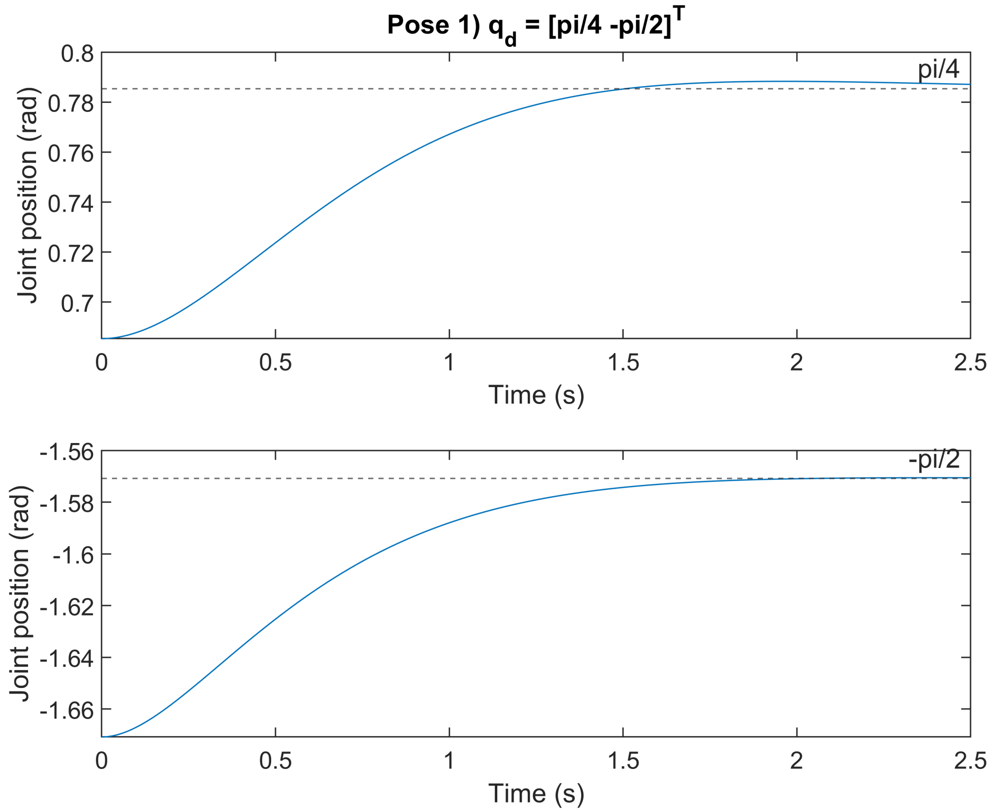
```
K_d
```

```
K_d = 2×2
   680     0
     0   680
```

b) Closed-loop response - Plots

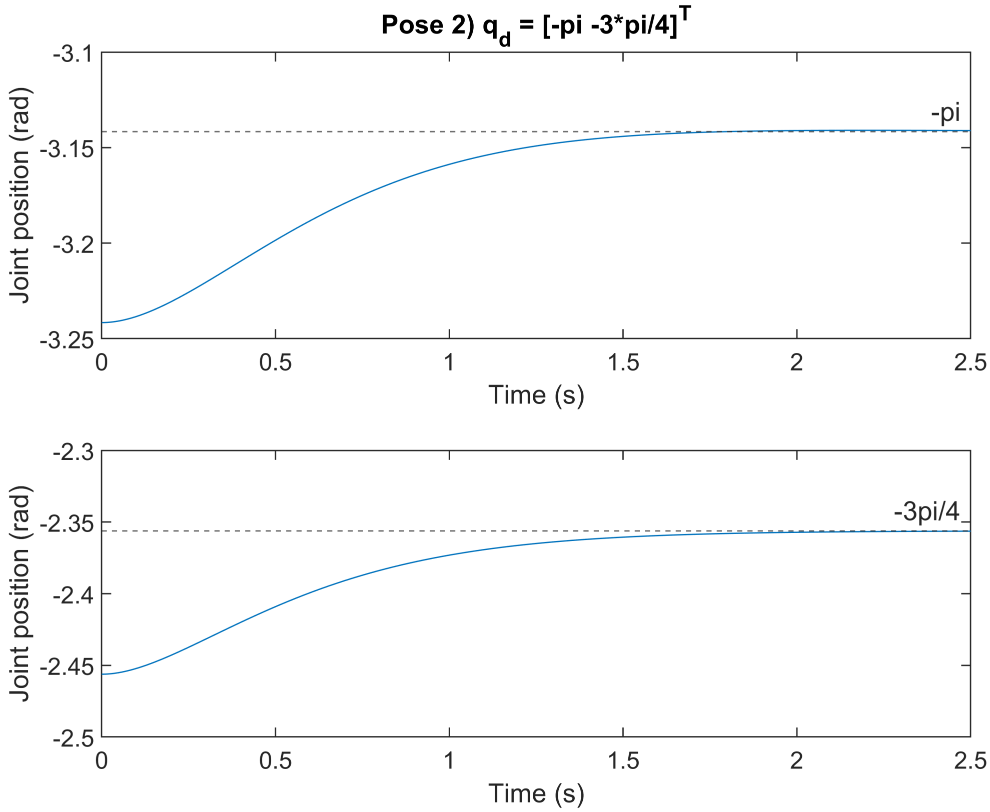
Pose 1) q_d = [pi/4 -pi/2]'

```
% q_1 = out.q;
figure();

subplot(2,1,1);
plot(out.tout,q_1(:,1));
title('Pose 1) q_d = [pi/4 -pi/2]^T');
yline(pi/4,'--','pi/4');
xlabel('Time (s)'); ylabel('Joint position (rad)');

subplot(2,1,2);
plot(out.tout,q_1(:,2));
yline(-pi/2,'--','-pi/2');
xlabel('Time (s)'); ylabel('Joint position (rad)');
```



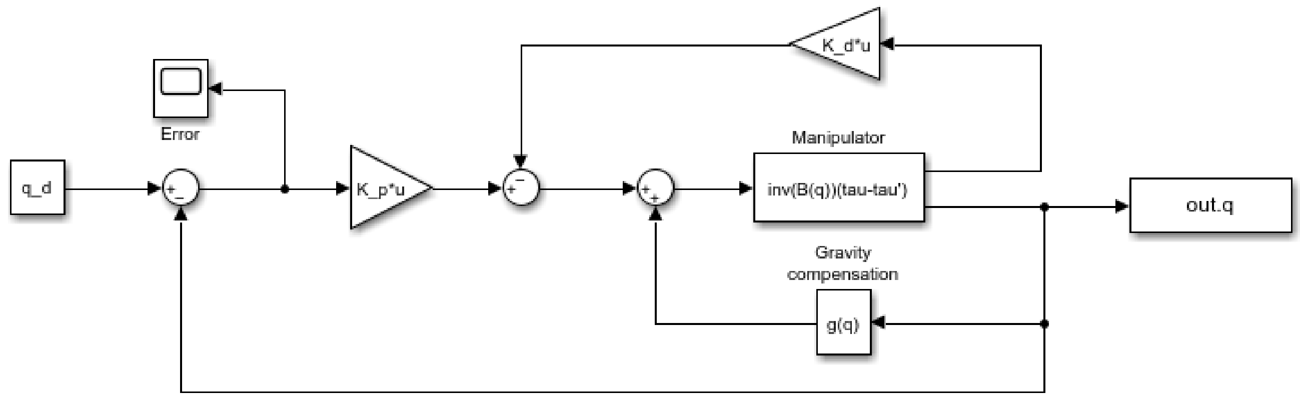
Pose 2) q_d = [-pi -3*pi/4]'

```
% q_2 = out.q;
figure();

subplot(2,1,1);
plot(out.tout,q_2(:,1));
title('Pose 2) q_d = [-pi -3*pi/4]^T');
yline(-pi,'--','-pi');
xlabel('Time (s)'); ylabel('Joint position (rad)');
ylim([-3.25 -3.1]);

subplot(2,1,2);
plot(out.tout,q_2(:,2));
yline(-3*pi/4,'--','-3pi/4');
xlabel('Time (s)'); ylabel('Joint position (rad)');
ylim([-2.5 -2.3]);
```



It is clear from the plots that controller drives each joint angle to the desired value within the allotted time.

Simulink Block Diagram



Error plot (Dashboard) – Pose 2