1. a) Forward Path.

$$\theta_r \cdot K_p \cdot \frac{1+sT_p}{s} \cdot \frac{k_m}{1+sT_m} \cdot \frac{1}{s} = \theta_m$$

So, $$\boxed{\frac{\theta_m}{\theta_r} = \frac{K_p k_m (1+sT_p)}{s^2 (1+sT_m)}}$$

b) Return Path $R(s)$

$$R(s) = k_{TP} \cdot \theta_m(s)$$

So, $$\boxed{\frac{R(s)}{\theta_m(s)} = k_{TP}}$$

c) Closed-loop input/output function $\frac{\theta_m(s)}{\theta_r(s)}$

$$(\theta_r - \theta_m k_{TP}) \cdot \underbrace{K_p \cdot \left(\frac{1+sT_p}{s}\right) \cdot \frac{k_m}{1+sT_m} \cdot \frac{1}{s}}_{\text{assume } C(s).} = \theta_m.$$

$$(\theta_r - \theta_m k_{TP})\, C(s) = \theta_m$$

$$\theta_r C(s) = \theta_m k_{TP} C(s) + \theta_m = \theta_m (1 + k_{TP} C(s))$$

$$\frac{\theta_m}{\theta_r} = \frac{C(s)}{1 + k_{TP} C(s)}.$$

Substituting $C(s)$ back and simplifying with formula,

$$\boxed{\frac{\theta_m}{\theta_r} = \frac{\frac{1}{k_{TP}}}{1 + \frac{s^2 (1+sT_m)}{k_{TP} K_p k_m (1+sT_p)}}}$$

d) $\left[(-k_{TP}\cdot\theta_m)\cdot K_P\cdot\left(\frac{1+sT_P}{s}\right)-\frac{DR_a}{k_t}\right]\frac{k_m}{1+sT_m}\cdot\frac{1}{s}=\theta_m$

$$\frac{(1+sT_m)s}{k_m}\theta_m-k_{TP}K_P\left(\frac{1+sT_P}{s}\right)\theta_m=\frac{DR_a}{k_t}$$

Simplifying, we get.

$$\frac{\theta_m(s)}{D(s)}=\frac{\frac{sR_a}{k_t K_P k_{TP}(1+sT_P)}}{1+\frac{s^2(1+sT_m)}{K_P k_{TP} k_m(1+sT_P)}}$$

2a) Joint control with position & velocity feedback.

From Siciliano 8.28,

$$\frac{\theta_m}{\theta_r} = \frac{1/k_{TP}}{1 + \frac{s k_{TV}}{K_P k_{TP}} + \frac{s^2}{k_m K_P k_{TP} K_V}}$$

Also, $k_m = \frac{1}{k_V} = \frac{1}{0.5} = 2$.

Substituting $k_{TV} = k_{TP} = 1$, $k_m = 2$, we get

$$\frac{\theta_m}{\theta_r} = \frac{1}{1 + \frac{s}{K_P} + \frac{s^2}{2 K_P K_V}}$$

$$\boxed{\frac{\theta_m}{\theta_r} = \frac{2 K_P K_V}{2 K_P K_V + 2 K_V s + s^2}}$$

b) $\zeta = 0.4$, $\omega_n = 20$ rad/sec.

From Siciliano 8.30, $K_V \underbrace{k_{TV}}_{=1} = \frac{2 \zeta \omega_n}{k_m}$.

So, $K_V = \frac{2(0.4)(20)}{2}$

$$\boxed{K_V = 8}$$

From siciliano, 8.31,

$$K_P k_{TP} K_V = \frac{\omega^2}{k_m} \Rightarrow K_P \cdot 1 \cdot 8 = \frac{20^2}{2} = 200$$

$$\boxed{K_P = 25.}$$

From siciliano 8.29, we get denominator

$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$. for pole, $D(s) = 0$.

$= s^2 + 2(0.4) \cdot 20 \cdot s + 400 = 0$

$= s^2 + 16s + 400 = 0$.

So, $\boxed{s = -8 \pm 18.33i}$.

c) From siciliano 8.33, we get.

$$X_R = K_p k_{tp} K_v$$
$$= 25 \cdot 1 \cdot 8$$

$$\boxed{X_R = 200}$$

$$T_m = \frac{R_a \cdot I_m}{k_v k_t} = \frac{0.3 \times 6}{0.5 \times 0.5} = \frac{1.8}{0.25} = \boxed{7.2 \text{ sec}}$$

From. 8.34, $T_R = \max\left\{T_m, \frac{1}{\zeta\omega_n}\right\}$ ($0.4 \to 20$)

$= \max\left\{7.2, \frac{1}{8}\right\}$

$$\boxed{T_R = 7.2 \text{ sec}}$$

To increase T_R, we can increase resistance of armature.

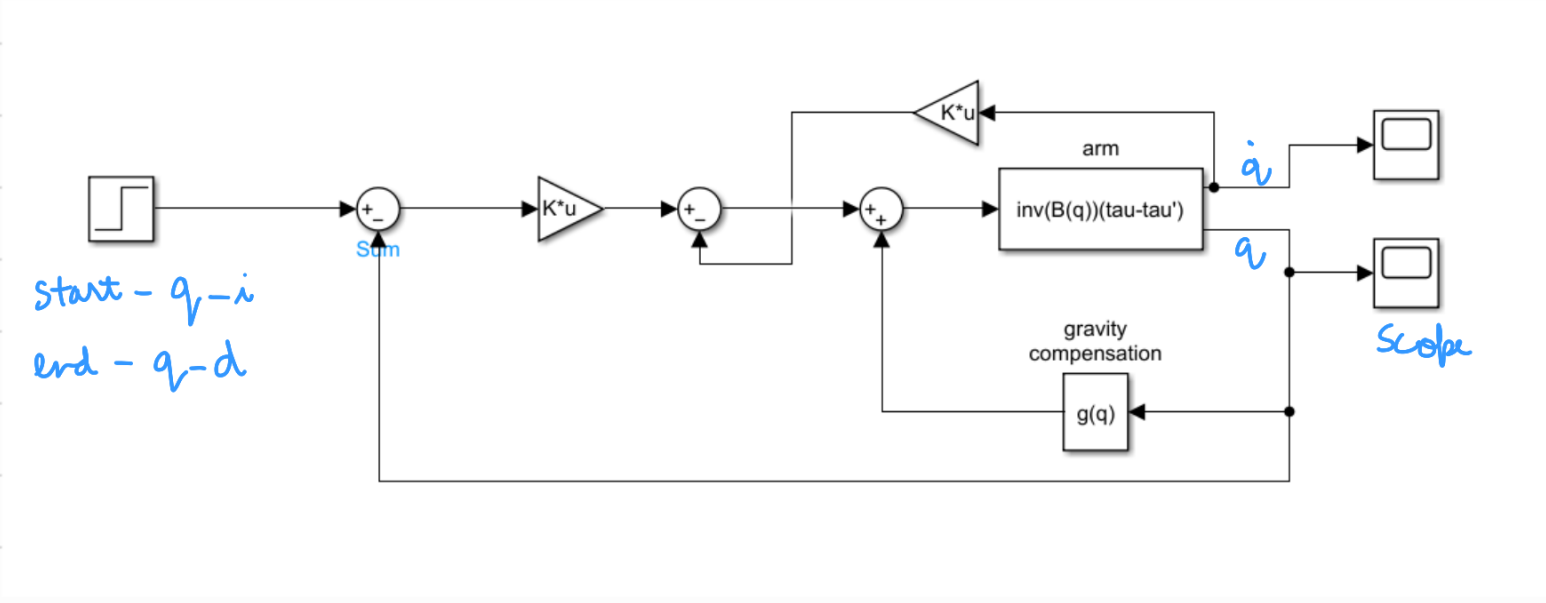
3.

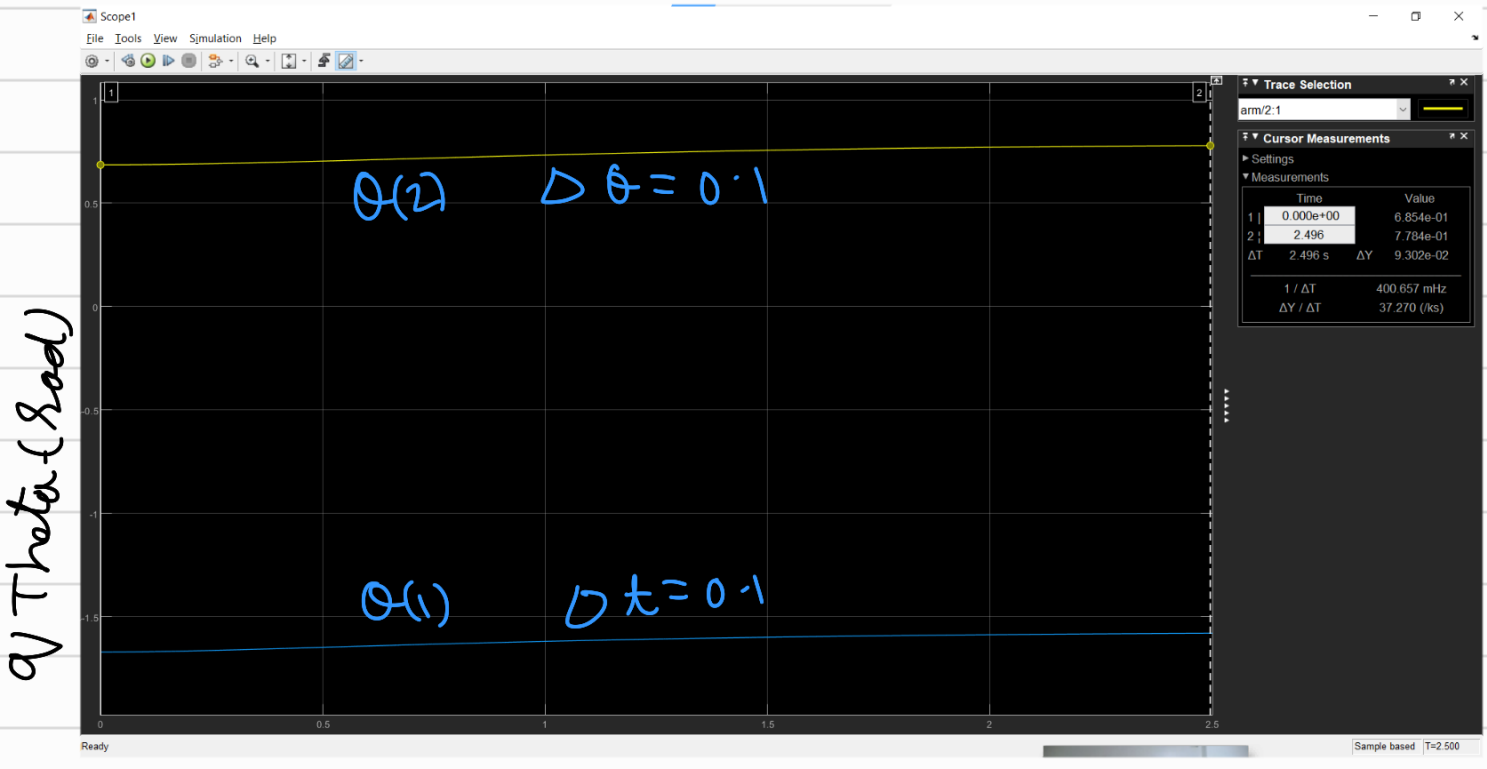
Code:

```
global a k_r1 k_r2 pi_m pi_l
% load manipulator dynamic parameters without load mass
 param;
 pi_l = pi_m;
% gravity acceleration
 g = 9.81;
% friction matrix
 K_r = [k_r1,0;0,k_r2];
 %from gear ratio, kr from params file.
 Fm1=0.01;Fm2=0.01;
 F_m = [Fm1,0;0,Fm2];
 F_v = K_r*F_m*K_r;
 % Fm=Kr^-1*Fv*Kr^-1; from Siciliano 8.21.
% sample time of controller
 Tc = [0.001]; %1ms given in question.
% controller gains
 kp_test = 500  ; kv_test = 500 ;
 K_p = [kp_test,0;0,kp_test];
 K_d = [kv_test,0;0,kv_test];

% desired position
 q_d = [pi/4;-pi/2]; %case 1
 %q_d = [-pi;-3*pi/4]; %case 2
% initial position
 q_i = q_d - [0.1;0.1];
% duration of simulation
```

```
t_d = [2.5]
% sample time for plots
Ts = Tc;
```

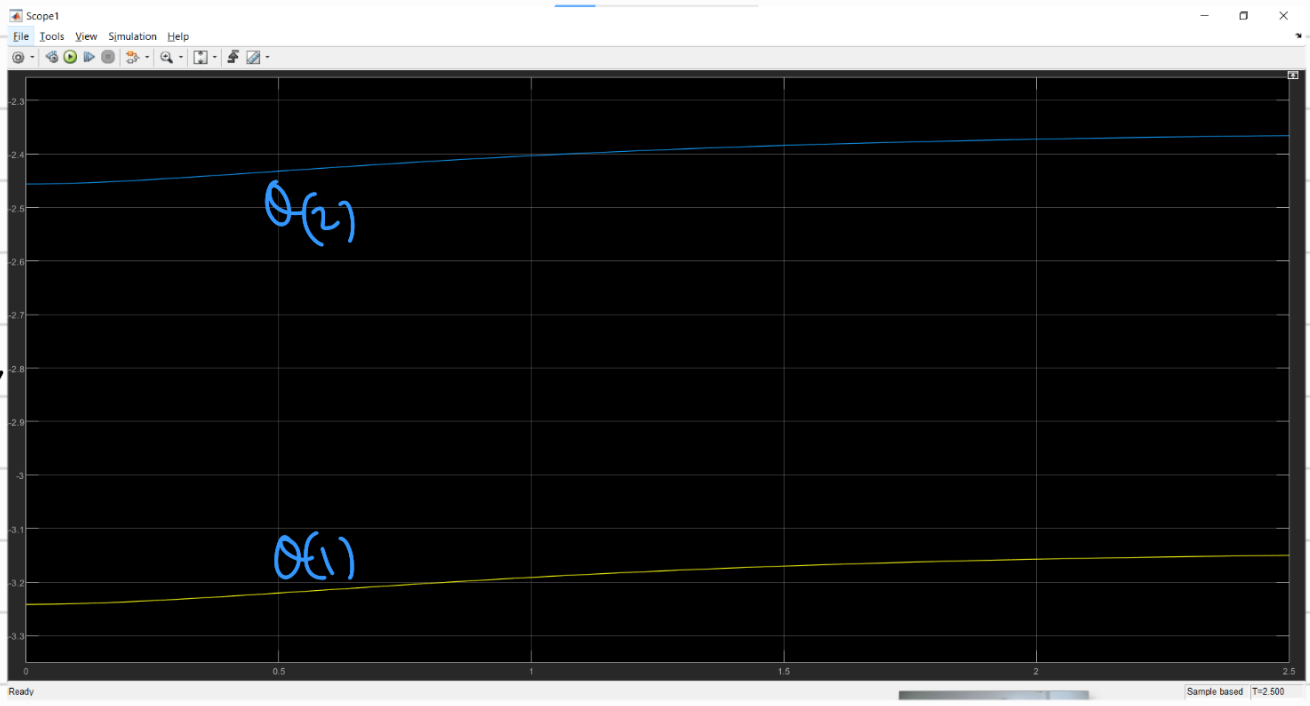




q Theta (Rad)

Time (sec)

Kp=500 , Kv=500 for both motors



a theta (rad)

time (sec)