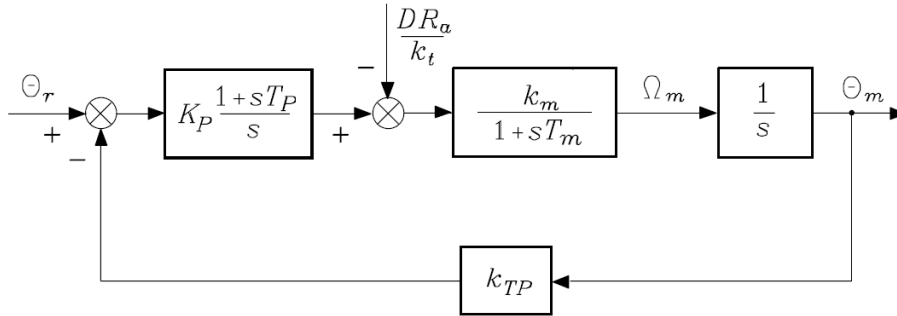


MAE C163C / C263C Homework #3

(Due via Gradescope by 11:59pm on Friday, 4/22)

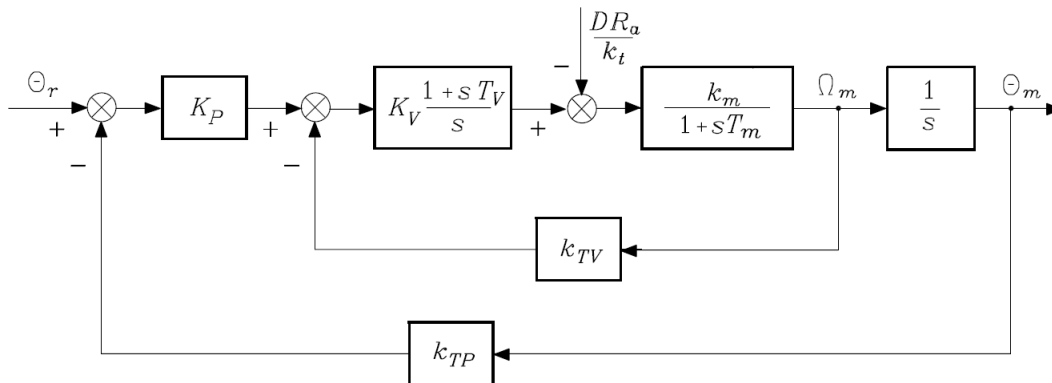
1. Block diagram algebra



The block diagram above describes drive control with position feedback. Report the transfer functions for the following:

- Forward path (feedforward)
- Return path (feedback)
- Closed-loop input/output transfer function $\frac{\Theta_m(s)}{\Theta_r(s)}$
- Closed-loop disturbance/output transfer function $\frac{\Theta_m(s)}{D(s)}$

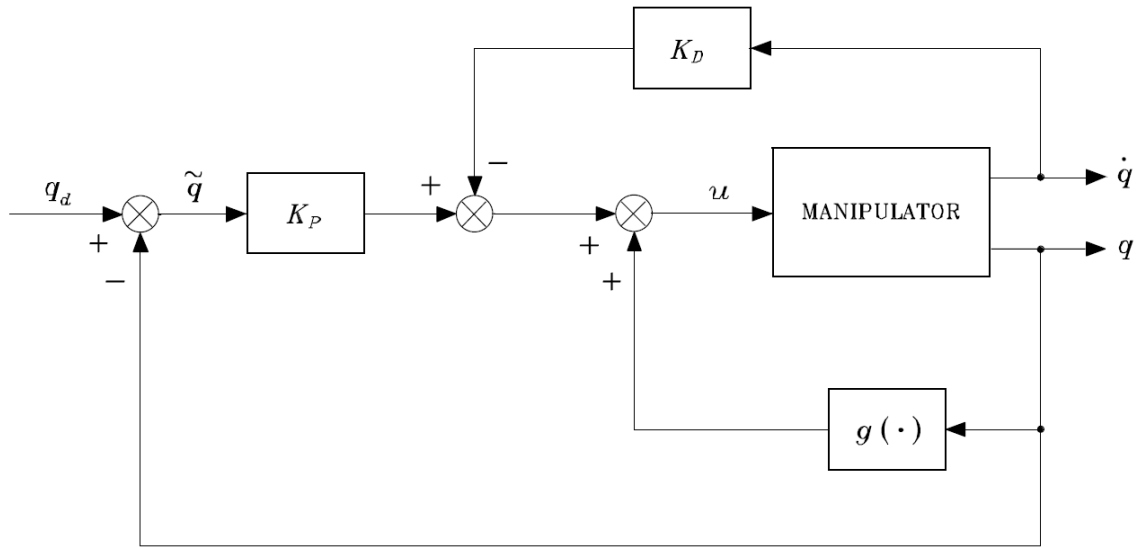
2. Independent joint control with position and velocity feedback



The given block diagram describes drive control with position and velocity feedback. Consider a single joint drive system with the following model parameters: $I_m = 6 \text{ kg-m}^2$, $R_a = 0.3 \Omega$, $k_t = 0.5 \text{ N-m/A}$, $k_v = 0.5 \text{ V-s/rad}$, $F_m = 0.001 \text{ N-m-s/rad}$, and unit transducer constants k_{TP} and k_{TV} . You will need to refer to Siciliano et al., Sec. 5.2.1 to assign values to the remaining motor model parameters. For ease of grading, set $T_V = T_m$.

- Report the symbolic closed-loop input/output transfer function. Design parameters K_P and K_V should be the only variables.
- Design a position and velocity feedback controller having a closed-loop response with damping ratio $\zeta = 0.4$ and natural frequency $\omega_n = 20 \text{ rad/s}$. Report your K_P and K_V , along with the poles of the closed-loop system.
- Report the disturbance rejection factor X_R and output recovery time T_R . Comment on what can be done to improve the output recovery time.

3. Joint space PD control with gravity compensation



The given block diagram describes joint space PD control with gravity compensation.

Consider a two-link planar arm with the following parameters:

$$a_1 = a_2 = 1 \text{ m} \quad \ell_1 = \ell_2 = 0.5 \text{ m} \quad m_{\ell_1} = m_{\ell_2} = 50 \text{ kg} \quad I_{\ell_1} = I_{\ell_2} = 10 \text{ kg} \cdot \text{m}^2$$

$$k_{r1} = k_{r2} = 100 \quad m_{m1} = m_{m2} = 5 \text{ kg} \quad I_{m1} = I_{m2} = 0.01 \text{ kg} \cdot \text{m}^2$$

The arm is assumed to be driven by two equal actuators with the following parameters:

$$F_{m1} = F_{m2} = 0.01 \text{ N} \cdot \text{m} \cdot \text{s/rad} \quad R_{a1} = R_{a2} = 10 \text{ ohm}$$

$$k_{t1} = k_{t2} = 2 \text{ N} \cdot \text{m/A} \quad k_{v1} = k_{v2} = 2 \text{ V} \cdot \text{s/rad}$$

- a) Design a closed-loop stable PD controller with gravity compensation in Simulink using the provided manipulator and $g(\cdot)$ Simulink .mdl files and main script .m file. Note that you can open each .mdl file provided, right-click on the subsystem, select “copy,” and then paste the subsystem into your own custom Simulink model. You are also being provided with a file of model parameters and a function file for calculating the inertia matrix $B(q)$.

Implement the control in discrete-time with a sampling time of 1 ms.

Your PD controller (comprised of a single set of K_P and K_D gain matrices) must work for both of the following desired postures: $\dot{q} = [\pi/4 \quad -\pi/2]^T$ and $q = [-\pi \quad -3\pi/4]^T$. Report your K_P and K_D gain matrices. There is no single correct answer. I will accept any single set of K_P and K_D gain matrices that achieves a stable steady state with minimal tracking errors for both desired postures within the time horizon specified in part *b*.

- b) Simulate the closed-loop response of the system and verify its closed-loop stability with plots. For each of the two desired postures, plot the time history of the two joint positions in rad in separate subfigures for a time horizon of 2.5 s. For each of the two desired postures, initialize each joint angle at $t = 0$ with a value that is perturbed from the desired posture by -0.1 rad. Indicate the desired joint angle in each subfigure by drawing a dashed horizontal line at the desired value and show that your controller drives each joint angle to the desired value within the allotted time.

For full credit, you must also submit the following:

- A .pdf or image-based version of your final `HW3_main.m` file. Files of the .m format cannot be uploaded to Gradescope and should not be e-mailed to instructors for grading.
- A screenshot of your final Simulink controller (high-level view is sufficient)
- Labeled plots