1. a) Forward Path.

$$\theta_r \cdot K_p \cdot \frac{1+sT_p}{s} \cdot \frac{k_m}{1+sT_m} \cdot \frac{1}{s} = \theta_m$$

So,
$$\frac{\theta_m}{\theta_r} = \frac{K_p k_m (1+sT_p)}{s^2(1+sT_m)}$$

b) Return Path $R(s)$

$$R(s) = k_{TP} \cdot \theta_m(s)$$

So,
$$\frac{R(s)}{\theta_m(s)} = k_{TP}$$

c) Closed-loop input/output function $\frac{\theta_m(s)}{\theta_r(s)}$

$$(\theta_r - \theta_m k_{TP}) \cdot \underbrace{K_p \cdot \left(\frac{1+sT_p}{s}\right) \cdot \frac{k_m}{1+sT_m} \cdot \frac{1}{s}}_{\text{assume } C(s).} = \theta_m.$$

$$(\theta_r - \theta_m k_{TP})\, C(s) = \theta_m$$

$$\theta_r\, C(s) = \theta_m k_{TP}\, C(s) + \theta_m = \theta_m(1 + k_{TP}\, C(s))$$

$$\frac{\theta_m}{\theta_r} = \frac{C(s)}{1 + k_{TP}\, C(s).}$$

Substituting $C(s)$ back and simplifying with formula,

$$\frac{\theta_m}{\theta_r} = \frac{\frac{1}{k_{TP}}}{1 + \frac{s^2(1+sT_m)}{k_{TP} K_p k_m (1+sT_p)}}$$

d) $\left(\left(-k_{TP} \cdot \theta_m\right) \cdot K_P \cdot \left(\frac{1+sT_P}{s}\right) - \frac{D\,R_a}{k_t}\right) \frac{k_m}{1+sT_m} \cdot \frac{1}{s} = \theta_m$

$$\frac{(1+sT_m)s}{k_m}\theta_m - k_{TP} K_P \left(\frac{1+sT_P}{s}\right)\theta_m = \frac{D\,R_a}{k_t}$$

Simplifying, we get.

$$\frac{\theta_m(s)}{D(s)} = \frac{\frac{s R_a}{k_t K_P k_{TP}(1+sT_P)}}{1 + \frac{s^2(1+sT_m)}{K_P k_{TP} k_m (1+sT_P)}}$$

2 a) Joint control with position & velocity feedback.

From Siciliano 8.28,

$$\frac{\theta_m}{\theta_r} = \frac{1/k_{TP}}{1 + \frac{s k_{TV}}{K_P k_{TP}} + \frac{s^2}{k_m K_P k_{TP} K_V}}$$

Also, $k_m = \frac{1}{k_v} = \frac{1}{0.5} = 2.$

Substituting $k_{TV} = k_{TP} = 1$, $k_m = 2$, we get

$$\frac{\theta_m}{\theta_r} = \frac{1}{1 + \frac{s}{K_P} + \frac{s^2}{2 K_P K_V}}$$

$$\frac{\theta_m}{\theta_r} = \frac{2 K_P K_V}{2 K_P K_V + 2 K_V s + s^2}$$

b) $\zeta = 0.4$, $\omega_n = 20$ rad/sec.

From Siciliano 8.30, $K_V \underbrace{k_{TV}}_{=1} = \frac{2 \zeta \omega_n}{k_m}.$

So, $K_V = \frac{\cancel{2}(0.4)(20)}{\cancel{2}}$

$K_V = 8$

From siciliano, 8.31,

$K_P k_{TP} K_V = \frac{\omega^2}{k_m} \Rightarrow K_P \cdot 1 \cdot 8 = \frac{20^2}{2} = 200$

$K_P = 25.$

From siciliano 8.29, we get denominator

$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$. for pole, $D(s) = 0$.

$= s^2 + 2(0.4) \cdot 20 \cdot s + 400 = 0$

$= s^2 + 16s + 400 = 0$.

So, $s = -8 \pm 18.33i$.

c) From siciliano 8.33, we get.

$X_R = K_P k_{TP} K_V$

$= 25 \cdot 1 \cdot 8$

$X_R = 200$.

$$T_m = \frac{R_a \cdot I_m}{k_v k_t} = \frac{\cancel{0.3} \times 6}{\cancel{0.5} \times 0.5} = \frac{1.8}{0.25} = 7.2 \text{ sec.}$$

From. 8.34, $T_R = \max\left\{ T_m, \frac{1}{\underbrace{\zeta}_{0.4}\underbrace{\omega_n}_{20}} \right\}$

$= \max\left\{ 7.2, \frac{1}{8} \right\}$

$T_R = 7.2$ sec.

To increase T_R, we can increase resistance of armature.

3.

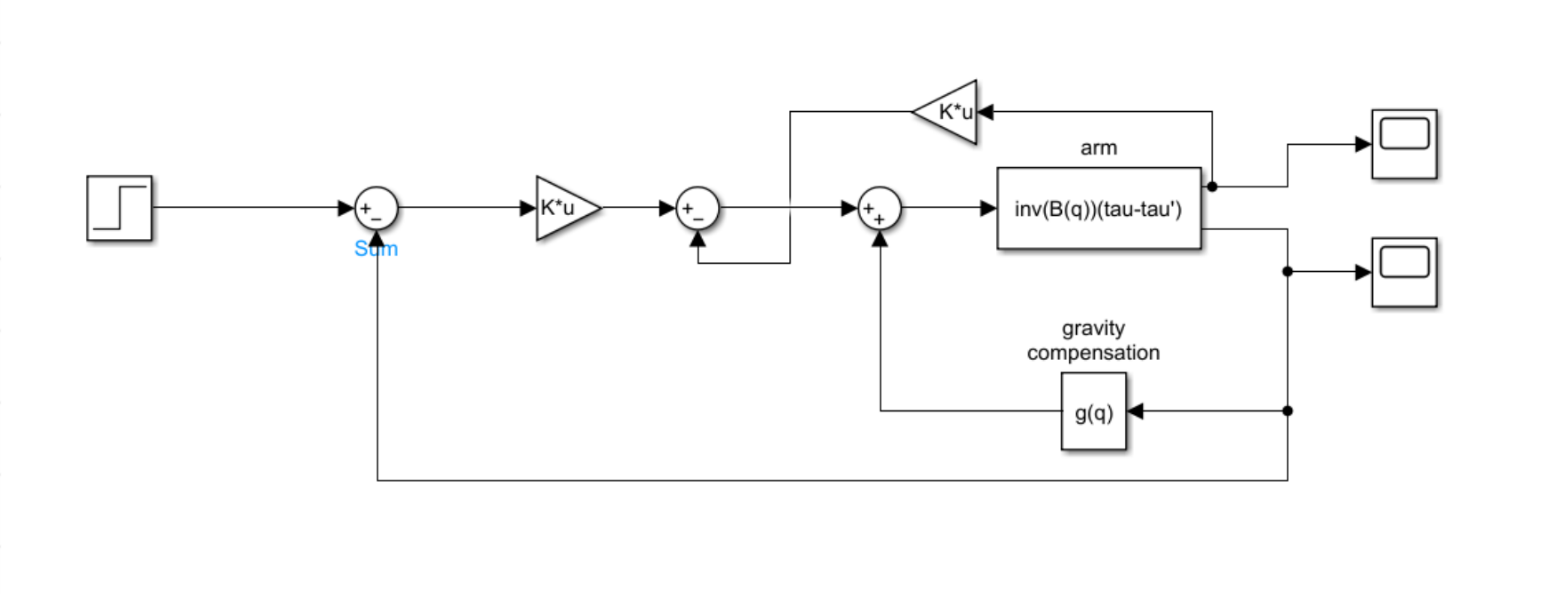
Code:

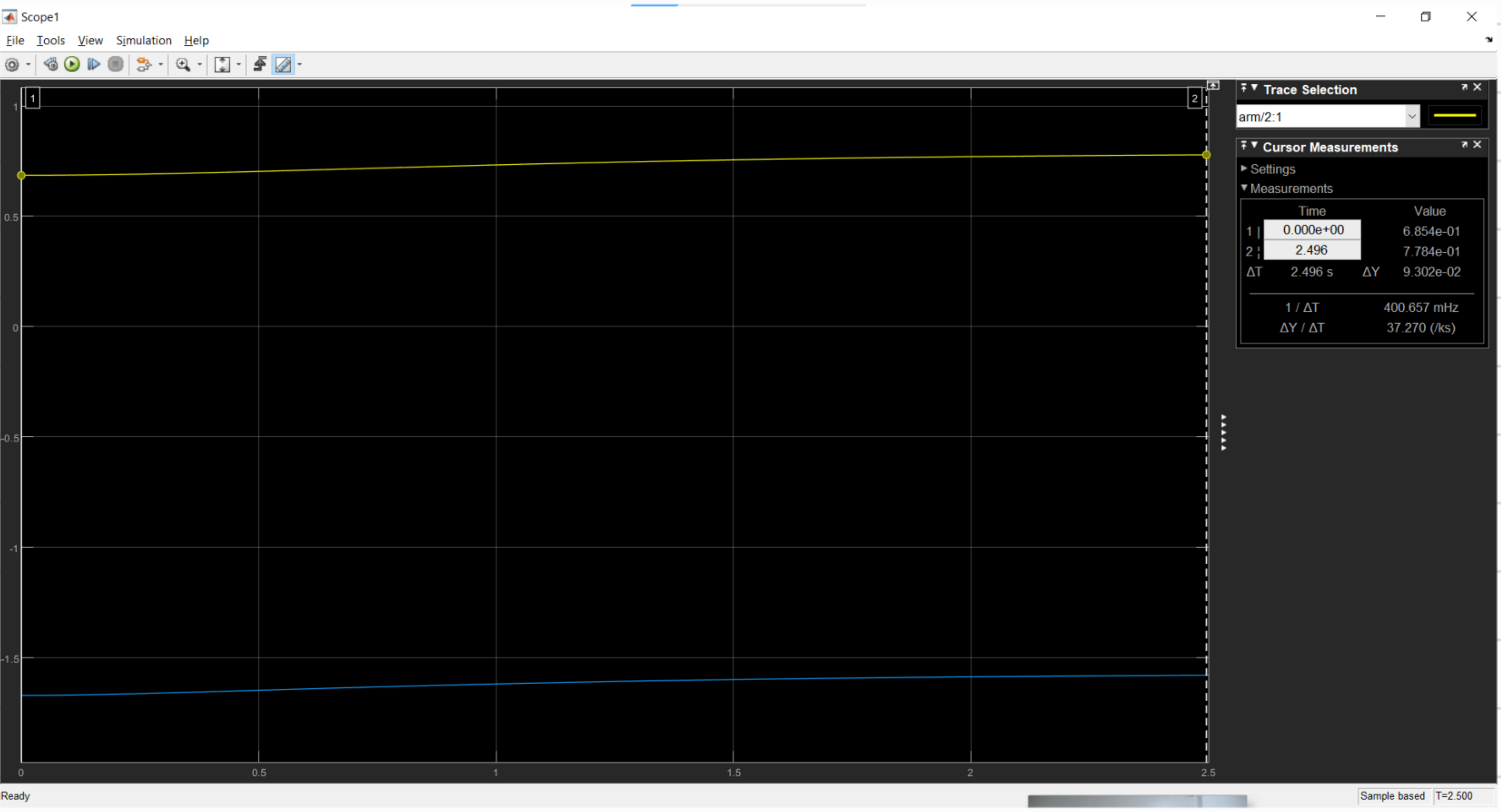
```
global a k_r1 k_r2 pi_m pi_l
% load manipulator dynamic parameters without load mass
  param;
  pi_l = pi_m;
% gravity acceleration
  g = 9.81;
% friction matrix
  K_r = [k_r1,0;0,k_r2];
  %from gear ratio, kr from params file.
  Fm1=0.01;Fm2=0.01;
  F_m = [Fm1,0;0,Fm2];
  F_v = K_r*F_m*K_r;
  % Fm=Kr^-1*Fv*Kr^-1; from Siciliano 8.21.
% sample time of controller
  Tc = [0.001]; %1ms given in question.
% controller gains
  kp_test = 500  ; kv_test = 500 ;
  K_p = [kp_test,0;0,kp_test];
  K_d = [kv_test,0;0,kv_test];

% desired position
  q_d = [pi/4;-pi/2]; %case 1
  %q_d = [-pi;-3*pi/4]; %case 2
% initial position
  q_i = q_d - [0.1;0.1];
% duration of simulation
```

t_d = [2.5]

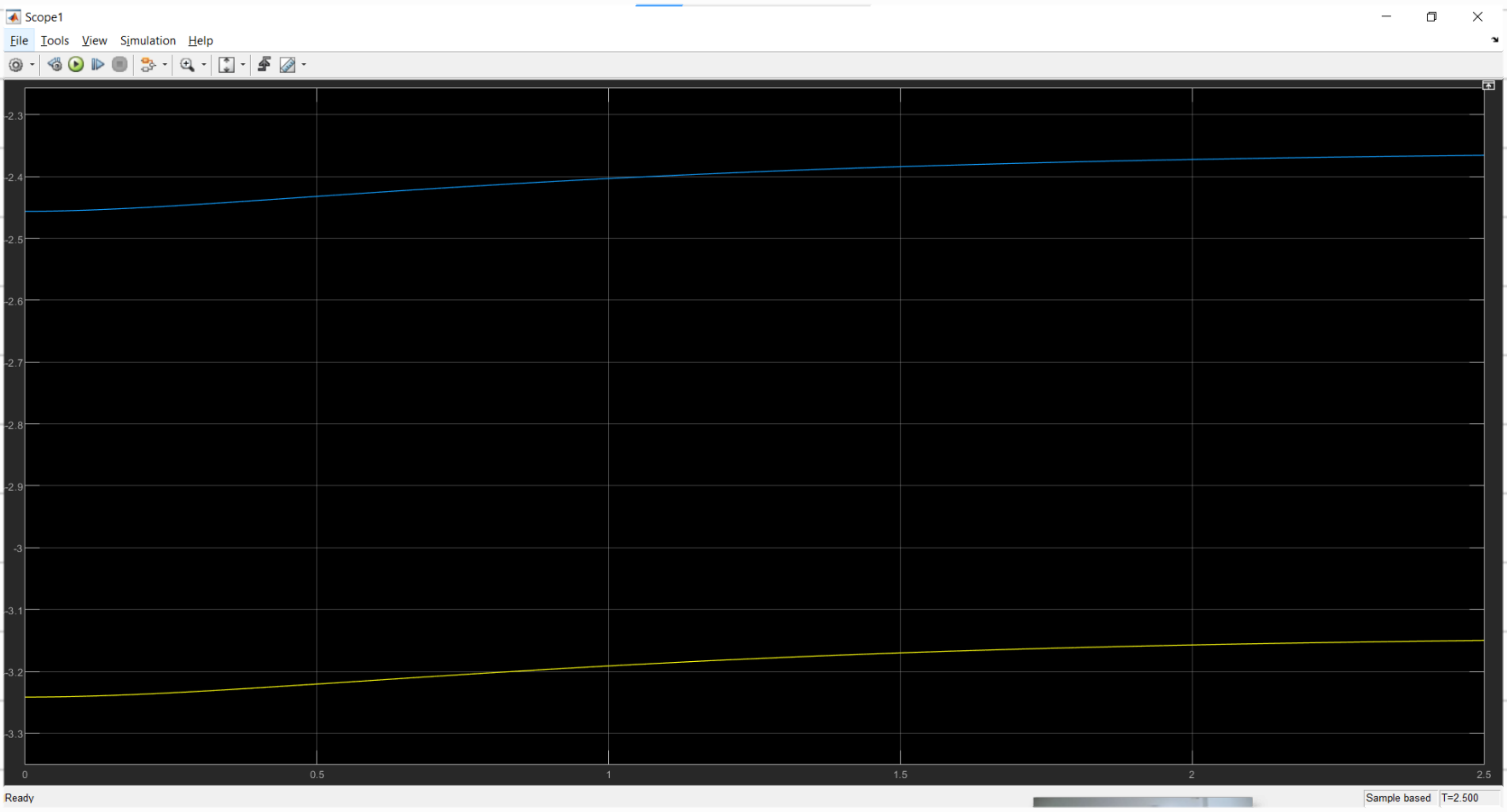
% sample time for plots

Ts = Tc;





Kp=500 , Kv=500 for both motors


Scope1
File Tools View Simulation Help
-2.3
-2.4
-2.5
-2.6
-2.7
-2.8
-2.9
-3
-3.1
-3.2
-3.3
0
0.5
1
1.5
2
2.5
Ready
Sample based
T=2.500