Q.2

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a) Forward Path.

$$\Rightarrow \theta_m = \theta_r \cdot K_p\left(\frac{1+ST_p}{S}\right)\frac{K_m}{(1+ST_m)}\left(\frac{1}{s}\right)$$

b) Return Path.

$$\Rightarrow H(s) = K_{TP}$$

c)

$\Rightarrow \theta_m = E_2 G_2 \qquad G_2 = \frac{1}{s}$

$\Rightarrow E_2 = (\theta_r - \theta_m K_{TP})(K_p)\left(\frac{1+sT_p}{s}\right)\left(\frac{K_m}{1+sT_m}\right)\left(\frac{1}{s}\right)$

$\Rightarrow \theta_m = (\theta_r - \theta_m K_{TP})(K_p)\left(\frac{1+sT_p}{s}\right)\left(\frac{K_m}{1+sT_m}\right)\left(\frac{1}{s}\right)$

Let $C_x = (K_p)\left(\frac{1+sT_p}{s}\right)\left(\frac{K_m}{1+sT_m}\right)\left(\frac{1}{s}\right)$

$\Rightarrow \theta_m = (\theta_r - \theta_m K_{TP}) C_x$

$\Rightarrow \theta_m + \theta_m C_x K_{TP} = \theta_r C_x$

$\Rightarrow \theta_m (1 + C_x K_{TP}) = \theta_r C_x$

$$\Rightarrow \frac{\theta_m}{\theta_r} = \frac{C_x}{C_x K_{TP} + 1}$$

$$\Rightarrow \frac{\theta_m}{\theta_r} = \frac{1/K_{TP}}{1 + \frac{1}{K_{TP} C_x}}$$

$$\Rightarrow \frac{\theta_m}{\theta_r} = \frac{1/K_{TP}}{1 + \frac{1}{K_{TP} K_p K_m \frac{(1+sT_p)}{s^2(1+sT_m)}}}$$

$$\Rightarrow \boxed{\frac{\theta_m(s)}{\theta_r(s)} = \frac{1/K_{TP}}{1 + \frac{s^2(1+sT_m)}{K_{TP} K_p K_m (1+sT_p)}}}$$

d) $\Rightarrow \theta_m \Rightarrow \cancel{Y(z)} = E(z)\,H(z)$ $\qquad H(z) = 1$

$$\Rightarrow \theta_m = E(z)\,H(z)$$

$$\Rightarrow E(z) = \left(-\theta_m K_{Tp} K_p \left(\frac{1+sT_p}{s}\right) - \frac{D R_a}{K_t}\right) \frac{K_m}{(1+sT_m)} \frac{1}{s}$$

$$\Rightarrow \theta_m = \left(-\theta_m K_{Tp} K_p \left(\frac{1+sT_p}{s}\right) - \frac{D R_a}{K_t}\right) \frac{K_m}{(1+sT_m)} \left(\frac{1}{s}\right)$$

$$\Rightarrow \boxed{\frac{\theta_m(s)}{D(s)} = \frac{\left(\frac{-R_a K_m}{K_t(1+sT_m)} \cdot \frac{1}{s}\right)}{\left(1 + K_{Tp} K_p \left(\frac{1+sT_p}{s}\right) \cdot \frac{K_m}{(1+sT_m)} \cdot \frac{1}{s}\right)}}$$

2.a)

$$\Rightarrow \theta_m = \left(\left(\theta_r - K_{TP}\,\theta_m\right) K_p \left(\frac{1}{s}\right) - \theta_m \cdot K_{TV}\right) K_V \left(\frac{1+sT_V}{s}\right)\left(\frac{K_m}{1+sT_m}\right)$$

$$\text{Let } C_x = (K_V)\left(\frac{1+sT_V}{s}\right)\left(\frac{K_m}{1+sT_m}\right)$$

$$\Rightarrow \theta_m = \left(\left(\theta_r - K_{TP}\,\theta_m\right) K_p \left(\frac{1}{s}\right) - \theta_m \cdot K_{TV}\right) C_x$$

$$\Rightarrow \theta_m + \theta_m \cdot K_{TV} \cdot C_x + \theta_m \cdot K_{TP} \cdot K_p \cdot C_x \left(\frac{1}{s}\right) = \theta_r \left(\frac{1}{s}\right)$$

$$\Rightarrow \frac{\theta_m}{\theta_r} = \frac{C_x \cdot K_p}{s + sK_{TV} \cdot C_x + K_{TP} \cdot K_p \cdot C_x}$$

$$\Rightarrow \frac{\theta_m}{\theta_r} = \frac{K_p}{\frac{s}{C_x} + sK_{TV} + K_{TP} \cdot K_p}$$

Since $T_m = T_V$, simplifying C_x.

$$C_x = K_V \frac{(1+sT_m)}{s} \frac{K_m}{(1+sT_m)}$$

$$= \frac{K_V K_m}{s}$$

$$\Rightarrow \frac{\theta_m}{\theta_r} = \frac{K_p}{\frac{s}{C_s} + sK_{TV} + K_{TP} \cdot K_p}$$

$$\Rightarrow \frac{\theta_m}{\theta_r} = \frac{K_p}{\frac{s^2}{K_v \cdot K_m} + sK_{TV} + K_{TP} \cdot K_p}$$

$$\Rightarrow \boxed{\frac{\theta_m}{\theta_r} = \frac{1/K_{TP}}{\frac{s^2}{K_v \cdot K_m \cdot K_{TP} \cdot K_p} + \frac{sK_{TV}}{K_{TP} \cdot K_p} + 1.}}$$

b) Given,

$I_m = 6\ kg\text{-}m^2$

$R_a = 0.3\ \Omega$

$K_t = 0.5\ N\text{-}m/A$

$K_v = 0.5\ V\text{-}s/rad$

$F_m = 0.001\ N\text{-}m\text{-}s/rad$

Given Siciliano, $K_m = 1/K_v$, $T_m = \frac{R_a \cdot I_m}{K_v \cdot K_t}$

$\Rightarrow$ we know,

$\Rightarrow K_V K_{TV} = \frac{2 \zeta \omega_n}{K_m}$ $\quad K_{TV} = 1$ [Siciliano 8.3 ...]

$\Rightarrow K_V = \frac{2 \times 0.4 \times 20}{0.5} \times 10^{2}\ rad^2/V\text{-}s^2$

$\Rightarrow \boxed{K_V = 32\ rad^2/V\text{-}s^2}$

$\Rightarrow K_p \cdot K_{TV} \cdot K_v = \dfrac{\omega_n^2}{K_m}$

$\Rightarrow 32 \times K_p \times 1 = \dfrac{(40)^2}{2}$

$\Rightarrow \boxed{K_p = 25}$

c) Disturbance rejection factor.

$\Rightarrow X_R = K_p \cdot K_v \cdot K_{TP}$.

$\Rightarrow X_R = 25 \times 32$

$\Rightarrow \boxed{X_R = 800}$.

d) $T_R = \max \left\{ T_m, \dfrac{1}{\zeta \omega_n} \right\}$

$= \max \left\{ 36/5, \dfrac{1}{8} \right\}$

$\boxed{T_R = 36/5}$

$T_m = \dfrac{R_a \cdot I_{am}}{K_v \cdot K_t}$

$= \dfrac{0.3 \times 6}{0.5 \times 0.5}$

$= 36/5$

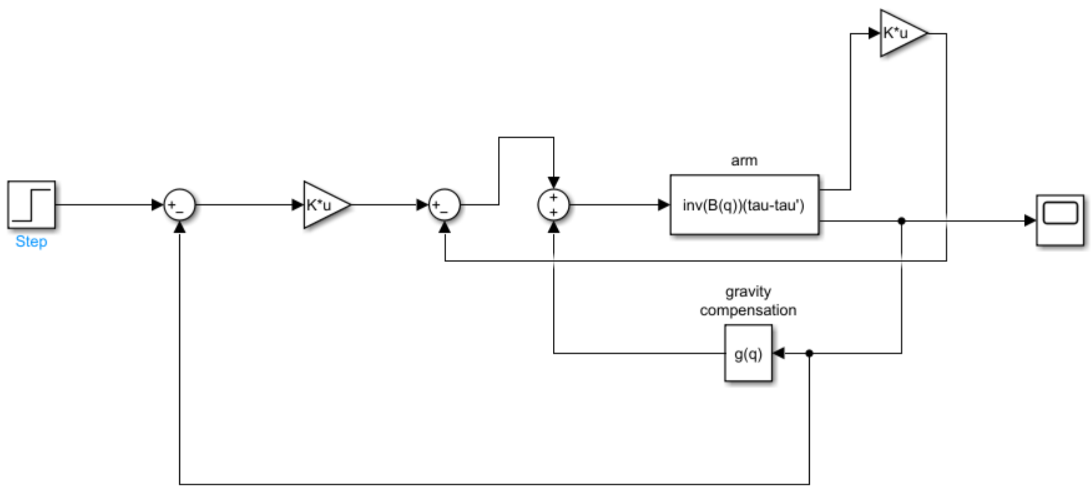
MATLAB CODE FOR POSITION 1:

```
%NIKUNJ SANGHAI
% Variable initialization

global a k_r1 k_r2 pi_m pi_l

% load manipulator dynamic parameters without load mass
  param;
  pi_l = pi_m;

% gravity acceleration
  g = 9.81;

% friction matrix
  K_r = [100 0; 0 100]
  F_m=[0.01 0;0 0.01]
  F_v = K_r*F_m*K_r

% sample time of controller
  Tc = [0.001]

% controller gains
  K_p = [500 0;0 500]
  K_d = [550 0;0 550]

% desired position
  q_d = [pi/4 -pi/2]
```

% Changing value here for position 2

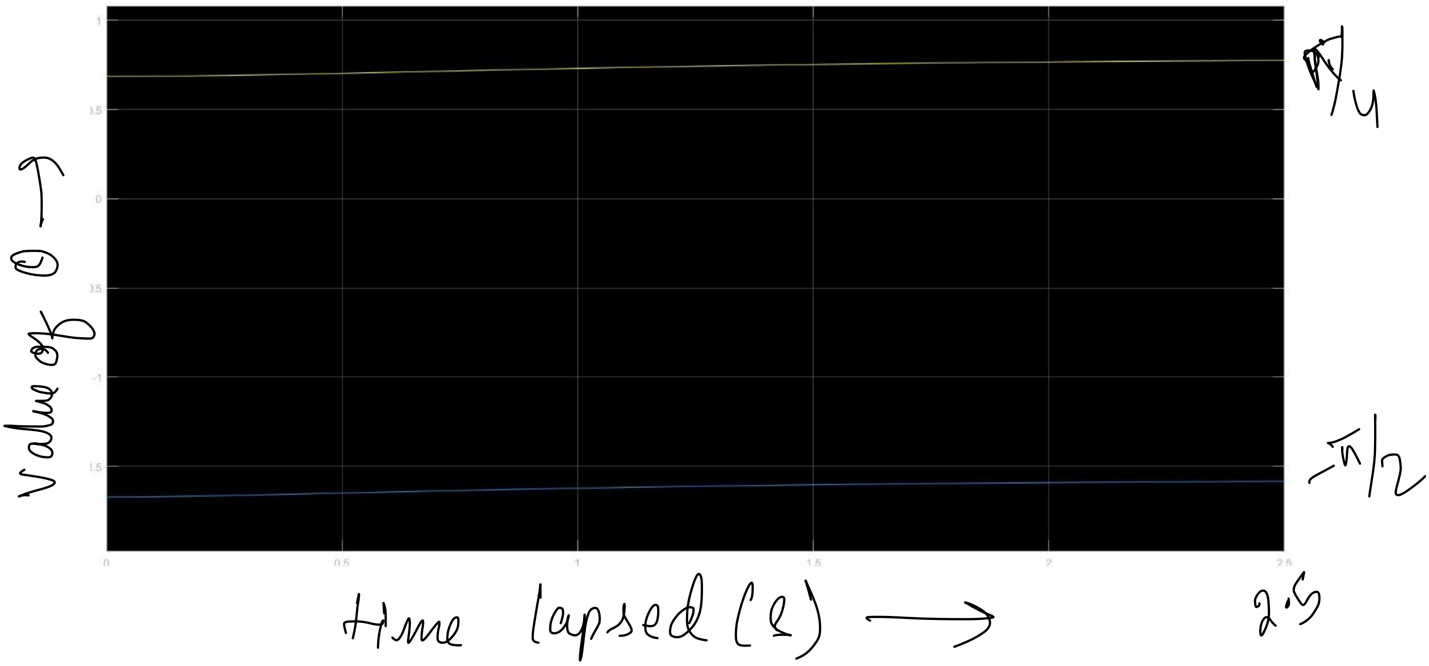
```
% initial position
  q_i = [pi/4-0.1 -pi/2-0.1]

% duration of simulation
  t_d = [2.5]

% sample time for plots
  Ts = Tc;
```

Snapshot Of Simulink Model:



Graphs for :

1)Position 1 [pi/4 -pi/2]



2)Position 2[-pi -3*pi/4]