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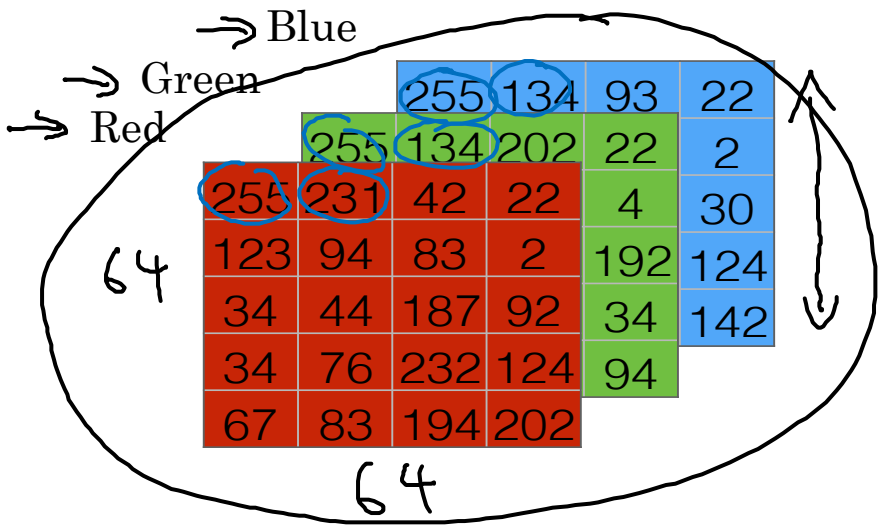
Basics of Neural Network Programming

Binary Classification

Binary Classification

→ 1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$64 \times 64 \times 3 = 12288$

$n = n_x = 12288$

$x \longrightarrow y$

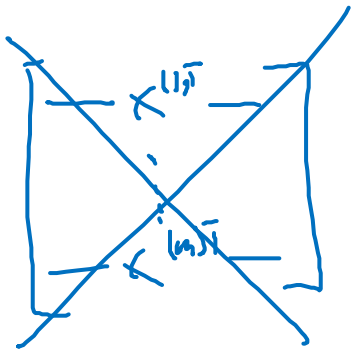
Notation

(x, y) $\quad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples : $\{(\underline{x}^{(1)}, \underline{y}^{(1)}), (\underline{x}^{(2)}, \underline{y}^{(2)}), \ldots, (\underline{x}^{(m)}, \underline{y}^{(m)})\}$

$m = m_{train}$ $\qquad m_{test} = \#\text{test examples.}$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \updownarrow n_x$$



$\longleftarrow m \longrightarrow$

$$\begin{bmatrix} - & x^{(1)T} & - \\ & \vdots & \\ - & x^{(m)T} & - \end{bmatrix}$$

$Y = [y^{(1)} \; y^{(2)} \; \cdots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

$X \in \mathbb{R}^{n_x \times m}$ $\qquad X.\text{shape} = (n_x, m)$

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Basics of Neural Network Programming

Logistic Regression

Logistic Regression

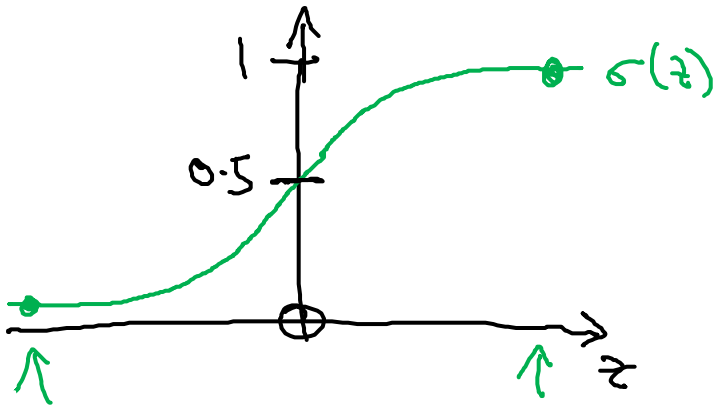
Given x, want $\hat{y} = P(y=1|x)$

$0 \le \hat{y} \le 1$

$x \in \mathbb{R}^{n_x}$

Parameters: $w \in \mathbb{R}^{n_x}$, $b \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1$, $x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix}$ $\begin{matrix} \} b \leftarrow \\ \\ \\ \\ \} w \leftarrow \end{matrix}$

$\sigma(z) = \frac{1}{1+e^{-z}}$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{BigNum}} \approx 0$

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Basics of Neural Network Programming

Logistic Regression cost function

Logistic Regression cost function

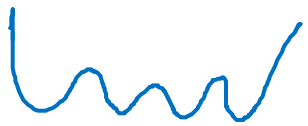
$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

$z^{(i)} = w^T x^{(i)} + b$

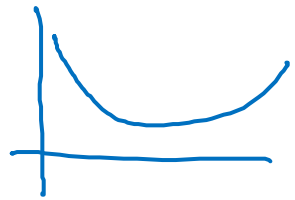
$x^{(i)}$
$y^{(i)}$
$z^{(i)}$
i-th example.

Given $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

Loss (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$



$\mathcal{L}(\hat{y}, y) = -(y \log \hat{y} + (1-y) \log(1-\hat{y}))$



If $y=1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y}$ ← want $\log \hat{y}$ large, want $\hat{y}$ large.

If $y=0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y})$ ← want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

Cost function: $J(w,b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^{m} \left[y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)}) \right]$

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Basics of Neural Network Programming

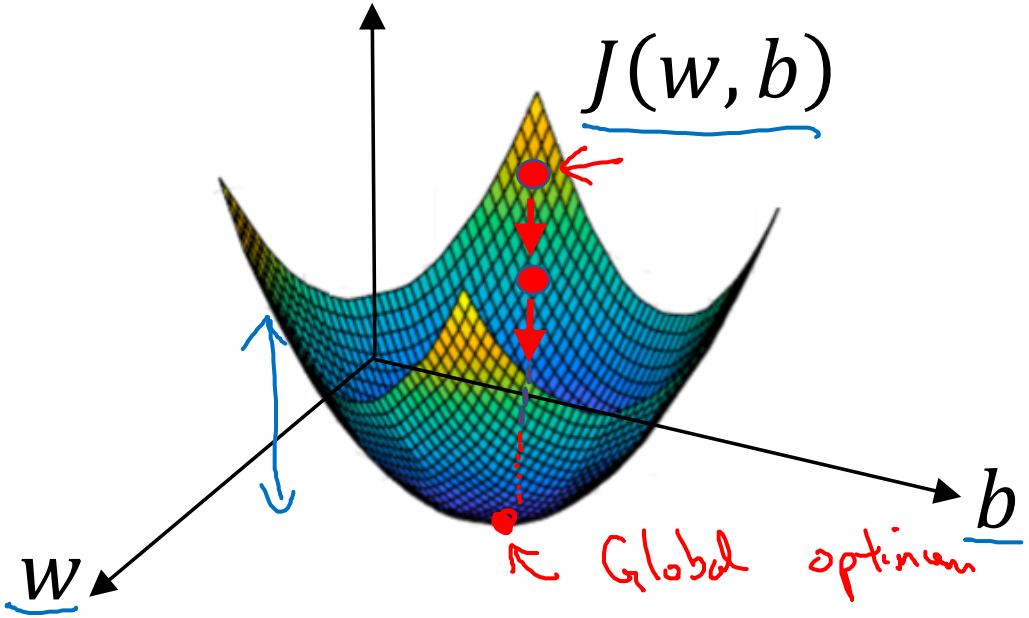
Gradient Descent

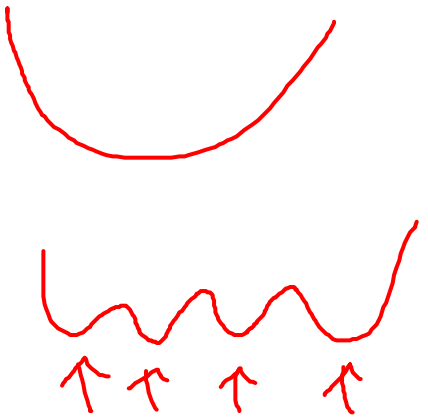
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

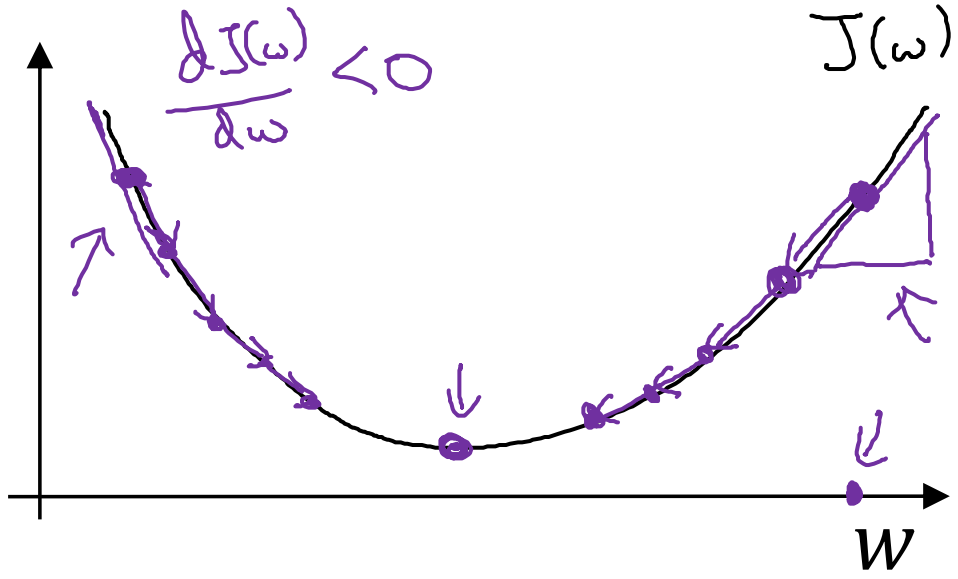
$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





Gradient Descent



Repeat {

$w := w - \alpha \frac{dJ(w)}{dw}$ ← learning rate

}

"dw"

$w := w - \alpha dw$

$\frac{dJ(w)}{dw} = ?$

$J(w,b)$

$w := w - \alpha \frac{\partial J(w,b)}{\partial w}$ → dw

$b := b - \alpha \frac{\partial J(w,b)}{\partial b}$ → db

∂ "partial derivative"

d ← J

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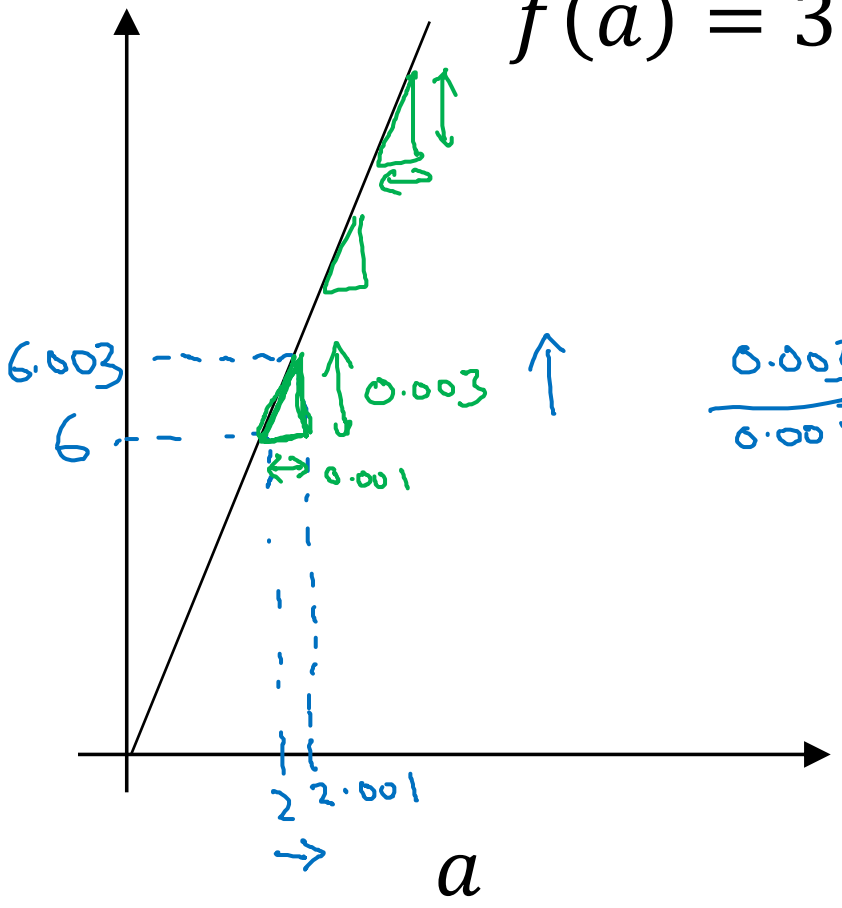
Basics of Neural Network Programming

Derivatives

Intuition about derivatives



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

→ $a = 2$ $f(a) = 6$

$a = 2.001$ $f(a) = 6.003$

slope (derivative) of $f(a)$ at $a = 2$ is 3

→ $a = 5$ $f(a) = 15$

$a = 5.001$ $f(a) = 15.003$

slope at $a = 5$ is also 3

$\frac{d f(a)}{da} = 3 = \frac{d}{da} f(a)$

0.001
0.000000001
0.000000000001

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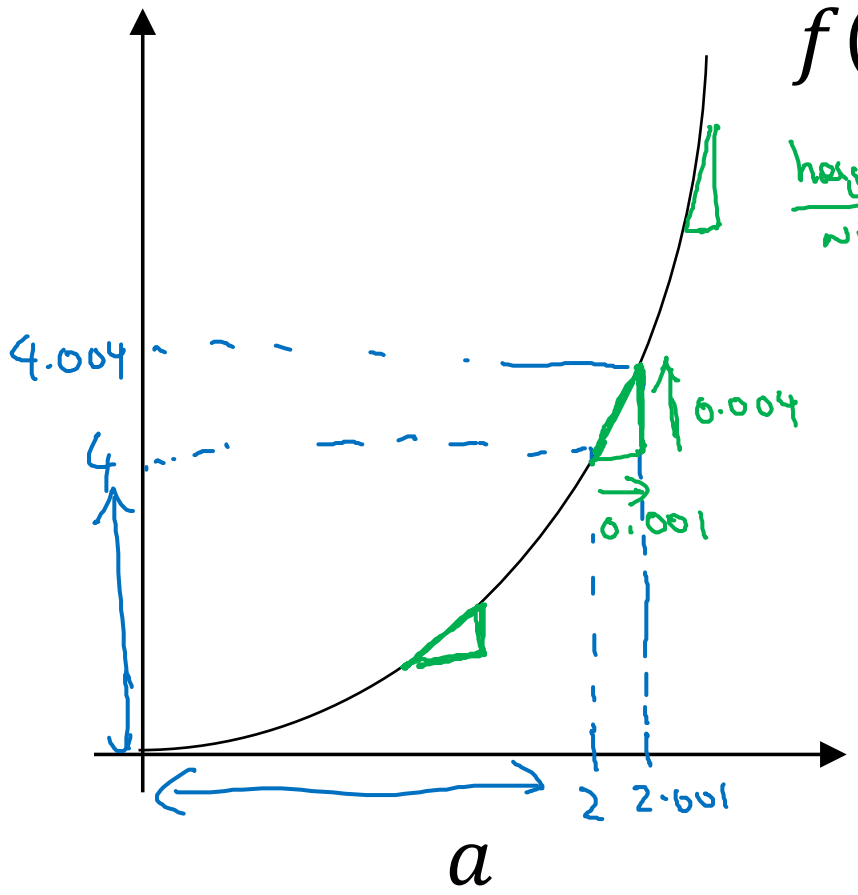
Basics of Neural Network Programming

More derivatives examples

Intuition about derivatives



$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

0.001 ←

$0.000000\ldots01$ ←

$a = 2$

$a = 2.001$

$f(a) = 4$

$f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of $f(a)$ at $a = 2$ is 4.

$\frac{d}{da} f(a) = 4$ when $a = 2$.

$a = 5$

$a = 5.001$

$f(a) = 25$

$f(a) \approx 25.010$

$\frac{d}{da} f(a) = 10$ when $a = 5$

$\frac{d}{da} f(a) = \frac{d}{da} a^2 = 2a$

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More derivative examples

$f(a) = a^2$

$\frac{d}{da} f(a) = \underbrace{2a}_{4}$

$a = 2$ $\quad f(a) = 4$

$a = 2.001$ $\quad f(a) \approx 4.004$

$f(a) = a^3$

$\frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$

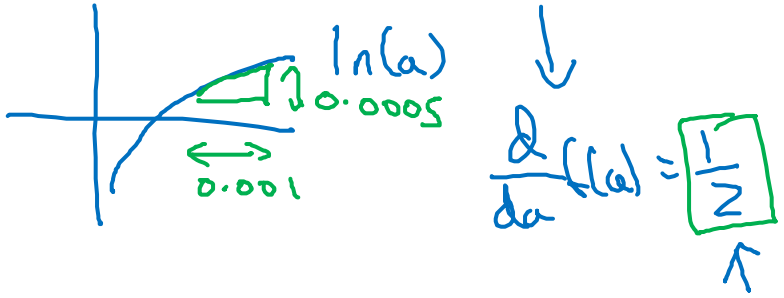
$a = 2$ $\quad f(a) = 8$

$a = 2.001$ $\quad f(a) \approx 8.012$

$f(a) = \log_e(a)$

$\ln(a)$

$\frac{d}{da} f(a) = \frac{1}{a}$



$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$

$a = 2$ $\quad f(a) \approx 0.69315$

$a = 2.001$ $\quad f(a) \approx 0.69365$

0.0005

0.0005

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Basics of Neural Network Programming

Computation Graph

Computation Graph

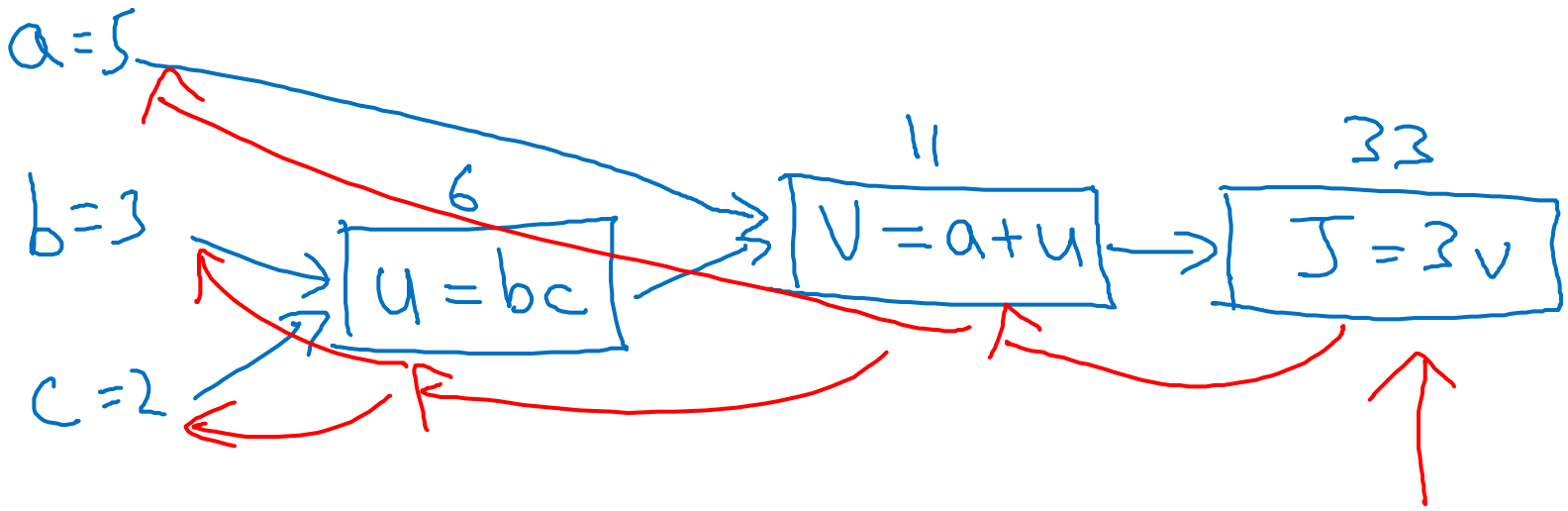
$J(a,b,c) = 3(a+bc) = 3(5+3\times 2) = 33$

u, v, J

$u = bc$

$v = a+u$

$J = 3v$



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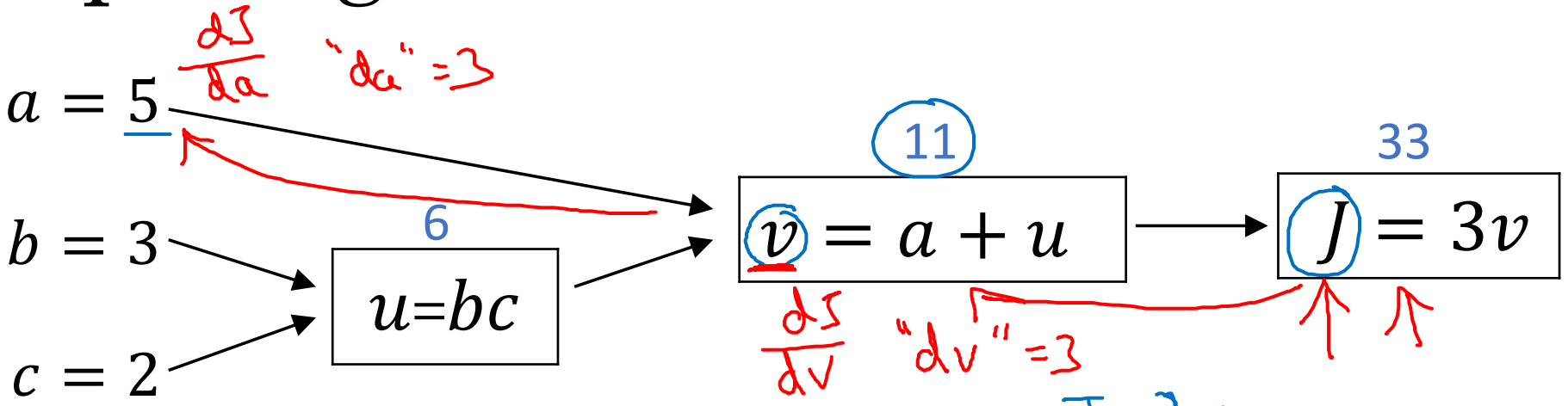
Basics of Neural Network Programming

Derivatives with a Computation Graph

Computing derivatives



$\frac{dJ}{dv} = ? = 3$

$\frac{dJ}{da} = 3 = \frac{dJ}{dv} \frac{dv}{da}$

3×1

$\frac{dv}{da} = 1$

$a \rightarrow v \rightarrow J$

$J = 3v$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$a = 5 \rightarrow 5.001$

$\rightarrow v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$f(a) = 3a$

$\frac{df(a)}{da} = \frac{df}{da} = 3$

$J = 3v$

$\frac{dJ}{dv} = 3$

$\frac{d\,\text{FinalOutputVar}}{d\,\text{var}}$

dJdvar

"dvar"

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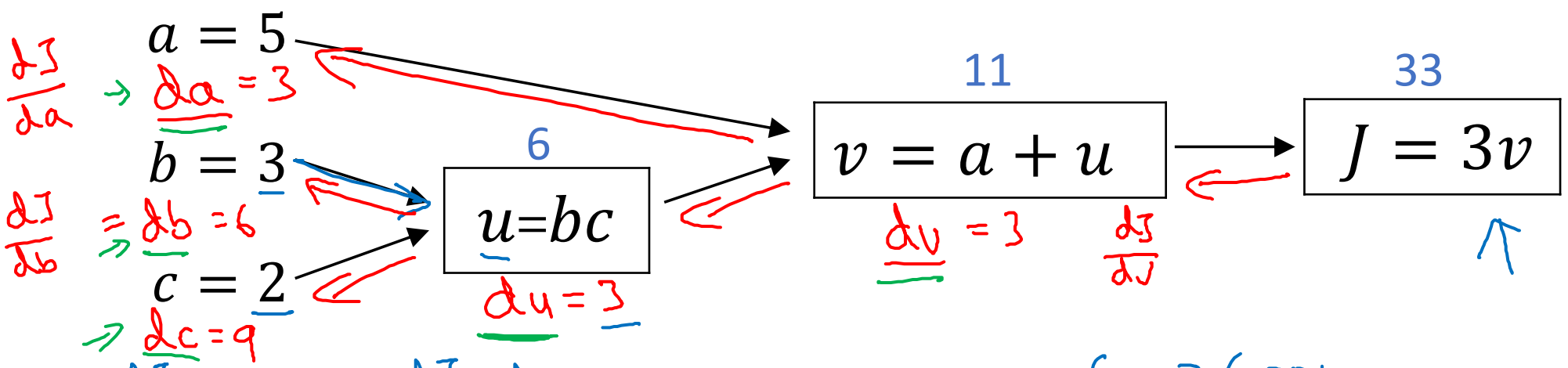
Computing derivatives



$$\frac{dJ}{du} = 3 = \frac{dJ}{dv} \cdot \frac{dv}{du}$$

($\frac{dJ}{dv}$: 3, $\frac{dv}{du}$: 1)

$$\frac{dJ}{db} = \frac{dJ}{du} \cdot \frac{du}{db} = 6$$

($\frac{dJ}{du} \rightarrow 3$, $\frac{du}{db} = 2$)

$$\frac{dJ}{da} = \frac{dJ}{du} \cdot \frac{du}{da} = 9$$

($\frac{dJ}{du} = 3 \times 3$)

$u = 6 \rightarrow 6.001$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$b = 3 \rightarrow 3.001$

$u = b \cdot c = 6 \rightarrow 6.002$

$J = 33.006$

$c = 2$

$.006$

$v = 11.002$

$J = 3v$

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Basics of Neural Network Programming

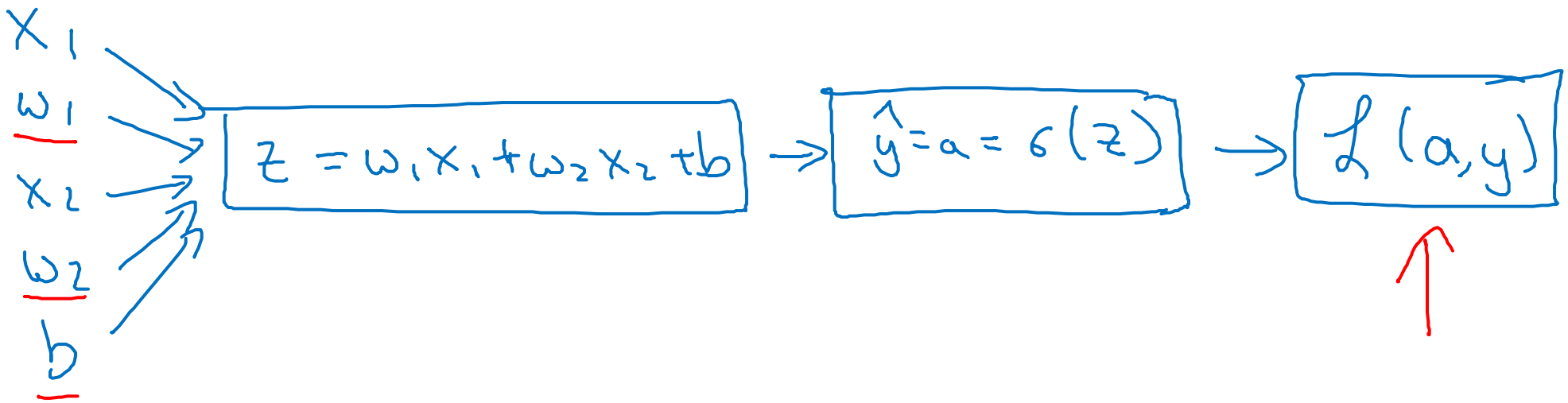
Logistic Regression Gradient descent

Logistic regression recap

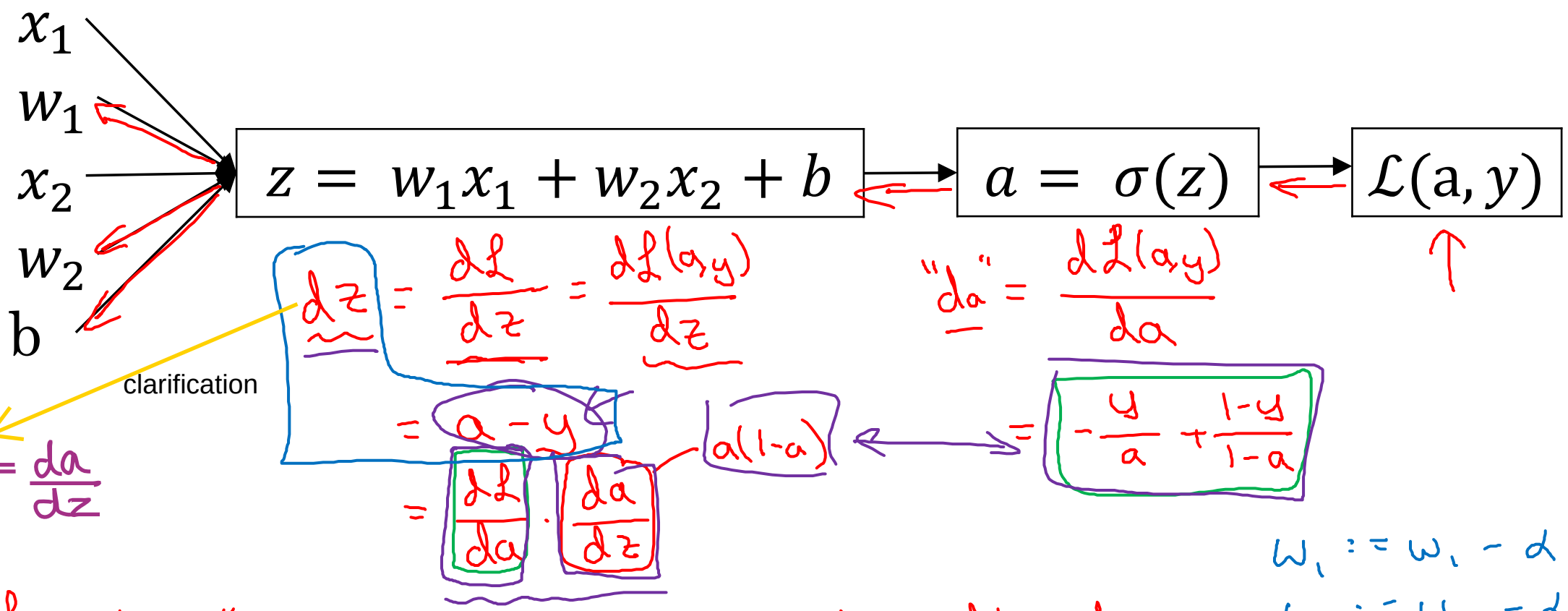
$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



Logistic regression derivatives



$\frac{\partial L}{\partial w_1} = "dw_1" = x_1 \cdot dz$ $\quad dw_2 = x_2 \cdot dz \quad db = dz$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})$$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$(x^{(i)}, y^{(i)})$

$dw_1^{(i)}$, $dw_2^{(i)}$, $db^{(i)}$

$$\frac{\partial}{\partial w_1} J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})$$

$dw_1^{(i)}$ — $(x^{(i)}, y^{(i)})$

Logistic regression on m examples

$J=0$; $dw_1=0$; $dw_2=0$; $db=0$

For $i=1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -\left[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})\right]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

dw_3 ... dw_n

$n=2$

$J /= m$

$dw_1 /= m$; $dw_2 /= m$; $db /= m$.

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$.

Vectorization

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Basics of Neural Network Programming

Vectorization

What is vectorization?

$z = w^T x + b$

$w = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ $x = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ $w \in \mathbb{R}^{n_x}$ $x \in \mathbb{R}^{n_x}$

Non-vectorized:

```
z = 0
for i in range(n-x):
    z += w[i] * x[i]
z += b
```

Vectorized

```
z = np.dot(w,x) + b
```

$w^T x$

→ GPU
→ CPU

} SIMD - single instruction multiple data.

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Basics of Neural Network Programming

More vectorization examples

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

$u = Av$

$u_i = \sum_j A_{ij} v_j$

```
u = np.zeros((n,1))
for i ...
    for j ...
        u[i] += A[i][j] * v[j]
```

```
u = np.dot(A,v)
```

Vectors and matrix valued functions

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.zeros((n,1))
for i in range(n):
    u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v)
```

```
np.log(v)
np.abs(v)
np.maximum(v,0)
v**2
1/v
```

Logistic regression derivatives

$J = 0$, ~~$dw_1 = 0$, $dw_2 = 0$~~, $db = 0$

$dw = np.zeros((n_x, 1))$

for $i = 1$ to n:

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -\left[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\right]$

$dz^{(i)} = a^{(i)}(1 - a^{(i)})$

for $i = 1 \ldots n_x$:

~~$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$~~

~~$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$~~

$n_x = 2$

$dw \mathrel{+}= x^{(i)} dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

$J = J/m$, ~~$dw_1 = dw_1/m$, $dw_2 = dw_2/m$~~, $db = db/m$

$dw /= m$.

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Basics of Neural Network Programming

Vectorizing Logistic Regression

Vectorizing Logistic Regression

$$z^{(1)} = w^T x^{(1)} + b$$
$$a^{(1)} = \sigma(z^{(1)})$$

$$z^{(2)} = w^T x^{(2)} + b$$
$$a^{(2)} = \sigma(z^{(2)})$$

$$z^{(3)} = w^T x^{(3)} + b$$
$$a^{(3)} = \sigma(z^{(3)})$$

$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$ (n_x, m) $\mathbb{R}^{n_x \times m}$

$w^T = \begin{bmatrix} - & x^{(1)} & - \\ - & x^{(2)} & - \\ & \vdots & \\ - & x^{(n_x)} & - \end{bmatrix}$

$Z = \begin{bmatrix} z^{(1)} & z^{(2)} & \cdots & z^{(m)} \end{bmatrix} = w^T X + \underbrace{[b \; b \cdots b]}_{1 \times m} = \begin{bmatrix} w^T x^{(1)} + b & w^T x^{(2)} + b & \cdots & w^T x^{(m)} + b \end{bmatrix}$

$Z = np.dot(w.T, X) + b$ $(1,1)$ $\mathbb{R}$ "Broadcasting"

$A = [a^{(1)} \; a^{(2)} \cdots a^{(m)}] = \sigma(Z)$

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Basics of Neural Network Programming

Vectorizing Logistic Regression's Gradient Computation

Vectorizing Logistic Regression

$dz^{(1)} = a^{(1)} - y^{(1)}$ $dz^{(2)} = a^{(2)} - y^{(2)}$ ....

$dZ = [dz^{(1)}\ dz^{(2)} \cdots dz^{(m)}]$ ←

$1 \times m$

$A = [a^{(1)} \cdots a^{(m)}]$. $Y = [y^{(1)} \cdots y^{(m)}]$

→ $dZ = A - Y = [a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \quad \cdots]$

→ $dw = 0$

$dw += X^{(1)} dz^{(1)}$

$dw += X^{(2)} dz^{(2)}$

⋮

$dw /= m$

$db = 0$

$db += dz^{(1)}$

$db += dz^{(2)}$

⋮

$db += dz^{(m)}$

$db /= m.$

$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)}$

$= \frac{1}{m}$ np.sum(dz)

$dw = \frac{1}{m} X dz^T$

$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \cdots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$

$= \frac{1}{m} [x^{(1)} dz^{(1)} + \cdots + x^{(m)} dz^{(m)}]$

$n \times 1$

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Implementing Logistic Regression

$J = 0,\ dw_1 = 0,\ dw_2 = 0,\ db = 0$

for i = 1 to m:

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J\ += -\left[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})\right]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1\ += x_1^{(i)} dz^{(i)}$

$dw_2\ += x_2^{(i)} dz^{(i)}$

$dw\ += x^{(i)} * dz^{(i)}$

$db\ += dz^{(i)}$

$J = J/m,\ dw_1 = dw_1/m,\ dw_2 = dw_2/m$

$db = db/m$

for iter in range(1000):

$Z = w^T X + b$

$= np.dot(w.T, X) + b$

$A = \sigma(Z)$

$dZ = A - Y$

$dw = \frac{1}{m} X dZ^T$

$db = \frac{1}{m} np.sum(dZ)$

$w := w - \alpha\, dw$

$b := b - \alpha\, db$

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Basics of Neural Network Programming

Broadcasting in Python

Broadcasting example

Calories from Carbs, Proteins, Fats in 100g of different foods:

	Apples	Beef	Eggs	Potatoes
Carb	56.0	0.0	4.4	68.0
Protein	1.2	104.0	52.0	8.0
Fat	1.8	135.0	99.0	0.9

$= A$ (3,4)

59 cal

$\frac{56}{59} \approx 94.9\%$

Calculate % of calories from Carb, Protein, Fat. Can you do this without explicit for-loop?

```
cal = A.sum(axis = 0)
percentage = 100*A/(cal.reshape(1,4))
```

(3,4) / (1,4)

Broadcasting example

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \begin{bmatrix}100\\100\\100\\100\end{bmatrix} \cancel{100}$$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}100 & 200 & 300\\100 & 200 & 300\end{bmatrix}$$

(m,n) $(2,3)$ $(1,n) \rightsquigarrow (m,n)$ $(2,3)$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}100 & 100 & 100\\200 & 200 & 200\end{bmatrix} =$$

(m,n) $(m,1)$ $\rightsquigarrow$ (m,n)

General Principle

(m, n) matrix

$+ \; - \; * \; /$

$(1, n) \rightsquigarrow (m, n)$

$(m, 1) \rightsquigarrow (m, n)$

$(m, 1) + \mathbb{R}$

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 \\ 102 \\ 103 \end{bmatrix}$

$[1 \; 2 \; 3] + 100 = [101 \quad 102 \quad 103]$

Matlab/Octave: bsxfun

deeplearning.ai

Basics of Neural Network Programming

Explanation of logistic regression cost function (Optional)

Logistic regression cost function

$\hat{y} = \sigma(w^T x + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$

Interpret $\hat{y} = p(y=1|x)$

If $y=1$: $p(y|x) = \hat{y}$

If $y=0$: $p(y|x) = 1 - \hat{y}$

Logistic regression cost function

If $y = 1$: $p(y|x) = \hat{y}$

If $y = 0$: $p(y|x) = 1 - \hat{y}$

$p(y|x)$

$p(y|x) = \hat{y}^{y} (1-\hat{y})^{(1-y)}$

If $y=1$: $p(y|x) = \hat{y}^{1} (1-\hat{y})^{0} = \hat{y}$

If $y=0$: $p(y|x) = \hat{y}^{0} (1-\hat{y})^{(1-0)} = 1 \times (1-\hat{y}) = 1-\hat{y}$

$\uparrow \log p(y|x) = \log \hat{y}^{y} (1-\hat{y})^{(1-y)} = y \log \hat{y} + (1-y) \log (1-\hat{y})$

$= -\mathcal{L}(\hat{y}, y) \downarrow$

Andrew Ng

Cost on m examples

$$\log p(\text{labels in training set}) = \log \prod_{i=1}^{m} p(y^{(i)} \mid x^{(i)})$$

$$\log p(\dots) = \sum_{i=1}^{m} \log p(y^{(i)} \mid x^{(i)})$$

$$= -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

Maximum likelihood estimation

Cost: (minimize)

$$J(w,b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$