

College of Engineering, Pune.
B.Tech II Year
November 2022, Odd Semester

TEST - 1

Linear Algebra and Uni-variate Calculus

Duration-1.5 hours Marks-30

All Questions Compulsory

Use of Programmable Calculator Forbidden

Q.1) Solve the following:

(a) $5y(xy - 8)dy + (y^3 + 8y)dy = 0$

(b) $(2 \tan y + 2x^2)dx - (x \sin y)dy = 0$

[3x2=6]

Q.2) Find a homogeneous linear second order ordinary differential equation whose solution is the set of all straight lines in the xy -plane.

[2]

Q.3) Which of the following forms sub-spaces. Provide or prove counter examples

$$\{(x, y) \in \mathbb{R}^2 \mid x = y + 2\}$$

$$\{(x, y, z) \in \mathbb{R}^3 \mid x = y, 2z = z\}$$

[3x2=6]

Q.4) If x^2 and 1 are solutions of $yy'' - xy' = 0$ then so is any linear combination of these. State true or false and justify.

[3]

Q.5) Find a linear ordinary differential equation for which the function $e^{-x} \cos 2x$ and $e^{-x} \sin 2x$ are linearly independent solutions.

[3]

Q.6) Find the Inverse :

$$A = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -5 & 5 \\ -2 & 0 & -1 \end{bmatrix}$$

[5]

Q.7) Determine the values of a and b:

i. No-solution. ii. Infinite number of solutions iii. Unique solution

$$2x - y + 4z = 6 \tag{1}$$

$$x + y - 2z = 0 \tag{2}$$

$$3x + 2y + 1z = 3 \tag{3}$$

$$[5]$$

Q.6) Find Inverse:

$$A = \begin{bmatrix} 3 & 0 & 0 & -1 \\ -1 & 0 & 3 & 0 \\ a1 & b & c4 & 3 \\ -1 & 0 & 3 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 0 & 0 & -1 \\ -1 & 0 & 3 & 0 \\ a1 & b & c4 & 3 \\ -1 & 0 & 3 & 2 \end{bmatrix}$$

$$[2 \times 4 = 8]$$

0.1 Assignment 2: Mathematical Expressions in Latex:

Equations *Expressions* in Latex.

Equation 1:

$$2x^2 + 4 = 6. \quad (4)$$

Equation 2:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0.$$

Multiplication:

$$a \cdot b$$

Fraction:

$$\frac{a}{b}$$

Definite Integral:

$$\int_a^b f(x) dx. f(x) \text{ og } dx$$

Summation:

$$\sum_{i=1}^n f(x_i) \Delta x$$

Limits:

$$\lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Determinants:

$$|-5| = 5$$

Root:

$$\sqrt{x+1}$$

Vector:

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \hat{a} \text{ og } \overrightarrow{AB}$$

Sets:

$$\{x \in \mathbb{R} \mid 2 \leq x < 5\}$$

Sets:

$$f(x) = \begin{cases} x^2 & x > 2, \\ x - 1 & x \leq 2 \end{cases}$$

Bracket Fraction:

$$\left(\frac{a}{b}\right)$$

Matrices:

$$A = \begin{bmatrix} 6 & 9 & 3 \\ 5 & 2 & 1 \\ 4 & 8 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Mathematical Symbols:

$$\pm \quad \infty \quad \leq \quad \geq \quad \circ \quad \in \quad \notin \quad \neq \quad \bullet \quad \Leftrightarrow \quad \Updownarrow \quad \times \quad \angle$$

Trigonometry, Logarithm and Exponential :

$$\sin(x) \quad \cos(x) \quad \tan(x) \quad \ln(x) \quad \log(x) \quad \exp(x)$$

Formula Derivation

Identities:

$$\begin{aligned} (a + b)^2 &= (a + b)(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned}$$

Problem Statement :

Find a and b redAssume the Values, \LaTeX .

$$bluea^2 + b^2 = c^2 \text{ kg}$$

$$a + b = 4,56e4kg.m^2.s^{-3}.$$