ASSIGNMENT-4

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Ans! Given,

$$x + \alpha y = 5 \text{ — eq 1}$$
$$x + 2y = \alpha \text{ — eq 2}$$
$$y - 2z = 6 \text{ — eq 3}$$

(a) if $\alpha = 3$

by putting $\alpha = 3$

$$x + 3y = 5 \text{ — eq 1}$$
$$x + 2y = 3 \text{ — eq 2}$$
$$y - 2z = 6 \text{ — eq 3}$$

eq 4 → eq1 − 1 × eq2

eq 4 → $(x + 3y) - (x + 2y) = (5) - (3)$

$$\boxed{y = 2}$$

Put $y = 2$ in eq 2

$$x + 2(2) = 3$$
$$x = 3 - 4$$
$$\boxed{x = -1}$$

Put $y = 2$ in eq 3

$$(2) - 2z = 6$$
$$-2z = 6 - 2$$
$$\boxed{z = -2}$$

$$\boxed{\text{Solution} \rightarrow (x = -1, y = 2, z = -2)}$$

(b) if $\alpha = 2$

$$x + 2y = 5 \quad \text{— eq 1}$$
$$x + 2y = 2 \quad \text{— eq 2}$$
$$y - 2z = 6 \quad \text{— eq 3}$$

both eq 1 and eq 2 both have $x+2y$ in the left side. but different value on right and side (5 and 2 respectively). This lead to contradiction:

$$5 \neq 2$$

$$\boxed{\text{no solution exist if } \alpha = 2, \text{ as the system is inconsistent}}$$

(c) if $x = 3$ (Find all solutions)

Solving y in terms of α by putting $x = 3$ in eq 1

$$\alpha y = 5 - 3$$
$$\alpha y = 2$$
$$y = \frac{2}{\alpha} \text{ (assuming } \alpha \neq 0)$$

Putting $y = \frac{2}{\alpha}$ into eq^n. 2

$$3 + 2\left(\frac{2}{\alpha}\right) = \alpha$$
$$3 + \frac{4}{\alpha} = \alpha$$
$$\frac{3\alpha + 4}{\alpha} = \alpha$$
$$3\alpha + 4 = \alpha^2$$

$$\alpha^2 - 3\alpha - 4 = 0$$

Solving this quadratic for α using the quadratic formula:

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)}$$

$$\alpha = \frac{3 \pm \sqrt{9 + 16}}{2}$$

$$\alpha = \frac{3 \pm \sqrt{25}}{2}$$

$$\alpha = \frac{3+5}{2}, \quad \alpha = \frac{3-5}{2}$$

$$\alpha = 4, \quad \alpha = -1$$

If $\alpha = 4$, $x = 3$

Putting $\alpha = 4$ into $y = \frac{2}{\alpha}$

$$y = \frac{2}{4} = \frac{1}{2}$$

$$y = \frac{1}{2}$$

Putting $y = \frac{1}{2}$ into eq 3

$$\frac{1}{2} - 2z = 6$$

$$-2z = 6 - \frac{1}{2}$$

$$-2z = \frac{11}{2}$$

$$z = -\frac{11}{4}$$

So, one sol^n is:-

$$\boxed{x = 3, y = \frac{1}{2}, z = -\frac{11}{4}, \alpha = 4}$$

If $\alpha = -1$

Putting $\alpha = 1$ in $y = \frac{2}{\alpha}$

$$y = \frac{2}{-1} = -2$$

$$y = -2$$

Putting $y = -2$ into eq^n 3 to find z

$$-2 - 2z = 6$$

$$-2z = 8$$

$$z = -4$$

$$\boxed{x = 3, y = -2, z = -4, \alpha = -1}$$

Solution set:-

$$\{y, z, \alpha\} = \{(\tfrac{1}{2}, -\tfrac{11}{4}, 4), (-2, -4, -1)\}$$

$$\boxed{\{y, z, \alpha\} = \{(\tfrac{1}{2}, -\tfrac{11}{4}, 4), (-2, -4, -1)\}}$$

Ans 2.

$$A = \begin{bmatrix} 4 & 0 & \omega \\ 1 & -2 & -1 \\ -3 & \omega & 1 \end{bmatrix}$$

A matrix is not inventible if its determinant is zero.

The determinant of A (det(A)), can be calculated using cofactor expansion along the first row:

$$\det(A) = 4\times\begin{vmatrix} -2 & -1 \\ \omega & 1 \end{vmatrix} - 0\times\begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix} + \omega\times\begin{vmatrix} 1 & -2 \\ -3 & \omega \end{vmatrix}$$

$$\det(A) = 4\begin{vmatrix} -2 & -1 \\ \omega & 1 \end{vmatrix} - 0 + \omega\times\begin{vmatrix} 1 & -2 \\ -3 & \omega \end{vmatrix}$$

Now calculated co-factor of 4 and ω

co-factor of 4:- $\begin{vmatrix} -2 & -1 \\ \omega & 1 \end{vmatrix} = (-2)(1) - (-1)(\omega) = -2+\omega$

$\omega - 2$

co-factor of ω:- $\begin{vmatrix} 1 & -2 \\ -3 & \omega \end{vmatrix} = (1)(\omega) - (-2)(-3) = \omega - 6$

Now,

$$\det(A) = 4\times(\omega-2) + \omega\times(\omega-6)$$

$$\det(A) = 4\omega - 8 + \omega^2 - 6\omega$$

$$\det(A) = \omega^2 - 2\omega - 8$$

For noninventible matrix $\det(A) = 0$

$$\omega^2 - 2\omega - 8 = 0$$

This is a quadratic equation in w, solving using the quadratic formula :-

$$w = \frac{(-2) \pm \sqrt{(-2)^2 - 4.(1).(-8)}}{2(1)}$$

$$w = \frac{2 \pm \sqrt{4+32}}{2}$$

$$w = \frac{2 \pm \sqrt{36}}{2}$$

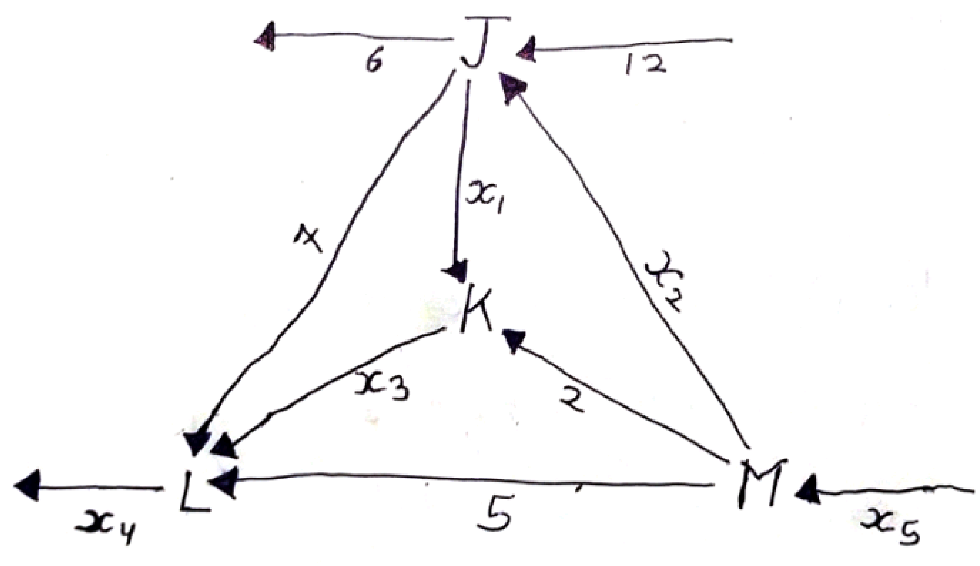
$$w = \frac{2+6}{2}, \quad w = \frac{2-6}{2}$$

$$w = 4, \quad w = -2$$

The values of w for which A are non-inverstable are:-

$$\boxed{w = \{4, -2\}}$$

Ans3.



J, K, L, M are junctions. Inflow is the ~~flow~~ water entering into junction and outflow is the water leaving the junction.

For Junction J :-

$$Inflow = 12 + x_2$$

$$Outflow = 6 + 7 + x_1$$

$\Rightarrow$ we not flow of water entering - flow of water leaving is equal to 0. For all the junctions.

$$Inflow - Outflow = 0$$

$$(12 + x_2) - (6 + 7 + x_1) = 0$$

$$x_2 - x_1 - 1 = 0$$

$$x_2 - x_1 = 1 \text{ — For J}$$

For Junction K :-

$$Inflow = x_1 + 2$$

$$outflow = x_3$$

$$Inflow - outflow = 0$$

$$(x_1 + 2) - (x_3) = 0$$

$$x_1 - x_3 = -2$$

$$x_3 - x_1 = 2 \text{ — For K}$$

For Junction L :-

$$Inflow = 7 + x_3 + 5$$

$$Outflow = x_4$$

$$Inflow - Outflow = 0$$

$$(7 + x_3 + 5) - (x_4) = 0$$

$$x_3 - x_4 = -12 \text{ — For L}$$

For Junction M:-

$$\text{Inflow} = x_5$$

$$\text{Outflow} = x_2 + 2 + 5$$

$$\text{Inflow} - \text{Outflow} = 0$$

$$(x_5) - (x_2 + 2 + 5) = 0$$

$$x_5 - x_2 = 7 \text{ — For Junction M}$$

=> Now our 4 equation for all 4 Junctions are:-

$$x_2 - x_1 = 1 \text{ — For J}$$

$$x_3 - x_1 = 2 \text{ — For K}$$

$$x_3 - x_4 = -12 \text{ — For L}$$

$$x_5 - x_2 = 7 \text{ — For M}$$

=> Now Representing the above system of linear equation in matrix representation by using $Ax = b$

here, A is the coefficient matrix

x is the column matrix of variables

b represent the column matrix of constants.

$$Ax = b$$

∴ By using above 4 equations of (J, K, L, M):-

$$\boxed{Ax = b \Rightarrow \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -12 \\ 7 \end{bmatrix}}$$

Above is the expression of linear eqs system in Matrix form $Ax = b$.

Ans 4. Given Augmented Matrix :-

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 1 & -2 & 0 \\ 1 & 2 & -1 & 1 \end{array}\right]$$

To convert it in reduced row echelon form (RREF) and solve for x_1, x_2, x_3

Step 1:- Eliminate the entry in a_{31}
Make a_{31} entry equal to 0:

$$R_3 \rightarrow R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 1 & -2 & 0 \\ 1-1 & 2-3 & -1-0 & 1-4 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 1 & -2 & 0 \\ 0 & -1 & -1 & -3 \end{array}\right]$$

Step 2:- Eliminate the entry in a_{32}
Make a_{32} entry equal to 0:

$$R_3 \rightarrow R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 1 & -2 & 0 \\ 0+0 & -1+1 & -1-2 & -3+0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

Step 3:- Scale the row 3 value to get a leading 1
make a_{33} entry equal to 1 by dividing row 3 by -3:

$$R_3 \rightarrow \frac{R_3}{-3}$$

$$\begin{bmatrix} 1 & 3 & 0 & | & 4 \\ 0 & 1 & -2 & | & 0 \\ \frac{0}{-3} & \frac{0}{-3} & \frac{-3}{-3} & | & \frac{-3}{-3} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 0 & | & 4 \\ 0 & 1 & -2 & | & 0 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

Step 4:- Eliminate the entry at a_{23}

make a_{23} equal to 0 by adding 2 times of row 3 in row 2:

$$R_2 \rightarrow R_2 + 2R_3$$

$$\begin{bmatrix} 1 & 3 & 0 & | & 4 \\ 0+0 & 1+0 & -2+2 & | & 0+2 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 0 & | & 4 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

Step 5 :- Eliminate the entry at a_{12}

make a_{12} equal to 0 by subracting 3 times of row 2 from row 1

$$R_1 \rightarrow R_1 - 3R_2$$

$$\begin{bmatrix} 1-0 & 3-3 & 0-0 & | & 4-6 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

Final, reduced row echelon form :-

$$[A|b] = \begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

Now, matrix is in reduced row echelon form, we can read off the solution :-

$$x_1 = -2, \quad x_2 = 2, \quad x_3 = 1$$

Then, the solution set of the system is :-

$$\boxed{x = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}}$$