Assignment - II

Ans 1. Let Echindna = E, Goanna = G and Koala = K

Only E = 15 people = Represented by + in venn diagram

Only G = 11 people = Represented by - in venn diagram

Only K = 8 people = Represented by * in venn diagram

$N((E \cap G) \setminus K) = 24$ people = Represented by Δ in venn diagram

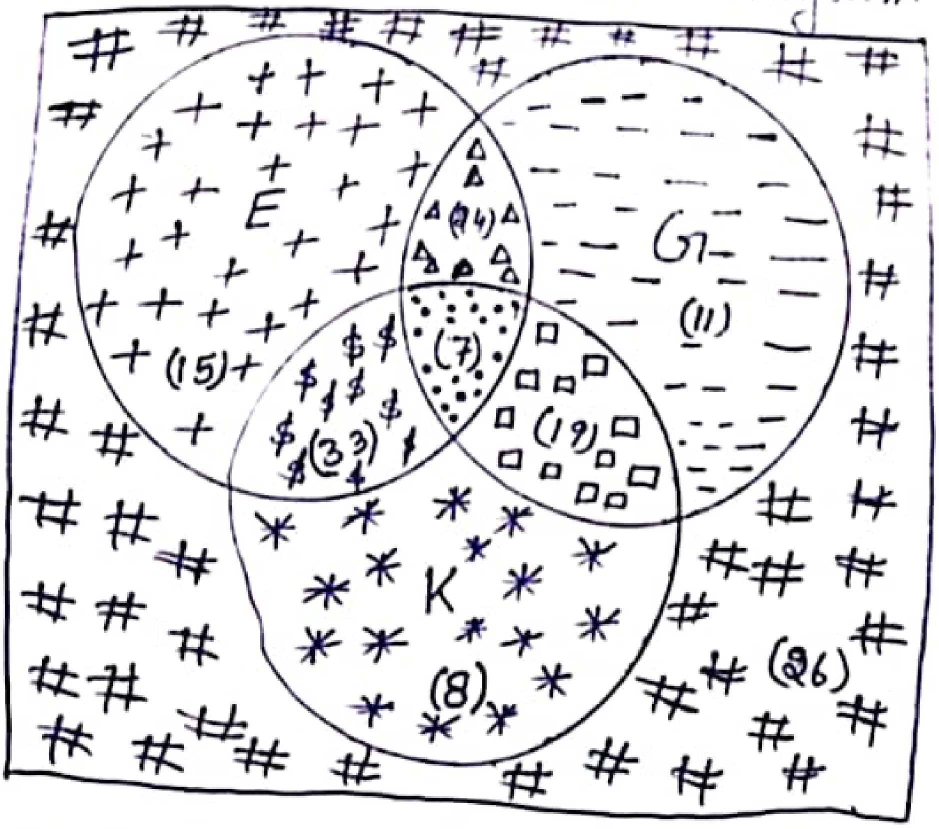
$N((E \cap K) \setminus G) = 33$ people = Represented by \$ in venn diagram

$N((G \cap K) \setminus E) = 19$ people = Represented by $\square$ in venn diagram

$N(E \cap G \cap K) = 7$ people = Represented by • in venn diagram

$N(\overline{E \cup G \cup K}) = 26$ people = Represented by # in venn diagram

(a)



⇐ Venn diagram

Total no. of people in a park $= 143$

(b) (i) $N(G)$ no. of people saw Goanna $= G(\text{only}) + |(E \cap G) \setminus K| + |(G \cap K) \setminus E| + |(E \cap G \cap K)|$

$$N(G) = 11 + 24 + 19 + 7 = 61$$

$P(G)$ Probability of seeing Goanna $= \dfrac{N(G)}{\text{Total no. of people in park}}$

$$P(G) = \frac{61}{143} = 0.426573 \approx 0.4266$$

$$\boxed{P(G) \approx 0.4266}$$

(ii) $N(|(E \cup G) \setminus K|)$ no. of people saw E or G but not K :-

$$N(|(E \cup G) \setminus K|) = E(\text{only}) + G(\text{only}) + |(E \cap G) \setminus K|$$

$$N(|(E \cup G) \setminus K|) = 15 + 11 + 24 = 50$$

$$P(|(E \cup G) \setminus K) = \frac{N(|(E \cup G) \setminus K|)}{\text{Total no. of people in a park}}$$

$$P(|(E \cup G) \setminus K|) = \frac{50}{143} = 0.349650 \approx 0.3497$$

$$\boxed{P(|(E \cup G \setminus K|) \approx 0.3497}$$

(iii) $P(K|E)$ Probability of seeing Koala given that they saw echinda → This is the conditional probability

$$P(K|E) = \frac{\text{no. of } (K \cap E)}{\text{no. of } (E)} = \frac{N(|(K \cap E) \setminus G|) + N(K \cap E \cap G)}{\text{only}(E) + N(K \cap E \cap G) + N(|(E \cap K) \setminus G|) + N(|(E \cap G) \setminus K|)}$$

$$P(K|E) = \frac{33 + 7}{15 + 7 + 33 + 24} = \frac{40}{79}$$

$$P(K|E) = \frac{40}{79} = 0.506329 \approx 0.5063$$

$$\boxed{P(K|E) \approx 0.5063}$$

(iv) $P((E\setminus G)|(E\cup G\cup K))$ saw an echinda but not a Goanna given that they saw atleast one of the species of animal :-

$$P((E\setminus G)|(E\cup G\cup K)) = \frac{\text{no. of } ((E\setminus G)\cap(E\cup G\cup K))}{\text{no. of } (E\cup G\cup K)}$$

$$P((E\setminus G)|(E\cup G\cup K)) = \frac{\text{no. of } (E\setminus G)}{\text{no. of } (E\cup G\cup K)}$$

$$P((E\setminus G)|(E\cup G\cup K)) = \frac{\text{Only}(E) + N(|(E\cap K)\setminus G|)}{\text{Universal} - N(\overline{E\cup G\cup K})}$$

$$P((E\setminus G)|(E\cup G\cup K)) = \frac{15+33}{143-26} = \frac{48}{117} = 0.410256$$

$$\boxed{P((E\setminus G)|(E\cup G\cup K)) \approx 0.4103}$$

Ans 2. n = standard Pizza $= 11$

4 without Tomato — 7 with Tomato

discount on Pizza → 6 different days each week

Given, arrangement doesn't matter

(ii) no. restriction

Formula of combination for no restriction:-

$^{n+r-1}C_r$

(n) no. of pizza in restaurant = 11

(r) no. of pizza selected = 6

discounted pizza can be repeated in a week

no. of ~~way~~ chocies for no restriction on which pizza are discounted (no restriction):-

no restriction = $^{n+r-1}C_r = {}^{11+6-1}C_6 = {}^{16}C_6$

no restriction = $^{16}C_6 = \frac{16!}{6!\,11!} = \frac{\cancel{16}^{4} \times \cancel{15} \times \cancel{14} \times 13 \times \cancel{12} \times \cancel{11!}}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{11!}}$

$\boxed{\text{no restriction} = 8008}$

(b) They want to discount 6 different pizza (6 different):-

6 different ⇒ Select 6(r) pizza from 11(n)

discount 6 different = $^nC_r = {}^{11}C_6 = \frac{n!}{r!(n-r)!}$

discount 6 different = $^{11}C_6 = \frac{11!}{6!\,5!}$

discount 6 different = $\frac{11!}{6!\,5!} = \frac{11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$\boxed{\text{discount 6 different pizza} = 462}$

(c) discount 6 different pizza but the restaurant want to sell discounted pizza without Tomato on 1 or 2 night :-

For this there are 2 case.

Case 1:- Discount pizza without tomato on 1 night

pizza without tomato = 4, pizza with tomato = 7

Case 1 = select 1 pizza from 4 X select 5 pizza from 7

$$\text{Case } 1 = {}^4C_1 \times {}^7C_5 = \frac{4!}{1! 3!} \times \frac{7!}{5! \times 2!} = \frac{4 \times \cancel{3!}}{1 \times \cancel{3!}} \times \frac{7 \times \overset{3}{\cancel{6}} \times \cancel{5!}}{\cancel{5!} \times \cancel{2} \times 1}$$

$$\text{Case } 1 = 4 \times 21 = 84$$

Case 2:- Discount pizza with tomato on 2 night

pizza without tomato = 4, pizza with tomato = 7

Case 2 = select 2 pizza from 4 X Select 4 pizza from 7

$$\text{Case } 2 = {}^4C_2 \times {}^7C_4 = \frac{4!}{2! 2!} \times \frac{7!}{4! 3!} = \frac{\overset{2}{\cancel{4}} \times 3 \times \cancel{2!}}{\cancel{2} \times \cancel{2!}} \times \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times \cancel{2} \times 1}$$

$$\text{case } 2 = 6 \times 35 = 210$$

$$\text{Case } 2 = 210$$

now, we will add the both the cases for getting answer

pizza without tomato on 1 or 2 night = case 1 + case 2

pizza without tomato on 1 or 2 night = 84 + 210

$$\boxed{\text{pizza without tomato on 1 or 2 night} = 294}$$

Ans 3 (a) Given,

$$\sum_{n=1}^{\infty}\left(\frac{3y+1}{5}\right)^n$$, find y value for which series diverges

$$\sum_{n=1}^{\infty}\left(\frac{3y+1}{5}\right)^n = \left(\frac{3y+1}{5}\right)+\left(\frac{3y+1}{5}\right)^2+\left(\frac{3y+1}{5}\right)^3+ \cdots \left(\frac{3y+1}{5}\right)^{\infty}$$

Above series is geometric series

where, $r = \frac{a_2}{a_1} = \frac{\left(\frac{3y+1}{5}\right)^2}{\left(\frac{3y+1}{5}\right)} = \frac{3y+1}{5}$

$$r = \frac{3y+1}{5}$$

For a geometric series to converge

$$|r| < 1$$

∴ For convergence,

$$\left|\frac{3y+1}{5}\right| < 1$$

For mod inequality:-

$$-1 < \frac{3y+1}{5} < 1$$

$$-5 < 3y+1 < 5$$

$$-5-1 < 3y < 5-1$$

$$\frac{-6}{3} < y < \frac{4}{3}$$

$$-2 < y < \frac{4}{3}$$

the. series converge when y is in range :- $-2<y<\frac{4}{3}$

$\therefore$ series will diverge when y is outside the interval

series diverge :- $y \leq -2$ or $y \geq \frac{4}{3}$

$$\boxed{\sum_{n=1}^{\infty}\left(\frac{3y+1}{5}\right)^n \text{ this series diverge :- } y \in (-\infty,-2] \cup [\tfrac{4}{3},\infty)}$$

(b) $\sum_{n=1}^{\infty} b_n$, where $\underbrace{\sum_{n=1}^{N} b_n = \frac{N+1}{N}}_{\text{partial sum}}$, $N \geq 1$ (Given)

For checking above infinite series converge we firstly use vanishing criterian test

Vanishing Criterian Test :- If the limit of the terms of a series as n approaches to infinity is not equal to 0, then the series diverges, but if it is equal to 0 then series may or may not converge.

$\Rightarrow$ So for above series if by appling vanishing criterian test limit approaches 0 as n approaches infinity then we have to do another test. If it do not approaches 0 it mean it is divergeing

$$\sum_{n=1}^{\infty} b_n \Rightarrow \sum_{n=1}^{N} b_n = \frac{N+1}{N}$$

checking for limit when n approaches to ∞

$$\sum_{n=1}^{\infty} b_n = \lim_{n=1}^{\infty}\left(\frac{N+1}{N}\right) = \lim_{n=1}^{\infty}\left(1+\frac{1}{N}\right)$$

as $n \to \infty \Rightarrow 1+0 = 1$

as $n \to \infty$ limit of the series doesn't approaches to 0 means not equal to 0.

Therefore our series diverge

$\boxed{\text{By appling vanishing criterion we conclude that above series \underline{diverges}}}$