Assignment 1.

Maths.

Name: SHUBHARTHAK SANGHARASHA.

Student ID: a1944839.

Q1: Applying fermi estimation to this question.

applying following principles of estimation

assuming :-

Assume that room is rectangular

Room dimensions :- length = 11 meters

width = 7 meters

height - 4 meters

Radius of golf ball = 2.5cm (0.025m)

Mass of golf ball = 40grams (0.040kg)

Packing efficiency of sphere = 65%.

Above all values are assumptions

Now, calculation to estimate the total mass of ball filling the room

Step 1: Calculate the volume of the room

formula → Volume of the room = L×W×H.

Volume of room = 11 × 7 × 4m (cuboid)

Volume of room = 308 m

Step 2: Volume of single ball

formula = Volume of golf ball = $\frac{4}{3} \times \pi \times r^3$.

volume of golf ball = $\frac{4}{3} \times 3.14 \times (0.025)^3$

volume of golf ball $\approx 6.54 \times 10^{-5}\,m^3$.

Step 3. Estimation of no. of golf balls that can fit in the room:

we use packing efficiency here:-

which we assume as 65%.

Effictive volume = $0.65 \times 308\,m^3$

$= 202.3\,m^3$

Step 4: Estimated no. of golf balls that can fit into the room

Number of golf balls = $\dfrac{\text{Effictive volume}}{\text{Volume of golf ball}}$

$= \dfrac{202.3\,m^3}{6.54 \times 10^{-5}\,m^3}$

$=$ 3,061,162 balls.

Step 5 : Estimated mass of balls in room

Total mass = Number of golf balls × Mass of each golf ball.

$= 3,061,162 \times 0.040$

$\boxed{\text{Total mass.} = 122,446.48 \text{ kg.}}$ estimated balls mass in halle

Q2 : Given statement : $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}$ ST $x = 2y+1$

Explanation in words :-

For every integer x, there exist a rational number y such that $x = 2y+1$

Or we can also say that, for all integer x, we can find one or more rational numbers y which when doubled and add 1 to it gives x.

given equation :- $x = 2y+1$, where $x \in \mathbb{Z}$ and $y \in \mathbb{Q}$

Lets describe the value of y in term of x :-

$$2y = x - 1$$

$$y = \frac{x-1}{2} \quad \text{--- (1)}$$

For y to be a rational number it should be represented as p/q form where $q \neq 0$.

In equation number (1) we can see that it is presented in p/q form where $q \neq 0$

Therefore y must be an rational number for every x integer.

Conclusion:

The statement is true because for every integer x, there exist a rational number. $y \left(y = \frac{x-1}{2}\right)$ that satisfies $x = 2y + 1$.

Q3 : Given equation :- $\frac{3+2x^2}{5}$, $x \geq 0$

Finding $f^{-1}(x)$:-

Step 1: Replace $f(x)$ with y

$$y = \frac{3+2x^2}{5}$$

Step 2: Define x in term of y

$$5y = 3+2x^2$$

$$5y-3 = 2x^2$$

$$\frac{5y-3}{2} = x^2$$

$$\frac{\sqrt{5y-3}}{\sqrt{2}} = x$$

$$x = \frac{\sqrt{5y-3}}{\sqrt{2}} = \sqrt{\frac{\cancel{5y-3}}{\cancel{\sqrt{2}}}} = \sqrt{\frac{5y-3}{2}}$$

Step 3: Writing inverse function (replace x with $f^{-1}(x)$ and y with x)

$$\boxed{f^{-1}(x) = \sqrt{\frac{5x-3}{2}}}$$

We know that range of a $f(x)$ is the domain of $f^{-1}(x)$.

Lets find the range of $f(x)$

$$f(x) = \frac{3+2x^2}{5} \; ; \; x \geq 0$$

min value of $f(x)$ comes at $x=0$

$$f(0) = \frac{3+2(0)^2}{5} = \frac{3}{5}$$

As x starts to ∞, because x has no higher bound.

At $x = \infty$; $f(x)$ also tends to ∞

so, range of $f(x)$ is $[\frac{3}{5}, \infty)$

which correspond to a domain of $f^{-1}(x)$

Therefore, $f^{-1}(x) = \sqrt{\frac{5x-3}{2}}$, with domain $[\frac{3}{5}, \infty)$

<u>Q4</u>: Given 3 sets:-

$$A = \{\frac{1}{2}, 1, 2, 3\}$$

$$B = \{x \in Q / 3x+1 \in N\}$$

$$C = (0, 1] \in R.$$

Firstly determine the set B:-

$$B = \{x \in Q \mid 3x+1 \in N\}$$

It means. x belong to a rational number set. such that $(3x+1)$ belongs to a natural number.

$3x+1 = n$, where n is natural number.

$$3x = n-1$$

$$x = \frac{n-1}{3}$$ If $n = 1, 2, 3 \ldots n$ then

$$x = \{0, \frac{1}{3}, \frac{2}{3}, \frac{3}{3} \ldots \frac{n-1}{3}\}$$

(a) $A \cap C$

$A = \left\{\frac{1}{2}, 1, 2, 3\right\}$; $C = (0, 1] \in R$.

$A \cap C$: contains the elements which are present in both sets A and C

means :- $0 <$ elements of set $A \leq 1$

Therefore $A \cap C = \left\{\frac{1}{2}, 1\right\}$ because it's come under above range

$$\boxed{A \cap C = \left\{\frac{1}{2}, 1\right\}}$$

(b) $C | B$

$C | B$ is set difference. It state that elements should present in C but not in B

$C | B = C - B \cap C$.

$(B \cap C)$ elements in C which are also present in B are :-

$$B = \left\{ \frac{n-1}{3}, \text{ where } n \in N \right\}$$

$$C = \{ x, 0 < x \leq 1, x \in R \}$$

$$B \cap C \Rightarrow 0 \left(\frac{n-1}{3} \right) \leq 1, \text{ where } n \in N$$

Therefore $B \cap C = \left\{ \frac{1}{3}, \frac{2}{3}, 1 \right\}$

$$C | B = C - B \cap C$$

$$C|B = C(0 < x \leq 1) - \left\{ \frac{1}{3}, \frac{2}{3}, 1 \right\}$$

where $x \in R$

$$\boxed{C|B = \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 1\right)}$$

(c) $A \cup (B \cap C)$

above we find the $B \cap C$ which is $\left\{ \frac{1}{3}, \frac{2}{3}, 1 \right\}$

and A is $\left\{\frac{1}{2}, 1, 2, 3\right\}$

$A \cup (B \cap C) = \left\{\frac{1}{2}, 1, 2, 3\right\} \cup \left\{\frac{1}{3}, \frac{2}{3}, 1\right\}$

$$\boxed{A \cup (B \cap C) = \left\{\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1, 2, 3\right\}.}$$