Solomon Marcus

Professor Solomon Marcus is affiliated to the Department of Mathematics, University of Bucharest, Romania, where he was successively, student 1945, instructor 1951, assistant professor 1955, associate professor 1964, professor 1966, and emeritus professor 1991. Research in set theory, real analysis, general topology, where he is quoted mainly for his results related to measure vs. Baire category, differentiation, Darboux property, Jensen convexity, quasicontinuity, determinant and stationary sets, symmetry of sets and functions, Riemann integrability in topological spaces, Hamel bases. In the late fifties and early sixties Professor Marcus became one of the initiators of mathematical and computational linguistics, proposing algebraic, logical and set-theoretic models for some fundamental linguistic categories in phonology, morphology and syntax. Later, he extended his interest to poetry, being one of the founders of mathematical poetics. Then, he became active in the semiotics of artificial languages (including also programming languages). Quoted by more than a thousand authors, he gave invited lectures and had temporary positions in most European countries, in U.S.A., Canada, Brazil, New Zealand and other countries. He is a member of the editorial boards of several journals of mathematics, computer science, linguistics, poetics and semiotics. Among his former students and pupils one can find many well known names in the fields of mathematics and computer science.

Bridging Linguistics and Computer Science, via Mathematics

From Poetry to Mathematical Analysis

When I was fifteen, I was fascinated by poetry. Mathematics was still a territory remaining to be discovered (school mathematics seems to be still today rather a failure). In the fall of 1944 I began a new life: I survived the second world war, I was classified the first among 156 candidates at the French type high-school final examination called "baccalaureat" in a city of a devastated Romania and I got the right to be accepted by any institute of higher education of the country with no further examination. I had a very short time to take a decision. I arrived with many complications (the war consequences were still visible) in Bucharest, the capital of Romania, and I began to visit various faculties and look at the programmes they offered. I was almost twenty. During the summer of 1944, I had my first contact with the true mathematics, reading a presentation of the non-Euclidean geometries. I was very impressed. I paid special attention to the announcements I saw at the Faculty of Sciences, Department of Mathematics. From the list of courses to be taught I remember: infinitesimal calculus, mathematical logic, topology, theory of functions, abstract algebra, analytic, projective and differential geometry, number theory. Although most of these words were unknown to me, I had a feeling that they refer to a universe responding to my intellectual needs. Like poetry some years earlier, words such as "infinitesimal" and "topology" gave me the impression that they refer to some aspects hidden to common, everyday observation. My assumption was that mathematics, again like poetry, refers to a second reality, to be discovered or invented. I became a student in mathematics, but poetry, my first love, remained for ever an essential component of my intellectual background. This belief was strengthen by my professor of "abstract algebra", Dan Barbilian, one of the greatest Romanian poets. The attribute "abstract" may seem today pleonastic when associated with "algebra", but at that time a rhetorical stress was necessary to call attention on the opposition with traditional, algorithmic and numerical algebra. Under the influence of my teachers Nicolescu, Stoilow, Froda and of the teacher of my teachers, Pompeiu, all with PhD at Sorbonne-Paris, guided by the great masters of French mathematical analysis, I chose real analysis, set theory and topology as my main field of research, attracted by the counter-intuitive, sometimes pathological phenomena such as the continuous functions nowhere differentiable. I published in this field of pure mathematics about a hundred articles, many of which are still used by the most active authors in the field (see the basic monographs devoted to it, as well as the specialized journal "Real Analysis Exchange"). Looking at them from today's perspective, one could consider them as a preliminary step to the "fractal geometry of nature" developed in the seventies by Benoit Mandelbrot.

Itineraries to Language

However, it is known that some of the most difficult problems considered in set theory, real analysis and topology in the first decades of our century proved to be related to some aspects of the foundations of mathematics and mathematical logic. On the other hand, the latter fields showed already from the beginning of our century (see Axel Thue's combinatorial systems and David Hilbert's formal systems) their strong connections with what we call today a structure of formal language. Chronologically however, my first step towards bridging mathematics and languages was a different one. Already in the late fifties, I learned of the emergence of the new field called "automatic translation", simultaneously developed in West and East. Some engineers in electronics and computers, sometimes associated with linguists, were optimistic in trying to build algorithms of translation from a natural language into another one, under the more or less explicit assumption that translation is mainly a lexical and syntactic activity that can be easily arranged in an algorithmic form and then transformed in a computer program. I was not attracted by such projects; it happened, however, that as an additional activity to the projects of automatic translation, various attempts of formalization of some basic aspects of the morphology and syntax of natural languages were developed. Among the authors of these formal models were A.N. Kolmogorov, V.A. Uspenskii, R.L. Dobrusin, O.S. Kulagina and A.V. Gladkii in the former Soviet Union, A. Sestier, Y. Lecerf, P. Ihm in Western Europe, David Hays in U.S.A., J. Lambek in Canada, Y. Bar-Hillel, H. Gaifman, M. Perles, E. Shamir in Israel, A. Trybulec in Poland, M. Novotny and L. Nebesky in Czechoslovakia, J. Kunze in Germany. Some of these authors adopted an approach based on set theory, free semigroups and algebra of binary relations, others were oriented towards ideas coming from the Polish school of logic (Ajdukiewicz, Lesniewski). A common denominator of all of them was their linguistic background. Each of them exploited some linguistic ideas, methods and/or concepts. Linguistics was for long-time dominated by various schools of structuralism. In the early fifties, two schools were dominant: American distributionalism (Zellig S. Harris, Charles Hockett) and Danish glossematics (Louis Hjelmslev), but the classical heritage of Ferdinand de Saussure and of Prague Linguistic School in Europe, of Leonard Bloomfield and Edwin Sapir in U.S.A. was still very important. I became interested in all these developments, trying to bridge linguistics and mathematics. Harris' view, exploiting the contextual behaviour of various linguistic units, was near to the spirit of the theory of free semigroups, while Hjelmslev conceived linguistics as a kind of algebra. I was also influenced by some trends in Romanian structural linguistics and I have to quote in this respect at least two names: Emanuel Vasiliu and Paula Diaconescu.

Continuous and Discrete Mathematics

It was not so easy for me to adapt to all these developments. My mathematical training and activity were oriented almost exclusively towards continuous mathematics, almost all of my research articles and my teaching at the University were devoted to it, while the mathematics of language was conceived, more or less explicitly, as a chapter of discrete mathematics. Algebra, combinatorics, mathematical logic were not so familiar to me and I had to change in some respects my habits of thinking, in order to adapt them to the study of language. Only in a next step I realized that these two kinds of mathematics, continuous and discrete, share an important common problem, the study of infinity by finite means, and they interact so much that we cannot separate them. My experience in mathematical analysis, including topology and set theory, proved to be a good source of inspiration in the field of language. Several (joint) papers I published in this respect have as their starting point some phenomena in continuous mathematics related, for instance, to symmetry or to convexity. The analogy between continuous and discrete phenomena remains a basic strategy, which still deserves to be used in the study of languages, with a great chance of interesting results. However, more important than its technicality, mathematics proved to be useful in the field of languages by its way of thinking. Despite its very controversial nature, the mathematical way of thinking is first of all characterized by its step by step procedure, each step being rigorously and explicitly based on the previous steps. This fact can be easily confirmed by a very simple experiment: take two books, one of mathematics, the other of geography, for instance: if we delete from each of them the first 20 pages, the remaining part in the former will be almost completely non-intelligible, while in the latter it will be to a large extent intelligible. The mathematical thinking is looking for explicitness and has the tendency to become marathonic; it clearly separates what is given from what is to be found; it proceeds from the simplest (apparently trivial) things and moves, step by step, to less simple things. Complexity is increasing gradually. At each step, we pack the already acquired concepts and results, by means of an adequate symbolism, in order to keep our language within reasonable limits of complexity.

Linguistic Structuralism and Automatic Translation through the Glasses of Mathematics

Having in front of me these two lines of development: linguistic structuralism and construction of algorithms for the automatic translation of languages, both dominated by the aim to formalize linguistic phenomena, I realized that the best help mathematics could give to make efficient their interaction is to read the basic ideas of linguistic structuralism through the glasses of what I described above as the mathematical way of thinking, by taking into account the mathematical models coming from the field of automatic translation. This project was realized in 1967 in two books, the first in French, the second in English; while in the former the main attention was directed towards linguistic structuralism, in the latter we tried to order in a uniform framework the European mathematical models inspired by automatic translation. To give some examples in this respect, we realized that the Trubetzkoy-Cantineau system of oppositions, Hjelmslev's system of relations and Harris' system of different types of distributions are isomorphic, so they can be captured in a unique framework, which in its turn permits to enlarge Dobrushin's model of elementary grammatical categories, in order to represent the phenomena of contextual ambiguity in their most general form (Marcus ed., 1981, 1983). Various types of syntactic projectivity introduced in Western Europe and in U.S.A. were captured in a unique framework and articulated with the graph-theoretic model of a grammatical proposition, due to M.l. Beleckii, V.M. Grigorjan and l.D. Zaslavskii. Two basic different approaches to the concept of phoneme, one based on binary distinctive features, the other on phonemic sequences, received their clear formal status, with the surprising fact that the relation between two sequences belonging to the same phoneme is not transitive. Different approaches to the concept of morpheme, the set-theoretic models of part of speech and of grammatical case were critically examined. One of the most interesting concept emerging from the preliminaries to automatic translation, the concept of syntactic configuration of various orders, received a uniform presentation, including all its variants (Kulagina, Gladkii, Novotny) and leading to some new ones (further ideas and systematization are due to Maria Semeniuk-Polkowska).

The Impact of some Political and Ideological Factors

Local conditions in Romania were difficult in the emergence period of information sciences, mainly in the fifties and in the sixties. Communication with scientists from Western countries were very reduced, while the necessary scientific journals and books available in Romanian universities were to a large extent those from Eastern Europe. We had to wait for the Russian translations of the most important Western books and articles. But in some respect we were lucky, because the policy of the communist party was to include the computer revolution as a basic component of the communist society. Romanian scholars such as the mathematician Gr. C. Moisil and the linguist Al. Rosetti knew how to take advantage of this opportunity. They articulated their efforts to make possible the early development of mathematical and computational linguistics at the University of Bucharest. In 1962, they founded the journal "Cahiers de Linguistique Théorique et Appliquée", specially devoted to this line of research. They also introduced regular courses on mathematical and computational linguistics at the University of Bucharest, which were chronologically among the first in the world in this respect. Professor Moisil, the founder of the Romanian school of mathematical logic, with fundamental contributions related to non-classical logics, was very efficient in attracting young mathematicians at the crossroad of logic, linguistics and computer science. He challenged us with new topics and problems, many of them very provocative. To give only one example in this respect, I will mention the following fact: Short time before his death, in May 1973, he told me, during a walk, that Gabriel Sudan, a former PhD student of Hilbert, is the author, before Ackermann, of an example of a recursive function which is not primitive recursive. He promised to give me more details with another occasion, which however never arrived. I told this fact to Cristian Calude, then one of my best students, and, after a long investigation, he succeeded to discover the corresponding result in an article by Sudan, whose title and introduction did not at all promise to hide this fact. Now, Ackermann and Sudan are both recognized as authors of a first example (Sudan's example is different from that proposed by Ackermann) of a recursive function which is not primitive recursive. But, as it usually happens, the search is richer than the object found. This exercise was enough for Calude to discover the pleasure of research in the field of recursiveness, which became one of his main fields of interest, as he confessed to me.

Chomsky Grammars. From Linguistics to Computer Science

In the late fifties, Chomskian linguistics emerged, immediately recognized as a revolution in the scientific humanities of the XXth century. The book "Syntactic Structures" published by Mouton (The Hague) in 1957 was initially received as a purely linguistic event, claiming a new, generative-transformational approach to language, as opposed to the analytical one, promoted by comparative-historical linguistics and by classical structural linguistics. Instead of accepting as given the well-formed strings of English and analysing their structure, Chomsky sees human language as the result of the activity of a hypothetical machine, called "generative-transformational grammar". Linguistics became for Chomsky a chapter of cognitive psychology, elaborating hypothetical-explanatory models of human linguistic competence. Despite the fact that all motivations brought by Chomsky were of linguistic nature, most of his pioneering articles were published in non-linguistic journals such as "IRE Transactions on Information Theory" and "Information and Control" and in a non-linguistic handbook such as "Handbook of Mathematical Psychology". How should we interpret this fact? Did Chomsky suggest in this way that his approach could be relevant to Shannon's information theory? This link remained so far rather weak. The only articles supporting this link are those in collaboration with G.A. Miller and they refer more or less explicitly to Markov processes. More convincing is the choice of a handbook of mathematical psychology (1963) for the publication of two other articles, because this choice was in conformity with his view of linguistics as a branch of cognitive psychology. Today, it appears very ironical the fact that Chomsky avoided to publish in journals of computer science (there are only two exceptions in this respect, one of them being his famous article in collaboration with M.P. Schutzenberger), despite the tremendous relevance of his grammars for the theory of programming languages and, generally, for theoretical computer science. Indeed, S. Ginsburg and H.G. Rice proved in 1961 that the Chomskian context-free grammars are equivalent to Backus normal forms defining the syntax of the programming language ALGOL 60, while R.W. Floyd has shown that, taking into account some semantic aspects of ALGOL 60, some context-sensitive rules which are not context-free are necessary too. Similar situations appear with virtually any programming language. The discovery of this fact made from Chomsky hierarchy of grammars and languages the basic tool to investigate the syntax and the semantics of programming languages; what is understood by the theory of programming languages is to a large extent the theory of formal languages, conceived in respect to Chomsky hierarchy. The standard presentation of this theory, in its form recognized today is due to Arto Salomaa (1973). The years 1960–1961 represent a turning moment in the evolution of Chomskian grammars; initially motivated by studies of natural languages, context-free and context-sensitive grammars prove their relevance to programming languages, as we pointed out above, while their role in linguistics is weak, being considered, to a large extent, inadequate for natural languages. In a second step, in the eighties, the question of non-context-freeness of natural languages was raised once more, due to the fact that the initial argument proposed by Chomsky, as well as some other arguments against the context-freeness of natural languages, are no longer accepted. In this way, context-free and context-sensitive grammars keep their importance to both natural and programming languages. However, this fact, generally accepted in theoretical computer science and particularly in computational linguistics, seems to be still ignored by many linguists.

Is Formal Linguistics a Pilot Science?

Going back to the period of the sixties, I remember that Chomsky's approach was popular among linguists mainly by the transformational component of his grammars, added to what was called a phrase structure grammar. However, in the field of computational linguistics a more balanced view prevailed, leaving room to both analytical and generative-transformational aspects. Beginning with the sixties, I tried to order all directions leading to a bridge between linguistics, computer science and mathematics and I identified one coming from linguistics (linguistic structuralism and various fields of applied linguistics), another from mathematics (free monoids, free semigroups, various combinatorial problems going from Gauss to Langford and the well known Thue combinatorial systems, number theory such as Conway's iterative reading of numbers), a third from logic (what we call today a "formal language" and a "formal grammar" can be associated with a Hilbert formal system as well as with any variant of modeling the idea of computation, such as Turing machine, Markov normal algorithm, recursive function etc.), another from automata theory (Rabin-Scott's article published in "IBM Journal of Res. and Dev.", in 1959 was the most accurate in this respect) and, related to it, from biology (mainly S.C. Kleene's "Representation of events in nerve nets and finite automata", 1956) and molecular genetics (Marcus, 1974). I remember that already in 1946 Claude Levi-Strauss proposed the slogan "linguistics as a pilot science" and I used it as a title for one of my articles (Marcus, 1969, 1974). My main idea was to consider not linguistics, but formal linguistics as a pilot science, involving also the association of the left hemisphere of the brain with sequential structures (mainly language and logic). I supposed that Roman Jakobson had a similar idea in mind when he stated that linguistics is the mathematics of social sciences. It happened that in the same year when I published my article "Linguistics as a pilot science" another article, with the same title, was published by Joseph Greenberg, the famous specialist in linguistic universals, arguing (implicitly polemical with Levi-Strauss and Jakobson) that the respective slogan is wrong. He had in view some failures of classical structuralism in its attempts to transfer from linguistics into other social sciences some concepts, methods and results. Examples were selected just in order to illustrate failures, while successes were ignored. My strategy was just the opposite: to point out the important successes (such as molecular genetics and programming languages); moreover, I argued that the function of a pilot science appears under the presupposition that linguistics increases its degree of formalization, a fact ignored by Greenberg and by other critics supportive of the mentioned slogan (for instance, Walter Koch).

Towards Contextual Grammars

My first monograph "Lingvistica matematică" was published in 1963. I realized only later that it was perhaps the first attempt to put in a systematic form the possibility to bridge linguistics and mathematics. Probably, this was the reason that it was required for translation in French, English, Russian and Czech, but I used this opportunity to publish in these languages completely new versions; the French version, "Introduction mathématique à la linguistique structurale" was published by Dunod, Paris, in 1967; the English version, "Algebraic Linguistics", appeared at Academic Press, New York in 1967; the Russian version, "Teoretiko mnojestvennye modeli jazykov", appeared in 1970 at Ed. Nauka, Moscow; the Czech version, "Algebraicke modely jazyka", appeared in 1969 at Ed. Academia, Prague. In 1964, I published in Romanian "Gramatici şi automate finite", devoted exclusively to different variants of regular grammars (type 3 in Chomsky hierarchy). A special section of this book was devoted to some characterizations of regular languages in terms of analytical models, but I did not push this study further. However, it became clear in the sixties that several authors were involved in bridging analytical and generative (Chomskian) models, A.V. Gladkii and M. Novotny being perhaps the most important of them. At the same time, some alternative generative models were developed with significant impact in computational linguistics (dependency grammars by David G. Hays), in logic (categorical grammars, by H. Gaifman, preceded by the syntactic calculus proposed by Joachim Lambek), in cellular biology (Lindenmayer systems, later with impact in other fields such as computer graphics). Under the influence of these facts, I tried to build a generative counterpart of contextual analysis and I proposed, at the International Conference on Computational Linguistics in Senga Saby (near Stockholm), in 1968, a new type of generative device I called "contextual grammars". The name was not appropriate, because it was confused with context-sensitive grammars. My intention was, in the spirit of that time, to connect the idea of contextual analysis (developed by Dobrusin, Sestier, Jurgen Kunze and myself) and that of a generative device; in other words, to transform contextual analysis in a generative machine. All concepts involved in this enterprise emerged from the structural analysis of natural languages. Contextual grammars were conceived just as one more way to bridge the analytical and the generative approach to the grammar of a natural language. However, in my article published in 1968 I avoided any explicit linguistic motivation of contextual grammars, giving directly their technical definition. Perhaps this is the reason why this formalism had no linguistic impact; in exchange, it became attractive for many theoretical computer scientists. Several surveys of this research were published, from time to time, by Gheorghe Păun, one of the main contributors in this field; his most recent survey "Marcus Contextual Grammars" has appeared at Kluwer Academic Publisher (Dordrecht, Holland, 1997). In the meantime, I devoted a special chapter to the linguistic aspects of contextual grammars in the second volume of the "Handbook of Formal Languages" (eds. G. Rozenberg, A. Salomaa), published by Springer in 1997. The link between contextual grammars and natural languages, for a long time ignored, is now a topic of increasing interest, as it can be seen from a series of articles considering various extensions of contextual grammars and the way they challenge other types of grammars, in respect to their linguistic relevance.

Texts, Contexts, Intertexts and Hypertexts

Contextual grammars also have an interesting mathematical aspect. Strings and contexts interact symmetrically, within the framework of a Galois connection. Despite this fact, in all already considered variants of contextual grammars the symmetry between strings and contexts is not respected. In order to fill this gap, I proposed some new variants, where the same grammar is able to generate both strings and contexts; moreover, I tried to articulate texts, contexts, intertexts and hypertexts, taking into account the development of text theory in both its linguistic-literary variant (for instance that given by Teun A. van Dijk) and its mathematical computer science variant (see the series of articles published by A. Ehrenfeucht and G. Rozenberg, some times in collaboration with P. Ten Pas and/or H.J. Hoogeboom). The intertext originated in M. Bakhtin's dialogic principle: each text is in a more or less explicit dialogue with other texts. Although the idea of intertext appeared in the framework of literary studies, it seems that the corresponding phenomenon is more and better visible in scientific texts (see the quotations and the bibliographic references). Hypertexts emerge from the need to transgress the sequentiality of usual texts, making possible all paradigmatic connections related to the elements of a text; this possibility is a consequence of recent computational capabilities developed in the field of hypermedia. Ultimately, this line of research leads to the need to develop suitable types of contextual grammars, where arbitrary strings are replaced by texts, while contexts are restricted to ordered pairs of texts. It is important to observe that "text" became in the last decades a real universal paradigm, leading to the vision of the world as a text. This fact brings a new argument in favor of the status of linguistics as a pilot science. As a matter of fact, already in the field of molecular genetics the idea of a text emerges from the need to consider higher levels of organization of DNAs and proteins. The natural trend of quantification of our field of knowledge leads to the need to find a basic alphabet of irreducible units and to use the sequential combinatorial capacities of these units—it seems that this process cannot avoid the fundamental pattern of our mother tongue.

Contextual Grammars and Chaitin-Kolmogorov Complexity. Another Perspective

Contextual grammars are naturally linked to reading processes and, through them, to Chaitin-Kolmogorov complexity. In this respect we are inspired by

Walter J. Savitch (1993), who was concerned mainly with Chomskian grammars. However, we claim that contextual grammars are more suitable for a natural and simple definition of algorithmic complexity, due to their intrinsic character (the absence of any auxiliary symbol). The intuitive idea in our approach is the remark that we read any (finite) text by extending it to an infinite one (this fact could be a way to interpret Umberto Eco's "opera aperta"). This happens because infinity is more structured than finiteness. Infinity can be understood only by an explicit rule and expliciteness requires, in its turn, simplicity. The most convincing example in this respect is the fact that we need n rules to generate the set $A(n)$ of the first n natural numbers, but we need only two rules to generate the set A of all natural numbers (this fact is true for both Chomskian grammars and contextual grammars). This means that the set $A(n)$ accepts a reading that is shorter than its size n (= cardinal of $A(n)$), as soon as we interpret the extension A of $A(n)$ as a reading of $A(n)$; indeed, the size of A is equal to 2. We adopted here the convention to define the size of a grammar as the number of its rules; it happens that for $A(n)$ this number is also the cardinal of $A(n)$. So, we are lead to the following general problem: given a finite language $L(1)$ on the alphabet V and a class H of generative devices on V (a device G in H consists of a finite set of rules by means of which we generate a language on V), we define a reading of $L(1)$ with respect to H to be any infinite language L generated by G in H, such that the finite language L_n of those strings in L whose length is not larger than the length n of the longest string in $L(1)$ is just $L(1)$. For instance, in the above example A is a reading of $A(n)$. Suppose we can define in a reasonable way the size of any device in H. In this case, the size of any language generated by a device in H is defined as the smallest possible size of such a device. The size of a finite language can be defined independently of the class H, either as its cardinal number or as the length of its longest string. This is the intrinsic size of the given finite language. The size is said to be H-consistent if any finite language can be generated by a device in H and its intrinsic size is identical to the size resulting from the definition above with respect to H. Now the interesting problem is to compare the intrinsic size of a given finite language $L(1)$ with the sizes of the possible readings of $L(1)$ in respect to H. Does there always exist a reading of $L(1)$ with respect to H? The following two conditions are sufficient for the existence of such a reading: a) any finite language on V has a grammar in H; b) the universal language on V has a grammar in H. These conditions are fulfilled, for instance, when H is any class of grammars in Chomsky hierarchy or when H is the class of simple contextual grammars. We will assume the existence in H of at least one reading of $L(1)$. However, the possibility of absence of any reading of $L(1)$ with respect to some choices of H deserves attention, as a possible approach to the lack of coherence or to the lack of cohesion of a text. Let us denote by size (reading of $L(1)$) the smallest possible size of a reading of $L(1)$. If it is larger than the size of $L(1)$, then we consider $L(1)$ as partially readable with respect to H. The degree of unreadability is given by the difference: size (reading of $L(1)$) - size ($L(1)$). Let us observe that this difference, if it is strictly positive, takes usually only small values; this is the case when the conditions a) and b) considered above are satisfied. In this case we can work with a grammar G in H obtained by adding to the rules used to generate $L(1)$ the necessary rules to generate the universal language on V, from which we have eliminated the strings whose lengths are not larger than the maximum length of strings in $L(1)$. If size (reading of $L(1)$) = size ($L(1)$), then $L(1)$ has the highest possible complexity with respect to H and it is close to the limit of readability (it corresponds, mutatis mutandis, to the status of a random string considered as the case of maximum algorithmic complexity in the Chaitin-Kolmogorov approach). However, in most situations size (reading of $L(1)$) is strictly smaller than size ($L(1)$) and the difference size ($L(1)$) - size (reading of $L(1)$) is a measure of the efficiency of the reading process of $L(1)$ with respect to H.

Reading via Simple Contextual Grammars

Now let us consider as H the class of simple contextual grammars on V. Such a grammar is composed of a finite set L_1 of strings and a finite set C_1 of contexts on V. Among various possibilities to define the size of such a grammar G, we choose size $G = \max(s, c)$, where s is the length of the longest string in L_1 and c is the length of the longest context in C_1 (the length of the context $< x, y >$ is the sum of the lengths of x and y). An alternative possibility is to take size of G to be $s + c$. Let us recall that the language generated by G is the smallest language L containing L_1 such that if $x \in L$ and $< u, v > \in C_1$, then $uxv \in L$. Let us take the example of the language $L_n = \{a_1, a_2, \ldots, a_n\}$ on $V = \{a\}$. It has the intrinsic size equal to n while the grammar $G =< V, \{a\}, \{< \lambda, a >\} >$ (where λ is the null string) has the size equal to 1 and it is a reading of L_n, because $L(G) = \{a_n \mid n = 1, 2, \ldots\}$; so, the efficiency of the reading process is equal to $n - 1$. If $L_1 = \{a_1, a_3\}$, then the intrinsic size of L_1 is 3, while the grammar $G =< \{a\}, L_1, \{< \lambda, \lambda >, < \lambda, a_2 >\} >$ has the size equal to 3 and leads to a reading of L_1 with size (G) = size (L_1) =3. However, there is a more efficient reading of L_1, given by the grammar $< \{a\}, \{a\}, \{< \lambda, \lambda >, < \lambda, a_2 >\} >$, whose size is equal to 2, while no reading of L_1 and of size 1 exists; so, the efficiency of the reading process is here equal to $3 - 2 = 1$. Combinatorial arguments may lead to the characterization of those finite languages on $\{a\}$ accepting an efficient reading, but the problem becomes more difficult when the cardinal of the alphabet takes values larger than one. Further steps are necessary to extend this approach to more general types of contextual grammars.

Back to Poetry

The situation we described as being at the limit of the possibility of a reading process brings back the case of poetry, having just the aim to realize maximum of semantic density by means of a minimal expression. In other words, in an ideal piece of poetry no abbreviation is possible, nothing can be eliminated, modified or added. The highest complexity associated with a piece of poetry goes in a happy marriage to its hidden simplicity; but the latter is obtained only as a result of the reading process. Talking about poetry, it is the right moment to recall the beginning of this story. We never forget our first love and its name was, for me, poetry. As a matter of fact, language and linguistics were a way to reach a deeper understanding of poetry for me. My first approach to the edification of a field that could deserve to be called "mathematical poetics" happened in the late sixties and the resulting monograph "Poetica matematică" published in Romanian in 1970, knew a German improved edition in 1973, at Athenaeum Verlag, Frankfurt/Main. Links between poetry and mathematics are known for a long time, but most investigations in this respect were related mainly to quantitative-statistical aspects. My aim was to build a theory of the poetic language seen as a continuous, topological structure, in contrast with the discrete, algebraic nature of the scientific (mathematical) language. My guide was the tradition represented by George D. Birkhoff (the father of ergodic theory), Matila C. Ghyka and Pius Servien, all three deserving to be considered as founders of the new field of mathematical aesthetics. Birkhoff, with his famous measure of the beauty of an artistic work, given by the ratio O/C, where O is the order and C is the complexity of the considered work, had to be reinterpreted in the light of today's information theory and still waits to be reconsidered in the perspective of Chaitin-Kolmogorov algorithmic complexity. Ghyka's approach via the common denominator of organic growth and artistic creativity through the glasses of the golden proportion and of Fibonacci sequences becomes now a particular case of the generative approach, via different types of grammars and machines (mainly the so-called picture grammars). Pius Servien idea to replace the traditional opposition between the poetic language and the ordinary one with the opposition poetic-scientific was a starting point. Our first approach to this opposition was reconsidered and improved in several steps (mainly in our book "Art and Science", in Romanian, Ed. Eminescu, Bucureşti, 1986), culminating with our plenary lecture at the Fifth Congress of the Hellenic Semiotic Society (Thessaloniki, May 1997), where we proposed a way to place all the oppositions between poetry and science within the framework of their similarities. The formal language point of view was adopted in the study of most creative processes, according to our slogan "formal linguistics as a pilot science". In a sequence of articles and books, I tried (in most cases in collaboration with some of my colleagues) to point out the hidden creative machine (usually, under the form of various types of grammars in Chomsky hierarchy) of a work of art, for instance, in the study of fairy-tales (S. Marcus, ed., "La semiotique formelle du folklore", Klincksieck, Paris, 1978), of theater (S. Marcus, ed., "The formal study of drama", special issues of the journal "Poetics", Amsterdam, 1976 and 1984), of visual arts (S. Marcus, ed., "Semiotica matematică a artelor vizuale", in Romanian, Ed. Ştiinţifică, Bucureşti, 1981), of music (see the sequence of articles in "Romanian Review", in the eighties). A special word deserves the mathematical linguistic approach to theater. The starting idea was to consider a Boolean matrix associated to a theatrical play; the matrix has m lines and n columns, where m is the number of the characters and n is the number of scenes in the play. At the intersection of line i and column j (i between 1 and m, j between 1 and n) we place the digit 1 if the ith character appears in the jth scene and the digit 0 in the contrary case. A very rich information is obtained by adequately processing this matrix. Due to its simplicity and its operational and intuitive nature, the proposed tool became very popular among researchers in the field of theatrical semiotics, being quoted, used and improved by many authors.

Transdisciplinarity. Between Risk and Chance

Trying to bridge such heterogeneous fields made my social life difficult enough. Mainly in the fifties and in the sixties, some of my colleagues mathematicians and computer scientists were reluctant towards my enterprise involving several fields usually considered far away from each other. When you try to connect two fields, you need to be in both of them, but you risk to be considered in none of them. Fortunately, in the sixties I was already the author of a large number of research articles in the field of mathematical analysis, well received by the experts, so my status as a mathematician was beyond doubt. Traditionally, mathematicians conceive their field in two variants: theoretical (pure) and applied, the latter being related mainly to mechanics, physics and chemistry. Pure and applied mathematics are considered in interaction and it is a great honor for a theorist to be able to obtain results with some relevance in physics. Gradually, biology too was accepted as a field where mathematics could have some impact. But, already in the XlXth century, mathematics showed its relevance for economics and to some extent, for psychology too. However, for the predominant mentality, humanities remain in a kind of incompatibility with exact sciences. My first attempt to bridge linguistics with mathematics under the label "mathematical linguistics" came as a shock for many people, good to be exploited in the newspapers and by other media, but not to be trusted. This happened despite the fact that structural linguistics, very popular among linguists, prepared the way to mathematization (structuralism was always a preliminary step to mathematical modeling). Paradoxically, it seems that this shock contributed to the social success of mathematical linguistics, which became in the meantime "computational linguistics". In the sixties, the skepticism was also nourished by the generally recognized failure of the huge projects of automatic translation. However, it happened that, beyond their initial motivation, the mathematical and computational models of languages proved to have an important linguistic impact (to give only one example, syntactic projectivity became a basic tool in the study of dependency grammars). On the other hand, as we have already pointed out, Chomskian models, initially conceived exclusively for linguistics, became a basic tool in the study of programming languages. Now we can add that the new field of computational biology is also using the experience of formal generative grammars. Such unexpected evolutions show that the reasons to connect linguistics and computer science (via mathematics) are multiple and very deep. Let us recall that important approaches in historical linguistics use the DNA metaphor, while the latter is a term of reference for non-conventional computation, and its understanding is via formal linguistic models. Computational linguistics is now a basic component of Artificial Intelligence. Journals such as "Linguistics and Philosophy", "Theoretical Linguistics", "Computational Linguistics", "Mathématiques, Informatique et Sciences Humaines" and many others contain regularly articles involving a mathematical and computational formalism. So, mathematics no longer needs some rhetorical means to reach the attention of specialists of language and of computer science. A similar process occurred in the field of poetics. Journals such as "Poetics", "Leonardo", "Computers and the Humanities", "Cahiers de Linguistique Théorique et Appliquée", "Symmetry Culture and Science" publish regularly articles bridging art, mathematics and computer science. The project of unification of knowledge, started already by the Vienna Circle, in the twenties and in the thirties of our century, is now strongly stimulated by the development of information sciences and particularly by theoretical computer science.

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